Progress In Electromagnetics Research, PIER 40, 55–69, 2003

SPATIAL CORRELATION FUNCTIONS FOR FIELDS IN THREE-DIMENSIONAL RAYLEIGH CHANNELS

Ph. De Doncker

Université Libre de Bruxelles
CP 165/5 1, Av. Roosevelt 50
1050 Bruxelles, Belgium

Abstract—Starting from a continuous plane-wave representation of the electric and magnetic fields, spatial auto- and cross-correlation functions for field components and their modulus are derived in the three-dimensional Rayleigh channel case. It is shown that existing results, generally relying on two-dimensional or isotropic models, can significantly differ from those obtained thanks to a three-dimensional approach.

1. INTRODUCTION

Spatial correlation for fields plays an important role in wireless communication systems and it has been extensively studied in the framework of outdoor mobile networks. In the early works [1, 2], the correlation functions for fields were investigated either as a function of time while the mobile unit moves, or as a function of base station position. The waves impinging the receiver were in both cases assumed to have equal energy (Rayleigh channel) and to propagate in the receiver plane (two-dimensional hypothesis), leading to the well-known Bessel correlation function $J_0(kd)$ at the mobile unit (k is the signal wave number and d the spatial lag) [1–3].

In recent years, further attention has been paid to spatial correlation due to the multi-element antenna systems development and performance analysis of various communication systems has been carried out relying on the same two-dimensional assumption [4–7]. This hypothesis seems however too restrictive in situations where the receiver is uniformly surrounded by scatterers, as in closed systems, and, till now, only few attention has been given to three-dimensional Rayleigh channel correlations. Closed systems are mostly encountered in electromagnetic compatibility where the interactions of waves with electronic devices are studied in enclosures or reverberating chambers [8], but it seems obvious that in certain circumstances the usual two-dimensional assumption could also be misleading in communication system analysis, namely in indoor environment.

In the three-dimensional case, the existing studies generally do not take into account the wave polarization, leading to a general isotropic $\sin kd/kd$ spatial correlation [8–10]. As will be shown later, the spatial correlation functions for field components are not isotropic, so that using approximate correlation functions could have an important impact, for instance on multi-element linear antenna system design. To the author's knowledge, the only work dealing with polarization has been carried out by Hill in the reverberating chamber framework [9, 11, 12] but it does not give a complete overview of the problem and its formalism is restricted to reverberating chamber analysis. The aim of our study is to collect all existing results in a unified formalism, and to provide new correlation functions to fill in the gaps in the existing literature. In the first part of this paper, Hill's reverberating chamber formalism will be adapted to the most general Rayleigh channel study. Next, step by step, all the correlation functions for the field components will be derived to obtain a full picture of the problem. In the next sections, a time dependence $e^{j\omega t}$ is assumed and suppressed.

2. INTEGRAL REPRESENTATION OF THE RAYLEIGH CHANNEL

Let $\mathcal{D}$ be a source-free region where the mean electric and magnetic fields level can be supposed constant. In an indoor or outdoor communication channel, or in an enclosure or reverberating chamber, the fields are made up of many waves arriving from the various scatterers surrounding $\mathcal{D}$, so that it is reasonable on physical grounds to write at any point the fields as a discrete sum of propagating plane waves [1]. Such a sum representation does not lend itself well to simple closed-form expressions for correlation functions and for sake of simplicity, it is more advisable to pass to the continuous limit, defining an integral representation for the electric field in $\mathcal{D}$ as:

$$\vec{E} = \int_{\Omega} \vec{F}(\Omega) e^{-j\vec{k}\cdot\vec{r}} d\Omega \tag{1}$$

where the integration is performed over all real angles $\Omega = (\theta, \phi)$ and where $\vec{F}(\Omega)$ is the complex plane wave spectrum of $\vec{E}$. If $\hat{k}$ is the unit vector pointing in the $\vec{k}$ direction and noting η the free-space impedance, a similar expression for the magnetic field can be inferred from Maxwell equations:

$$\vec{H} = \frac{1}{\eta} \int_{\Omega} \hat{k} \times \vec{F}(\Omega) e^{-j\vec{k}\cdot\vec{r}} d\Omega \tag{2}$$

It is important to note that in (1) and (2), waves coming from all angles around $\mathcal{D}$ are considered.

All the spatial characteristics of $\vec{E}$ and $\vec{H}$ can be deduced from the plane wave spectrum $\vec{F}(\Omega)$ and from the spatial dependence $e^{-j\vec{k}\cdot\vec{r}}$. Equation (1) corresponds to one realization of $\vec{E}$, and to deduce general statistical characteristics, a set of realizations is to be considered. In the remaining of this text, the ensemble average over these realizations will be denoted $\langle\ \rangle$. To each realization corresponds a given plane wave spectrum, and, under these assumptions, $\vec{F}(\Omega)$ is a random function whose statistical properties have to be defined.

In a spherical coordinate system, $\vec{F}(\Omega)$ can be written as

$$\vec{F}(\Omega) = F_\theta(\Omega)\vec{1}_\theta + F_\phi(\Omega)\vec{1}_\phi \tag{3}$$

where both F_θ and F_ϕ are complex

$$\begin{aligned} F_\theta &= F_{\theta r} + jF_{\theta i} \\ F_\phi &= F_{\phi r} + jF_{\phi i} \end{aligned} \tag{4}$$

Following [1] and [11], the plane wave spectrum properties can be deduced from the Rayleigh channel assumption. First, since the electric field is made up of many waves having random phases, in a Rayleigh channel, $\langle \vec{E} \rangle = 0$, which implies

$$\langle F_\theta \rangle = \langle F_\phi \rangle = 0 \tag{5}$$

Next, each wave results from multiple independent bounces on the surrounding obstacles so that two waves arriving from different directions are supposed to be uncorrelated:

$$\begin{aligned} \langle F_{\theta r}(\Omega_1) F_{\theta r}(\Omega_2) \rangle &= \langle F_{\theta i}(\Omega_1) F_{\theta i}(\Omega_2) \rangle \\ &= \langle F_{\phi r}(\Omega_1) F_{\phi r}(\Omega_2) \rangle \\ &= \langle F_{\phi i}(\Omega_1) F_{\phi i}(\Omega_2) \rangle = C\delta(\Omega_1 - \Omega_2) \end{aligned} \tag{6}$$

Since in a Rayleigh channel all the waves are supposed to have equal energy, C is independent of Ω, and with (1) it can be linked to the electric field mean power $\langle |\vec{E}|^2 \rangle = E_0^2$:

$$C = \frac{E_0^2}{16\pi} \tag{7}$$

Finally, the real and imaginary parts of the plane wave spectrum and its θ and ϕ components are supposed to be uncorrelated:

$$\begin{aligned} \langle F_{\theta r}(\Omega_1) F_{\theta i}(\Omega_2) \rangle &= \langle F_{\phi r}(\Omega_1) F_{\phi i}(\Omega_2) \rangle \\ &= \langle F_{\phi r}(\Omega_1) F_{\theta r}(\Omega_2) \rangle \\ &= \langle F_{\phi i}(\Omega_1) F_{\theta i}(\Omega_2) \rangle \\ &= \langle F_{\phi r}(\Omega_1) F_{\theta i}(\Omega_2) \rangle \\ &= \langle F_{\phi i}(\Omega_1) F_{\theta r}(\Omega_2) \rangle = 0 \end{aligned} \tag{8}$$

Equations (5)–(8) are the usual basic properties of Rayleigh channels written in terms of $\vec{F}(\Omega)$. From these equations, two other useful expressions can be deduced:

$$\langle F_\theta(\Omega_1) F_\phi^*(\Omega_2) \rangle = 0 \tag{9}$$

and

$$\langle F_\theta(\Omega_1) F_\theta^*(\Omega_2) \rangle = \langle F_\phi(\Omega_1) F_\phi^*(\Omega_2) \rangle = 2C\delta(\Omega_1 - \Omega_2) \tag{10}$$

where $*$ denotes complex conjugates.

The statistical properties of the fields can be inferred from (5)–(10). We will here focus on the spatial correlation function which is defined for two complex random variables u and v by [13]

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{\left\langle \left(u(\vec{r}_1) - \langle u(\vec{r}_1)\rangle\right)\left(v^*(\vec{r}_2) - \langle v^*(\vec{r}_2)\rangle\right)\right\rangle}{\sqrt{\left\langle |u - \langle u\rangle|^2\right\rangle \left\langle |v - \langle v\rangle|^2\right\rangle}} \tag{11}$$

We will see later that the spatial correlation function will only depend on the distance $d = |\vec{r}_1 - \vec{r}_2|$ (eventually taken along one particular axis) and it will be noted $\rho(d)$.

3. THE SPATIAL CORRELATION FUNCTIONS

3.1. Spatial Correlation for Fields

In a first step, let us consider the spatial auto-correlation function for the electric field ($\langle \vec{E} \rangle = 0$):

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{\langle \vec{E}(\vec{r}_1) \cdot \vec{E}^*(\vec{r}_2)\rangle}{\langle |\vec{E}|^2 \rangle} \tag{12}$$

Using (1):

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{1}{E_0^2} \int_{\Omega_1} \int_{\Omega_2} \langle \vec{F}(\Omega_1) \cdot \vec{F}^*(\Omega_2)\rangle e^{-j(\vec{k}_1 \cdot \vec{r}_1 - \vec{k}_2 \cdot \vec{r}_2)} d\Omega_1 d\Omega_2 \tag{13}$$

With the help of (10) it can be shown that

$$\langle \vec{F}(\Omega_1) \cdot \vec{F}^*(\Omega_2)\rangle = 4C\delta(\Omega_1 - \Omega_2) \tag{14}$$

and (13) becomes

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{1}{4\pi} \int_{\Omega} e^{-j\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)} d\Omega \tag{15}$$

Without lost of generality it can be supposed that $\vec{r}_1 - \vec{r}_2 = d\vec{1}_z$ and

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{1}{4\pi} \int_{\Omega} e^{jkd\cos\theta} d\Omega \tag{16}$$

so that the classical result is derived:

$$\rho(d) = \frac{\sin kd}{kd} \tag{17}$$

The same kind of procedure can be used to derive the spatial correlation for the magnetic field starting from (2). The same correlation function is obtained and (17) will be considered in the remaining of the text as the reference function with which the other spatial correlation functions will be compared. The correlation length l_c is defined as the distance d corresponding to the first zero of the spatial correlation function. For the fields, we see that this correlation length $l_c = 0.5\lambda$.

3.2. Spatial Correlation for Spherical Components

From (1), the θ component of $\vec{E}$ is given by

$$E_\theta = \int_\Omega F_\theta(\Omega) e^{-j\vec{k}\cdot\vec{r}} d\Omega \tag{18}$$

Its square modulus mean value is:

$$\langle E_\theta(\vec{r}) E_\theta^*(\vec{r}) \rangle = \int_{\Omega_1} \int_{\Omega_2} \langle F_{\theta_1}(\Omega_1) F_{\theta_2}^*(\Omega_2) \rangle e^{-j(\vec{k}_1 - \vec{k}_2)\cdot\vec{r}} d\Omega_1 d\Omega_2 \tag{19}$$

so that, using the same procedure for E_ϕ and with (10)

$$\langle |E_\theta|^2 \rangle = \langle |E_\phi|^2 \rangle = \frac{E_0^2}{2} \tag{20}$$

Let us now derive the spatial correlation function for E_θ

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{1}{E_0^2/2} \int_{\Omega_1} \int_{\Omega_2} \langle F_{\theta_1}(\Omega_1) F_{\theta_2}^*(\Omega_2) \rangle e^{-j(\vec{k}_1\cdot\vec{r}_1 - \vec{k}_2\cdot\vec{r}_2)} d\Omega_1 d\Omega_2 \tag{21}$$

using (10):

$$\rho(\vec{r}_1, \vec{r}_2) = \frac{1}{4\pi} \int_\Omega e^{-j\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)} d\Omega \tag{22}$$

By symmetry consideration, it can be supposed without lost of generality that $\vec{r}_1 - \vec{r}_2 = d\vec{1}_z$

$$\rho(d) = \frac{1}{4\pi} \int_\Omega e^{jkd\cos\theta} d\Omega \tag{23}$$

and finally, for the θ and ϕ components of the electric field, the spatial correlation function is the same as for the fields:

$$\rho(d) = \frac{\sin kd}{kd} \tag{24}$$

The same expression can be obtained for the magnetic field, and the classical expression (17) is valid for both the fields and their spherical components.

3.3. Spatial Correlation for Cartesian Components

In a first step, let us derive the spatial correlation function for the real and imaginary parts of one cartesian component of $\vec{E}$. By (1), all three components are equivalent, and without lost of generality, the spatial correlation function for the real and imaginary parts of E_z will be derived. For instance, according to (1), the real part is given by

$$E_{zr}(\vec{r}) = -\int_{\Omega} F_{\theta r} \sin\theta \cos\vec{k}\cdot\vec{r} + F_{\theta i} \sin\theta \sin\vec{k}\cdot\vec{r}\, d\Omega \tag{25}$$

so that using (6)–(8) and by symmetry:

$$\langle E_{zr}^2 \rangle = \langle E_{zi}^2 \rangle = \frac{E_0^2}{16\pi} \int_{\Omega} \sin^2\theta d\Omega = \frac{E_0^2}{6} \tag{26}$$

The spatial correlation function is now given by

$$\begin{aligned} \rho(\vec{r}_1, \vec{r}_2) &= \frac{1}{E_0^2/6} \langle E_{zr}(\vec{r}_1) E_{zr}(\vec{r}_2) \rangle \\ &= \int_{\Omega_1}\int_{\Omega_2} \langle F_{\theta_1 r} F_{\theta_2 r} \rangle \sin\theta_1 \sin\theta_2 \cos\vec{k}_1\cdot\vec{r}_1 \cos\vec{k}_2\cdot\vec{r}_2 d\Omega_1 d\Omega_2 \\ &\quad + \int_{\Omega_1}\int_{\Omega_2} \langle F_{\theta_1 i} F_{\theta_2 i} \rangle \sin\theta_1 \sin\theta_2 \sin\vec{k}_1\cdot\vec{r}_1 \sin\vec{k}_2\cdot\vec{r}_2 d\Omega_1 d\Omega_2 \end{aligned} \tag{27}$$

According to (6):

$$\begin{aligned} \rho(\vec{r}_1, \vec{r}_2) &= \frac{3}{8\pi} \int_{\Omega} \sin^2\theta \cos\vec{k}\cdot\vec{r}_1 \cos\vec{k}\cdot\vec{r}_2 d\Omega \\ &\quad + \frac{3}{8\pi} \int_{\Omega} \sin^2\theta \sin\vec{k}\cdot\vec{r}_1 \sin\vec{k}\cdot\vec{r}_2 d\Omega \\ &= \frac{3}{8\pi} \int_{\Omega} \sin^2\theta \cos\big(\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)\big) d\Omega \end{aligned} \tag{28}$$

The choice of one particular cartesian component E_z breaks down the global spherical symmetry, and it is no more possible to derive a unique isotropic correlation function. It is necessary to distinguish correlation along the particular z-axis with correlation in the xy-plane.

3.3.1. Correlation in the xy-plane

In this case, the correlation must be isotropic in this plane, and without lost of generality it can be supposed that $\vec{r}_1 - \vec{r}_2 = d\vec{1}_y$ for instance.

(28) then becomes:

$$\begin{aligned}\rho(d) &= \frac{3}{8\pi}\int_{\Omega}\sin^2\theta\cos(kd\sin\theta\sin\phi)d\Omega \\ &= \frac{3}{8\pi}\int_0^{\pi}\int_0^{2\pi}\sin^3\theta\cos(kd\sin\theta\sin\phi)d\theta d\phi\end{aligned} \tag{29}$$

This integral computation is quite lengthy and it will not reported here for conciseness. Basically $\sin^3\theta$ can be developed in terms of $\sin\theta$ and $\sin 3\theta$ and the following identities have to be used [14]:

$$\begin{aligned}\int_0^{2\pi}\cos(z\sin\phi)d\phi &= 2\pi J_0(z) \\ \int_0^{\pi} J_0(z\sin\theta)\sin\theta d\theta &= \sqrt{\frac{2\pi}{z}}J_{1/2}(z) \\ \int_0^{\pi} J_0(2z\sin\theta)\sin 3\theta d\theta &= -\pi J_{3/2}(z)J_{-3/2}(z)\end{aligned} \tag{30}$$

where J_ν is the Bessel function of order ν.

Finally, it is possible to show that the spatial correlation function for the real or imaginary part of E_z in the xy-plane is given by

$$\rho(d) = \frac{3}{2}\frac{\sin kd}{kd}\left(1 - \frac{1}{(kd)^2}\right) + \frac{3}{2}\frac{\cos kd}{(kd)^2} \tag{31}$$

As shown on Figure 1, this correlation function closely follows the reference function (17) but the correlation length is now $l_c = 0.43\lambda$.

3.3.2. Correlation along the z-axis

Along the z-axis, $\vec{r}_1 - \vec{r}_2 = d\vec{1}_z$ and (28) becomes

$$\begin{aligned}\rho(d) &= \frac{3}{8\pi}\int_{\Omega}\sin^2\theta\cos(kd\cos\theta)d\Omega \\ &= \frac{3}{8\pi}\int_0^{\pi}\int_0^{2\pi}\sin^3\theta\cos(kd\cos\theta)d\theta d\phi\end{aligned} \tag{32}$$

The computation of $\rho(d)$ is again quite tedious and after a few manipulations, the spatial correlation function for the real or imaginary part of E_z along the z-axis is given by

$$\rho(d) = \frac{3}{(kd)^2}\left(\frac{\sin kd}{kd} - \cos kd\right) \tag{33}$$

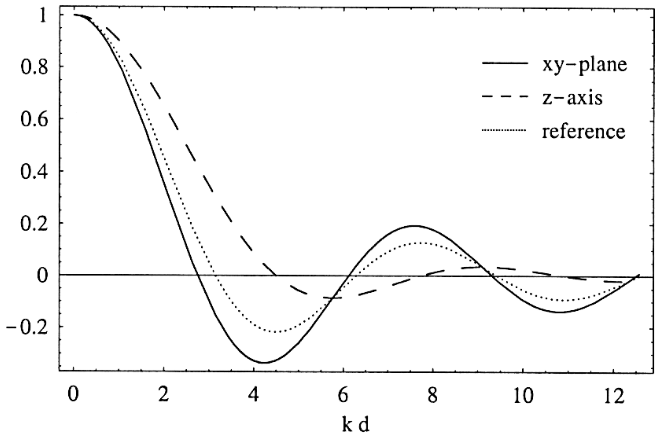

Figure 1. Correlation functions for E_{zr}, E_{zi} and E_z compared to the reference solution (17).

This function is also drawn on Figure 1. It has of course the same oscillating behaviour as the reference function but the correlation length is $l_c = 0.71\lambda$, 65% higher than in the xy-plane! Moreover it decays much more rapidly to zero than the classical $\sin kd/kd$ solution.

3.3.3. Correlation for E_z

It is now possible to derive the correlation function for the complex component E_z. According to (1):

$$E_z(\vec{r}) = -\int_{\Omega} (F_{\theta r} + jF_{\theta i}) e^{-j\vec{k}\cdot\vec{r}} \sin\theta d\Omega \tag{34}$$

Using (6)–(8) and by symmetry consideration

$$\begin{aligned} \langle E_z \rangle = \langle E_y \rangle = \langle E_x \rangle = 0 \\ \langle |E_z|^2 \rangle = \langle |E_y|^2 \rangle = \langle |E_x|^2 \rangle = \frac{E_0^2}{3} \end{aligned} \tag{35}$$

so that after a few manipulations without any particular difficulty, the correlation function for E_z has the expression

$$\begin{aligned} \rho(\vec{r}_1, \vec{r}_2) &= \frac{1}{E_0^2/3} \langle E_z(\vec{r}_1) E_z^*(\vec{r}_2) \rangle \\ &= \frac{3}{8\pi} \int_{\Omega} \sin^2\theta \cos\left(\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)\right) d\Omega + j\frac{3}{8\pi} \int_{\Omega} \sin^2\theta \sin\left(\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)\right) d\Omega \end{aligned} \tag{36}$$

The second integral is null both for correlation in the xy-plane and along the z-axis. The correlation function for E_z has the same expression (28) as for the real or imaginary parts, so that (31) and (33) drawn on Figure 1 are also valid for the complex components.

3.3.4. Correlation for $|E_z|$

In a Rayleigh channel, both the real and imaginary parts of each field component satisfy a Gaussian distribution with zero mean and equal variances. Moreover, since $|E_z|$ satisfy a Rayleigh distribution, (35) implies that

$$\langle |E_z| \rangle = \langle |E_y| \rangle = \langle |E_x| \rangle = \sqrt{\frac{\pi}{12}} E_0 \tag{37}$$

In this case, the correlation function for the modulus of a component is given by [8, 13]:

$$\rho_m(d) = \frac{2/\pi\left(\rho(d)\arcsin(\rho(d)) + \sqrt{1-\rho^2(d)}\right) - \pi/4}{1-\pi/4} \tag{38}$$

where ρ_m is the correlation function for the modulus and ρ the correlation function for the component (31) or (33). This correlation function computed using (31) and (33) is drawn on Figure 2. It presents two major properties: first it does not oscillate around zero (as is observed in the 2D case [1]) and next, it decays very rapidly near the origin. The correlation lengths are $l_c = 0.21\lambda$ in the xy-plane and $l_c = 0.31\lambda$ along the z-axis.

3.3.5. Correlation for $|E_z|^2$

The correlation function for the square modulus of a component is given by [8]:

$$\rho_{sm}(d) = \rho^2(d) \tag{39}$$

where ρ_{sm} is the correlation function for the square modulus and ρ the correlation function for the component (31) or (33). This correlation function computed using (31) and (33) is drawn on Figure 3. Its oscillations are very low compared to the previous correlation functions. The correlation lengths are $l_c = 0.43\lambda$ in the xy-plane and $l_c = 0.71\lambda$ along the z-axis as for the complex components. Along the z-axis, ρ_{sm} rapidly decays near zero and beyond 0.6λ, $|E_z|^2$ values are mostly uncorrelated.

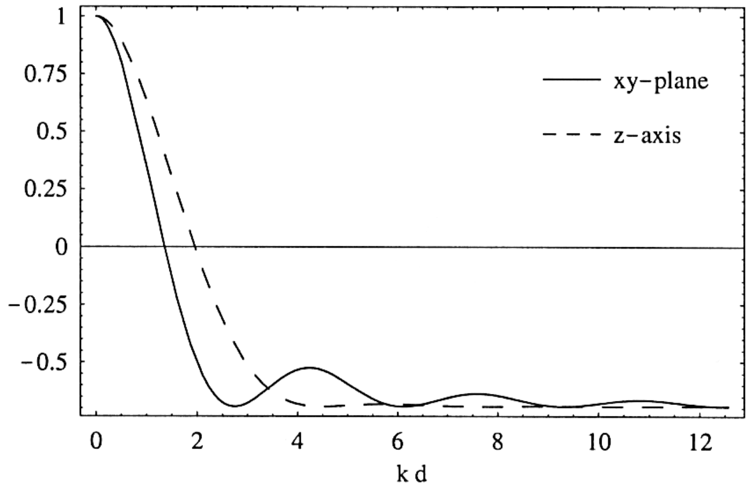

Figure 2. Correlation functions for $|E_z|$.

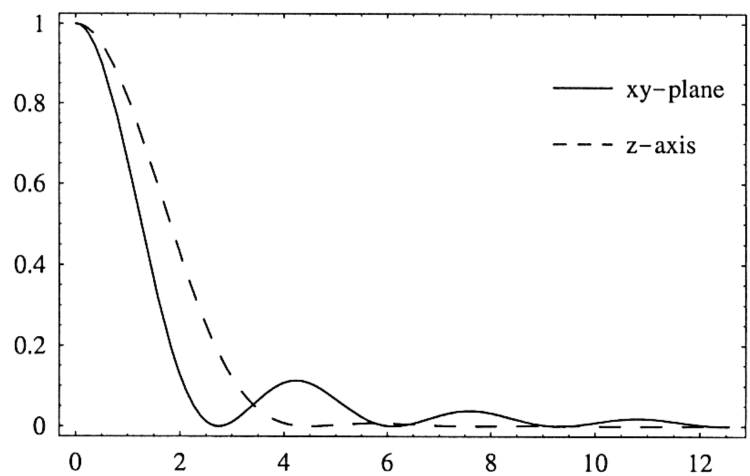

Figure 3. Correlation functions for $|E_z|^2$.

3.3.6. Correlation for the Magnetic Field

According to (2), since the unit vector $\hat{k}$ points in the radial direction, the magnetic field can be expressed as

$$\vec{H} = \frac{1}{\eta} \int_{\Omega} (F_{\phi} \vec{1}_{\theta} - F_{\theta} \vec{1}_{\phi}) e^{-j\vec{k} \cdot \vec{r}} d\Omega \tag{40}$$

It is essentially the same expression as for the electric field, except a scaling factor which disappears in the correlation function normalization and a swap of F_θ and F_ϕ. Since perfect symmetry is assumed in the θ and ϕ components of $\vec{E}$ in a Rayleigh channel, this permutation does not have any impact on the spatial characteristics of $\vec{H}$ and the correlation functions derived for $\vec{E}$ can be used without any change for $\vec{H}$. This differs from the 2D approach where the symmetry between $\vec{E}$ and $\vec{H}$ is artificially broken by choosing a particular invariance axis with respect to which the fields are to be chosen transverse or longitudinal [1].

3.4. Cross-Correlations

To derive the cross-correlation functions for the field components, it is easier to start with the expression for the cross-correlation between two field components along the $\hat{s}_1$ and $\hat{s}_2$ directions separated by an angle γ [11]:

$$\rho(\hat{s}_1, \hat{s}_2) = \frac{\langle E_{s_1}(\vec{r}) E_{s_2}^*(\vec{r}) \rangle}{\sqrt{\langle |E_{s_1}|^2 \rangle \langle |E_{s_2}|^2 \rangle}} = \cos\gamma \tag{41}$$

It can be deduced that the correlation between any pair of orthogonal cartesian components of the same field ($\vec{E}$ or $\vec{H}$) is always null. In a similar way, using (1), (2) and the uncorrelation between F_θ and F_ϕ (9), the correlation between pairs of spherical orthogonal components of $\vec{E}$ and $\vec{H}$ also vanishes:

$$\langle E_\theta H_\phi^* \rangle = \langle E_\phi H_\theta^* \rangle = \langle E_\theta E_\phi^* \rangle = \langle H_\theta H_\phi^* \rangle = 0 \tag{42}$$

Let us now examine the cross-correlation between the cartesian components of $\vec{E}$ and $\vec{H}$. Considering or instance the z components of $\vec{E}$ and $\vec{H}$:

$$\begin{aligned} E_z &= -\int_{\Omega} F_\theta \sin\theta e^{-j\vec{k}\cdot\vec{r}} d\Omega \\ H_z &= -\frac{1}{\eta} \int_{\Omega} F_\phi \sin\theta e^{-j\vec{k}\cdot\vec{r}} d\Omega \end{aligned} \tag{43}$$

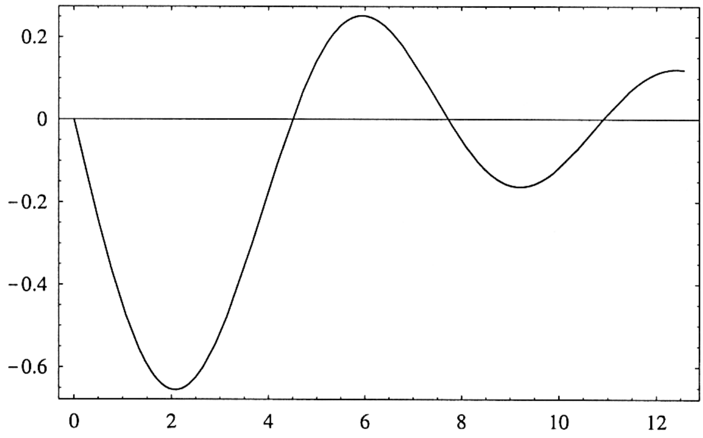

Figure 4. Cross-correlation between E_z and H_x along the y-axis.

Using (9) and by symmetry consideration:

$$\langle E_z H_z^* \rangle = \langle E_y H_y^* \rangle = \langle E_x H_x^* \rangle = 0 \tag{44}$$

so that the spatial correlation between similar components of $\vec{E}$ and $\vec{H}$ is always null, in any direction. To derive the correlation function for dissimilar components, let us for instance consider the x component of $\vec{H}$. With the help of (9):

$$\langle E_z H_x^* \rangle = -\frac{1}{\eta} \int_{\Omega_1} \int_{\Omega_2} \langle F_{\theta_1} F_{\theta_2}^* \rangle \sin\theta_1 \sin\phi_2 e^{-j(\vec{k}_1\cdot\vec{r}_1 - \vec{k}_2\cdot\vec{r}_2)} d\Omega_1 d\Omega_2 \tag{45}$$

so that the correlation function is given by

$$\rho(\vec{r}_1, \vec{r}_2) = -\frac{3}{8\pi} \int_{\Omega} \sin\phi \sin^2\theta e^{-j\vec{k}\cdot(\vec{r}_1 - \vec{r}_2)} d\Omega \tag{46}$$

If the spatial separation $\vec{r}_1 - \vec{r}_2$ lies on the x- or z-axis the integral over ϕ vanishes, but for $\vec{r}_1 - \vec{r}_2 = d\vec{1}_y$, it gives a first order Bessel function, so that after a few manipulations, in this case:

$$\rho(d) = \frac{3j}{2} \left(\frac{\cos kd}{kd} - \frac{\sin kd}{(kd)^2} \right) \tag{47}$$

The imaginary part of this correlation function is drawn on Figure 4. As found in the 2D case [1], $\rho(d)$ is null at the origin and then oscillates around zero. The second zero correlation appears for a lag $d = 0.71\lambda$. It is important to note that in contrary to the 2D case, the correlation only exists along the axis orthogonal to both components. Otherwise, two orthogonal cartesian components of $\vec{E}$ and $\vec{H}$ are uncorrelated.

4. CONCLUSION

In order to fill-in the gaps in the existing literature, the spatial correlation functions for fields have been studied in the 3D Rayleigh channel case. To obtain easy-to-handle closed forms expressions for these functions, a continuous plane-wave spectrum representation of the fields has been chosen. The auto- and cross-correlation functions for the field components, their modulus and square modulus have been derived from this representation. It has been shown that the global fields or their spherical components satisfy the classical $\sin kd/kd$ correlation function while the spatial correlation functions for the cartesian components are anisotropic. In this last case, it has been shown that the existing 2D or isotropic spatial correlation functions involve important differences compared to the full 3D auto- or cross-correlation functions, so that they must be used with care in practical implementations.

REFERENCES

1. Jakes, W. C., *Microwave Mobile Communications*, Wiley, New York, 1974.
2. Lee, W. C., *Mobile Communications Design Fundamentals*, Wiley, New York, 1993.
3. Saunders, S. R., *Antennas and Propagation for Wireless Communication Systems*, Wiley, New York, 1999.
4. Fuhl, J., A. F. Molisch, and E. Bonek, "Unified channel model for mobile radio systems with smart antennas," *IEE Proc. Radar Sonar Navig.*, Vol. 145, 32–41, Feb. 1998.
5. Chuah, C., D. N. C. Tse, J. M. Kahn, and R. A. Valenzuela, "Capacity scaling in MIMO wireless systems under correlated fading," *IEEE Trans. Infor. Theor.*, Vol. 48, 637–650, March 2002.
6. Shiu, D., G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Comm.*, Vol. 48, 502–512, March 2000.
7. Salz, J. and J. H. Winters, "Effect of fading correlation on adaptative arrays in digital mobile radio," *IEEE Trans. Vehic. Tech.*, Vol. 43, 1049–1057, Nov. 1994.

8. Lehman, T. H., “A statistical theory of electromagnetic fields in complex cavities,” *Interaction Notes*, Note 494, May 1993.
9. Hill, D. A., “Spatial correlation function for fields in a reverberation chamber,” *IEEE Trans. Electromagn. Compat.*, Vol. 37, 138, Feb. 1995.
10. Pnini, R. and B. Shapiro, “Intensity fluctuations in closed and open systems,” *Phys. Rev. E*, Vol. 54, 1032–1035, Aug. 1996.
11. Hill, D. A., “Plane wave integral representation for fields in reverberation chambers,” *IEEE Trans. Electromagn. Compat.*, Vol. 40, 209–217, Aug. 1998.
12. Hill, D. A., “Linear dipole response in a reverberation chamber,” *IEEE Trans. Electromagn. Compat.*, Vol. 41, 365–368, Nov. 1999.
13. Papoulis, A., *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, Auckland, 1965.
14. Gradshteyn, I. S. and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic press, New York, 1965.