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MATHEMATICAL MODELING OF ELECTROMAGNETIC WAVE SCATTERING BY WAVY PERIODIC BOUNDARY BETWEEN TWO MEDIA

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Abstract—The extension of C method, combined with idea of Tikhonov's regularization is proposed. The regularizing algorithm for numerical solution of electromagnetic wave diffraction by the boundary of dielectric media is developed. This algorithm is based on the solution of the system linear algebraic equations of C method as subject of regularizing method of A. N. Tikhonov. The numerical calculations of scattered field in the case of E-polarization are presented. The efficiency and reliability of the method for the solution of the problems of boundary shape reconstruction have been proved and demonstrated numerically for several situations.

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1. INTRODUCTION

Mathematical modeling for boundaries between two media (terrain surface, ocean) has a large history and bibliography [1–4].

An efficient remote terrain and ocean monitoring requires solving of the following problems. The first, direct one, is the development of electromagnetic models of electromagnetic waves scattering by surfaces of various media. The second, inverse one, is based on these models and has to provide estimation and remote control of the relief of ocean and earth, namely their properties relying on information about certain characteristics of scattered electromagnetic fields.

It is clear that an efficient and robust solution to direct problem mentioned above is of principal importance. Although huge body of papers treating this complicated problem, there is evident lack of solutions which are based on rigorous approaches.

We propose here the robust and clear in implementation method that present certain modification of known C method [5–10] for solving the problem of electromagnetic wave scattering by rather arbitrary shaped surface. This approach makes a reliable base for solution of recognition problem: the reconstruction of surface profile and material parameters of media from known data of scattered electromagnetic field.

The major objective of present paper is consideration of the principal methodological issues, which can testify an accuracy and efficiency of the solution, and serve as keys to the successful utilization of the method suggested in real life experiments and devises. The preliminary study demonstrates high performance and reliability of the approach for rather wide scope of problems of material parameters and surface relief recognition and reconstruction.

2. DIRECT PROBLEM

We consider two dimensional diffraction problem for time-harmonic E polarized (electric field density vector is parallel to axis $0x$) plane waves by the arbitrary profile boundary of two media with relative permittivities ε_1 and ε_2 and permeability μ (Fig. 1). The boundary line between two media is described by function $z = a\,(y)$ with period

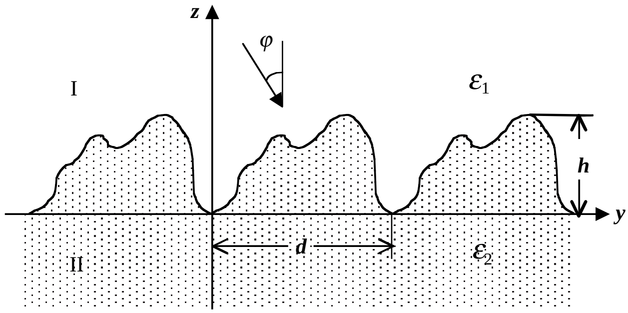

Figure 1. The profile of boundary between two media.

d, maximal deviation from axis $0y$ is equal to h (see Fig. 1). Incident plane wave propagates in the media with permittivity ε_1 with angle of incidence φ, which is counted as shown in Fig. 1. Time factor is chosen as $e^{-i\omega t}$. The excitation fields has the form

$$\begin{aligned} H_y^i &= Ae^{i\frac{2\pi}{\lambda}\sqrt{\varepsilon_1\mu}(y\sin(\varphi)-z\cos(\varphi))} \\ H_y^i &= -\sqrt{\frac{\varepsilon_1}{\mu}}\cos(\varphi)\,E_x^i \\ H_z^i &= -\sqrt{\frac{\varepsilon_1}{\mu}}\sin(\varphi)\,E_x^i \end{aligned}$$

Here λ is a wavelength in vacuum.

It is necessary to find out the diffraction filed that has to meet fallowing requirements:

1. Maxwell equations;
2. Radiation conditions at infinity;
3. Transparency boundary conditions, requiring continuity of tangential components of total field in the boundary;
4. The quasi periodic conditions (Floquet conditions);
5. The condition of energy boundness in any finite domain.

It can be proved, (see, for example [11] and [12]) that conditions 1–5 guarantee the unique-ness of the diffraction problem solution. For further convenience we introduce the following variables:

$$\bar{z} = \bar{\kappa}z, \quad \bar{y} = \bar{\kappa}y, \quad \bar{\kappa} = 2\pi/d.$$

Then Maxwell equations for diffraction field acquire the form

$$\begin{cases} \dfrac{\partial E^s_{x_n}}{\partial \bar{z}} = ik\mu H^s_{y_n} \\ -\dfrac{\partial E^s_{x_n}}{\partial \bar{y}} = ik\mu H^s_{z_n} \\ -ik\varepsilon_n E^s_{x_n} = \dfrac{\partial H^s_{z_n}}{\partial \bar{y}} - \dfrac{\partial H^s_{y_n}}{\partial \bar{z}} \end{cases} \qquad n = 1, 2; \tag{1}$$

where $\kappa = d/\lambda$, values $n = 1$ and $n = 2$ refer to the first and second media. The equation defining the boundary between media in new variables can be presented in the following form

$$\bar{z} = A_0 a(\bar{y}), \quad A_0 = 2\pi h/d,$$

where $a(\bar{y})$ is a periodic function with period equals 2π such that $0 \le a(\bar{y}) \le 1$.

For the sake of simplicity we consider the case $\varphi = 0$. All derivations for the case $\varphi \neq 0$ can be obtained in the similar way.

Subjecting diffraction field to boundary conditions, we derive

$$\begin{aligned} -i\kappa_1 e^{-i\kappa_1 A_0 a(\bar{y})} + \frac{\partial E^s_{x_1}}{\partial \bar{z}} - A_0 \dot{a}(\bar{y}) \frac{\partial E^s_{x_1}}{\partial \bar{y}} &= \frac{\partial E^s_{x_2}}{\partial \bar{z}} - A_0 \dot{a}(\bar{y}) \frac{\partial E^s_{x_2}}{\partial \bar{y}} \\ e^{-i\kappa_1 A_0 a(\bar{y})} + E^s_{x_1} &= E^s_{x_2} \end{aligned} \tag{2}$$

where $\kappa_1 = \kappa\sqrt{\varepsilon_1 \mu}$. Notation like $\dot{a}(y)$ means here and below the derivation in respect to the argument.

Following the conventional C method, we introduce new variables $v = \bar{y},\ u = \bar{z} - A_0 a(\bar{y})$, which transform equation (1) into the form

$$\begin{cases} \left(\dfrac{\partial}{\partial v} - A_0 \dot{a}(v) \dfrac{\partial}{\partial u}\right) G_{1n} + G_{2n} = 0, \\ -\left(\dfrac{\partial^2}{\partial u^2} + \kappa^2 \varepsilon_n \mu\right) G_{1n} + \left(\dfrac{\partial}{\partial v} - A_0 \dot{a}(v) \dfrac{\partial}{\partial u}\right) G_{2n} = 0, \end{cases} \quad n = 1, 2; \tag{3}$$

where $G_{1n} = E^s_{xn},\ G_{2n} = i\kappa\mu H^s_{zn}$. The equation, describing boundary, simplifies itself into $u = 0$ and transforms the boundary condition into equations

$$\begin{aligned} -i\kappa_1 e^{(-i\kappa_1 A_0 a(v))} \quad &+ \quad \left(1 + A_0^2 \dot{a}^2\right) \frac{\partial G_{11}}{\partial u} - A_0 \dot{a} \frac{\partial G_{11}}{\partial v} \\ &= \quad \left(1 + A_0^2 \dot{a}^2\right) \frac{\partial G_{12}}{\partial u} - A_0 \dot{a} \frac{\partial G_{12}}{\partial v} \end{aligned} \tag{4}$$

$$e^{(-i\kappa_1 A_0 a(v))} + G_{11}(v,0) = G_{12}(v,0)$$

The further derivations are connected with transformation of (3) and (4) into infinite system of linear algebraic equations with respect to unknown coefficients, which are the coefficients of expansions of functions G_{11}, G_{12} over the system of eigen functions of relevant spectral problems of C method.

3. SPECTRAL PROBLEM OF C-METHOD

We are seeking for the solutions of (3) having the form

$$G_n = e^{i\rho_n u} g_{mn}(v), m, n = 1, 2 \tag{5}$$

where ρ_n are the spectral parameters of C method. After substitution of (5) into (3) we arrive to

$$\begin{cases} \dot{g}_{1n} - i\rho A_0 \dot{a}(v) g_{1n} + g_{2n} = 0 \\ \dot{g}_{2n} - i\rho A_0 \dot{a}(v) g_{2n} + (\rho_n^2 - \kappa^2 \varepsilon_n \mu) g_{1n} = 0 \end{cases} \qquad n = 1, 2 \tag{6}$$

It is easy to see that functions g_{1n} and g_{2n} satisfy the equation

$$-\ddot{g}_{mn} + 2i\rho A_0 \dot{a}\dot{g}_{mn} + \left[i\rho A_0 \ddot{a} + \rho^2 A_0^2 \dot{a}^2 + \rho - k^2 \varepsilon_n \mu \right] g_{mn} = 0 \tag{7}$$

As a solution to (3) has to be periodic function with respect to variable v, the periodic condition is to be fulfilled by functions $g_{mn}(v)$:

$$g_{mn}(0) = g_{mn}(2\pi), \quad \dot{g}_{mn}(0) = \dot{g}_{mn}(2\pi) \tag{8}$$

Hence, it is necessary to find out the values of spectral parameter p providing non trivial solutions to equation (7) meeting the periodicity condition (8). The solution to this problem can be constructed in the following way. We shall seek for functions $g_{mn}(v)$ as an expansion to Fourier series:

$$g_{mn}(v) = \sum_{p=-\infty}^{\infty} F_p^{mn} e^{ipv} \tag{9}$$

Substituting (9) into (6) we obtain the infinite system of equations that we can present in matrix form

$$X - A_n(\rho) X = 0 \tag{10}$$

where

$$X = \left\| \begin{matrix} F_{1n} \\ F_{2n} \end{matrix} \right\|, \quad F_{nm} = \left(F_p^{nm} \right)_{p=-\infty}^{\infty}$$

$$A_n(\rho) = \begin{bmatrix} A(\rho) & -iD \\ i\gamma_n^2 D & A(\rho) \end{bmatrix}, \quad A(\rho) = \|A_{qp}\|_{q,p=-\infty}^{\infty}$$

$$D = \|D_{qp}\|_{q,p=-\infty}^{\infty}, \quad D_{qp} = \begin{cases} 1, p = q = 0 \\ \dfrac{\delta_{qp}}{p}, p \neq 0 \end{cases}$$

and δ_{qp} is Kroneker delta, $\gamma_n^2 = \kappa^2 \varepsilon_n \mu - \rho^2$. The entries A_{qp} are expressed via Fourier coefficients of function $\dot{a}(v)$:

$$A_{qp} = \begin{cases} 1, p = q = 0 \\ \rho A_0 \dot{a}_{-p}, q = 0 \\ \rho A_0 \dfrac{\dot{a}_{q-p}}{q}, q \neq 0 \end{cases}$$

Matrices $A(\rho)$ and D produce compact operators in space l_2, which are analytically depending on spectral parameter p. Thus, matrices $A_n(\rho)$ from (10) also produce compact operator in space $l_2 \times l_2$, and each $A_n(\rho)$ is analytical operator-function of parameter p.

It can be proved that for $\rho \neq \pm\sqrt{\kappa^2 \varepsilon_n \mu - p^2}$, where $p = 0, \pm 1, \pm 2 ...$, the bounded operator $(\mathrm{I} - A_n(\rho))^{-1}$ exists. The set of values p, providing the existence of nontrivial solutions to equation (10), is countable, isolated and of finite multiplicity. This follows from Fredholm's theorem [13] about analytical operator-functions. For numerical solution of (10) we applied truncation method. This is correct for operator-function $A_n(\rho)$ is compact.

Now, having values of spectral parameter p and corresponding to p eigen vectors X, one can construct the functions $e^{i\rho u} g_{mn}(v)$, satisfying the system of equations (3).

4. THE SOLUTION TO THE DIFFRACTION PROBLEM. REGULARIZING ALGORITHM

As it has been stated above, the set of spectral parameter corresponding to both media ($n = 1, 2$) is not more than countable and isolated set.

Let $U_n = (\rho_{mn})_{m=1}^{\infty,} n = 1, 2$ is the set of spectral parameters such that $\mathrm{Re}(\rho_{m1}) + \mathrm{Im}(\rho_{m1}) \geq 0$ and $\mathrm{Re}(\rho_{m2}) + \mathrm{Im}(\rho_{m2}) \leq 0$. Then, according to C-method, functions G_{11} and G_{12}, describing diffraction fields in the first and second media correspondingly, can be presented in the form

$$G_{11} = \sum_{m\in V_1} C_{m1} e^{i\rho_{m1}u} g_{m1}(\nu), u \geq 0$$
$$G_{12} = \sum_{m\in V_1} C_{m2} e^{i\rho_{m2}u} g_{m2}(\nu), u \leq 0 \tag{11}$$

Note, that the choice of sets $U_n,\ n = 1, 2$ is dictated by necessity for diffraction field to meet radiation conditions in corresponding domains - half spaces $u \geq 0$ and $u \leq 0$ respectively. Now, we substitute expansion (11) into boundary conditions (4), and accounting (9), we obtain the finite system of linear algebraic equations with unknowns $(C_{nm})_{m=-\infty}^{\infty}, n = 1, 2,$

$$\sum_{m\in U_1} F_n^{m1} C_{m1} - \sum_{m\in U_2} F_n^{m2} C_{m2} = -L_n(-A_0\kappa_1) \tag{12}$$
$$\sum_{m\in U_1} G_n^{m1} C_{m1} - \sum_{m\in U_2} G_n^{m2} C_{m2} = \kappa_1 L_n(-A_0\kappa_1)$$
$$n = 0, \pm 1, \pm 2 \ldots \tag{13}$$

Here

$$G_n^{mp} = \frac{\kappa^2 \varepsilon_p \mu - n^2}{\rho_{mp}} F_n^{mp} + nA_0 \sum_{s=-\infty}^{+\infty} \dot{a}_{n-s} F_s^{mp}$$
$$L_n(\gamma) = \frac{1}{2\pi} \int_0^{2\pi} \exp\{i\gamma a(\nu) - in\nu\} d\nu \tag{14}$$
$$\dot{a}_n = \frac{1}{2\pi} \int_0^{2\pi} \dot{a}(\nu) \exp\{-in\nu\} d\nu$$

We can also rewrite (12) in matrix form:

$$Fx = B, \quad F = \begin{bmatrix} F_1 & -F_2 \\ G_1 & -G_2 \end{bmatrix}$$
$$F_p = \|F_n^{mp}\|_{m=1,n=-\infty}^{\infty}, \quad G_p = \|G_n^{mp}\|_{m=1,n=-\infty}^{\infty}$$
$$B = \left\| \begin{matrix} B_1 \\ B_2 \end{matrix} \right\|, \quad B_1 = (-L_n(-\kappa_1 A_0))_{n=-\infty}^{\infty} \tag{15}$$
$$B_2 = -\kappa_1 B_1, \quad x = \left\| \begin{matrix} C_1 \\ C_2 \end{matrix} \right\|$$

Analyses of matrix entries of (12) made it clear that operator equation (14) is an equation of the first kind. That is why the direct usage of truncation method for numerical solution of (14) is undesirable because of well known instability problem arising. That is why some regularizing procedure is absolutely necessary. We suggest to solve operator equation (14) applying Tikhonov's regularization [14].

The formal schema of regularizing procedure includes the following steps. Owing to the fact that original diffraction problem has unique solution, equation (14) has unique solution also. Suppose that instead of explicit values of F and B we know their approximate values $\tilde{F}$ and $\tilde{B}$, namely

$$\sup_{\|x\|=1} \left\| \tilde{F}x - Fx \right\| \leq h, \quad \left\| \tilde{B} - B \right\| \leq \delta$$

where h and δ are known input data of the algorithm.

As $\tilde{F}$ and $\tilde{B}$ we can take corresponding truncated matrixes in (12). Tikhonov's regularization method suggest the search of elements x_α providing minimum to smoothing functional

$$\Phi_\alpha (x_\alpha) = \left\| \tilde{F}x_\alpha - \tilde{B} \right\|^2 + \alpha \|x_\alpha\|^2 \tag{16}$$

where $\alpha > 0$ is regularizing parameter, which is to be defined from the condition

$$\left\| \tilde{F}x_\alpha - \tilde{B} \right\|^2 = 2 \left(h \|x_\alpha\|^2 + \delta \right) \tag{17}$$

As a norm $\|\ldots\|$ in (15) one can choose the norm of corresponding finite-dimensional space. With such a choice the search of x_α from (15) is equivalent to the solution of the equation

$$\alpha x_\alpha + \tilde{F}^* \tilde{F} x_\alpha = \tilde{F}^* \tilde{B} \tag{18}$$

Here $\tilde{F}^*$ is the conjugate to $\tilde{F}$ operator.

Equation (17) has been solved numerically by means of truncation method. Parameter α has been chosen from condition (16). Numerical experiments proved high efficiency and considerable enforcing of stability of the suggested method of the solution to (14).

5. INVERSE PROBLEM

The developed above method of the solution to the direct diffraction problem and corresponding numerical algorithm form the efficient background for the following inverse problem.

The input data for the problem are complex amplitudes $R = (R_n(\lambda))_{n=-N}^{N}$ of reflected propagating waves, λ is a wave length. We suppose that this data are known in certain range $[\lambda_1, \lambda_2]$. Besides the period of boundary shape and dielectric parameters of media are also known. It is necessary do find out by these input data the function, defining the boundary of two media. Let $a = (a_n)_{n=-\infty}^{\infty}$ are Fourier coefficients of this boundary function. The solution

of operator equation (17) gives the mapping that associates set of $a = (a_n)_{n=-\infty}^{\infty}$ with set of complex amplitudes $R = (R_n(\lambda))_{n=-N}^{N}$. Thus, the non linear operator

$$F(a, \lambda) = R(\lambda), \lambda \in [\lambda_1, \lambda_2] \tag{19}$$

is defined on certain set of vectors $D_F \subset l_2$. Consequently, the mathematical posing of inverse problem consists in finding out the solution to (18) in such sense that residual $F(a, \lambda) - R(\lambda)$ is minimized in relevant metric (see (20)). Having found out the Fourier coefficients $a = (a_n)_{n=-\infty}^{\infty}$ from (18), we can derive the function, describing the boundary between two media. This can be done by means of stable procedure that is the summation of Fourier series with approximate in l_2 space metric coefficients [15].

Formally, the scheme of solution may be outlined as follows. Let $Y(\lambda)$ is the set of operator F values. Introduce on $Y(\lambda)$ the norm according to the following formula

$$\|R(\lambda)\|_1^2 = \sum_{-N}^{N} |R_n(\lambda)|^2 \frac{\cos\varphi_n}{\cos\varphi} \tag{20}$$

Here the following notations are used: φ_n are angles of diffracted field, φ is angle of the incident field. Consider the functional that is given in domain D_F of operator F definition:

$$\Phi(a) = \sum_{m=1}^{P} \left\| R_n^e(\lambda_m) - R_n^M(\lambda_m) \right\|_1^2 + \gamma \sum_{n=-Q}^{Q} |a_n|^2 \left(1 + n^{2R}\right) \tag{21}$$

where $\gamma > 0$ is the parameter of regularization, $R = 1$ (in general, $R \geq 1$ is a parameter of the functional), $\lambda_m \in [\lambda_1, \lambda_2]$, $a(y) = \sum_{n=-Q}^{Q} a_n e^{iny}$,. Vector $a_y = (a_{\gamma n})_{n=-Q}^{Q}$, which provides the functional (20) with minimum, is considered to be a solution to (18). Norm $\|\ldots\|_1$ is defined by formula (19). Vectors $R^e = (R(\lambda_m))_{m=1}^{P}$ are input data of inverse problem. They can be found from solution of direct problem (12) with given vector $a_y = (a_{\gamma n})_{n=-Q}^{Q}$.

The search of vector a_y is constructed by means of regularized quasi Newton's method with step adjustment, using only first derivatives. The minimum residual method (see [14,15]) is applied for the choice of regularizing parameter γ that is in compliance with given level of noise in input data $R^e(\lambda_m) = (R_n^e(\lambda_m))_{n=-N}^N$. On the bases of the approach developed, the numerical algorithms for the solving (18) and (19) have been implemented.

6. NUMERICAL EXPERIMENTS

Here we present several numerical illustrations for test problems, which have been performed by suggested approach and the corresponding algorithm implementation. Relying on the solution to equation (17), we simulated input data $R^e(\lambda_m) = (R_n^e(\lambda_m))_{n=-N}^N$, $m = 1, 2 \ldots P$ for two boundaries between media. We have chosen two types of boundary profile:

$$a_1(y) = h\left[0.5 + \frac{\pi^3 y}{6d}\left(\frac{2y}{d} - 1\right)\left(1 - \frac{y}{d}\right)\right]$$

and

$$a_2(y) = h\left[0.375 + 0.25\sin\left(\frac{2\pi y}{d}\right) + 0.125\cos\left(\frac{4\pi}{d}\right)\right]$$

that are periodically continued from interval $[0, d]$ onto interval $(-\infty, +\infty)$. Parameters d and h feet restriction $\frac{2\pi h}{d} \leq 1$. The wavelength of incident plane E polarized wave was varying within the range $0.5 \leq \frac{d}{\lambda} \leq 3.5$. Permittivity of the first medium has been chosen as $\varepsilon_1 = 1$ and of the second one as $\varepsilon_2 = 2.25$. Permeability of both media is $\mu = 1$. Functions $a_1(y)$ and $a_2(y)$ are chosen for they belong to two essentially different classes. Namely, function $a_2(y)$ is a finite series of its Fourier coefficients. In the contrary, function $a_1(y)$ is an infinite Fourier series, which Fourier coefficients have algebraic type of decaying only.

Results of numerical tests are presented in Fig. 2 and Fig. 3. Solid lines correspond to the ex-act functions $a_1(y)$ and $a_2(y)$. Dotted lines are the graphs of functions $a_1^R(y)$ and $a_2^R(y)$ that have been defined via input data $R^e(\lambda_m) = (R_n^e(\lambda_m))_{n=-N}^N$ according to above described algorithm. As they almost coincide with graphic accuracy, the deviations $h^{-1}\left(\left|a_1(y) - a_1^R(y)\right|\right)$ and $h^{-1}\left(\left|a_{21}(y) - a_2^R(y)\right|\right)$ are presented in the same figures. As it is clearly seen, the maximum absolute value of deviation is less than 10^{-3} for function $a_2(y)$ and 10^{-2} for $a_1(y)$. It worth to be emphasized that maximum absolute value of deviation essentially decreases with value of points P increases

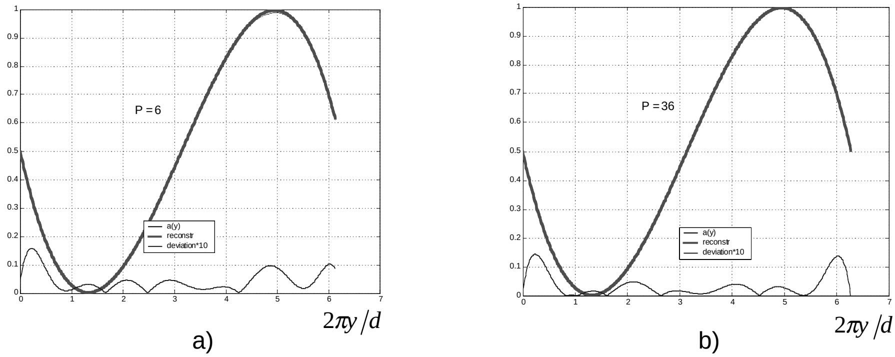

Figure 2. Reconstruction of boundary shape for the profile given by function $a_1(y) = h\left[0.5 + \frac{\pi^3 y}{6d}\left(\frac{2y}{d} - 1\right)\left(1 - \frac{y}{d}\right)\right]$ for different values P of given incident waves: $P = 6$ (Fig. a) and $P = 36$ (Fig. b), where $\varepsilon_2 = 2.25, 0.5 \leq d/\lambda \leq 3.5, 2\pi h/d = 0.4$.

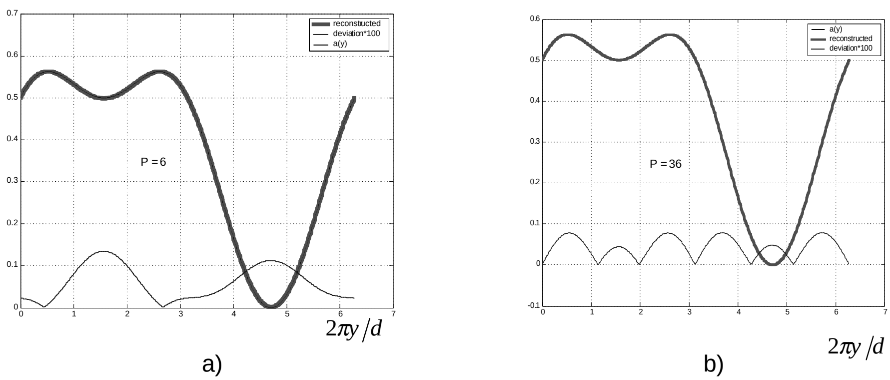

Figure 3. Reconstruction of boundary shape for the profile given by function $a_1(y) = h\left[0.375 + 0.25\sin\left(\frac{2\pi y}{d}\right) + 0.125\cos\left(\frac{4\pi}{d}\right)\right]$ for different values P of given incident waves: $P = 6$ (Fig. a) and $P = 36$ (Fig. b), where $\varepsilon_2 = 2.25, 0.5 \leq d/\lambda \leq 3.5, 2\pi h/d = 0.4$.

(we remind that P is a number of values of incident plane wave wavelengths, for which the input data $R^e(\lambda_m)$, $m = 1, 2 \ldots P$ have been calculated).

Basing on the numerical experiments have been performed and partially illustrated here, we can conclude that the approach suggested enables the reconstruction of the boundary functions of two media. When the relative level of noise in input data is about 10^{-3}, the relative error of the profile reconstruction is less than 10^{-2}.

7. CONCLUSION AND PERSPECTIVE

The extension of C method is suggested. It is made in two main directions. The first one is concerning direct two dimensional problem of time-harmonic plane wave diffraction by periodic boundary between two dielectric media. The key new step is Tikhonov's regularization involved in solution procedure. Such involving has given essential increasing of the stability of canonic C method.

The second extension is based on the first one, and is devoted to new area of C method application, namely, to inverse and ill posed problems of various media's boundary recognition and reconstruction. This extension, as well as the first one, includes ideas of Tikhonov's regularization as essential part of the method.

Numerical tests proved the efficiency and rather good stability of new algorithms suggested. A very similar technique can be used for reconstruction of material parameters of the media (which were supposed to be known in the present paper). The detailed description of such aimed algorithms will be the topic of our special publication.

Thus, the methods developed herein and, especially, the ideas lying in their background are looking very promising for the solution of wide area of applied problems of remote sensing and monitoring of earth and ocean.

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