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A GENETIC ALGORITHM/METHOD OF MOMENTS APPROACH TO THE OPTIMIZATION OF AN RF COIL FOR MRI APPLICATIONS — THEORETICAL CONSIDERATIONS

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Abstract—A Combined Genetic Algorithm and Method of Moments design methods is presented for the design of unusual near-field antennas for use in Magnetic Resonance Imaging systems. The method is successfully applied to the design of an asymmetric coil structure for use at 190 MHz and demonstrates excellent radiofrequency field homogeneity.

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1. INTRODUCTION

In this paper, a novel, full-wave, forward optimization technique based on a Genetic Algorithm (GA) and Method of Moments (MoM) is introduced for the design of Magnetic Resonance Imaging (MRI) transmitter/receiver radiofrequency (RF) coils. Our immediate goal is to construct a numerical technique whereby a conductor pattern can be optimized given a required time-harmonic, field distribution within a desired target region in the near-field of the structure.

In conventional MRI applications, a patient is placed in a strong and homogeneous static magnetic field, causing the otherwise randomly oriented magnetic moments of the protons, in water molecules within the body, to precess around the direction of the applied field. The part of the body in the homogeneous region of the magnet is then irradiated with (RF) energy, causing some of the protons to change their spin orientation. When the RF energy source is removed, the protons in the sample return to their original configuration, inducing measurable signal in a receiver coil tuned to the frequency of precession. This is the magnetic resonance (MR) signal. The useful RF components are those generated in at plane at 90 degrees to the direction of the static magnetic field. The same coil structure that generates the RF field can be used to receive the MR signal or a separate receiver coil place close to the patient may be used. In either case the coils are tuned to the Larmor precessional frequency ω_0 where $\omega_0 = \gamma B_0$ and γ is the gyromagnetic ratio for a specific nuclide and B_0 is the applied static magnetic field. It is important that within the region of interest (also known as the diameter of the sensitive region or DSV), the RF fields are of as constant an amplitude as possible.

As MRI progresses towards high frequency applications, organ specific RF coils (those tailored to the shape of regions of the body) must be designed with full-wave considerations in mind. This has not been the case in the past, as the field strength (and therefore operating frequency) of conventional RF coils has only warranted the use of quasi-static design methods [1 and references therein]. Organ specific designs require flexible geometric structures that are somewhat difficult to achieve using inverse methods. Although inverse methods have considerable promise in many applications [2–4], we explore the use of flexible forward methods for the optimization of an asymmetric RF coil, a difficult design problem.

The potential advantages of an asymmetric system for MRI [5], which include reduced patient claustrophobia, has initiated considerable interest in developing an RF coil for this application. Given the specified RF magnetic field homogeneity within the DSV, the proposed technique begins with the use of GA to obtain a quasi-static solution for the contours' geometry. Then a second GA step, which combines with a MoM algorithm, is implemented to search for a time-harmonic solution for the values and positions of the capacitors that produces the required current density and therefore field homogeneity. Finally, two input tuning and matching capacitors and a shield acting as a ground plane are included in the geometry. The EM software package FEKO [6] is employed for the adjustment of the tuning capacitors.

In this paper, the proposed technique is applied to optimize an asymmetric RF coil operating at 190 MHz (corresponding to ^{1}H resonances in a 4.5 Tesla MRI system). The geometry of the structure is shown in Figure 1. The coil is of cylindrical shape of length L and radius R; and the size and the geometry of the DSV is characterized by its radius r_o and axial displacement z_{os}.

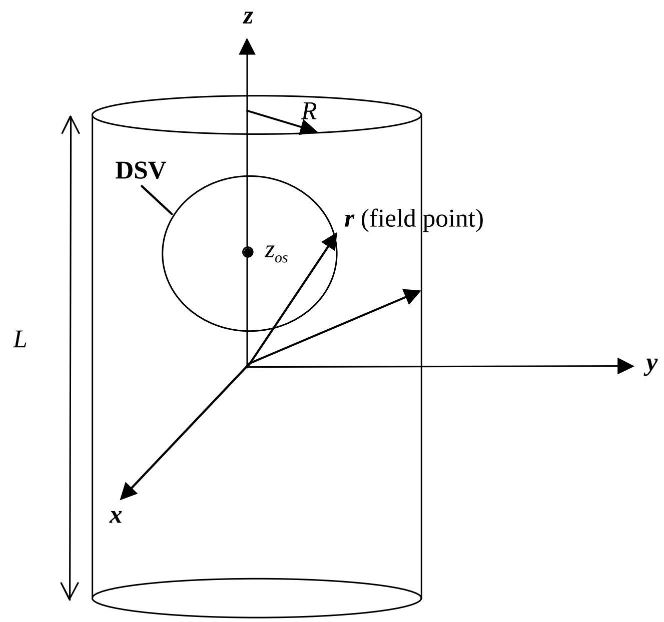


Figure 1. Geometry of the RF coil.

2. THEORY AND RESULTS

The configuration of [3] characterized by two symmetrical sets of contours is adopted in this paper. Let visualise the surface of the cylinder in the plane defined by $-90° \leq \phi \leq 90°$ and $-L/2 \leq z \leq L/2$, as the ζ-z plane illustrated in Figure 2. The position vector of any

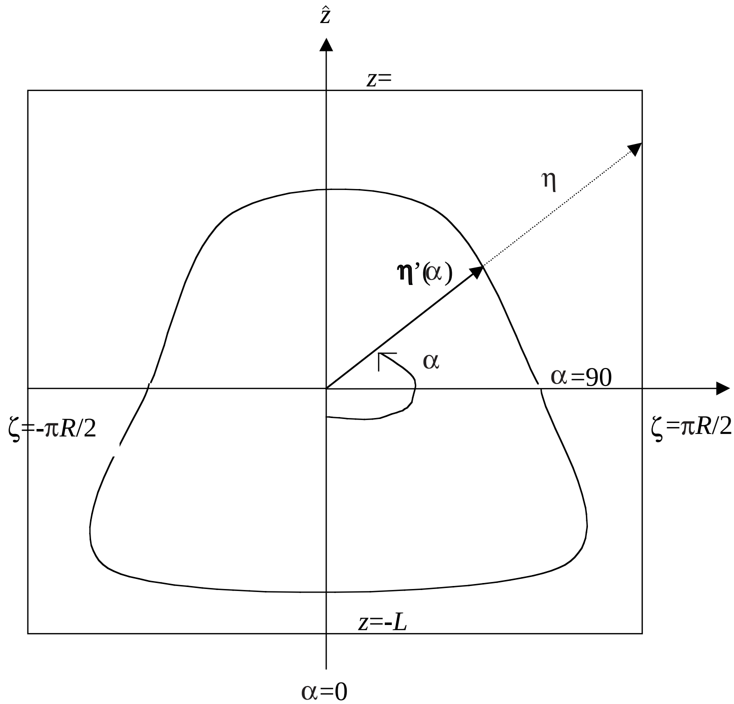


Figure 2. Layout of the surface of a half cylinder.

point on the loop, $\boldsymbol{\eta}'$, is defined by the corresponding radial and angular coordinates η and α. The maximum length allowed in the plane at a given α is defined by $\eta_{\max}(\alpha)$. The loop is then subdivided into N number of thin wire segments each separated by an angular displacement of $\Delta\alpha_n$ as shown. Since the loop is symmetrical about the z axis, the following cosine series can be used to express the radial position of any vertex as:

$$\eta'_n(a_M, \ldots, a_0) = a_0 + \sum_{i=1}^{M} a_i \cos\left(i\alpha'_n - \frac{\pi}{2}\right) \tag{1}$$

where $\{a_M, \ldots, a_0\}$ are the unknown coefficients of the series. Let us transform $\boldsymbol{\eta}'$ back to the corresponding $\boldsymbol{r}'$ in the cartesian coordinates where $\boldsymbol{r}' = R\cos\psi\hat{x} + R\sin\psi\hat{y} + z\hat{z}$ and $\psi = \eta'\cos(\alpha - \pi/2)/R$, and

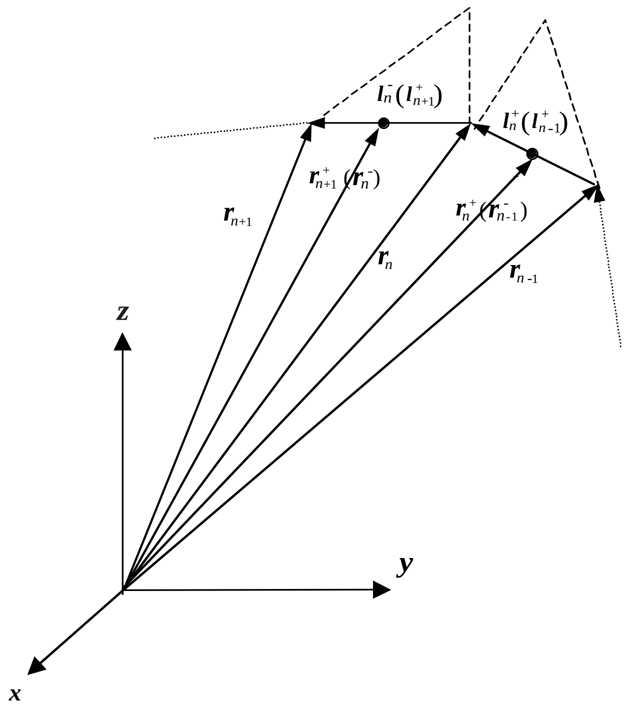


Figure 3. Definition of rooftop functions.

introduce the one-dimensional rooftop basis function $\boldsymbol{T}_n(\boldsymbol{r}')$ as follows:

$$\boldsymbol{T}_n(\boldsymbol{r}') = \begin{cases} 1 - \frac{\boldsymbol{r}_n - \boldsymbol{r}'}{|\boldsymbol{l}_n^+|} \frac{\boldsymbol{l}_n^+}{|\boldsymbol{l}_n^+|} & \boldsymbol{r}' \text{ on } \boldsymbol{l}_n^+ \\ 1 - \frac{\boldsymbol{r}' - \boldsymbol{r}_n}{|\boldsymbol{l}_n^-|} \frac{\boldsymbol{l}_n^-}{|\boldsymbol{l}_n^-|} & \boldsymbol{r}' \text{ on } \boldsymbol{l}_n^- \end{cases} \tag{2}$$

As shown in Figure 3 the rooftop function points in the anti-clockwise direction and reside on two segments (centred at $\boldsymbol{r}_n^+$ and $\boldsymbol{r}_n^-$) which are defined by the vectors $\boldsymbol{l}_n^+$ and $\boldsymbol{l}_n^-$ respectively.

The current in wire are expanded by a set of rooftop basis functions written as:

$$\boldsymbol{J}(\boldsymbol{r}') = \sum_{n=1}^{N} I_n \boldsymbol{T}_n(\boldsymbol{r}') \tag{3}$$

where I_n are the current coefficients and $\boldsymbol{r}'$ is the position of the current point source.

The vector potential at any point $\boldsymbol{r}$ in the cartesian coordinates is given by

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0 w}{4\pi} \int_C G(\boldsymbol{r}|\boldsymbol{r}')\boldsymbol{J}(\boldsymbol{r}')dl' \tag{4}$$

where w is the diameter of the wire and C is the perimeter of the loop; $G(\boldsymbol{r}|\boldsymbol{r}')$ is the free-space Green's function; and $\boldsymbol{J}(\boldsymbol{r}')$ is the current flowing in the loop given by (3).

Assuming the quasi-static case where current is uniformly distributed along the wire, we assign $I_j = 1$ and $G(\boldsymbol{r}|\boldsymbol{r}') = 1/|\boldsymbol{r} - \boldsymbol{r}'|$.

The transverse magnetic field is related to the vector potential by:

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \tag{5}$$

Substituting (3) and (4) into (5), we can express the transverse magnetic field B_x in terms of the coefficients $\{a_M, \ldots, a_0\}$ as

$$B_x(\boldsymbol{r}, \{a_M, \ldots, a_0\}) = \sum_{j=1}^{N} \frac{\partial}{\partial y} G(\boldsymbol{r}|\boldsymbol{r}')l_j \cdot \hat{z} - \frac{\partial}{\partial z} G(\boldsymbol{r}|\boldsymbol{r}')l_j \cdot \hat{y} \tag{6}$$

Let $\boldsymbol{r}_1, \ldots, \boldsymbol{r}_Q$ be the positions of the Q sample field points chosen within the DSV, we define the degree of homogeneity as f_0, or the difference between the maximum and minimum field at these points as:

$$\begin{aligned} f_0(a_M, \ldots, a_0) \; = \; & \max(B_x(\{\boldsymbol{r}_Q, \ldots, \boldsymbol{r}_1\}, \{a_M, \ldots, a_0\})) \\ & - \min(B_x(\{\boldsymbol{r}_Q, \ldots, \boldsymbol{r}_1\}, \{a_M, \ldots, a_0\})) \end{aligned} \tag{7}$$

This function is then to be optimized by the use of our GA code [7]. Initially a set of P chromosomes $(a_M, \ldots, a_0)$, or the population, are generated randomly. Each individual is encoded in binary form as chromosome of size $M \times S$ where S is the number of bits for each parameter. The number of possible values allowed for a_i is 2^S, and each parameter a_i is represented as $b_i^{(S)}, \ldots, b_i^{(1)}$ $(i = 1, \ldots, M;\ b^{(j)} \in \{0, 1\};\ j = 1, \ldots, S)$. Each chromosome is associated with a fitness value representing its ability to survive. Reproduction is then performed through the fitness-weighted selection process. Crossover and mutation are implemented and the next generation are then produced for the next cycle of generation. The algorithm terminates as the generation evolves to a stage where the chromosome of best fitness converges to a solution. The procedure is summarized in the diagram of Figure 4.

The RF coil has the following parameters: $R = 100\,\text{mm}$; $L = 320\,\text{mm}$; $r_0 = 40\,\text{mm}$; $z_{os} = 50\,\text{mm}$ with homogeneity specified as

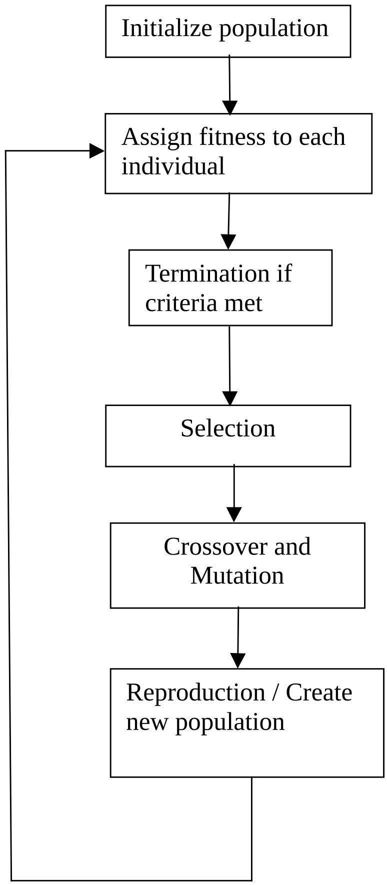


Figure 4. Fundamental structure of a genetic algorithm.

5% the difference between maximum and minimum B_x. The contour is constructed with a wire of radius $w = 1.1\,\text{mm}$. The first step is applying the GA code to obtain a quasi-static solution for the contour's layout. The major parameters in the algorithm were: $P = 50$; $M = 8$; $S = 16$. Each contour consists of only one loop which is defined by the parameters within the following range: $40\,\text{mm} < a_0 < 90\,\text{mm}$ and $-50\,\text{mm} < a_i < 50\,\text{mm}$, $i = 1, \ldots, M-1$. For certain sets of coefficients, the size of the contour at a given α_0 may exceed the maximum limit of $\eta'_{\max}(\alpha_0)$ (or $\eta' \geq \eta_{\max}$). In this case, the equality $\eta'(\alpha_0) = \eta_{\max}(\alpha_0)$ is implemented in our algorithm. A solution was generated after 30 generations and the specified homogeneity is achieved. Figure 5 shows the geometry of the contours and the resultant quasi-static B_x field in

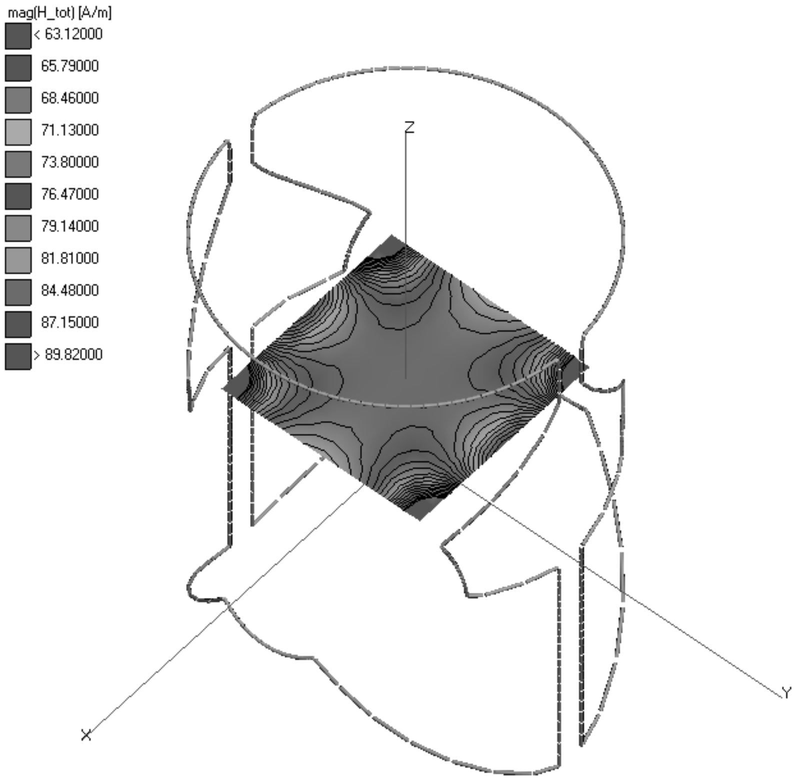
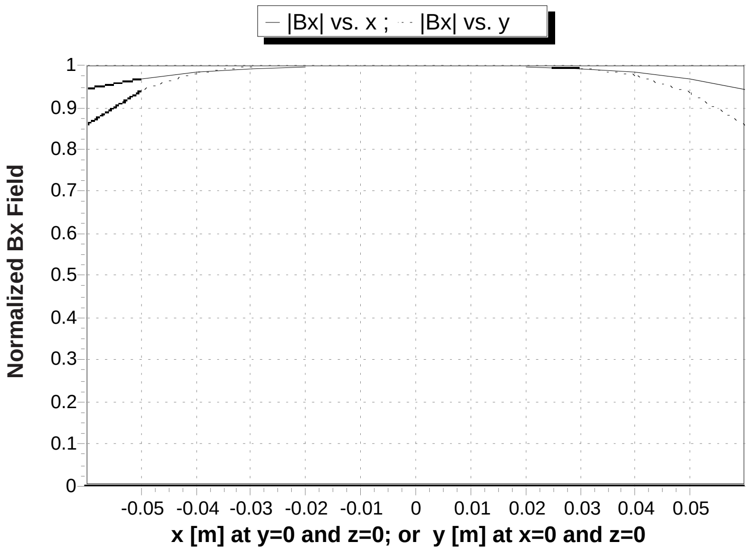


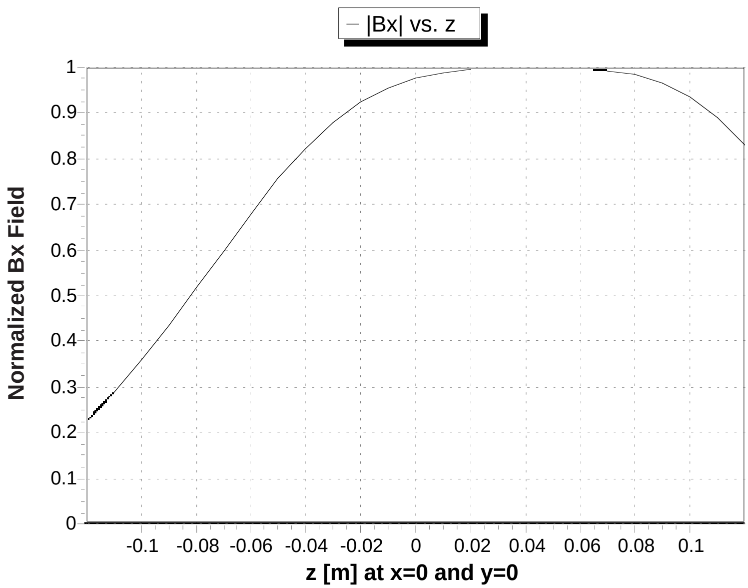
Figure 5. RF coil's configuration and the quasi-static the distribution of transverse magnetic field B_x in the $z = 50\,\mathrm{mm}$ plane.

the plane that cuts the centre of the DSV at $z_{os} = 50\,\mathrm{mm}$. Figure 6 shows the B_x field along the axis that cut the centre of DSV.

Let us consider the RF coil as illustrated by the circuit diagram of Figure 7. Capacitors are placed at various positions of the contours for the purpose of suppressing the variation of time-harmonic current along the wire. The function of the tuning capacitors C_s and C_p is to match the input impedance of the parallel loops Z'_{in} with the $50\,\Omega$ coaxial cable. The procedure begins with the use of a Genetic Algorithm/Method of Moments (GA/MoM) technique to obtain a full-wave solution for the positions and values of the capacitors $C_1, \ldots, C_J$ that lead to the required homogeneous fields. Once these capacitors are determined, a shield is included as the ground plane and C_s and C_p are tuned manually by the use of the electromagnetic software FEKO until $Z_{in} = 50\,\Omega$ or the return loss is less than $-20\,\mathrm{dB}$. Therefore, in



(a)



(b)

Figure 6. Distribution of normalized B_x, along (a) the x and y axis at $z = 50\,\mathrm{mm}$; (b) the z axis.

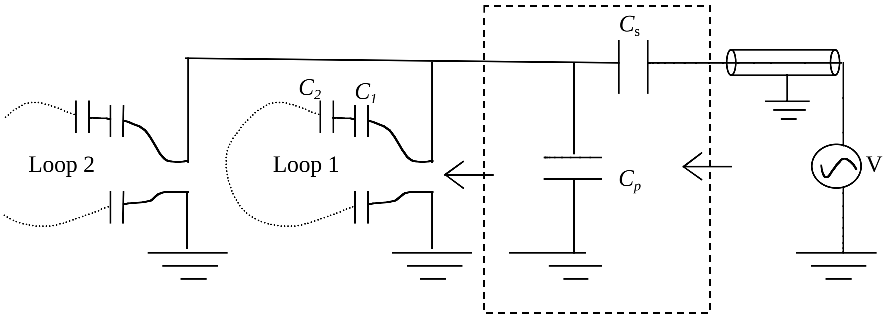


Figure 7. Circuit diagram of a RF coil in time-harmonic case.

the GA implementation, the input network is neglected with C_s and C_p assumed to be short and open-circuited respectively. The MoM formulation is based on the technique of [8] with the current in the loop expanded by a set of 1-D rooftops basis functions which lie along the centre of the wire. By the continuity equation and the Lorentz gauge, the electric field $\boldsymbol{E}$ at any point in space can then be expressed in terms of the current coefficients. Choosing the voltage excitation at a certain element and testing the tangential $\boldsymbol{E}$ field on the surface of wire with 1-D razor testing functions (between the centres of two adjacent segments), the following matrix equation can be obtained:

$$[Z]_{N\times N}[I]_{N\times 1} = [V]_{N\times 1} \tag{8}$$

where is $[Z]$ is the impedance matrix; $[I]$ and $[V]$ are the current and excitation vectors respectively. The position of the capacitors is represented by the corresponding the current element. For example, if a capacitor of impedance z_c is placed at current element m, the following change is implemented to the impedance matrix:

$$z_{mm} = z_{mm} + z_c \tag{9}$$

where z_{mm} is the diagonal matrix element at row m and column m.

Since the RF coil is symmetrical about the y-z plane, the capacitors and excitation on the contours must be identical. Suppose there are J capacitors per contour and their values and positions are defined by the variables c_j and e_j respectively, then in the GA code each individual consists of $2J$ variables or unknowns and is written as $\{_1, \ldots, c_J, e_1, \ldots, e_J\}$. If the number of possible c_j is 2^S and e_j is 2^U then the chromosome in binary form is written as:

$$\begin{aligned}\{c_1, \ldots, c_J, e_1, \ldots, e_J\} \equiv\ & \{(b_{c_J}^{(S)}, \ldots, b_{c_J}^{(1)}), \ldots\ldots, (b_{c_1}^{(S)}, \ldots, b_{c_1}^{(1)}), \\ & (b_{e_J}^{(U)}, \ldots, b_{e_J}^{(1)}), \ldots\ldots, (b_{e_1}^{(U)}, \ldots, b_{e_1}^{(1)})\}\end{aligned} \tag{10}$$

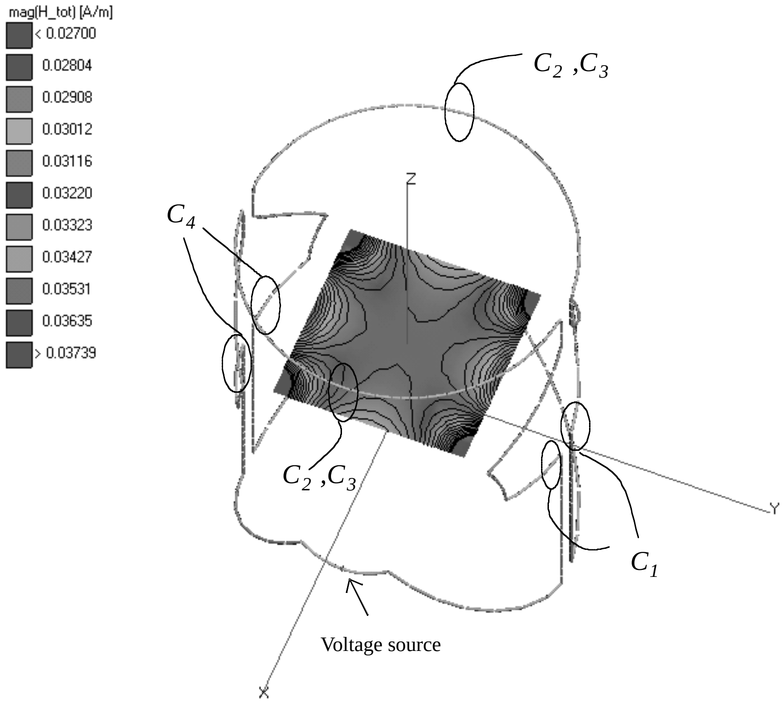
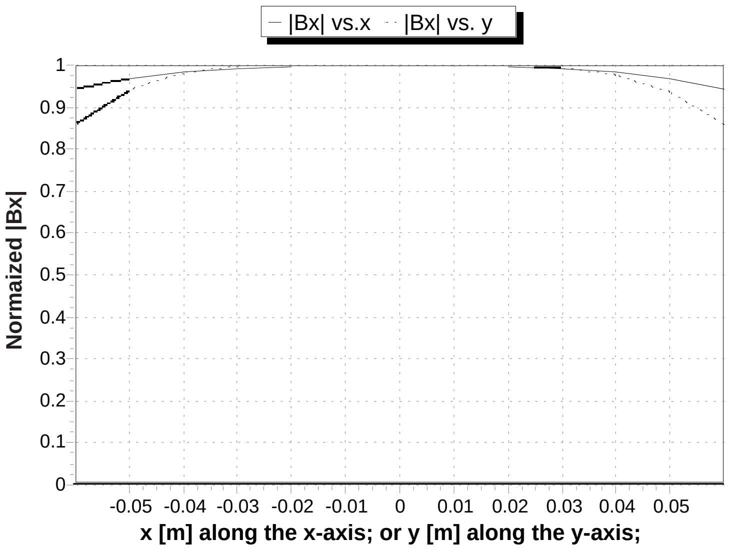


Figure 8. Time-harmonic solution for a centered excited RF coil computed by GA/MoM code; and distribution of transverse magnetic field B_x in the $z = 50$ mm plane.

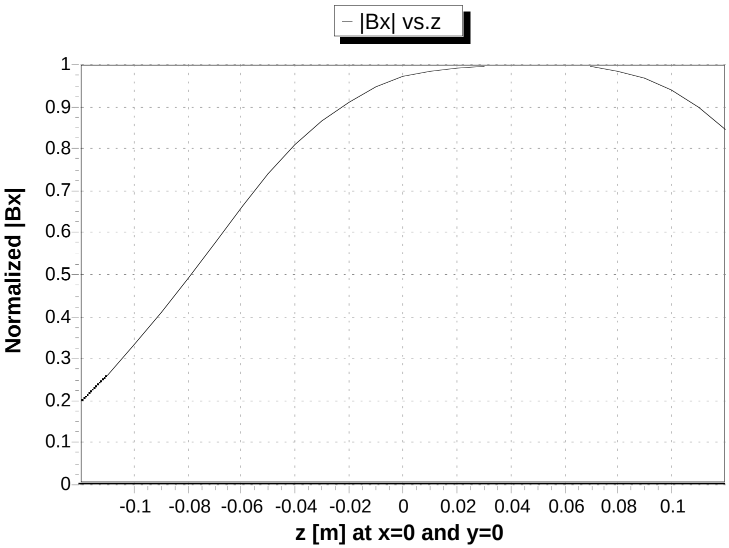
where $(b_{c_j}, b_{e_j} \in \{0, 1\})$; S and U are the number of bits for c_j or e_j respectively. Given a set of capacitors, the current coefficients obtained from the above equation i.e., $[I] = [Z]^{-1}[V]$ are used to calculate the corresponding magnetic field produced within the DSV. Give a set of field positions $(\boldsymbol{r}_1 \ldots, \boldsymbol{r}_Q)$ selected within the DSV, the function to be optimized by the GA is f_i which is written as the difference between the maximum and minimum of these magnetic fields:

$$\begin{aligned} f_1(c_J, \ldots, c_1, e_J, \ldots, e_1) \\ = \max(B_x(\{\boldsymbol{r}_Q, \ldots \boldsymbol{r}_1\}, \{c_J, \ldots, c_1, e_J, \ldots, e_1\})) \\ - \min(B_x(\{\boldsymbol{r}_Q, \ldots, \boldsymbol{r}_1\}, \{c_J, \ldots, c_1, e_J, \ldots, e_1\})) \end{aligned} \quad (11)$$

In our first test, the position of voltage sources is chosen at the lower part of the contours in the x-z plane (the plane that cuts the contour in halves). Because of the electrical symmetry about the x-z plane, capacitors on either side of the contour are mirror image of each other.



(a)



(b)

Figure 9. Distribution of normalized B_x, along (a) the x and y axis at $z = 50\,\text{mm}$; (b) the z axis fed by a time-harmonic voltage source at the center.

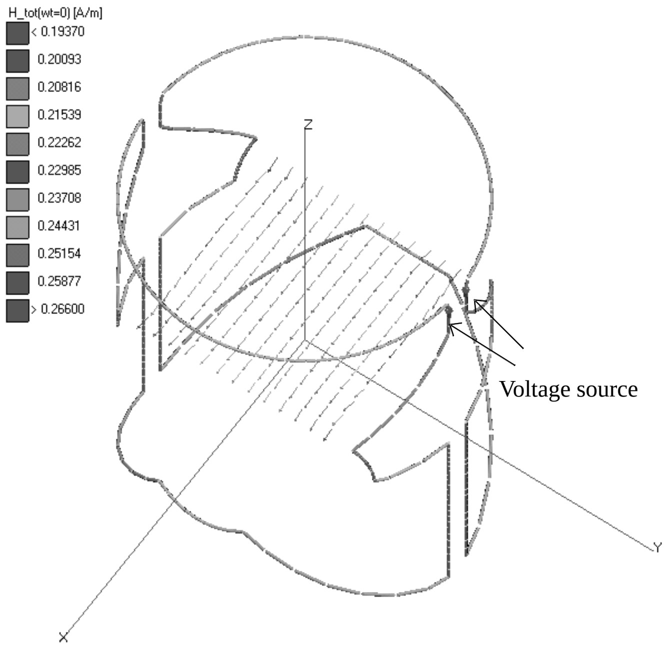
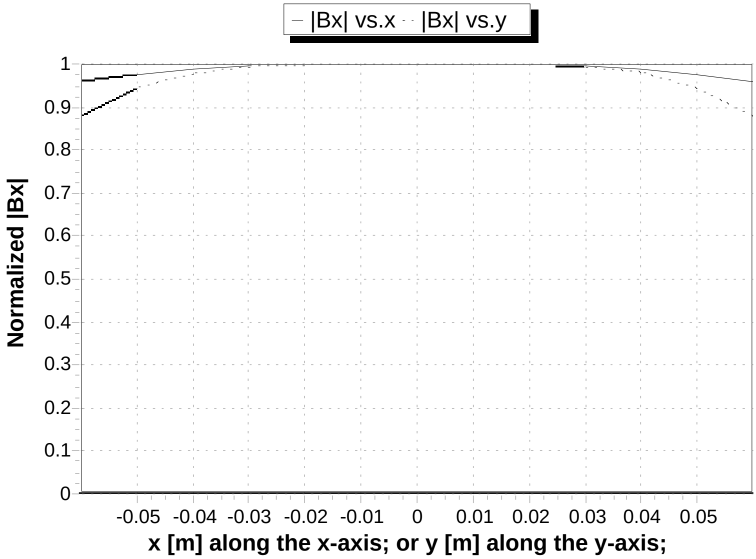


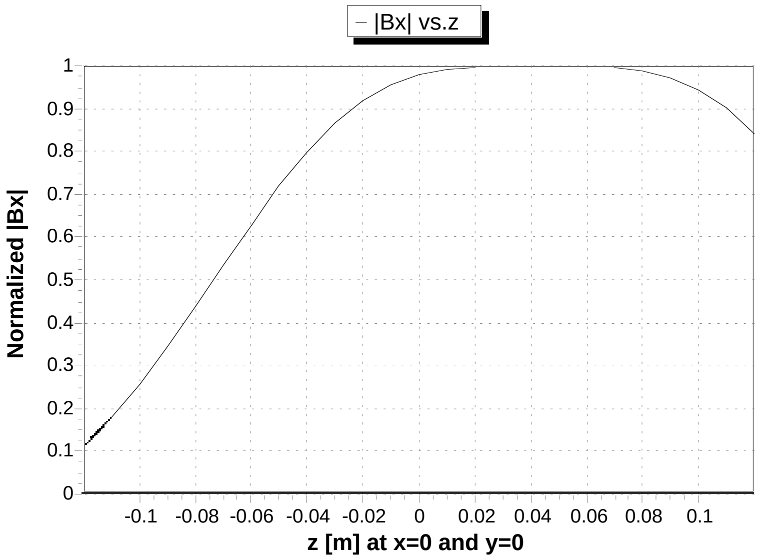
Figure 10. Time-harmonic solution for a side excited RF coil computed by GA/MoM code; and distribution of transverse magnetic field B_x in the $z = 50\,\mathrm{mm}$ plane.

The problem is reduced to optimizing the capacitors located on only one side of the contour, say $y > 0$, which are to be mapped onto the corresponding position in the other half, say $y < 0$. The number of unknown is decreased to J or half the number of capacitors on a contour. The GA code was implemented with $J = 2$ (i.e., no. of capacitors per contour 4); $S = 4$ and $U = 4$. Figure 8 and Figure 9 show the distributions of B_x at 190 MHz. It can be seen that the homogeneity is as good as that of the quasi-static case. However, in practice the configuration is very inconvenient to implement as two sets of input circuit are needed for the separate excitations.

In order to feed the contours at one single point through and through only one set of input capacitors, another RF coil was tested. In this configuration, the voltage source is chosen at a position (near the x-z plane) where the contours are at its closest distance to each other. The GA was run to optimize a solution for the 8 unknown capacitors. Again the homogeneity is not degraded from that of the quasi-static case as can be seen in the plots of Figure 10 and Figure 11. With the use of FEKO and with the inclusion of a shield and the input



(a)



(b)

Figure 11. Distribution of normalized B_x, along (a) the x and y axis at $z = 50\,\text{mm}$; (b) the z axis fed by a time-harmonic voltage source at the center. Eight capacitors are distributed.

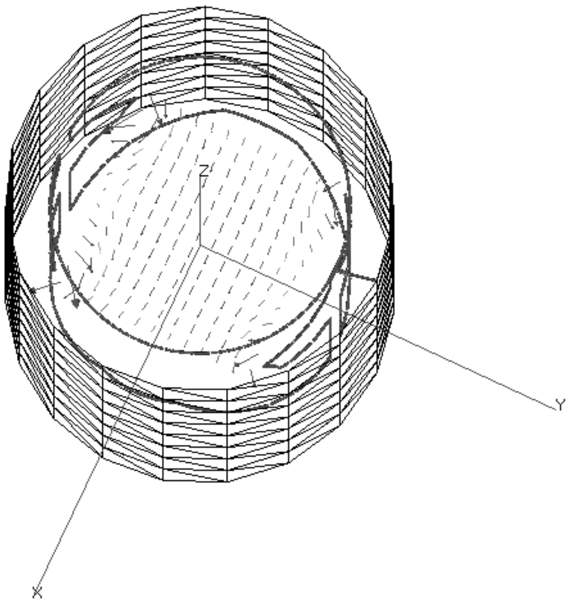


Figure 12. A side fed RF coil and distribution of transverse magnetic field B_x in the $z = 50\,\text{mm}$ plane.

circuit as illustrated in Figure 7, the input impedance Z_{in} was adjusted to approximately $50\,\Omega$ line. The RF coil and its field distribution are shown in Figure 12.

We have previously shown that coils modeled in this produce experimental results in prototypes that conform closely to predictions [3].

3. CONCLUSION

A novel genetic algorithm has been presented for optimizing the geometry of RF coils for MRI application. Because a rigorous full-wave technique based method of moments is used in the forward algorithm, the proposed method is very versatile with minimal limitation imposed by the geometrical and physical conditions of the structure. Despite the fact the optimized configuration is very simple,e consisting of only two symmetrical single-loop contours, the homogeneity of the coil is satisfactory. The novel method is a step forward towards developing a new asymmetric RF coil structures and those with unusual requirements. In the next stage of the development, the algorithm will be enhanced with the inclusion of multi-layered contours and a number of prototypes constructed.

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