Progress In Electromagnetics Research, PIER 33, 167–182, 2001

NULL STEERING OF LINEAR ANTENNA ARRAYS USING A MODIFIED TABU SEARCH ALGORITHM

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Abstract—A useful and flexible method based on the tabu search algorithm for the pattern synthesis of linear antenna arrays with the prescribed nulls is presented. Nulling of the pattern is achieved by controlling the amplitude-only and both the amplitude and phase of each array element. To show the versatility of the present method, some design specifications such as the sidelobe level, the null depth and the dynamic range ratio are considered by introducing a set of weighting factors in the cost function constructed for the tabu search algorithm. Several illustrative examples of Chebyshev pattern with the imposed single, multiple and broad nulls are given.

1. INTRODUCTION

The increasing pollution of the electromagnetic environment has prompted the study of array pattern nulling techniques [1–23]. These techniques are very important in radar, sonar and communication systems for minimising degradation in signal-to-noise ratio performance due to undesired interference. There has also been considerable interest in synthesising array patterns with broad nulls [19–23]. The broad nulls are needed when the direction of arrival of the unwanted interference may vary slightly with time or may not known exactly, and where a comparatively sharp null would require continuous steering for obtaining a reasonable value for the signal-to-noise ratio.

Most of the methods of null steering available in the literature includes controlling the excitation amplitude-only, the excitation phase-only, the element position only and the complex weights (both the amplitude and the phase). The methods of amplitude-only control utilize an array of attenuators to adjust the element amplitudes [5–7]. If the array elements possess even symmetry about the center of the array, the number of attenuators required and the computational time are halved. Amplitude-only control is also easy to implement and less sensitive to quantization error. The problem for excitation phase-only and element position only nulling methods is inherently nonlinear and can not be solved directly by an analytical method. By assuming that the phase perturbations are small, the nulling equations can be linearized [2], but it makes impossible to place nulls at symmetric location with respect to the main beam. Therefore, the methods based on nonlinear optimization techniques [8–10] and several iterative methods [11, 12] have been proposed for phase-only control of antenna arrays. The phase-only null synthesizing is attractive since in a phased array the required controls are available at no extra cost [3]. It can be achieved to place the nulls at symmetric direction with respect to the main beam by perturbing the element positions, however, it requires a mechanical driving system such as servomotors to place the desired locations of the array elements [10, 13–15]. Although the methods of nulling mentioned above have their relative merits, none of them can produce the most close pattern to the desired radiation pattern with specified properties. The most efficient but most costly choice is control of both amplitude and phase of each array element because it has greater degrees of freedom for the solution space [3, 16–18]. This allows a trade-off between the quality of the constrained pattern and the complexity of electronic control.

In this paper, a method based on the tabu search algorithm is presented to steer the single, multiple and broad-band nulls to the directions of interference by the amplitude-only and also both the amplitude and the phase of each array element. Tabu search algorithm (TSA) [24, 25] has been developed to be an effective and efficient scheme for combinatorial optimization that combines a hill-climbing search strategy based on a set of elementary moves and a heuristics to avoid to stops at sub-optimal points and the occurrence of cycles. One characteristic of tabu search is that it finds good near-optimal solutions early in the optimization run. It does not require initial guesses, not use derivatives, and, it is also independent of the complexity of the objective function considered. Applications of the TSA as an optimization procedure to the electromagnetic and antenna problems [26–31] are very newly and only a few compared to other heuristic optimization techniques such as the genetic and the simulated annealing algorithms. In previous works [30, 31], we successfully introduced the TSA to compute the resonant frequencies of triangular and circular microstrip antennas. In these works, first, a model for the effective dimension expression of the microstrip antenna was chosen, then the unknown coefficient values of the expression were optimised by the TSA.

The TSA used in this study is the modified tabu search algorithm (MTSA) proposed by Karaboga et al. [30]. The classical TSA [24, 25] uses a solution vector consisting of a string of bits. Thus, in solving a numerical problem, the transformation from binary to real numbers should be used. However, the MTSA uses a real-valued solution vector and adaptive mechanism for producing neighbors. This neighbor production mechanism enables us to find the most promising region of the search space. Because of these fascinating features, in this work, the MTSA is used for the pattern nulling.

2. FORMULATION

If the array element excitations are conjugate-symmetrical about the center of the linear array, the far field array factor of this array with an even number of uniformly spaced isotropic elements ($2N$) can be written as

$$F(\theta) = 2\sum_{k=1}^{N} (a_{kR}\cos\phi_k - a_{kI}\sin\phi_k) \tag{1}$$

with

$$\phi_k = \frac{2\pi}{\lambda} d_k \sin\theta \tag{2}$$

where d_k is the distance between position of the kth element and the array center, θ is the scanning angle from broadside, and a_{kR} and a_{kI} are the real and imaginary parts of the kth element excitation a_k, respectively. In the optimization process, the dynamic range ratio, the maximum sidelobe level and the desired null depth level are also considered by including a set of weighting factors in the cost function given below.

$$\begin{aligned} C \;=\; & w_1\,|F_o(\theta) - F_d(\theta)| + w_2|NLDL_o - NLDL_d| \\ & + w_3|MSLL_o - MSLL_d| + w_4|a_{\max}/a_{\min}| \end{aligned} \tag{3}$$

where $F_o(\theta)$, $F_d(\theta)$, $NLDL_o$, $NLDL_d$, $MSLL_o$, and $MSLL_d$ are, respectively, the pattern of MTSA, the desired pattern, the null depth level of the MTSA, the desired null depth level, the maximum sidelobe level of the MTSA, and the desired maximum sidelobe level. The maximum and minimum values of element excitations are denoted by $a_{\max}$ and $a_{\min}$, respectively. The weighting factors w_1, w_2, w_3 and w_4 should be selected by experience such that the cost function is capable of guiding potential solutions to obtain satisfactory array pattern performance with desired properties. To obtain the desired pattern with the prescribed nulls, the cost function given in eq. (3) will be minimised by the MTSA, which is described in the following section.

3. MODIFIED TABU SEARCH ALGORITHM

Tabu search is a general heuristic search procedure devised for finding a global optimum of a function which may be linear or non-linear. It is a form of iterative search and does not use derivative-based transition rules. The classical TSA uses a solution vector consisting of a string of bits. Thus, in solving a numerical problem, the transformation from binary to real numbers should be used. This process has two major disadvantages. The first disadvantage is that the process yields a large number of neighbors (e.g., too many evaluations) when the word chosen is very long. The second disadvantage is the difficulty with neighborhood processing. This difficulty is that while a neighbor of the solution vector (e.g., a string of bits) is obtained, the changing of the most significant bit does not produce a number near the present variable. So, this is not reasonable regarding the neighborhood. In order to overcome these difficulties, the MTSA has been proposed in our previous work [30].

A real-valued solution vector is used by the MTSA; thus, a new neighbor production mechanism is constructed. In this mechanism, the neighbors are chosen adaptively, adding an adaptive coefficient at each iteration. Due to the diversification principle, the coefficient is large at early iterations; therefore, the neighbors are chosen too far from the present solution. This neighbor production mechanism enables us to find the most promising region of the search space. After some iterations, the coefficient is getting smaller; thus, the intensive searching at the most promising region can be done.

The MTSA starts with an arbitrary solution created by a random number generator. In this particular problem, it is equivalent to starting with randomly generated values for the element excitations. A solution is represented with a vector of element excitation values and an associated set of neighbors. A neighbor is reached directly from the present solution by an operation called "move". A succession of moves is carried out to transform the arbitrary solution to an optimal one. The new solution is the highest evaluation move among the neighbors in terms of the performance value and tabu restrictions which exist to avoid new moves that were evaluated in earlier iterations.

The tabu search used in this paper employs an adaptive mechanism for producing neighbors. The neighbors of a present solution for the element excitations are created by the following procedure.

If $F(t) = (a_1, a_2, \ldots, a_k)$ is the solution vector at the tth iteration, two neighbours $F(n_1, n_2)$ of this solution of which the element excitation a_k is not in the tabu list are produced by:

$$F(n_1, n_2) = \text{Remain}\left(F(\overline{n}_1, \overline{n}_2), a_{\max}\right) \tag{4}$$

where

$$F(\overline{n}_1, \overline{n}_2) = \begin{cases} a_k + \Delta(t) & \text{for odd neighbors} \\ a_k - \Delta(t) & \text{for even neighbors} \end{cases} \tag{5}$$

with

$$\Delta(t) = c_1 \left[\frac{\text{LatestImprovementIteration}}{t^{c_2} + \text{LatestImprovementIteration}}\right]^{c_3} \tag{6}$$

In eq. (4), the "remain function" keeps the elements of solution within the desired range. In eq. (6), c_1 determines the initial magnitude of $\Delta(t)$, c_2 and c_3 control the change of $\Delta(t)$, and LatestImprovementIteration is the iteration number at which the latest improvement was obtained. The index, t, in $\Delta(t)$ represents the iteration number.

Tabu restrictions used here are based on the recency and frequency memory storing the information about the past steps of the search. The recency-based memory prevents cycles of length less than or equal to a predetermined number of iterations from occurring in the trajectory. The frequency-based memory keeps the number of changes of solution vector elements. If an element of the solution vector does not satisfy the following tabu restrictions, then it is accepted as tabu:

$$\text{Tabu Restrictions} = \begin{cases} \text{recency}(k) > \text{recency limit} \\ \qquad\qquad \text{or} \\ \text{frequency}(k) < \text{frequency limit} \end{cases} \tag{7}$$

To select the new solution from the neighbors, performance values of all neighbors are evaluated in the cost function given by eq. (3) and the non-tabu neighbor producing the highest improvement according to the present solution is then selected as the next solution. If there are some tabu-neighbors which are better than the best solution found so-far, then those tabu solutions are freed.

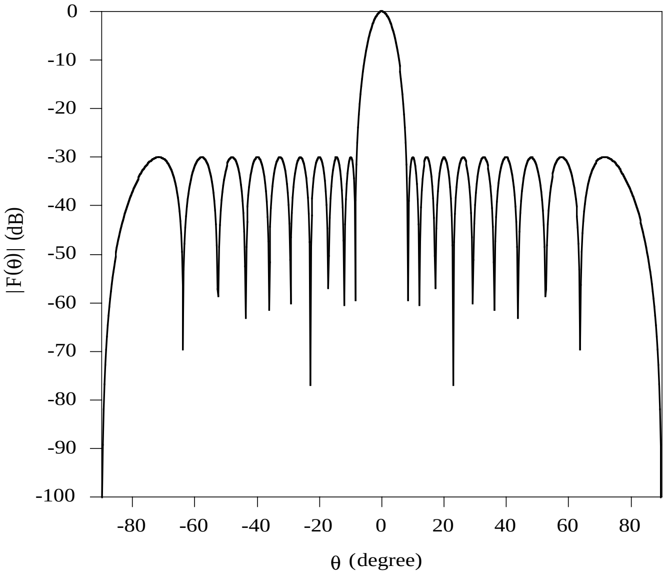

Figure 1. The initial 30-dB Chebyshev pattern.

4. NUMERICAL RESULTS

In order to illustrate the capabilities of the MTSA for steering single, multiple and broad-band nulls with the imposed directions by controlling the amplitude-only and both the amplitude and the phase, nine examples of a linear array with one-half wavelength spaced 20 isotropic elements have been performed. Initially, a 30-dB Chebyshev array pattern given in Fig. 1 for 20 equispaced elements with $\lambda/2$ interelement spacing is assumed. In the optimization process, the values of c_1, c_2, c_3, recency and frequency factors are chosen as 90 000, 3, 3, 1.5, and 2, respectively. The number of iterations is fixed to 600. This was sufficient to obtain satisfactory patterns with desired nulling performance on the average. The all calculations took almost 4 min on a personal computer with a Pentium III processor running at 700 MHz.

In the first application of the MTSA, three examples of the amplitude-only control are presented. The Chebyshev pattern with a single null imposed at $-20°$ is considered as the first example. The pattern is then obtained by the MTSA and illustrated in Fig. 2. The dynamic range ratios of this example and the initial Chebyshev pattern are 4.2 and 3.5, respectively. In order to show the effects of the weighting factors given in eq. (3) on the pattern, in the second example,

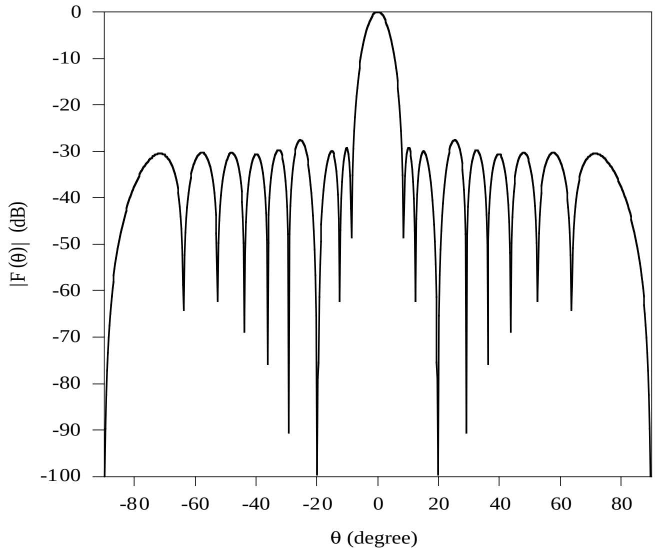

Figure 2. Radiation pattern obtained by controlling amplitude-only with one imposed null at $-20°$.

the dynamic range ratio is constrained to 3.6 by increasing only the value of the weighting factor w_4. The pattern having single null at $-20°$ with this constrained dynamic range ratio is shown in Fig. 3. As expected, the result of first example is better than that of the second example because the smaller amplitude range means smaller degrees of freedom for the solution space hence worse sidelobe and null depth performance. The null depths of Figs. 2 and 3 are 99.6 dB and 52.7 dB, respectively.

In the third example, the pattern with a broad null sector centered 30° with $\Delta\theta = 5°$ is considered. The resulting pattern is shown in Fig. 4. The desired broad null is achieved with a null depth of 113 dB at the center angle of 30°. In the three examples given above, since the array elements have even symmetry around the center of the array, a corresponding image nulls occurred at the other side of the main beam simultaneously. As a result of this assumption, the number of attenuators required is N for the array with $2N$ elements. Table 1 gives the element amplitudes obtained by using the MTSA for Figs. 2–4.

The following examples of the second application of the MTSA have been performed for the case of controlling both the amplitude and phase. To demonstrate the relative importance of sidelobe level and null depth by changing the weighting factors in the cost function,

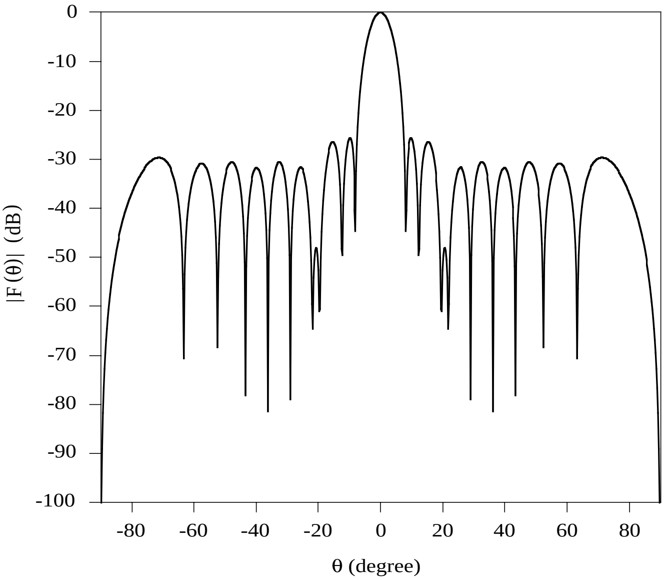

Figure 3. Radiation pattern obtained by controlling amplitude-only with one imposed null at $-20°$ and the constrained dynamic range of 3.6.

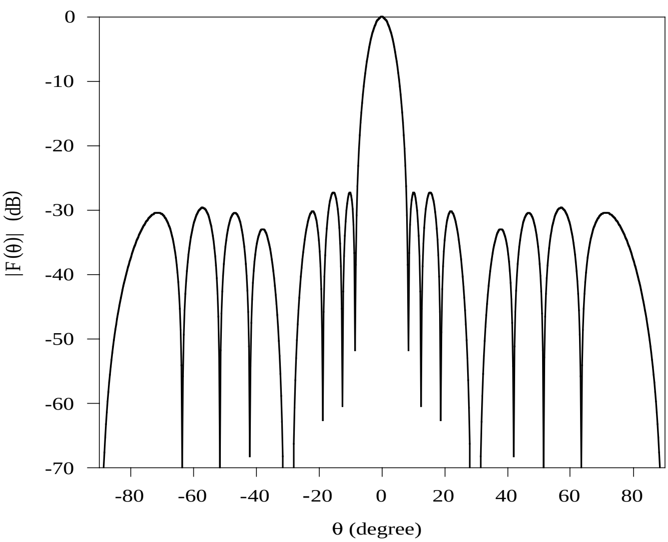

Figure 4. Radiation pattern obtained by controlling amplitude-only with a broad null sector centered $+30°$ with $\Delta\theta = 5°$.

Table 1. The element amplitudes computed by the MTSA for Figs. 2–4.

k	Element amplitudes		
	Fig. 2	Fig. 3	Fig. 4
±1	1.00000	1.00000	1.00000
±2	0.92957	0.95400	0.97010
±3	0.83554	0.85763	0.92263
±4	0.76751	0.78362	0.83132
±5	0.71990	0.72627	0.70817
±6	0.63215	0.66764	0.58014
±7	0.49578	0.55551	0.54721
±8	0.34967	0.42920	0.48164
±9	0.23787	0.27510	0.22778
±10	0.28537	0.29790	0.22881

the values of the $NLDL$ (w_2) and the $MSLL$ (w_3) are increased for the fourth and the fifth examples, respectively, while the other design parameters for these examples are the same as those of the first example. The corresponding patterns are shown in Figs. 5–6. The null depth and the maximum sidelobe level of Fig. 5 are 142 dB and -26.8 dB, respectively. However, the null depth and the maximum sidelobe level of Fig. 6 are 108 dB and -29.7 dB, respectively. These results apparently show that the trade-off of the relative importance between null depth and sidelobe level can be obtained by changing the weighting factors.

In Figs. 7 and 8, we have shown the nulling patterns with double nulls imposed at $-20°$ and $+40°$, and with triple nulls imposed at $-60°$, $-20°$ and $+40°$. It can be seen from Figs. 7 and 8 that all desired nulls are deeper than 85 dB. As a final example, the pattern having a broad null centered 30° with $\Delta\theta = 5°$ has been carried out by the MTSA and is shown in Fig. 9. A null depth of 70 dB is achieved over the spatial region of interest. The required element excitations obtained by MTSA for Figs. 5–9 have complex-conjugate symmetry about the center of the array and are listed in Table 2. It is also noted that the excitation coefficient values given in Tables 1–2 are normalized with respect to the excitation coefficient values of the center elements.

It is clear that the patterns in Figs. 5–9 are not symmetric with respect to the main beam. This is a consequence of the odd-symmetry of the element phases around the array center which, coupled with the even symmetry of the element amplitudes, results in a pattern that is not symmetric about the main beam peak at 0°. It should be also noted

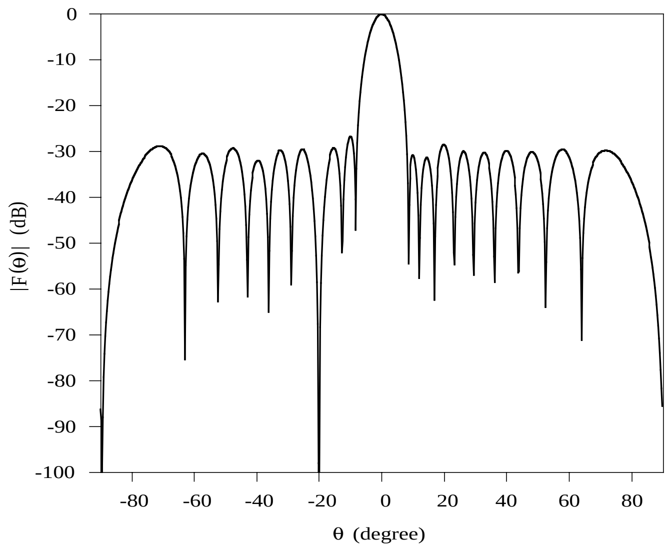

Figure 5. Radiation pattern obtained by controlling both the amplitude and phase with one imposed null at -20° and the constrained null depth.

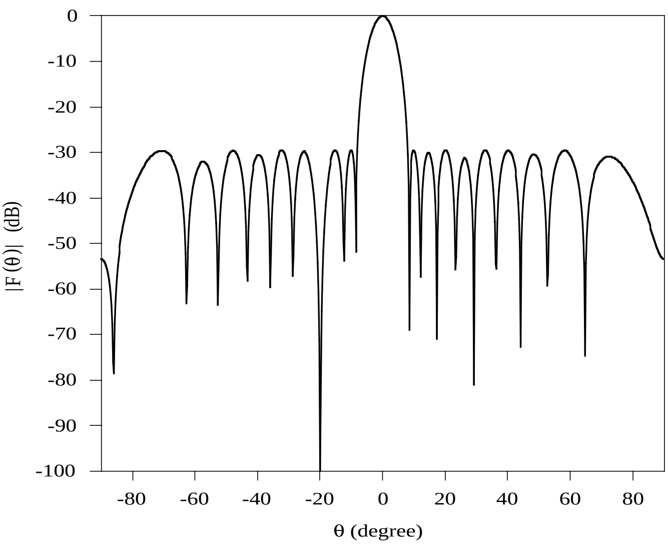

Figure 6. Radiation pattern obtained by controlling both the amplitude and phase with one imposed null at -20° and the constrained maximum sidelobe level.

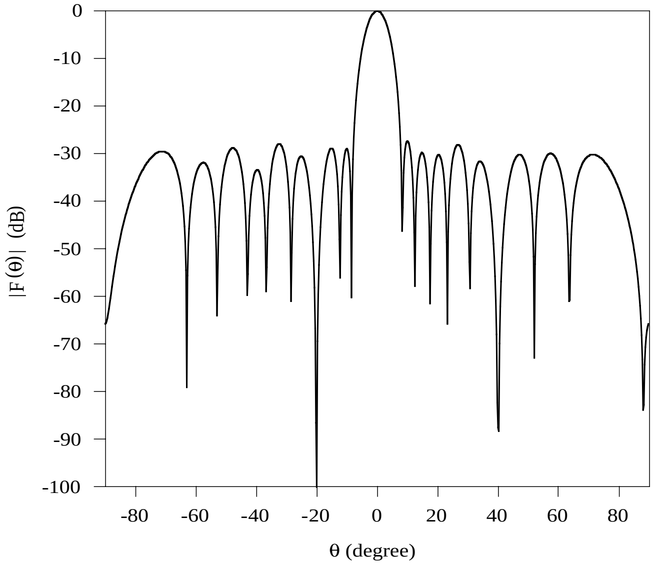

Figure 7. Radiation pattern obtained by controlling both the amplitude and phase with double imposed null at $-20°$ and $+40°$.

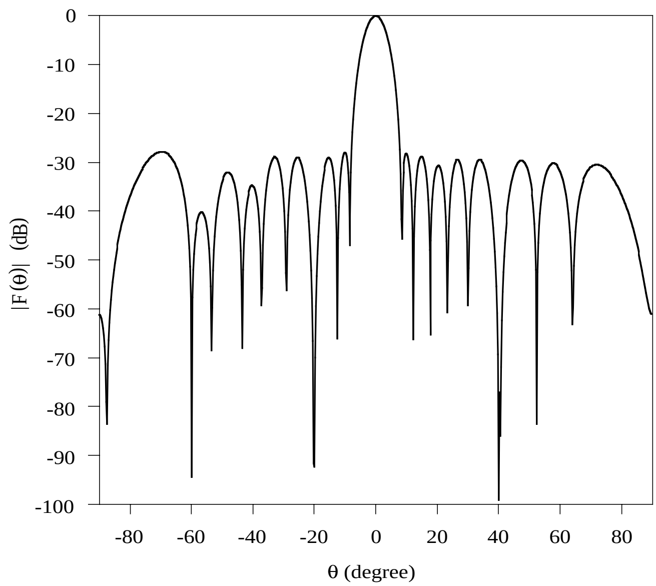

Figure 8. Radiation pattern obtained by controlling both the amplitude and phase with tripple imposed null at $-60°$, $-20°$ and $+40°$.

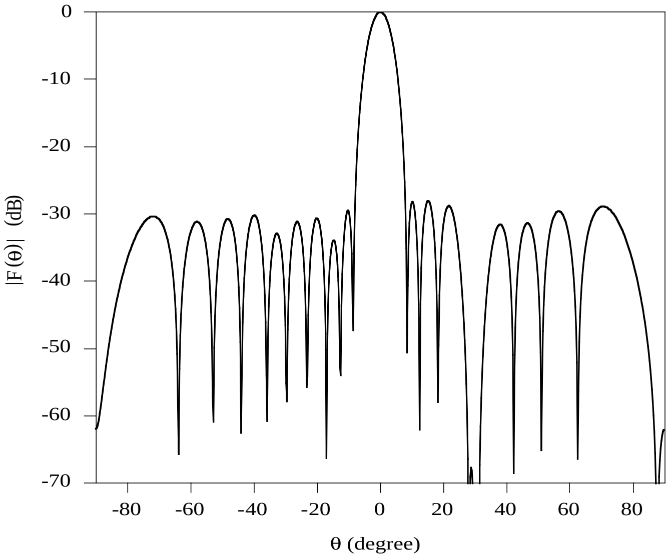

Figure 9. Radiation pattern obtained by controlling both the amplitude and phase with a broad null sector centered 30° with $\Delta\theta = 5°$.

Table 2. The complex weights computed by the MTSA for Figs. 5–9.

k	Complex weights				
	Fig. 5	Fig. 6	Fig. 7	Fig. 8	Fig. 9
± 1	$1.00000 \pm j0.0000$	$1.00000 \pm j0.0000$	$1.00000 \pm j0.0000$	$1.00000 \pm j0.0000$	$1.00000 \pm j0.0000$
± 2	$0.96710 \pm j0.0252$	$0.96440 \pm j0.0298$	$0.99840 \pm j0.0178$	$1.00680 \pm j0.0309$	$0.96540 \mp j0.0021$
± 3	$0.88993 \pm j0.0105$	$0.88753 \pm j0.0072$	$0.87753 \pm j0.0108$	$0.86893 \mp j0.0167$	$0.92643 \mp j0.0078$
± 4	$0.81812 \mp j0.0198$	$0.80622 \mp j0.0149$	$0.80222 \pm j0.0095$	$0.80792 \pm j0.0040$	$0.85232 \mp j0.0266$
± 5	$0.73877 \mp j0.0273$	$0.71847 \mp j0.0185$	$0.76267 \mp j0.0190$	$0.75077 \mp j0.0189$	$0.71487 \mp j0.0111$
± 6	$0.63934 \mp j0.0045$	$0.62964 \mp j0.0073$	$0.64344 \mp j0.0347$	$0.66054 \mp j0.0267$	$0.59624 \pm j0.0078$
± 7	$0.51181 \pm j0.0185$	$0.51331 \pm j0.0147$	$0.50221 \pm j0.0253$	$0.49321 \pm j0.0360$	$0.51231 \pm j0.0293$
± 8	$0.39134 \pm j0.0294$	$0.37514 \pm j0.0150$	$0.41284 \pm j0.0264$	$0.41044 \pm j0.0057$	$0.43074 \mp j0.0262$
± 9	$0.26228 \pm j0.0220$	$0.26108 \pm j0.0071$	$0.27278 \mp j0.0222$	$0.28508 \mp j0.0024$	$0.24468 \mp j0.0551$
± 10	$0.31331 \mp j0.0069$	$0.30091 \mp j0.0259$	$0.29181 \mp j0.0207$	$0.28241 \mp j0.0216$	$0.26411 \pm j0.0076$

that since the element excitations are conjugate-symmetrical about the center of the array the number of phase shifters to be used is $2N$, but the number of controllers for phase shifters is N due to odd symmetry property of the element phases.

It is evident from the Figs. 2–9 that this technique is capable of determining the element excitations for the array pattern with the single, multiple and broad nulls imposed at the directions of interference while the main beam and the sidelobes are quite close to the initial Chebyshev pattern. The half power beam width for nulling patterns by the MTSA is almost equal to that of initial Chebyshev pattern. The achieved null depths and the sidelobe levels have also very good performance. The results obtained by using both amplitude and phase are better than those of amplitude-only because it has greater degrees of freedom for the solution space than controlling only the amplitude.

The weighting factors used give the antenna designer greater flexibility and control over the actual pattern. The antenna designer should make a trade-off between the achievable and the desired pattern. By adjusting the weighting factors it is possible to obtain very reasonable approximations and trade-offs.

5. CONCLUSIONS

A method based on the tabu search algorithm for the pattern nulling is efficiently presented by controlling the amplitude-only and both the amplitude and the phase of each array element while keeping the pattern as close as possible to initial pattern. Numerical results illustrated show that the algorithm can obtain the patterns with satisfactory null depth, maximum sidelobe level and dynamic range ratio. It is worth noting that, although the algorithm proposed here is implemented to constrained synthesis for a linear array with isotropic half-wavelength spaced elements, one can see from the proposed technique it is not limited to this case. It can easily be implemented to nonisotropic-elements antenna arrays with different geometries for the design of various array patterns including superdirective, difference, and shaped-beam. Finally, it is hoped that this optimization approach can be helpful for antenna engineers as a simple, robust and flexible alternative to the other techniques used so far.

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