Progress In Electromagnetics Research, PIER 42, 219–232, 2003

A COUPLED-MODE THEORY-BASED ANALYSIS OF COUPLED MICROSTRIP LINES ON A FERRITE SUBSTRATE

M. Matsunaga

Department of Electrical and Electronic Engineering
Ehime University
3 Bunkyo-cho, Matsuyama, Ehime 790-8577, Japan

Abstract—Presented herein is a coupled-mode formulation for coupled microstrip lines on a magnetized ferrite substrate. The formulation discussed here is an extension of the coupled-mode theory for microstrip lines on an isotropic substrate. Since the magnetized ferrite exhibits a biaxial anisotropy in its permeability, the guided-wave fields in the magnetized ferrite are not subject to the conventional reciprocity relation for fields in an isotropic medium. Thus, a generalized reciprocity relation is first derived from two sets of guided-wave fields, which propagate in ferrite magnetized transversely along the strip surface. The reciprocity relation is then used to derive coupled-mode equations for coupled microstrip lines on a ferrite substrate. As a basic numerical example, the new formulation is applied to two coupled microstrip lines on a ferrite substrate.

1. INTRODUCTION

Multi-layered coupled-microstrip lines are being used widely at present in designing microwave integrated circuits. The use of coupled microstrip lines, especially, on substrates of magnetized ferrites has come to attract great attention because of magnetized ferrite's wide range of applications to nonreciprocal integrated devices. However, since magnetized ferrite exhibits a biaxial anisotropy in permeability, it is often extremely difficult to analyze the electromagnetic fields which propagate in it, hence there are few theoretical studies [1–4] of coupled microstrip lines on anisotropic substrates.

That which is presented in this paper is a coupled-mode formulation for coupled microstrip lines on a ferrite substrate magnetized perpendicular to the direction of electromagnetic wave propagating along strip surface — a formulation which is an extension of the coupled-mode theory for microstrip lines on an isotropic substrate [5–7]. It should be noted that the structure analyzed here is different from those treated in [8] and [9], which dealt with microstrip lines on a ferrite substrate magnetized longitudinally along strip lines. Therefore, an isolated microstrip has a nonreciprocal propagation characteristic, since the magnetized ferrite exhibits a biaxial anisotropy in its permeability. Thus, the guided-wave fields in the magnetized ferrite which analyzed here are not subject to the conventional reciprocity relation fields in an isotropic medium. First of all, a generalized reciprocity relation is derived for guided-wave fields which propagate in anisotropic media from two sets of Maxwell equations: one is satisfied by the guided-waves propagating in the $+z$ direction as shown in Fig. 1(a) and the other is satisfied by the guided-waves propagating in the $-z$ direction as shown in Fig. 1(b). This is an essential point of this paper. It should be noted that a coupled mode formulation is very important to design of microwave applications, since it is well known that the coupled-mode theory can extremely reduce the time of theoretical and numerical analyses of complicated coupled microstrip line structures such as multiple microstrip lines on multilayered or anisotropic substrates.

The reciprocity relation is used to obtain coupled-mode equations for the modal amplitudes which are peculiar to each isolated line on a magnetized ferrite substrate. Note here that when the formulation above is combined with the spectral domain method, the Galerkin moment method procedure may be used to calculate the coupling coefficients for each isolated microstrip. The present approach that we have developed is applied to the analysis of two coupled-microstrip lines on a magnetized ferrite substrate. Moreover, the numerical results

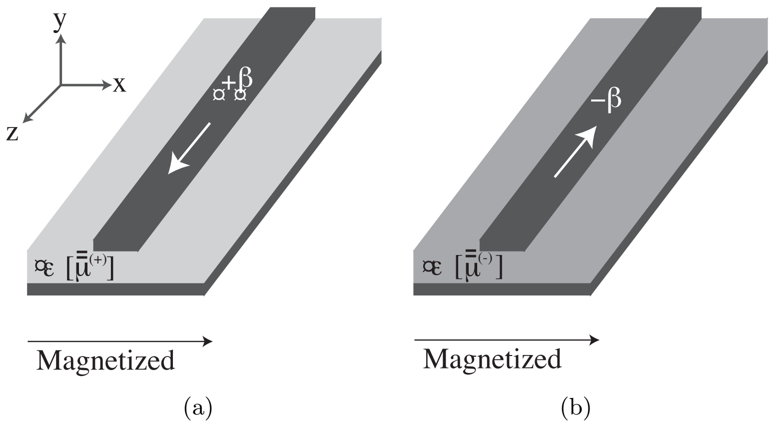

Figure 1. Two sets of guided-wave fields propagate in the ferrite. (a)The guided-wave ($\boldsymbol{E}^{(+)}$,$\boldsymbol{H}^{(+)}$,$\boldsymbol{J}^{(+)}$) propagate in the $+z$ direction, (b)The guided-wave ($\boldsymbol{E}^{(-)}$,$\boldsymbol{H}^{(-)}$,$\boldsymbol{J}^{(-)}$) propagate in the $-z$ direction.

for mode propagation constants are compared with those obtained by the Galerkin moment method procedure. It should be noted, therefore, that both results are in a good agreement.

2. GENERALIZED RECIPROCITY RELATION

The guided-wave fields in a ferrite magnetized in the x direction considered, the permeability tensor of the ferrite may be summarized by the following equations:

$$[\bar{\bar{\mu}}^{(+)}] = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & i\kappa \\ 0 & -i\kappa & \mu \end{bmatrix} \tag{1}$$

with

$$\mu = 1 - \frac{\omega_0 \omega_M}{\omega^2 - \omega_0^2}, \quad \kappa = -\frac{\omega \omega_M}{\omega^2 - \omega_0^2}. \tag{2}$$

Note that in the above equations $\omega_0 = -\gamma\mu_0 H_i$, $\omega_M = -\gamma\mu_0 M_i$, H_i is the internal dc magnetic field; M_i, the saturation magnetization; and γ, the gyromagnetic ratio.

The electric field $\boldsymbol{E}^{(+)}$, the magnetic field $\boldsymbol{H}^{(+)}$, and the current source $\boldsymbol{J}^{(+)}$ — all associated with the guided wave propagating in the

$+z$ direction in the magnetized ferrite — are defined as follows:

$$\boldsymbol{E}^{(+)} = \boldsymbol{e}^{(+)}(x, y)e^{-i\beta z} = [\boldsymbol{e}_t^{(+)}(x, y) + \hat{\boldsymbol{z}} e_z^{(+)}(x, y)]e^{-i\beta z} \tag{3}$$

$$\boldsymbol{H}^{(+)} = \boldsymbol{h}^{(+)}(x, y)e^{-i\beta z} = [\boldsymbol{h}_t^{(+)}(x, y) + \hat{\boldsymbol{z}} h_z^{(+)}(x, y)]e^{-i\beta z} \tag{4}$$

$$\boldsymbol{J}^{(+)} = \boldsymbol{j}^{(+)}(x, y)e^{-i\beta z} = [\boldsymbol{j}_t(x, y) + \hat{\boldsymbol{z}} j_z^{(+)}(x, y)]e^{-i\beta z}. \tag{5}$$

Note that $\boldsymbol{e}^{(+)}(x, y)$, $\boldsymbol{h}^{(+)}(x, y)$ and $\boldsymbol{j}^{(+)}(x, y)$ represent the eigenmode fields and current, and β denotes the mode propagation constant. Therefore, $\boldsymbol{E}^{(+)}$, $\boldsymbol{H}^{(+)}$ and $\boldsymbol{J}^{(+)}$ satisfy the following Maxwell equations:

$$\begin{cases} \nabla \times \boldsymbol{E}^{(+)} = -i\omega[\bar{\bar{\mu}}^{(+)}]\boldsymbol{H}^{(+)} \\ \nabla \times \boldsymbol{H}^{(+)} = i\omega\varepsilon\boldsymbol{E}^{(+)} + \boldsymbol{J}^{(+)} \end{cases} \tag{6}$$

in which the parameter ε is the permittivity of the ferrite. At the same time, the new set of fields and current source $\boldsymbol{E}^{(-)}$, $\boldsymbol{H}^{(-)}$ and $\boldsymbol{J}^{(-)}$, which are associated with the guided wave propagating in the $-z$ direction, and are defined using the respective components of $\boldsymbol{e}^{(+)}(x, y)$, $\boldsymbol{h}^{(+)}(x, y)$ and $\boldsymbol{j}^{(+)}(x, y)$ as follows:

$$\boldsymbol{E}^{(-)} = \boldsymbol{e}^{(-)}(x, y)e^{i\beta z} = [\boldsymbol{e}_t^{(+)}(x, y) - \hat{\boldsymbol{z}} e_z^{(+)}(x, y)]e^{i\beta z} \tag{7}$$

$$\boldsymbol{H}^{(-)} = \boldsymbol{h}^{(-)}(x, y)e^{i\beta z} = [-\boldsymbol{h}_t^{(+)}(x, y) + \hat{\boldsymbol{z}} h_z^{(+)}(x, y)]e^{i\beta z} \tag{8}$$

$$\boldsymbol{J}^{(-)} = \boldsymbol{j}^{(-)}(x, y)e^{i\beta z} = [\boldsymbol{j}_t^{(+)}(x, y) - \hat{\boldsymbol{z}} j_z^{(+)}(x, y)]e^{i\beta z}. \tag{9}$$

These fields and the current source ($\boldsymbol{E}^{(-)}$, $\boldsymbol{H}^{(-)}$, $\boldsymbol{J}^{(-)}$) will not satisfy the Maxwell equations in the same way as those for ($\boldsymbol{E}^{(+)}$, $\boldsymbol{H}^{(+)}$, $\boldsymbol{J}^{(+)}$), because a medium supporting the guided waves is anisotropic. Transformations (7)–(9) of the field variables substituted for (6), and the equations thus obtained compared with (6) term by term, it follows that the electric field $\boldsymbol{E}^{(-)}$, the magnetic field $\boldsymbol{H}^{(-)}$, and the current source $\boldsymbol{J}^{(-)}$ — all associated with the guided wave propagated in the $-z$ direction in the magnetized ferrite – satisfy the following Maxwell equations:

$$\begin{cases} \nabla \times \boldsymbol{E}^{(-)} = -i\omega[\bar{\bar{\mu}}^{(-)}]\boldsymbol{H}^{(-)} \\ \nabla \times \boldsymbol{H}^{(-)} = i\omega\varepsilon\boldsymbol{E}^{(-)} + \boldsymbol{J}^{(-)} \end{cases} \tag{10}$$

in which the matrix $[\bar{\bar{\mu}}^{(-)}]$ represents the permeability tensor of the ferrite magnetized in the $-x$ direction and is related to the original permeability tensor of the ferrite magnetized in the $+x$ direction $[\bar{\bar{\mu}}^{(+)}]$ as follows:

$$[\bar{\bar{\mu}}^{(-)}] = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu & -i\kappa \\ 0 & i\kappa & \mu \end{bmatrix} = [\bar{\bar{\mu}}^{(+)}]^T. \tag{11}$$

Let us assume that $\boldsymbol{E}$ and $\boldsymbol{H}$ are electric and magnetic fields produced by the current source $\boldsymbol{J}$ in a medium with $\varepsilon(y)$ and $[\bar{\bar{\mu}}(y)]$, and that $\boldsymbol{E}'$ and $\boldsymbol{H}'$ are electric and magnetic fields produced by the current source $\boldsymbol{J}'$ in another medium with $\varepsilon(y)$ and $[\bar{\bar{\mu}}'(y)]$. Each set of these fields thus satisfies the corresponding set of the Maxwell equations as follows:

$$\begin{cases} \nabla \times \boldsymbol{E} = -i\omega[\bar{\bar{\mu}}(y)] \cdot \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = i\omega\varepsilon(y)\boldsymbol{E} + \boldsymbol{J} \end{cases} \tag{12}$$

$$\begin{cases} \nabla \times \boldsymbol{E}' = -i\omega[\bar{\bar{\mu}}'(y)] \cdot \boldsymbol{H}' \\ \nabla \times \boldsymbol{H}' = i\omega\varepsilon(y)\boldsymbol{E}' + \boldsymbol{J}'. \end{cases} \tag{13}$$

Thus, the following equation may be derived from vector calculus done for (12) and (13):

$$\begin{aligned} \nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}' - \boldsymbol{E}' \times \boldsymbol{H}) = &- i\omega \boldsymbol{H}' \cdot \left([\bar{\bar{\mu}}(y)] - [\bar{\bar{\mu}}'(y)]^T\right) \cdot \boldsymbol{H} \\ &- \boldsymbol{E} \cdot \boldsymbol{J}' + \boldsymbol{E}' \cdot \boldsymbol{J}. \end{aligned} \tag{14}$$

When (14) is applied to a cylindrical geometry which is translationally invariant in the z direction, we obtain

$$\begin{aligned} &\frac{\partial}{\partial z} \int_S (\boldsymbol{E} \times \boldsymbol{H}' - \boldsymbol{E}' \times \boldsymbol{H}) \cdot \hat{\boldsymbol{z}} dxdy \\ &= -i\omega \int_S \boldsymbol{H}' \cdot ([\bar{\bar{\mu}}(y)] - [\bar{\bar{\mu}}'(y)]^T) \cdot \boldsymbol{H} dxdy \\ &+ \int_S \boldsymbol{E}' \cdot \boldsymbol{J} dxdy - \int_S \boldsymbol{E} \cdot \boldsymbol{J}' dxdy \end{aligned} \tag{15}$$

where S denotes the cross-sectional area in the transverse x-y plane. When the fields $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ are the guided fields $(\boldsymbol{E}^{(+)}, \boldsymbol{H}^{(+)}, \boldsymbol{J}^{(+)})$ and thus satisfy (6), and when the fields $(\boldsymbol{E}', \boldsymbol{H}', \boldsymbol{J}')$ are the guided fields $(\boldsymbol{E}^{(-)}, \boldsymbol{H}^{(-)}, \boldsymbol{J}^{(-)})$ and thus satisfy (10), the first term in the right hand side of (15) is canceled out, and (15) leads to

$$\frac{\partial}{\partial z} \int_S (\boldsymbol{E} \times \boldsymbol{H}' - \boldsymbol{E}' \times \boldsymbol{H}) \cdot \hat{\boldsymbol{z}} dxdy = \int_S \boldsymbol{E}' \cdot \boldsymbol{J} dxdy - \int_S \boldsymbol{E} \cdot \boldsymbol{J}' dxdy. \tag{16}$$

It should be noted that the above equation is a generalized reciprocity relation for two sets of guided wave fields in a magnetized ferrite. It might also be noted here that the above equation yields the same reciprocity relation for the two sets of guided-wave fields $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ and $(\boldsymbol{E}', \boldsymbol{H}', \boldsymbol{J}')$ as those [5] in the isotropic medium.

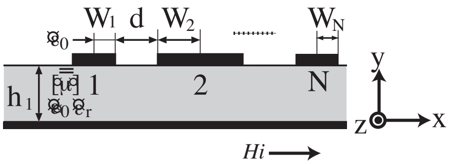

Figure 2. Cross section of N coupled microstrip lines on a Magnetized ferrite substrate.

3. COUPLED-MODE EQUATIONS

Using the reciprocity relation (16), the coupled-mode equations for N coupled microstrip lines on the substrate of magnetized ferrite whose permeability tensor is fixed at $[\bar{\bar{\mu}}^{(+)}]$ as shown in Fig. 2 can be formulated in the same way as in the case of equations for an isotropic substrate [5–7]. This illustrates, therefore, that if the eigenmode fields and current in the original coupled structure are applied to the first set of solutions $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ in (16), these solutions may be approximated to the following equations:

$$\begin{aligned}\boldsymbol{E} &= \sum_{\nu=1}^{N} a_\nu(z)\boldsymbol{e}_\nu^{(+)}(x, y)\\ &= \sum_{\nu=1}^{N} a_\nu(z)[\boldsymbol{e}_{\nu,t}^{(+)}(x, y) + \hat{\boldsymbol{z}} e_{\nu,z}^{(+)}(x, y)]\end{aligned} \tag{17}$$

$$\begin{aligned}\boldsymbol{H} &= \sum_{\nu=1}^{N} a_\nu(z)\boldsymbol{h}_\nu^{(+)}(x, y)\\ &= \sum_{\nu=1}^{N} a_\nu(z)[\boldsymbol{h}_{\nu,t}^{(+)}(x, y) + \hat{\boldsymbol{z}} h_{\nu,z}^{(+)}(x, y)]\end{aligned} \tag{18}$$

$$\boldsymbol{J} = \sum_{\nu=1}^{N} a_\nu(z)\boldsymbol{j}_\nu^{(+)}(x, y)$$

$$= \sum_{\nu=1}^{N} a_\nu(z)[\boldsymbol{j}_{\nu,t}^{(+)}(x,y) + \hat{\boldsymbol{z}} j_{\nu,z}^{(+)}(x,y)] \tag{19}$$

where $\boldsymbol{e}_\nu(x,y)$, $\boldsymbol{h}_\nu(x,y)$ and $\boldsymbol{j}_\nu(x,y)$ ($\nu = 1,2,...N$) are eigenmode functions for the fields and current propagating in the $+z$ direction along each of the N microstrip lines in isolation, and in which $a_\nu(z)$ is an unknown amplitude function. It should be noted that all components $(\boldsymbol{E}, \boldsymbol{H}, \boldsymbol{J})$ can be expanded with the same $a_\nu(z)$, since the expressions in (17)–(19) correspond to the modal expansions in terms of the N fundamental modes of each of the N isolated microstrip lines. Note that the following solutions may be obtained if both the eigenmode fields and the current, which propagate in the $-z$ direction along each N isolated microstrip line on a ferrite substrate which contains $[\bar{\bar{\mu}}'(y)] = [\bar{\bar{\mu}}^{(-)}]$, are applied to the second set of solutions $(\boldsymbol{E}', \boldsymbol{H}', \boldsymbol{J}')$ in (16):

$$\begin{aligned}\boldsymbol{E}' &= \boldsymbol{e}_\nu^{(-)}(x,y)e^{i\beta_\nu^{(0)}z}\\ &= [\boldsymbol{e}_{\nu,t}(x,y) - \hat{\boldsymbol{z}} e_{\nu,z}(x,y)]e^{i\beta_\nu^{(0)}z} \qquad (20)\\ \boldsymbol{H}' &= \boldsymbol{h}_\nu^{(-)}(x,y)e^{i\beta_\nu^{(0)}z}\\ &= [-\boldsymbol{h}_{\nu,t}(x,y) + \hat{\boldsymbol{z}} h_{\nu,z}(x,y)]e^{i\beta_\nu^{(0)}z} \qquad (21)\\ \boldsymbol{J}' &= \boldsymbol{j}_\nu^{(-)}(x,y)e^{i\beta_\nu^{(0)}z}\\ &= [\boldsymbol{j}_{\nu,t}(x,y) - \hat{\boldsymbol{z}} j_{\nu,z}(x,y)]e^{i\beta_\nu^{(0)}z} \qquad (22)\\ &\qquad (\nu = 1,2,...,N)\end{aligned}$$

where $\beta_\nu^{(0)}$ is the propagation constant of the isolated ν-th microstrip line.

Finally, if the (17)–(19) and (20)–(22), which are the expressions obtained for each of the N isolated microstrip lines, are substituted into the reciprocity relation (16), the following coupled-mode equations for the amplitude functions $a_\nu(z)$ ($\nu = 1,2,...,N$) are obtainable:

$$\frac{d}{dz}\boldsymbol{a} = -i[\boldsymbol{C}]\boldsymbol{a} \tag{23}$$

with

$$\boldsymbol{a} = [a_1\ a_2\ ...a_N]^T \tag{24}$$

$$[\boldsymbol{C}] = [\boldsymbol{M}]^{-1}[\boldsymbol{K}] \tag{25}$$

$$K_{\nu\mu} = \beta_\nu^{(0)} M_{\nu\mu} + Q_{\nu\mu} \tag{26}$$

$$M_{\nu\mu} = \frac{1}{2}(N_{\nu\mu} + N_{\mu\nu}) \tag{27}$$

$$N_{\nu\mu} = \frac{1}{2}\int_S [\boldsymbol{e}_\nu(x,y) \times \boldsymbol{h}_\mu(x,y)] \cdot \hat{\boldsymbol{z}} dxdy \tag{28}$$

$$Q_{\nu\mu} = -\frac{i}{4}\int_{l_\mu} [e_{\nu,x}(x,h_1)j_{\mu,x}(x) - e_{\nu,z}(x,h_1)j_{\mu,z}(x)]dx$$
$$(\nu, \mu = 1, 2, ..., N) \tag{29}$$

where l_μ denotes the cross-sectional contour of the μ-th line, and the eigenmode fields and current in the isolated lines are normalized so that $N_{\nu\nu}$ becomes 1. It should also be noted here that $Q_{\nu\nu} = 0$ since $e_{\nu,x}(x,y) = e_{\nu,z}(x,y) = 0$ on the surface of the ν-th line. As a result of obtaining the coupled-mode equations, an analysis of N coupled microstrip lines becomes just an analysis of each of the N isolated microstrip lines.

Solutions to the coupled mode equations (23) are the propagation constants of N coupled modes propagating in the forward and backward directions. Note here that although there are various numerical techniques [9], the eigenmode fields and currents for isolated microstrip lines — all have different values in forward and backward waves — which appear in equations (28) and (29) may be calculated by the spectral domain method combined with Galerkin moment method procedure [5–7]. However, in order to calculate eigenmode fields and a current for an isolated microstrip line, the fields equations for it should be obtained. Therefore, the process of obtaining the fields equations is shown in the following section.

4. FIELDS EQUATIONS FOR AN ISOLATED MICROSTRIP LINE

The electromagnetic fields in the magnetized ferrite with the relative permittivity ε_r and the permeability tensor $[\bar{\bar{\mu}}^{(+)}]$ satisfy the following Maxwell equations:

$$\begin{cases} \nabla \times \boldsymbol{E} = -i\omega\left[\bar{\bar{\mu}}^{(+)}\right]\boldsymbol{H} \\ \nabla \times \boldsymbol{H} = i\omega\varepsilon_0\varepsilon_r\boldsymbol{E} + \boldsymbol{J}. \end{cases} \tag{30}$$

Since x represents the principal axis, the fields may be expressed in terms of E_x and H_x. Therefore, several manipulations to (30) will lead to the following equations:

$$\nabla_t^2 E_x + \frac{\partial^2}{\partial x^2} E_x + k_0^2 \varepsilon_r \mu_{ef} E_x = -k_0 \frac{\kappa}{\mu} \frac{\partial}{\partial x} \bar{H}_x \tag{31}$$

$$\nabla_t^2 \bar{H}_x + \frac{1}{\mu} \frac{\partial^2}{\partial x^2} \bar{H}_x + k_0^2 \varepsilon_r \bar{H}_x = k_0 \varepsilon_r \frac{\kappa}{\mu} \frac{\partial}{\partial x} E_x. \tag{32}$$

And the transversal field components $\boldsymbol{E}_t = \hat{\boldsymbol{y}} E_y + \hat{\boldsymbol{z}} E_z$ and $\bar{\boldsymbol{H}}_t = \hat{\boldsymbol{y}} \bar{H}_y + \hat{\boldsymbol{z}} \bar{H}_z$ are expressed in terms of E_x and $\bar{H}_x$ as follows:

$$[\bar{\bar{\gamma}}] \cdot \boldsymbol{E}_t = -ik_0 [\bar{\bar{\mu}}_t] \cdot (\nabla_t \bar{H}_x \times \hat{\boldsymbol{x}}) + \frac{\partial}{\partial x} \nabla_t E_x \tag{33}$$

$$[\bar{\bar{\gamma}}] \cdot \bar{\boldsymbol{H}}_t = ik_0 \varepsilon_r (\nabla_t E_x \times \hat{\boldsymbol{x}}) + \frac{\partial}{\partial x} \nabla_t \bar{H}_x \tag{34}$$

with

$$\nabla_t = \hat{\boldsymbol{y}} \frac{\partial}{\partial y} + \hat{\boldsymbol{z}} \frac{\partial}{\partial z} \tag{35}$$

$$[\bar{\bar{\gamma}}] = \begin{bmatrix} k_0 \varepsilon_r \mu + \dfrac{\partial^2}{\partial x^2} & ik_0^2 \varepsilon_r \kappa \\ -ik_0^2 \varepsilon_r \kappa & k_0 \varepsilon_r \mu + \dfrac{\partial^2}{\partial x^2} \end{bmatrix} \tag{36}$$

$$[\bar{\bar{\mu}}_t] = \begin{bmatrix} \mu & i\kappa \\ -i\kappa & \mu \end{bmatrix} \tag{37}$$

where $\mu_{ef} = \mu - \frac{\kappa^2}{\mu}$, $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, $\bar{\boldsymbol{H}} = \eta_0 \boldsymbol{H}$, and in which the subscript t denotes the transverse y and z components of the indicated vectors. The z dependence of electromagnetic fields is assumed to be $e^{-i\beta z}$ where β is the propagation constant along the microstrip line. Here the Fourier transform with respect to the x coordinate for the electric and magnetic fields may be defined as follows:

$$\boldsymbol{E}(x, y, z) = \frac{1}{2\pi} e^{-i\beta z} \int_{-\infty}^{\infty} \tilde{\boldsymbol{E}}(\zeta, y) e^{-i\zeta x} d\zeta \tag{38}$$

$$\bar{\boldsymbol{H}}(x, y, z) = \frac{1}{2\pi} e^{-i\beta z} \int_{-\infty}^{\infty} \tilde{\bar{\boldsymbol{H}}}(\zeta, y) e^{-i\zeta x} d\zeta. \tag{39}$$

Therefore, (31) and (32) may be rewritten in the Fourier transformed domain as follows:

$$\left[\frac{\partial^2}{\partial y^2} + P\right]\tilde{E}_x = k_0 \frac{i\kappa}{\mu}\zeta \tilde{\tilde{H}}_x \tag{40}$$

$$\left[\frac{\partial^2}{\partial y^2} + Q\right]\tilde{\tilde{H}}_x = -k_0\varepsilon_r \frac{i\kappa}{\mu}\zeta \tilde{E}_x \tag{41}$$

where

$$P = k_0^2\varepsilon_r\mu_{ef} - \beta^2 - \zeta^2, \quad Q = k_0^2\varepsilon_r - \beta^2 - \frac{1}{\mu}\zeta^2\,. \tag{42}$$

If $\tilde{\tilde{H}}_x$ is eliminated from (40) and (41), the following Fourier-transformed Helmholtz equation for $\tilde{E}_x$ can be derived:

$$\left[\frac{\partial^4}{\partial y^4} + (P+Q)\frac{\partial^2}{\partial y^2} + (PQ - k_0\varepsilon_r\frac{\kappa^2}{\mu^2}\zeta^2)\right]\tilde{E}_x(\zeta, y) = 0. \tag{43}$$

The solution to (43) may be expressed in the following form:

$$\tilde{E}_x(\zeta, y) = A^+e^{-i\kappa_1 y} + A^-e^{i\kappa_1 y} + B^+e^{-i\kappa_2 y} + B^-e^{i\kappa_2 y} \tag{44}$$

with

$$\kappa_1 = \sqrt{\frac{P+Q}{2} + \sqrt{\frac{(P-Q)^2}{4} + k_0\varepsilon_r\frac{\kappa^2}{\mu^2}\zeta^2}}, \tag{45}$$

$$\kappa_2 = \sqrt{\frac{P+Q}{2} - \sqrt{\frac{(P-Q)^2}{4} + k_0\varepsilon_r\frac{\kappa^2}{\mu^2}\zeta^2}}\,, \tag{46}$$

where A^+, A^-, B^+ and B^- are unknown coefficients. And the substitution of (44) into (41) can lead to:

$$\begin{aligned}\tilde{\tilde{H}}_x(\zeta, y) =& \frac{k_0\varepsilon_r(i\kappa/\mu)\zeta}{\kappa_1^2 - Q}[A^+e^{-i\kappa_1 y} + A^-e^{i\kappa_1 y}] \\ &+ \frac{k_0\varepsilon_r(i\kappa/\mu)\zeta}{\kappa_2^2 - Q}[B^+e^{-i\kappa_2 y} + B^-e^{i\kappa_2 y}].\end{aligned} \tag{47}$$

Note that the expressions for $\tilde{E}_y$, $\tilde{E}_z$, $\tilde{\tilde{H}}_y$ and $\tilde{\tilde{H}}_z$ may now be deduced with ease from (33), (34), (44) and (47). In this case, the unknown coefficients above are determined by the application of the boundary conditions to the resulting field expressions. Thus, the dyadic Green functions, which relate $\tilde{E}_x(\zeta, h_1)$ and $\tilde{E}_z(\zeta, h_1)$ to the current densities $\tilde{\tilde{J}}_x(\zeta)$ and $\tilde{\tilde{J}}_z(\zeta)$, may be obtained in the spectral domain as follows:

$$\tilde{E}_x(\zeta, h_1) = \tilde{G}_{xx}(\zeta, h_1)\tilde{\tilde{J}}_x(\zeta) + \tilde{G}_{xz}(\zeta, h_1)\tilde{\tilde{J}}_z(\zeta) \tag{48}$$

$$\tilde{E}_z(\zeta, h_1) = \tilde{G}_{zx}(\zeta, h_1)\tilde{\tilde{J}}_x(\zeta) + \tilde{G}_{zz}(\zeta, h_1)\tilde{\tilde{J}}_z(\zeta). \tag{49}$$

Note that the detailed descriptions of $\tilde{G}_{xx}$, $\tilde{G}_{xz}$, $\tilde{G}_{zx}$ and $\tilde{G}_{zz}$ are omitted here because of the limited space available in this paper. Finally, equations (48) and (49) are solved to determine $\tilde{\boldsymbol{E}}$, $\tilde{\tilde{\boldsymbol{H}}}$ and $\tilde{\tilde{\boldsymbol{J}}}$ using the Galerkin moment method in the spectral domain. The solutions thus obtained are substituted into (28) and (29) to calculate the elements of the coupling matrix $[\boldsymbol{C}]$ in the spectral domain.

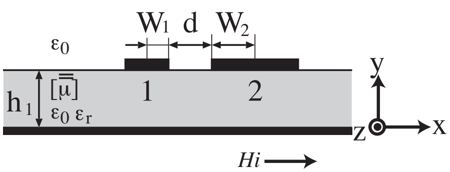

Figure 3. Cross section of two coupled microstrip lines on a magnetized ferrite substrate.

5. NUMERICAL RESULTS

As an example of numerical computations, we considered two coupled microstrip lines on a ferrite substrate as shown in Fig. 3, where the height of the substrate, the width of the microstrip lines, the relative permittivity, internal dc magnetic field, and saturation magnetization of the ferrite are $h_1 = 0.254\,\text{mm}$, $w_1 = w_2 = 0.254\,\text{mm}$ $\varepsilon_r = 12.6$, $\mu_0 H_i = 0.0275\,\text{T}$ and $\mu_0 M_i = 0.275\,\text{T}$ respectively. It is noted that these examples of parameters are chosen as those in [2]. Figure 4 shows the frequency dependence of the normalized propagation constants β/k_0 of the two coupled-modes for the forward wave and the backward wave when the separation distance between the microstrip lines is $d = 0.5\,\text{mm}$. In this figure the results obtained by Galerkin moment method procedure are also plotted. It is noted that both results are in a good agreement. The normalized propagation constants β/k_0 at the frequency $f = 10\,\text{GHz}$ are shown in Fig. 5 as functions of the separation distance d. It should be also noted that those results of the present analysis are in a good agreement with the results which obtained by Galerkin moment method.

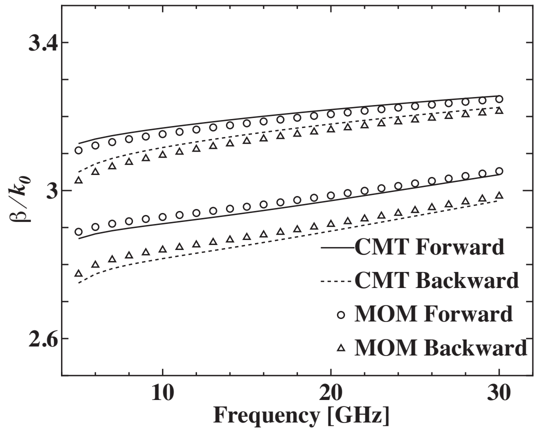

Figure 4. Forward and backward normalized propagation constants of two identical microstrip lines as functions of frequency, where $w_1 = w_2 = 0.254\,\text{mm}$, $h_1 = 0.254\,\text{mm}$, $\varepsilon_r = 12.6$, $d = 0.5\,\text{mm}$, $\mu_0 H_i = 0.0275\,\text{T}$, and $\mu_0 M_i = 0.275\,\text{T}$. Note that CMT refers to the coupled-mode theory which presented here, and MOM refers to the Galerkin moment method.

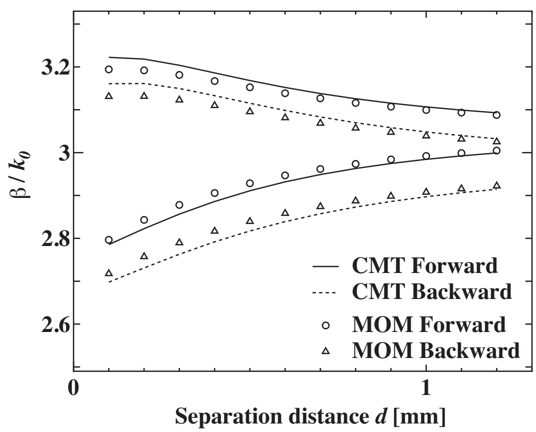

Figure 5. Forward and backward normalized propagation constants of two identical microstrip lines as functions of the separation distance d, where $w_1 = w_2 = 0.254\,\text{mm}$, $h_1 = 0.254\,\text{mm}$, $\varepsilon_r = 12.6$, $f = 10\,\text{GHz}$, $\mu_0 H_i = 0.0275\,\text{T}$ and $\mu_0 M_i = 0.275\,\text{T}$. Note that CMT refers to the coupled-mode theory which presented here, and MOM refers to the Galerkin moment method.

6. CONCLUSION

A coupled-mode formulation for coupled microstrip lines on a magnetized ferrite substrate has been presented. The proposed coupled-mode formulation is an efficient analytical and numerical technique, which yields the approximate dispersion characteristics of coupled microstrip lines on a ferrite magnetized transversely along the propagation direction of guided-wave fields with high enough accuracy. The guided-wave fields in a magnetized ferrite are not subject to a conventional reciprocity relation like the fields propagating in an isotropic medium. However, when guided-wave fields propagating in the opposite direction in ferrite magnetized in the reverse direction of the target structure are introduced as another set of guided-wave fields for a reciprocity relation, this creates a generalized reciprocity relationship between the guided-wave fields in the target structure and those in a complementary configuration. Therefore, a generalized reciprocity relation has been used to derive coupled-mode equations for coupled microstrip lines on a ferrite substrate magnetized in a transverse direction along strips. In conclusion, the extension of the coupled-mode theory to a formulation of the coupled microstrip line on a magnetized ferrite has been done, even if guided-wave fields in the magnetized ferrite demonstrate nonreciprocal nature of mode propagation in forward and backward directions. It is also noted that further extension of the present formulation to multiple microstrip lines on a multi-layered ferrite substrate is straightforward.

ACKNOWLEDGMENT

The author gratefully acknowledges the Sasakawa Scientific Research Grant from the Japan Science Society.

REFERENCES

1. Mesa, F. and M. Horno, "Application of the spectral domain method for the study of surface slow waves in nonreciprocal planar structures with a multilayered gyroelectric substrate," *IEE Proc. H*, Vol. 140, 193–200, 1993.
2. Hsia, I. Y., H. Y. Yang, and N. G. Alexopoulos, "Basic properties of microstrip circuit elements on nonreciprocal substrate-superstrate structures," *Journal of Electromagnetic Waves and Applications*, Vol. 5, 465–476, 1991.
3. Kitazawa, T., "Variational method for planar transmission lines

with anisotropic magnetic media," *IEEE Trans. Microwave Theory & Tech.,* Vol. 37, 1749–1754, 1989.

4. Pregla, R., "Method of lines for the analysis of multilayered gyrotropic waveguide structures," *IEE Proc. H,* Vol. 140, 183–192, 1993.
5. Yasumoto, K. and M. Matsunaga, "Coupled-mode analysis of coupled multiple microstrip lines," *IEICE Trans. Electronics*, Vol. E80-C, 340–345, 1997.
6. Matsunaga, M., M. Katayama, and K. Yasumoto, "Coupled-mode analysis of line parameters of coupled microstrip lines," *Progress in Electromagnetic Research*, Vol. PIER-24, 1–18, 1999.
7. Yasumoto, K., "Coupled-mode formulation of multilayered and multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-44, 585–590, 1996.
8. Xie, K. and L. E. Davis, "Nonreciprocity and the optimum operation of ferrite coupled lines," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-48, 562–573, 2000.
9. Mazur, J., Piotr, and A. Cwikla, "Coupled-mode analysis of ferrite microstrip lines," *IEEE Microwave Guided Wave Lett.*, Vol. 9, 300–302, 1999.