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EIGENFUNCTIONAL EXPANSION OF DYADIC GREEN'S FUNCTIONS IN GYROTROPIC MEDIA USING CYLINDRICAL VECTOR WAVE FUNCTIONS

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Abstract—This paper presents a novel eigenfunction expansion of the electric-type dyadic Green's function for an unbounded gyrotropic medium in terms of the cylindrical vector wave functions. The unbounded Green dyadics are formulated based on the Ohm-Rayleigh method, orthogonality of the vector wave functions, and the newly formulated curl and divergence of dyadic identities. The irrotational part of the Green's function is obtained from the residual theorem. Unlike some of the published work where some assumptions are made prior to the formulation, the irrotational dyadic Green's function in this paper is formulated rigorously based on the idea given by Tai.

1. INTRODUCTION

The dyadic Green's functions (DGFs) technique [1, 2], and [3] has been widely used to investigate the electromagnetic boundary problems for more than 20 years. Although the dyadic Green's functions can be obtained in closed form for only a few simple geometries, the compact formulations and solutions of some electromagnetic problems they offer make their use extremely attractive. The dyadic Green's functions of canonical problems may be constructed in several ways. One of the common approaches is to express the Greens functions for defining electromagnetic vector potentials or fields in terms of the Fourier transform, whereas another approach is to represent the Green's functions for defining electromagnetic fields in terms of coordinates vectors and from a set of appropriate electric and magnetic vector potentials. Among all the available approaches, the vector wave function expansion approach is most widely employed to derive the dyadic Green's functions [1, 3], and [3].

Although the DGFs in isotropic media have been well-documented in the last three decades, a complete formulation of the DGFs in various media using the eigenfunction expansion technique has not been achieved so far. For example, for the past three decades, the dyadic Greens functions in anisotropic media have been derived [4–12] using one of the three methods, namely the Fourier transform technique, the method of angular spectrum expansion and the transmission matrix method. However, there is still no available result for the DGF of gyrotropic media in terms of cylindrical vector wave functions; this hence motivates the present work. Throughout this work, a time dependence of $e^{-i\omega t}$ is assumed and suppressed in the formulation.

2. DGFS FOR UNBOUNDED GYROTROPIC MEDIA

A gyrotropic medium is described by the following constitutive relations

$$\begin{aligned} \boldsymbol{D} &= \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} + \xi \boldsymbol{H} && \text{(1a)} \\ \boldsymbol{B} &= \zeta \boldsymbol{E} + \overline{\boldsymbol{\mu}} \cdot \boldsymbol{H}, && \text{(1b)} \end{aligned}$$

where ξ and ζ are, respectively, the electric-magnetic and magnetic-electric mutual-coupling parameters or dielectric parameters of bi-isotropic media, and the permittivity and permeability tensors are defined as

$$\overline{\boldsymbol{\epsilon}} = \begin{bmatrix} \epsilon_t & -i\epsilon_a & 0 \\ i\epsilon_a & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}, \tag{2a}$$

$$\overline{\boldsymbol{\mu}} = \begin{bmatrix} \mu_t & -i\mu_a & 0 \\ i\mu_a & \mu_t & 0 \\ 0 & 0 & \mu_z \end{bmatrix}. \tag{2b}$$

Substituting (1) into the source-incorporated Maxwell's equations leads to

$$\nabla \times (\overline{\boldsymbol{\alpha}} \cdot \nabla \times \boldsymbol{E}) - i\omega \nabla \times \left(\overline{\boldsymbol{\beta}} \cdot \boldsymbol{E}\right) + i\omega \overline{\boldsymbol{\gamma}} \cdot \nabla \times \boldsymbol{E} - \omega^2 \overline{\boldsymbol{\delta}} \cdot \boldsymbol{E} = i\omega \boldsymbol{J} \tag{3}$$

where for simplicity, we define

$$\begin{aligned} \overline{\boldsymbol{\alpha}} &= \overline{\boldsymbol{\mu}}^{-1}, && \text{(4a)} \\ \overline{\boldsymbol{\beta}} &= \zeta \overline{\boldsymbol{\mu}}^{-1}, && \text{(4b)} \\ \overline{\boldsymbol{\gamma}} &= \xi \overline{\boldsymbol{\mu}}^{-1}, && \text{(4c)} \\ \overline{\boldsymbol{\delta}} &= \overline{\boldsymbol{\epsilon}} - \xi\zeta \overline{\boldsymbol{\mu}}^{-1}. && \text{(4d)} \end{aligned}$$

2.1. General Formulation of Unbounded DGFs

The electric field can thus be expressed in terms of the DGF and current source distribution as

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega \int_{V'} \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}') dV', \tag{5}$$

where V' denotes the volume occupied by the exciting current source. Similarly, substituting (5) into (3) leads to

$$\nabla \times \left(\overline{\boldsymbol{\alpha}} \cdot \nabla \times \overline{\boldsymbol{G}}\right) - i\omega \nabla \times \left(\overline{\boldsymbol{\beta}} \cdot \overline{\boldsymbol{G}}\right) + i\omega \overline{\boldsymbol{\gamma}} \cdot \nabla \times \overline{\boldsymbol{G}} - \omega^2 \overline{\boldsymbol{\delta}} \cdot \overline{\boldsymbol{G}} = \overline{\boldsymbol{I}} \delta(\boldsymbol{r} - \boldsymbol{r}'), \tag{6}$$

where $\overline{\boldsymbol{I}}$ and $\delta(\boldsymbol{r} - \boldsymbol{r}')$ denote the dyadic identity and Dirac delta function, respectively.

According to the well-known Ohm-Rayleigh method, the source term in (6) can be expanded in terms of the solenoidal and irrotational cylindrical vector wave functions in cylindrical coordinates system. Thus, we have

$$\begin{aligned} \overline{\boldsymbol{I}} \delta(\boldsymbol{r} - \boldsymbol{r}') &= \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \left[\boldsymbol{M}_n(h,\lambda) \boldsymbol{A}_n(h,\lambda) \right. \\ &\quad \left. + \boldsymbol{N}_n(h,\lambda) \boldsymbol{B}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda) \boldsymbol{C}_n(h,\lambda) \right], \end{aligned} \tag{7}$$

where $\boldsymbol{M}_n(h,\lambda)$ and $\boldsymbol{N}_n(h,\lambda)$ are the solenoidal, and $\boldsymbol{L}_n(h,\lambda)$ is the irrotational, cylindrical vector wave functions while λ and h are the spectral longitudinal and radial wave numbers, respectively. The solenoidal and irrotational cylindrical vector wave functions are defined [3] as

$$\boldsymbol{M}_n(h,\lambda) = \nabla \times \left[\Psi_n(h,\lambda) \hat{\boldsymbol{z}} \right], \tag{8a}$$

$$\boldsymbol{N}_n(h,\lambda) = \frac{1}{k_\lambda} \nabla \times \boldsymbol{M}_n(h,\lambda), \tag{8b}$$

$$\boldsymbol{L}_n(h,\lambda) = \nabla \left[\Psi_n(h,\lambda) \right], \tag{8c}$$

where $k_\lambda = \sqrt{\lambda^2 + h^2}$, and the generating function is given by

$$\Psi_n(h,\lambda) = J_n(\lambda\rho) e^{i(n\phi + hz)}. \tag{9}$$

The vector expansion coefficients, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$ in (7), are to be determined from the orthogonality relationships among the cylindrical vector wave functions which are given by [3]:

$$\begin{aligned} & \int_0^{2\pi} d\phi \int_0^\infty \rho d\rho \int_{-\infty}^\infty dz \boldsymbol{M}_n(h,\lambda) \cdot \boldsymbol{M}_{-n'}(-h',-\lambda') \\ &= \int_0^{2\pi} d\phi \int_0^\infty \rho d\rho \int_{-\infty}^\infty dz \boldsymbol{N}_n(h,\lambda) \cdot \boldsymbol{N}_{-n'}(-h',-\lambda') \\ &= 4\pi^2 \lambda \delta(\lambda - \lambda') \delta(h - h') \delta_{nn'}, \end{aligned} \tag{10a}$$

$$
\begin{aligned}
&\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz \boldsymbol{L}_n(h,\lambda) \cdot \boldsymbol{L}_{-n'}(-h',-\lambda') \\
&= 4\pi^2 \frac{(\lambda^2+h^2)}{\lambda} \delta(\lambda-\lambda')\delta(h-h')\delta_{nn'},
\end{aligned} \tag{10b}
$$

and

$$
\begin{aligned}
&\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz \boldsymbol{M}_n(h,\lambda) \cdot \boldsymbol{N}_{-n'}(-h',-\lambda') \\
&= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz \boldsymbol{N}_n(h,\lambda) \cdot \boldsymbol{L}_{-n'}(-h',-\lambda') \\
&= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz \boldsymbol{L}_n(h,\lambda) \cdot \boldsymbol{M}_{-n'}(-h',-\lambda') \\
&= 0.
\end{aligned} \tag{10c}
$$

Therefore, by taking the scalar product of (7) with $\boldsymbol{M}_{-n'}(-h',-\lambda')$, $\boldsymbol{N}_{-n'}(-h',-\lambda')$ and $\boldsymbol{L}_{-n'}(-h',-\lambda')$ each at a time, the vector expansion coefficients are given by:

$$
\boldsymbol{A}_n(h,\lambda) = \frac{1}{4\pi^2\lambda} \boldsymbol{M}'_{-n}(-h,-\lambda), \tag{11a}
$$

$$
\boldsymbol{B}_n(h,\lambda) = \frac{1}{4\pi^2\lambda} \boldsymbol{N}'_{-n}(-h,-\lambda), \tag{11b}
$$

$$
\boldsymbol{C}_n(h,\lambda) = \frac{\lambda}{4\pi^2(\lambda^2+h^2)} \boldsymbol{L}'_{-n}(-h,-\lambda), \tag{11c}
$$

where the prime notation of the cylindrical vector wave functions denotes the expressions at the source point $\boldsymbol{r}'$.

The dyadic Green's function can thus be expanded [3] as:

$$
\begin{aligned}
\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = &\int_0^{\infty} d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} [\boldsymbol{M}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda) \\
&+\boldsymbol{N}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda)\boldsymbol{c}_n(h,\lambda)],
\end{aligned} \tag{12}
$$

where the vector expansion coefficients $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$ and $\boldsymbol{c}_n(h,\lambda)$ are obtained by substituting (12) and (7) into (6), which the dyadic

Greens function must satisfy. Noting the properties of the vector wave functions

$$\boldsymbol{M}_n(h,\lambda) = \frac{1}{k_\lambda}\boldsymbol{\nabla}\times\boldsymbol{N}_n(h,\lambda), \tag{13a}$$

$$\boldsymbol{N}_n(h,\lambda) = \frac{1}{k_\lambda}\boldsymbol{\nabla}\times\boldsymbol{M}_n(h,\lambda), \tag{13b}$$

$$\boldsymbol{\nabla}\times\boldsymbol{L}_n(h,\lambda) = 0; \tag{13c}$$

and their vector or tensor relations

$$\boldsymbol{\nabla}\times[\overline{\boldsymbol{g}}\cdot\boldsymbol{M}_n(h,\lambda)] = hg_a\boldsymbol{M}_n(h,\lambda)+k_\lambda g_t\boldsymbol{N}_n(h,\lambda), \tag{14a}$$

$$\boldsymbol{\nabla}\times[\overline{\boldsymbol{g}}\cdot\boldsymbol{N}_n(h,\lambda)] = \frac{1}{k_\lambda}(h^2g_t+\lambda^2g_z)\boldsymbol{M}_n(h,\lambda)+hg_a\boldsymbol{N}_n(h,\lambda), \tag{14b}$$

$$\boldsymbol{\nabla}\times[\overline{\boldsymbol{g}}\cdot\boldsymbol{L}_n(h,\lambda)] = ih(g_z-g_t)\boldsymbol{M}_n(h,\lambda)-ik_\lambda g_a\boldsymbol{N}_n(h,\lambda), \tag{14c}$$

$$\overline{\boldsymbol{g}}\cdot\boldsymbol{M}_n(h,\lambda) = g_t\boldsymbol{M}_n(h,\lambda)+\frac{h}{k_\lambda}g_a\boldsymbol{N}_n(h,\lambda) +\frac{i\lambda^2}{k_\lambda^2}g_a\boldsymbol{L}_n(h,\lambda), \tag{14d}$$

$$\overline{\boldsymbol{g}}\cdot\boldsymbol{N}_n(h,\lambda) = \frac{h}{k_\lambda}g_a\boldsymbol{M}_n(h,\lambda)+\frac{h^2g_t+\lambda^2g_z}{k_\lambda^2}\boldsymbol{N}_n(h,\lambda) +\frac{ih\lambda^2}{k_\lambda^3}(g_t-g_z)\boldsymbol{L}_n(h,\lambda), \tag{14e}$$

$$\overline{\boldsymbol{g}}\cdot\boldsymbol{L}_n(h,\lambda) = -ig_a\boldsymbol{M}_n(h,\lambda)+\frac{ih}{k_\lambda}(g_z-g_t)\boldsymbol{N}_n(h,\lambda) +\frac{1}{k_\lambda^2}(h^2g_z+\lambda^2g_t)\boldsymbol{L}_n(h,\lambda); \tag{14f}$$

we end up with

$$\begin{aligned}&\int_0^\infty d\lambda\int_{-\infty}^{\infty}dh\sum_{n=-\infty}^{\infty}\{k_\lambda\left(\boldsymbol{\nabla}\times\overline{\boldsymbol{\alpha}}\cdot\boldsymbol{Na}+\boldsymbol{\nabla}\times\overline{\boldsymbol{\alpha}}\cdot\boldsymbol{Mb}\right)\\ &-i\omega\left(\boldsymbol{\nabla}\times\overline{\boldsymbol{\beta}}\cdot\boldsymbol{Ma}+\boldsymbol{\nabla}\times\overline{\boldsymbol{\beta}}\cdot\boldsymbol{Nb}+\boldsymbol{\nabla}\times\overline{\boldsymbol{\beta}}\cdot\boldsymbol{Lc}\right)\\ &+i\omega k_\lambda\left(\overline{\boldsymbol{\gamma}}\cdot\boldsymbol{Na}+\overline{\boldsymbol{\gamma}}\cdot\boldsymbol{Mb}\right)-\omega^2\left(\overline{\boldsymbol{\delta}}\cdot\boldsymbol{Ma}+\overline{\boldsymbol{\delta}}\cdot\boldsymbol{Nb}+\overline{\boldsymbol{\delta}}\cdot\boldsymbol{Lc}\right)\Big\}\\ =\ &\int_0^\infty d\lambda\int_{-\infty}^{\infty}dh\sum_{n=-\infty}^{\infty}\{\boldsymbol{MA}+\boldsymbol{NB}+\boldsymbol{LC}\}\end{aligned} \tag{15}$$

where and subsequently, the notations $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{M}$, $\boldsymbol{N}$ and $\boldsymbol{L}$ represent $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$, $\boldsymbol{c}_n(h,\lambda)$, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, $\boldsymbol{C}_n(h,\lambda)$, $\boldsymbol{M}_n(h,\lambda)$, $\boldsymbol{N}_n(h,\lambda)$ and $\boldsymbol{L}_n(h,\lambda)$ respectively.

By taking the anterior scalar product of (15) with $\boldsymbol{M}_{-n'}(-h',-\lambda')$, $\boldsymbol{N}_{-n'}(-h',-\lambda')$ and $\boldsymbol{L}_{-n'}(-h',-\lambda')$, respectively, and making use of the identities shown in (14), and by performing the integration over the entire space, we can formulate the equations satisfied by the unknown vectors and the known scalar and vector parameters in a matrix form as given below:

$$[\boldsymbol{\Omega}][\boldsymbol{X}] = [\boldsymbol{\Theta}], \tag{16}$$

where $[\boldsymbol{\Omega}]$ is a 3×3 matrix given by

$$[\boldsymbol{\Omega}] = [\boldsymbol{\Omega}_1\boldsymbol{\Omega}_2\boldsymbol{\Omega}_3] \tag{17}$$

with

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} h^2\alpha_t + \lambda^2\alpha_z - i\omega h(\beta_a-\gamma_a) - \omega^2\delta_t \\ hk_\lambda\alpha_a + \dfrac{i\omega}{k_\lambda}\left(h^2\gamma_t + \lambda^2\gamma_z - k_\lambda^2\beta_t + i\omega h\delta_a\right) \\ -\dfrac{\omega h\lambda^2}{k_\lambda^2}(\gamma_t-\gamma_z) - \dfrac{i\omega^2\lambda^2}{k_\lambda^2}\delta_a \end{bmatrix}, \tag{18a}$$

$$\boldsymbol{\Omega}_2 = \begin{bmatrix} hk_\lambda\alpha_a - \dfrac{i\omega}{k_\lambda}\left(h^2\beta_t + \lambda^2\beta_z - k_\lambda^2\gamma_t - i\omega h\delta_a\right) \\ k_\lambda^2\alpha_t - i\omega h(\beta_a-\gamma_a) - \dfrac{\omega^2}{k_\lambda^2}\left(h^2\delta_t + \lambda^2\delta_z\right) \\ -\dfrac{\omega\lambda^2}{k_\lambda}\gamma_a - \dfrac{ih\omega^2\lambda^2}{k_\lambda^3}(\delta_t-\delta_z) \end{bmatrix}, \tag{18b}$$

$$\boldsymbol{\Omega}_3 = \begin{bmatrix} \omega h(\beta_z-\beta_t) + i\omega^2\delta_a \\ -\omega k_\lambda\beta_a - \dfrac{i\omega^2 h}{k_\lambda}(\delta_z-\delta_t) \\ -\dfrac{\omega^2}{k_\lambda^2}\left(h^2\delta_z + \lambda^2\delta_t\right) \end{bmatrix}, \tag{18c}$$

and $[\boldsymbol{X}]$ and $[\boldsymbol{\Theta}]$ are two column vectors given respectively by

$$[\boldsymbol{X}] = \begin{bmatrix} \boldsymbol{a}_n(h,\lambda) \\ \boldsymbol{b}_n(h,\lambda) \\ \boldsymbol{c}_n(h,\lambda) \end{bmatrix}, \tag{19a}$$

$$[\boldsymbol{\Theta}] = \begin{bmatrix} \boldsymbol{A}_n(h,\lambda) \\ \boldsymbol{B}_n(h,\lambda) \\ \boldsymbol{C}_n(h,\lambda) \end{bmatrix}, \tag{19b}$$

By solving for the inverse of (17), the vector expansion coefficients for the DGF can be shown to be as follows:

$$\boldsymbol{a}_n(h,\lambda) = \frac{1}{\Gamma}\left[\alpha_1 \boldsymbol{A}_n(h,\lambda) - \beta_1 \boldsymbol{B}_n(h,\lambda) + \frac{\gamma_1}{\omega}\boldsymbol{C}_n(h,\lambda)\right], \quad (20a)$$

$$\boldsymbol{b}_n(h,\lambda) = \frac{1}{\Gamma}\left[-\alpha_2 \boldsymbol{A}_n(h,\lambda) - \beta_2 \boldsymbol{B}_n(h,\lambda) + \frac{\gamma_2}{\omega}\boldsymbol{C}_n(h,\lambda)\right], \quad (20b)$$

$$\boldsymbol{c}_n(h,\lambda) = \frac{1}{\Gamma}\left[\frac{\alpha_3}{\omega} \boldsymbol{A}_n(h,\lambda) - \frac{\beta_3}{\omega} \boldsymbol{B}_n(h,\lambda) + \frac{\gamma_3}{\omega^2}\boldsymbol{C}_n(h,\lambda)\right], \quad (20c)$$

where the coefficients α_1, α_2, α_3, β_1, β_2, β_3, γ_1, γ_2,γ_3 and Γ are as shown in Appendix B.

The unbounded dyadic Green's function can be written as

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = & \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\Gamma\lambda}\Big\{\boldsymbol{M}_n(h,\lambda)\Big[\alpha_1 \boldsymbol{M}'_{-n} \\ & (-h,-\lambda) - \beta_1 \boldsymbol{N}'_{-n}(-h,-\lambda) + \frac{\gamma_1\lambda^2}{k_\lambda^2\omega}\boldsymbol{L}'_{-n}(-h,-\lambda)\Big] \\ & +\boldsymbol{N}_n(h,\lambda)\Big[-\alpha_2 \boldsymbol{M}'_{-n}(-h,-\lambda) - \beta_2 \boldsymbol{N}'_{-n}(-h,-\lambda) \\ & + \frac{\gamma_2\lambda^2}{k_\lambda^2\omega}\boldsymbol{L}'_{-n}(-h,-\lambda)\Big] + \boldsymbol{L}_n(h,\lambda)\left[\frac{\alpha_3}{\omega}\boldsymbol{M}'_{-n}(-h,-\lambda)\right. \\ & \left.\left. -\frac{\beta_3}{\omega}\boldsymbol{N}'_{-n}(-h,-\lambda) + \frac{\gamma_3\lambda^2}{k_\lambda^2\omega^2}\boldsymbol{L}'_{-n}(-h,-\lambda)\right]\right\}. \end{aligned} \quad (21)$$

Hence, the dyadic Green's function for an unbounded gyrotropic medium is now represented explicitly in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions, as given in (21).

In order to simplify (21), the residue theorem is needed, and we must first extract the part in (21) which does not satisfy the Jordan lemma as was done in [1]. To do so, we write

$$\boldsymbol{L}_n(h,\lambda) = \boldsymbol{L}_{nt}(h,\lambda) + \boldsymbol{L}_{nz}(h,\lambda), \quad (22a)$$

$$\boldsymbol{L}'_{-n}(-h,-\lambda) = \boldsymbol{L}'_{-nt}(-h,-\lambda) + \boldsymbol{L}'_{-nz}(-h,-\lambda), \quad (22b)$$

$$\boldsymbol{N}_n(h,\lambda) = \boldsymbol{N}_{nt}(h,\lambda) + \boldsymbol{N}_{nz}(h,\lambda), \quad (22c)$$

$$\boldsymbol{N}'_{-n}(-h,-\lambda) = \boldsymbol{N}'_{-nt}(-h,-\lambda) + \boldsymbol{N}'_{-nz}(-h,-\lambda), \quad (22d)$$

$$\boldsymbol{L}_{nt}(h,\lambda) = -\frac{ik_\lambda}{h}\boldsymbol{N}_{nt}(h,\lambda), \quad (22e)$$

$$\boldsymbol{L}'_{-nt}(-h,-\lambda) = -\frac{ik_\lambda}{h}\boldsymbol{N}'_{-nt}(-h,-\lambda), \tag{22f}$$

$$\boldsymbol{L}_{nz}(h,\lambda) = \frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda), \tag{22g}$$

$$\boldsymbol{L}'_{-nz}(-h,-\lambda) = -\frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}'_{nz}(-h,-\lambda), \tag{22h}$$

where the subscripts t and z denote the transverse vector t-components and the z-vector components, respectively, of the two functions $\boldsymbol{L}_n(h,\lambda)$ and $\boldsymbol{N}_n(h,\lambda)$. The coefficient Γ is re-written in the following form in order to perform the λ integration:

$$\Gamma = k_\lambda^4\left(-\epsilon_t\mu_t+\zeta_t\xi_t\right)\left(\lambda^2-\lambda_1^2\right)\left(\lambda^2-\lambda_2^2\right) \tag{23}$$

where

$$\lambda_1 = \sqrt{\frac{1}{2\epsilon_t\mu_t-2\zeta_t\xi_t}\left[p_\lambda-\sqrt{p_\lambda^2-q_\lambda}\right]}, \tag{24a}$$

$$\lambda_2 = \sqrt{\frac{1}{2\epsilon_t\mu_t-2\zeta_t\xi_t}\left[p_\lambda+\sqrt{p_\lambda^2-q_\lambda}\right]}, \tag{24b}$$

with the coefficients p_λ and q_λ as shown in Appendix B.

In terms of these functions, (21) can be rewritten in the form

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = &\int_0^\infty d\lambda\int_{-\infty}^\infty dh\sum_{n=-\infty}^\infty\frac{1}{4\pi^2\lambda\left(\lambda^2-\lambda_1^2\right)\left(\lambda^2-\lambda_2^2\right)}\\ &\cdot\left[\tau_1\boldsymbol{M}_n(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)+\tau_2\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)\right.\\ &+\tau_3\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)+\tau_4\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)\\ &+\tau_5\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)+\tau_6\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda\\ &+\tau_7\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)+\tau_8\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\\ &\left.+\tau_9\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\right],\end{aligned} \tag{25}$$

where the coefficients for τ_1 to τ_9 are as shown in Appendix B.

2.2. Analytical Evaluation of the λ Integral

Using a similar idea as shown in [1], the irrotational dyadic Green's function can be obtained from (7) as

$$\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\delta(\boldsymbol{r}-\boldsymbol{r}') = \int_0^\infty d\lambda\int_{-\infty}^\infty dh\sum_{n=-\infty}^\infty\frac{1}{4\pi^2\lambda}\frac{k_\lambda^2}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda). \tag{26}$$

With some algebraic manipulations, we can split (25) into

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = &-\int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda}\frac{k_\lambda^2}{\omega^2\epsilon_z\lambda^2}\cdot \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\\ &+\int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda\left(\lambda^2-\lambda_1^2\right)\left(\lambda^2-\lambda_2^2\right)}\\ &\cdot\left[\tau_1\boldsymbol{M}_n(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)+\tau_2\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)\right.\\ &+\tau_3\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)+\tau_4\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)\\ &+\tau_5\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)+\tau_6\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda\\ &+\tau_7\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)+\tau_8\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\\ &\left.+\tau_{10}\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\right]. \end{aligned} \tag{27}$$

The first integration term in (27) is due to the contribution from the irrotational vector wave functions while the second integration term is due to the contribution from the solenoidal vector wave functions and can be evaluated by making use of the residue theorem in λ-plane (Appendix A). After some mathematical manipulations, we arrived at the final unbounded dyadic Greens function for a gyrotropic medium as $\rho \gtrless \rho'$

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = &-\frac{1}{\omega^2\epsilon_z}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}\delta(\boldsymbol{r}-\boldsymbol{r}') \pm \frac{i}{4\pi}\int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty}\frac{1}{2(\lambda_1^2-\lambda_2^2)}\\ &\sum_{j=1}^{2}\frac{(-1)^{j+1}}{\lambda_j^2}\left\{\begin{array}{l}\boldsymbol{M}_n^{(1)}(h,\lambda_j)\boldsymbol{P}'_{-n}(-h,-\lambda_j)\\ \boldsymbol{M}_n(h,-\lambda_j)\boldsymbol{P}'^{(1)}_{-n}(-h,\lambda_j)\end{array}\right.\\ &+\boldsymbol{Q}_n^{(1)}(h,\lambda_j)\boldsymbol{M}'_{-n}(-h,-\lambda_j)+\boldsymbol{Q}_n(h,-\lambda_j)\boldsymbol{M}'^{(1)}_{-n}(-h,\lambda_j)\\ &+\boldsymbol{U}_n^{(1)}(h,\lambda_j)\boldsymbol{N}'_{-nt}(-h,-\lambda_j)+\boldsymbol{U}_n(h,-\lambda_j)\boldsymbol{N}'^{(1)}_{-nt}(-h,\lambda_j)\\ &\left.\begin{array}{l}+\boldsymbol{V}_n^{(1)}(h,\lambda_j)\boldsymbol{N}'_{-nz}(-h,-\lambda_j)\\ +\boldsymbol{V}_n(h,-\lambda_j)\boldsymbol{N}'^{(1)}_{-nz}(-h,\lambda_j)\end{array}\right\}, \end{aligned} \tag{28}$$

where the superscript (1) of the vector wave functions denotes the first-kind cylindrical Hankel function $H_n^{(1)}(\lambda\rho)$. The vector wave functions $\boldsymbol{P}'_{-n,-h}(-\lambda_j)$, $\boldsymbol{Q}_{n,h}(\lambda_j)$, $\boldsymbol{U}_{n,h}(\lambda_j)$ and $\boldsymbol{V}_{n,h}(\lambda_j)$ are given, respectively, by

$$\begin{aligned} \boldsymbol{P'}_{-n}(-h,-\lambda_j) &= \varphi_1 \boldsymbol{M'}_{-n}(-h,-\lambda_j) + \varphi_2 \boldsymbol{N'}_{-nt}(-h,-\lambda_j) \\ &\quad + \varphi_3 \boldsymbol{N'}_{-nz}(-h,-\lambda_j), \qquad (29a) \\ \boldsymbol{Q}_n(h,\lambda_j) &= \varphi_4 \boldsymbol{N}_{nt}(h,\lambda_j) + \varphi_5 \boldsymbol{N}_{nz}(h,\lambda_j), \qquad (29b) \\ \boldsymbol{U}_n(h,\lambda_j) &= \varphi_6 \boldsymbol{N}_{nt}(h,\lambda_j) + \varphi_7 \boldsymbol{N}_{nz}(h,\lambda_j), \qquad (29c) \\ \boldsymbol{V}_n(h,\lambda_j) &= \varphi_8 \boldsymbol{N}_{nt}(h,\lambda_j) + \varphi_9 \boldsymbol{N}_{nz}(h,\lambda_j). \qquad (29d) \end{aligned}$$

where the coefficients φ_1 to φ_9 are shown in Appendix C.

3. CONCLUSION

This paper presents a complete eigenfunction expansion of the dyadic Green's functions for an unbounded gyrotropic medium. The unbounded dyadic Green's function in the gyrotropic medium is obtained based on the Ohm-Rayleigh method, together with the vector and tensor relationships as shown in (14) which greatly simplify the formulation. It should be pointed out that the irrotational dyadic Green's function is directly formulated from the theoretical derivation instead of those obtained indirectly from assumption based on the known solutions. The results obtained here could be used to further obtain the scattering Green's dyadics for a layered cylindrical structure where the scattering superposition method is applied. The method presented here could also be used to obtain more general results for other media. Certainly, practical applications of the results presented herein can be applied to analyse electromagnetic radiation due to various antennas in the gyrotropic medium. Design of these antennas can be conducted so as to control the antenna property in the medium whose dielectric parameters can also be changed to serve purposes of practical requirements.

APPENDIX A. INTEGRATION OF λ

To this end, we write

$$\begin{aligned} \boldsymbol{M}_n(h,\lambda) &= (\boldsymbol{\nabla}_t \times \hat{\boldsymbol{z}})\Psi_n(h,\lambda), \qquad (A1a) \\ \boldsymbol{M'}_{-n}(-h,-\lambda) &= (\boldsymbol{\nabla}_t \times \hat{\boldsymbol{z}})\Psi'_{-n}(-h,-\lambda), \qquad (A1b) \\ \boldsymbol{N}_n(h,\lambda) &= \boldsymbol{N}_{nt}(h,\lambda) + \boldsymbol{N}_{nz}(h,\lambda) \\ &= (\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \hat{\boldsymbol{z}})\frac{1}{k_\lambda}\Psi_n(h,\lambda). \qquad (A1c) \end{aligned}$$

Noting that $\nabla\times\nabla\times\hat{\boldsymbol{z}} = ih\nabla - \nabla^2\hat{\boldsymbol{z}} = ih\nabla_t - \nabla_t^2\hat{\boldsymbol{z}}$ where the subscript t denotes the transverse gradient operator. Then,

$$\boldsymbol{N}_{nt}(h,\lambda) = (ih\nabla_t)\frac{1}{k_\lambda}\Psi_n(h,\lambda), \tag{A2a}$$

and

$$\boldsymbol{N}_{nz}(h,\lambda) = (-\nabla_t^2\hat{\boldsymbol{z}})\frac{1}{k_\lambda}\Psi_n(h,\lambda). \tag{A2b}$$

similarly

$$\boldsymbol{N}'_{-nt}(-h,-\lambda) = (-ih\nabla'_t)\frac{1}{k_\lambda}\Psi'_{-n}(-h,-\lambda), \tag{A3a}$$

and

$$\boldsymbol{N}'_{-nz}(-h,-\lambda) = (-\nabla_t'^2\hat{\boldsymbol{z}})\frac{1}{k_\lambda}\Psi_{-n}(-h,-\lambda), \tag{A3b}$$

where $\Psi_n(h,\lambda)$ is given by (9). In actuality, the differentiations are performed before the integration. But in this case, it may be simpler to perform the $d\lambda$ integration before taking the derivative operations inside $\boldsymbol{M}_n(h,\lambda)$, $\boldsymbol{N}_{nt}(h,\lambda)$ and $\boldsymbol{N}_{nz}(h,\lambda)$. Hence after exchanging the order of $d\lambda$ integration and differentiation, typical integrals involving $\boldsymbol{M}_n(h,\lambda)$, $\boldsymbol{N}_{nt}(h,\lambda)$ and $\boldsymbol{N}_{nz}(h,\lambda)$ terms in (28) are of the form

$$I = \int_0^\infty d\lambda \frac{f(\lambda)J_n(\lambda\rho)J_{-n}(-\lambda\rho')}{\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)}. \tag{A4}$$

With

$$J_n(\lambda_\rho) = \frac{1}{2}\left[H_n^{(1)}(\lambda\rho) + H_n^{(2)}(\lambda\rho)\right],$$

we thus have

$$I = \frac{1}{2}\lim_{\delta\to 0}\left[\int_\delta^\infty d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_{-n}^{(1)}(-\lambda\rho')}{\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)} + \int_\delta^\infty d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_{-n}^{(2)}(-\lambda\rho')}{\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)}\right]. \tag{A5}$$

Here, the limit is introduced because a pole at $\lambda = 0$ now exists in each of the integrands due to the Hankel functions. Furthermore, letting $\lambda =$

$e^{-i\pi}\lambda'$ and using the reflection formulas $H_n^{(2)}(e^{-i\pi}\lambda\rho) = (-1)^n H_n^{(1)}(\lambda\rho)$ and $J_n(-\lambda\rho) = (-1)^n J_n(\lambda\rho)$, we have

$$\begin{aligned} I &= \frac{1}{2}\lim_{\delta\to 0}\left[\int_{\delta}^{\infty} d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_n^{(1)}(\lambda\rho')}{\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)}\right. \\ &\quad \left. + \int_{-\infty}^{-\delta} d\lambda' \frac{f(\lambda')J_n(\lambda'\rho)H_n^{(1)}(\lambda'\rho')}{\lambda'(\lambda'^2-\lambda_1^2)(\lambda'^2-\lambda_2^2)}\right]. \\ &= \frac{1}{2}P.V.\int_{-\infty}^{\infty} d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_n^{(1)}(\lambda\rho')}{\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)}, \end{aligned} \tag{A6}$$

where $P.V.$ represents a principal value of integral. Notice that in (A6), poles exist at $\lambda = \pm\lambda_{1,2}$ which may be on the real axis. But again with the introduction of some small loss, these poles are displaced from the real axis. Moreover, a residue contribution can be added to (A6) at the origin to make it a complete contour integral. In other words,

$$I = \frac{1}{2}\int_C d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_n^{(1)}(\lambda\rho')}{\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)} - \frac{f(0)}{2|n|}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}\frac{1}{\lambda_1^2\lambda_2^2}, \tag{A7}$$

where the following relation has been utilized:

$$\lim_{\lambda\to 0}\left[J_n(\lambda^<\rho)H_n^{(1)}(\lambda^>\rho)\right] = -\frac{i}{|n|\pi}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}.$$

In (A7), the last term is resultant from the residue contribution which has been included in the first term to make C a continuous contour.

Thus, we have for $\rho > \rho'$ the following formula:

$$I = \pi i\sum_{j=1}^{2}\frac{(-1)^{j+1}f(\lambda_j)J_n(\lambda_j\rho')H_n^{(1)}(\lambda_j\rho)}{2\lambda_j^2(\lambda_1^2-\lambda_2^2)}. \tag{A8}$$

A similar operation on $\rho < \rho'$ will result in:

$$I = -\pi i\sum_{j=1}^{2}(-1)^{j+1}\frac{f(\lambda_j)J_n(-\lambda_j\rho)H_n^{(1)}(-\lambda_j\rho')}{2\lambda_j^2(\lambda_1^2-\lambda_2^2)}, \tag{A9}$$

since $f(\lambda) = f(-\lambda)$.

The term due to the residue contribution from the origin $\lambda = 0$ in (28) tends to vanish as a consequence of

$$(\boldsymbol{\nabla}_t \times \hat{\boldsymbol{z}})(\boldsymbol{\nabla}_t' \times \hat{\boldsymbol{z}}) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = (i\hat{\boldsymbol{\rho}} - \hat{\boldsymbol{\phi}})(\hat{\boldsymbol{\phi}} - i\hat{\boldsymbol{\rho}}) \frac{n^2}{\rho^>\rho^<} \left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \tag{A10a}$$

$$(\boldsymbol{\nabla}_t \times \hat{\boldsymbol{z}})(-ih\boldsymbol{\nabla}_t') \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = ih(i\hat{\boldsymbol{\rho}} - \hat{\boldsymbol{\phi}})(\hat{\boldsymbol{\rho}} + i\hat{\boldsymbol{\phi}}) \frac{n^2}{\rho^>\rho^<} \left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \tag{A10b}$$

$$(ih\boldsymbol{\nabla}_t)(\boldsymbol{\nabla}_t' \times \hat{\boldsymbol{z}}) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = ih(\hat{\boldsymbol{\rho}} + i\hat{\boldsymbol{\phi}})(-i\hat{\boldsymbol{\rho}} + \hat{\boldsymbol{\phi}}) \frac{n^2}{\rho^>\rho^<} \left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \tag{A10c}$$

$$(\boldsymbol{\nabla}_t \times \hat{\boldsymbol{z}})(-\boldsymbol{\nabla}_t'^2 \hat{\boldsymbol{z}}) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = 0, \tag{A10d}$$

$$(-\boldsymbol{\nabla}_t^2 \hat{\boldsymbol{z}})(\boldsymbol{\nabla}_t' \times \hat{\boldsymbol{z}}) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = 0, \tag{A10e}$$

$$(ih\boldsymbol{\nabla}_t)(-ih\boldsymbol{\nabla}_t') \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = -h^2(\hat{\boldsymbol{\rho}} + i\hat{\boldsymbol{\phi}})(\hat{\boldsymbol{\rho}} + i\hat{\boldsymbol{\phi}}) \frac{n^2}{\rho^>\rho^<} \left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \tag{A10f}$$

$$(ih\boldsymbol{\nabla}_t)(-\boldsymbol{\nabla}_t'^2 \hat{\boldsymbol{z}}) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = 0, \tag{A10g}$$

$$(-\boldsymbol{\nabla}_t^2 \hat{\boldsymbol{z}})(-ih\boldsymbol{\nabla}_t) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = 0, \tag{A10h}$$

$$(-\boldsymbol{\nabla}_t^2 \hat{\boldsymbol{z}})(-\boldsymbol{\nabla}_t'^2 \hat{\boldsymbol{z}}) \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = 0, \tag{A10i}$$

where we assume that $\boldsymbol{\nabla}_t^2 \left(\frac{\rho^<}{\rho^>}\right)^{|n|} = 0$ for $\rho \neq \rho'$. This is expected on physical ground because it is an unphysical field with $\lambda = 0$ and $h \neq 0$. In fact, this field does not satisfy the dispersion relationship.

APPENDIX B. COEFFICIENTS OF INTERMEDIATE RESULTS

$$\begin{aligned} \alpha_1 \;=\; & k_\lambda^4 \Big\{ -\lambda^2 \mu_z \left(\epsilon_t \mu_t - \zeta\xi\right) + h^2 \left(-\epsilon_z \mu_t \mu_z + \zeta \mu_t \xi\right) \\ & + ih\mu_a \left(\zeta - \xi\right)\left(-\epsilon_z \mu_z + \zeta\xi\right)\omega + \left(-\epsilon_z \mu_z + \zeta\xi\right) \\ & \left[\epsilon_t \left(\mu_a - \mu_t\right)\left(\mu_a + \mu_t\right) + \zeta \mu_t \xi\right] \omega^2 \Big\}, \end{aligned} \tag{B1a}$$

$$\begin{aligned}
\beta_1 &= -k_\lambda^3 \Big\{ ih^3\mu_a\left(\epsilon_z\mu_z-\zeta\xi\right)+h^2\mu_t\left(\zeta-\xi\right)\left(-\epsilon_z\mu_z+\zeta\xi\right)\omega \\
&\quad +\lambda^2\left[\xi\left(\zeta^2\mu_t+\epsilon_a\mu_a\mu_z-\zeta\mu_z\xi\right)\right. \\
&\quad \left.+\,\epsilon_t\left(\zeta\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\mu_z\xi\right)\right]\omega \\
&\quad +ih\left[\lambda^2\mu_a\left(\epsilon_t\mu_z-\zeta\xi\right)+\left(\epsilon_z\mu_z-\zeta\xi\right)\right. \\
&\quad \left.\left(\epsilon_a\left(-\mu_a^2+\mu_t^2\right)+\zeta\mu_a\xi\right)\omega^2\right]\Big\}, \qquad \text{(B1b)}\\
\gamma_1 &= k_\lambda^4\Big\{-h\zeta\left(h^2+\lambda^2\right)\left(\mu_t-\mu_z\right)-i\left[\zeta^2\left(h^2+\lambda^2\right)\mu_a\right. \\
&\quad \left.+\left(h^2\left(\epsilon_t-\epsilon_z\right)\mu_a+\epsilon_a\left(h^2+\lambda^2\right)\mu_t\right)\mu_z\right]\omega \\
&\quad -h\left[\epsilon_z\mu_t\mu_z\left(\zeta-\xi\right)+\xi\left(\epsilon_a\mu_a\mu_z+\zeta\left(\mu_t-\mu_z\right)\xi\right)\right. \\
&\quad \left.+\,\epsilon_t\left(\zeta\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\mu_z\xi\right)\right]\omega^2 \\
&\quad +i\left(\epsilon_z\mu_z-\zeta\xi\right)\left[\epsilon_a\left(-\mu_a^2+\mu_t^2\right)+\zeta\mu_a\xi\right]\omega^3\Big\}, \qquad \text{(B1c)}\\
\alpha_2 &= -ik_\lambda^3\Big\{ih^3\mu_a\left(\epsilon_z\mu_z-\zeta\xi\right)+h^2\mu_t\left(\zeta-\xi\right)\left(-\epsilon_z\mu_z+\zeta\xi\right)\omega \\
&\quad -\lambda^2\left[\epsilon_a\zeta\mu_a\mu_z+\zeta\xi\left(-\zeta\mu_z+\mu_t\xi\right)\right. \\
&\quad \left.+\epsilon_t\left(\zeta\mu_t\mu_z+\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\xi\right)\right]\omega \\
&\quad +ih\left[\lambda^2\mu_a\left(\epsilon_t\mu_z-\zeta\xi\right)+\left(\epsilon_z\mu_z-\zeta\xi\right)\right. \\
&\quad \left.\left(\epsilon_a\left(-\mu_a^2+\mu_t^2\right)+\zeta\mu_a\xi\right)\omega^2\right]\Big\}, \qquad \text{(B1d)}\\
\beta_2 &= k_\lambda^2\Big\{h^4\mu_t\left(\epsilon_z\mu_z-\zeta\xi\right)-\lambda^4\left(\epsilon_t\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_t\xi\right) \\
&\quad +ih^3\mu_a\left(\zeta-\xi\right)\left(\epsilon_z\mu_z-\zeta\xi\right)\omega+ih\lambda^2\left(\zeta-\xi\right) \\
&\quad \cdot\left[\epsilon_a\left(\mu_a^2+\mu_t\left(-\mu_t+\mu_z\right)\right)+\mu_a\left(\epsilon_t\mu_z-\zeta\xi\right)\right]\omega \\
&\quad +\lambda^2\mu_z\left[\left(\epsilon_a+\epsilon_t\right)\left(\mu_a+\mu_t\right)-\zeta\xi\right]\left[\left(-\epsilon_a+\epsilon_t\right)\left(\mu_a-\mu_t\right)+\zeta\xi\right]\omega^2 \\
&\quad +h^2\left[\lambda^2\left(-\epsilon_z\mu_a^2+\epsilon_z\mu_t^2+\epsilon_t\mu_t\mu_z-2\zeta\mu_t\xi\right)\right. \\
&\quad \left.+\left(\epsilon_z\mu_z-\zeta\xi\right)\left(\epsilon_t\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_t\xi\right)\omega^2\right]\Big\}, \qquad \text{(B1e)}\\
\gamma_2 &= ik_\lambda^3\Big\{i\zeta\left(h^2+\lambda^2\right)^2\mu_a+h\left[\epsilon_t\lambda^2\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\right. \\
&\quad -h^2\epsilon_t\mu_t\mu_z-k_\lambda^2\left(\zeta^2\left(\mu_t-\mu_z\right)+\epsilon_a\mu_a\mu_z\right) \\
&\quad \left.+\epsilon_z\left(\lambda^2\left(-\mu_a^2+\mu_t^2\right)+h^2\mu_t\mu_z\right)\right]\omega-i\left[\lambda^2\left[\zeta\left(\epsilon_t\mu_a\right.\right.\right. \\
&\quad \left.\left.+\epsilon_a\mu_t\right)\mu_z+\epsilon_a\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\xi-\zeta\mu_a\xi^2\right]
\end{aligned}$$

$$
\begin{aligned}
&+h^2\left[\epsilon_a\zeta\left(\mu_a^2-\mu_t\left(\mu_t-2\mu_z\right)\right)-\epsilon_a\mu_t\mu_z\xi\right.\\
&\left.\left.+\mu_a\left(-\epsilon_z\zeta\mu_z+\epsilon_t\mu_z\left(2\zeta-\xi\right)+\epsilon_z\mu_z\xi-\zeta\xi^2\right)\right]\right]\omega^2\\
&+h\left[\left(\epsilon_a^2+\epsilon_t\left(-\epsilon_t+\epsilon_z\right)\right)\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\mu_z\right.\\
&-\zeta\left[\epsilon_t\left(\mu_a^2-\mu_t\left(\mu_t-2\mu_z\right)\right)+\left(2\epsilon_a\mu_a-\epsilon_z\mu_t\right)\mu_z\right]\xi\\
&\left.\left.+\ \zeta^2\left(-\mu_t+\mu_z\right)\xi^2\right]\omega^3\right\}, \qquad \text{(B1f)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_3 \;=\;& k_\lambda^2\lambda^2\left\{hk_\lambda^2\left(-\mu_t+\mu_z\right)\xi+i\left[\left(h^2\left(\epsilon_t-\epsilon_z\right)\mu_a\right.\right.\right.\\
&\left.\left.+\epsilon_a k_\lambda^2\mu_t\right)\mu_z+k_\lambda^2\mu_a\xi^2\right]\omega-h\left[\epsilon_a\zeta\mu_a\mu_z\right.\\
&+\left(\epsilon_t-\epsilon_z\right)\zeta\mu_t\mu_z+\left[\epsilon_t\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\right.\\
&\left.\left.+\ \zeta^2\left(\mu_t-\mu_z\right)+\epsilon_z\mu_t\mu_z\right]\xi\right]\omega^2-i\left(\epsilon_z\mu_z\right.\\
&\left.\left.-\zeta\xi\right)\left[\epsilon_a\left(-\mu_a^2+\mu_t^2\right)+\zeta\mu_a\xi\right]\omega^3\right\}, \qquad \text{(B1g)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_3 \;=\;& ik_\lambda\lambda^2\left\{-ik_\lambda^4\mu_a\xi-h\left[\epsilon_z\lambda^2\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\right.\right.\\
&-h^2\epsilon_z\mu_t\mu_z+\epsilon_t\left(\lambda^2\left(-\mu_a^2+\mu_t^2\right)+h^2\mu_t\mu_z\right)\\
&\left.+k_\lambda^2\left(\epsilon_a\mu_a\mu_z+\mu_t\xi^2-\mu_z\xi^2\right)\right]\omega+i\left[\epsilon_a\zeta\lambda^2\left(\mu_a\right.\right.\\
&-h^2\zeta\left(\epsilon_t\mu_a-\epsilon_z\mu_a+\epsilon_a\mu_t\right)\mu_z\left.-\mu_t\right)\left(\mu_a+\mu_t\right)\\
&-\left[\lambda^2\left(\zeta^2\mu_a-\left(\epsilon_t\mu_a+\epsilon_a\mu_t\right)\mu_z\right)+h^2\left(\zeta^2\mu_a+\left(-2\epsilon_t+\epsilon_z\right)\mu_a\mu_z\right.\right.\\
&\left.\left.\left.+\epsilon_a\left(-\mu_a^2+\mu_t^2-2\mu_t\mu_z\right)\right)\right]\xi\right]\omega^2+h\left[\left(\epsilon_a^2+\epsilon_t\left(-\epsilon_t+\epsilon_z\right)\right)\right.\\
&(\mu_a-\mu_t)(\mu_a+\mu_t)\mu_z\ -\zeta\left[\epsilon_t\left(\mu_a^2-\mu_t\left(\mu_t-2\mu_z\right)\right)\right.\\
&\left.\left.\left.+\left(2\epsilon_a\mu_a-\epsilon_z\mu_t\right)\mu_z\right]\xi+\zeta^2\left(-\mu_t+\mu_z\right)\xi^2\right]\omega^3\right\}, \qquad \text{(B1h)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_3 \;=\;& k_\lambda^2\left\{k_\lambda^4\left(\lambda^2\mu_t+h^2\mu_z\right)+\left[\ h^4\mu_z\left(\ \zeta^2-2\epsilon_a\mu_a-2\epsilon_t\mu_t\right.\right.\right.\\
&\left.+\xi^2\ \right)+h^2\lambda^2\left(\left(-2\epsilon_a\mu_a-\epsilon_z\mu_t\right)\mu_z\ +\zeta^2\left(\mu_t+\mu_z\right)\right.\\
&\left.+\epsilon_t\left(\mu_a^2-\mu_t\left(\mu_t+2\mu_z\right)\right)+\left(\mu_t+\mu_z\right)\xi^2\ \right)\\
&\left.+\lambda^4\left[\ \epsilon_z\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\left(\zeta^2-\epsilon_t\mu_z\ \ +\xi^2\right)\right]\right]\omega^2\\
&+ih\left[\epsilon_a\lambda^2\ \left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)-\left(2h^2\left(\epsilon_t\mu_a+\epsilon_a\mu_t\right)\right.\right.\\
&\left.+\lambda^2\left(\left(\epsilon_t+\epsilon_z\right)\mu_a\ +\epsilon_a\mu_t\right)\right)\mu_z\left.\right]\left(\zeta-\xi\right)\omega^3\\
&+[h^2\mu_z((\epsilon_a-\epsilon_t)(\mu_a-\mu_t)-\zeta\xi)
\end{aligned}
$$

$$
\begin{aligned}
& ((\epsilon_a+\epsilon_t)(\mu_a+\mu_t)-\zeta\xi)-\lambda^2(\epsilon_z\mu_z \\
& -\zeta\xi)(\epsilon_t(\mu_a-\mu_t)(\mu_a+\mu_t)+\zeta\mu_t\xi)]\omega^4\}, \qquad \text{(B1i)}\\
p_\lambda \;=\; & -h^2\left(\epsilon_z\mu_t+\epsilon_t\mu_z-2\zeta\xi\right)+ih\left[\epsilon_t\mu_a-\epsilon_z\mu_a\right.\\
& \left.+\epsilon_a\left(\mu_t-\mu_z\right)\right]\left(\zeta-\xi\right)\omega-\{\;\epsilon_a^2\mu_t\mu_z-\epsilon_t^2\mu_t\mu_z\\
& +\epsilon_a\mu_a\left(\zeta^2+\xi^2\right)-\zeta\xi\left(\zeta^2-\epsilon_z\mu_t+\xi^2\right)+\epsilon_t\left[\;\zeta^2\mu_t\right.\\
& \left.+\epsilon_z\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_z\xi+\mu_t\xi^2\;\right]\}\,\omega^2, \qquad \text{(B1j)}\\
q_\lambda \;=\; & 4\left(\epsilon_t\mu_t-\zeta\xi\right)\left(\epsilon_z\mu_z-\zeta\xi\right)\{\;h^4+h^2\left(\;\zeta^2-2\epsilon_a\mu_a\right.\\
& \left.-2\epsilon_t\mu_t+\xi^2\;\right)\omega^2-2ih\left(\epsilon_t\mu_a+\epsilon_a\mu_t\right)\left(\zeta-\xi\right)\omega^3\\
& +\left[\left(\epsilon_a-\epsilon_t\right)\left(\mu_a-\mu_t\right)-\zeta\xi\right]\left[\left(\epsilon_a+\epsilon_t\right)\left(\mu_a+\mu_t\right)-\zeta\xi\right]\omega^4\}\,, \qquad \text{(B1k)}\\
\tau_1 \;=\; & \frac{1}{\epsilon_t\mu_t-\zeta\xi}\,\{\;\lambda^2\mu_z\left(\epsilon_t\mu_t-\zeta\xi\right)+h^2\mu_t\left(\epsilon_z\mu_z-\zeta\xi\right)\\
& +ih\mu_a\left(\zeta-\xi\right)\left(\epsilon_z\mu_z-\zeta\xi\right)\omega+\left(\epsilon_z\mu_z-\zeta\xi\right)\left[\epsilon_t\left(\mu_a\right.\right.\\
& \left.\left.-\mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_t\xi\;\right]\omega^2\;\}\,, \qquad \text{(B1l)}\\
\tau_2 \;=\; & -\frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{k_\lambda\Big[\lambda^2\left(\zeta^2\mu_a+\epsilon_a\mu_t\mu_z\right)\omega\\
& +h^2\mu_a\left(-\epsilon_z\mu_z+\zeta\xi\right)\omega-\left(\epsilon_z\mu_z-\zeta\xi\right)\left[\epsilon_a\left(-\mu_a^2+\mu_t^2\right)+\zeta\mu_a\xi\right]\omega^3\\
& -ih\left[\zeta\lambda^2\left(\mu_t-\mu_z\right)-\mu_t\left(\zeta-\xi\right)\left(-\left(\epsilon_z\mu_z\right)+\zeta\xi\right)\omega^2\right]\Big]\Big\}, \qquad \text{(B1m)}\\
\tau_3 \;=\; & \frac{1}{(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{-ik_\lambda\Big[h^2\zeta\left(\mu_t-\mu_z\right)+ih\left(\zeta^2\mu_a+\epsilon_t\mu_a\mu_z\right.\\
& \left.+\epsilon_a\mu_t\mu_z-\zeta\mu_a\xi\right)\omega+\Big[\xi\left(\zeta^2\mu_t+\epsilon_a\mu_a\mu_z-\zeta\mu_z\xi\right)\\
& +\epsilon_t\left(\zeta\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\mu_z\xi\right)\Big]\omega^2\Big]\Big\}, \qquad \text{(B1n)}\\
\tau_4 \;=\; & \frac{1}{h^2(\epsilon_t\mu_t-\zeta\xi)\omega^2}\,\Big\{k_\lambda^2\,\Big[-\lambda^4\mu_t+\left(\epsilon_z\mu_z-\zeta\xi\right)\omega^2\\
& \cdot\left[h^2\mu_t+ih\mu_a\left(\zeta-\xi\right)\omega+\left(\epsilon_t\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_t\xi\right)\omega^2\right]\\
& +\lambda^2\Big[-h^2\mu_z+ih\mu_a\left(\zeta-\xi\right)\omega-\left(\epsilon_z\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)\right.\\
& \left.+\mu_t\left(\zeta^2-\epsilon_t\mu_z+\xi^2\right)\right)\omega^2\Big]\Big]\Big\}, \qquad \text{(B1o)}\\
\tau_5 \;=\; & -\frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega}\,\Big\{k_\lambda\,\Big[h^2\mu_a\left(-\epsilon_z\mu_z+\zeta\xi\right)\omega\\
& +\lambda^2\left(\epsilon_a\mu_t\mu_z+\mu_a\xi^2\right)\omega-\left(\epsilon_z\mu_z-\zeta\xi\right)\left[\epsilon_a\left(-\mu_a^2+\mu_t^2\right)\right.\\
& \left.+\zeta\mu_a\xi\;\right]\omega^3+ih\Big[\lambda^2\left(\mu_t-\mu_z\right)\xi+\mu_t\left(\zeta\right.
\end{aligned}
$$

$$-\xi\ )\left(-\left(\epsilon_z\mu_z\right)+\zeta\xi\right)\omega^2\Big]\Big]\Big\}, \tag{B1p}$$

$$\begin{aligned}\tau_6 &= \frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_\lambda^2\Big[h\lambda^2\mu_t+h^3\mu_z+i\mu_a\left(\zeta\lambda^2+h^2\xi\right)\omega \\ &\quad +h\left[\zeta^2\mu_z-\left(\epsilon_a\mu_a+\epsilon_t\mu_t\right)\mu_z-\zeta\mu_t\xi+\mu_t\xi^2\right]\omega^2 \\ &\quad -i\left[\zeta\left(\epsilon_t\mu_a+\epsilon_a\mu_t\right)\mu_z+\epsilon_a(\mu_a-\mu_t)(\mu_a+\mu_t)\,\xi-\zeta\mu_a\xi^2\right]\omega^3\Big]\Big\}, \end{aligned}\tag{B1q}$$

$$\begin{aligned}\tau_7 &= \frac{1}{(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{k_\lambda\Big[ih^2\left(\mu_t-\mu_z\right)\xi+h\left[\epsilon_t\mu_a\mu_z\right. \\ &\quad \left.+\,\epsilon_a\mu_t\mu_z+\mu_a\xi\left(-\zeta+\xi\right)\right]\omega+i\left[\epsilon_a\zeta\mu_a\mu_z\right. \\ &\quad \left.+\,\zeta\xi\left(-\zeta\mu_z+\mu_t\xi\right)+\epsilon_t\left(\zeta\mu_t\mu_z+\mu_a^2\xi-\mu_t^2\xi\right)\right]\omega^2\Big]\Big\}, \end{aligned}\tag{B1r}$$

$$\begin{aligned}\tau_8 &= \frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_\lambda^2\Big[h\lambda^2\mu_t+h^3\mu_z-i\mu_a\left(h^2\zeta+\lambda^2\xi\right)\omega \\ &\quad +h\left[\zeta^2\mu_t-\zeta\mu_t\xi+\mu_z\left(-\epsilon_a\mu_a-\epsilon_t\mu_t+\xi^2\right)\right]\omega^2+i\Big[\mu_a\Big(-\zeta^2 \\ &\quad +\epsilon_t\mu_z\Big)\,\xi+\epsilon_a\left(\zeta\left(\mu_a\ -\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\mu_z\xi\ \ \right)\Big]\,\omega^3\Big]\Big\}, \end{aligned}\tag{B1s}$$

$$\begin{aligned}\tau_9 &= \frac{1}{\lambda^2(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_\lambda^2\Big[-h^4\mu_z-\lambda^2\left[\epsilon_t\left(\mu_a\right.\right. \\ &\quad \left.\left.-\ \mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_t\xi\right]\omega^2-\mu_z\left[\left(\epsilon_a-\epsilon_t\right)\left(\mu_a\right.\right. \\ &\quad \left.\left.-\ \mu_t\right)-\zeta\xi\right]\left[\left(\epsilon_a+\epsilon_t\right)\left(\mu_a+\mu_t\right)-\zeta\xi\right]\omega^4 \\ &\quad -ih\left(\zeta-\xi\right)\omega\left[\lambda^2\mu_a-2\left(\epsilon_t\mu_a+\epsilon_a\mu_t\right)\mu_z\omega^2\right] \\ &\quad +h^2\left[-\lambda^2\mu_t-\mu_z\left(\zeta^2-2\epsilon_a\mu_a-2\epsilon_t\mu_t+\xi^2\right)\omega^2\right]\Big]\Big\}, \end{aligned}\tag{B1t}$$

$$\begin{aligned}\tau_{10} &= \frac{1}{\epsilon_z\lambda^2(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_\lambda^2\Big[\epsilon_t\lambda^4\mu_t+h^2\epsilon_t\lambda^2\mu_z \\ &\quad -h^4\zeta\xi-2h^2\zeta\lambda^2\xi-\zeta\lambda^4\xi-ih\lambda^2\left(\epsilon_t\mu_a+\epsilon_a\left(\mu_t\right.\right. \\ &\quad \left.\left.-\mu_z\right)\right)\left(\zeta-\xi\right)\omega+\Big[\lambda^2\Big(\zeta^2\left(\epsilon_a\mu_a+\epsilon_t\mu_t\right) \\ &\quad +\left(\epsilon_a-\epsilon_t\right)\left(\epsilon_a+\epsilon_t\right)\mu_t\mu_z\Big)-\zeta\Big[h^2\Big[\zeta^2-2\left(\epsilon_a\mu_a\right. \\ &\quad \left.+\epsilon_t\mu_t\right)\Big)+\lambda^2\left(\zeta^2-\epsilon_t\mu_z\right)\Big]\,\xi+\lambda^2\left(\epsilon_a\mu_a+\epsilon_t\mu_t\right)\xi^2 \\ &\quad -\zeta k_\lambda^2\xi^3\Big]\,\omega^2+2ih\zeta\left(\epsilon_t\mu_a+\epsilon_a\mu_t\right)\left(\zeta-\xi\right)\xi\omega^3 \\ &\quad -\zeta\xi\left(\left(\epsilon_a-\epsilon_t\right)\left(\mu_a-\mu_t\right)-\zeta\xi\right)\left(\left(\epsilon_a+\epsilon_t\right)\left(\mu_a+\mu_t\right)-\zeta\xi\right)\omega^4\Big]\Big\}. \end{aligned}\tag{B1u}$$

APPENDIX C. COEFFICIENTS FOR THE DGF

$$\begin{aligned}
\varphi_1 &= \frac{1}{\epsilon_t\mu_t-\zeta\xi}\Big\{\lambda_j^2\mu_z\left(\epsilon_t\mu_t-\zeta\xi\right)+h^2\mu_t\left(\epsilon_z\mu_z-\zeta\xi\right)\\
&\quad +ih\mu_a\left(\zeta-\xi\right)\left(\epsilon_z\mu_z-\zeta\xi\right)\omega+\left(\epsilon_z\mu_z-\zeta\xi\right)\left[\epsilon_t\left(\mu_a\right.\right.\\
&\quad \left.\left.-\,\mu_t\right)\left(\mu_a+\mu_t\right)+\zeta\mu_t\xi\,\right]\omega^2\Big\}, && \text{(C1a)}\\
\varphi_2 &= -\frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{k_{\lambda_j}\Big[\lambda_j^2\left(\zeta^2\mu_a+\epsilon_a\mu_t\mu_z\right)\omega\\
&\quad +h^2\mu_a\left(-\epsilon_z\mu_z+\zeta\xi\right)\omega-\left(\epsilon_z\mu_z-\zeta\xi\right)\Big[\epsilon_a\Big(-\mu_a^2\\
&\quad +\,\mu_t^2\Big)+\zeta\mu_a\xi\Big]\omega^3-ih\Big[\zeta\lambda_j^2\left(\mu_t-\mu_z\right)\\
&\quad -\mu_t\left(\zeta-\xi\right)\left(-\left(\epsilon_z\mu_z\right)+\zeta\xi\right)\omega^2\Big]\Big]\Big\}, && \text{(C1b)}\\
\varphi_3 &= \frac{1}{(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{-ik_{\lambda_j}\big[\,h^2\zeta\left(\mu_t-\mu_z\right)+ih\Big(\zeta^2\mu_a\\
&\quad +\,\epsilon_t\mu_a\mu_z+\epsilon_a\mu_t\mu_z-\zeta\mu_a\xi\Big)\omega+\Big[\xi\Big(\zeta^2\mu_t+\epsilon_a\mu_a\mu_z\\
&\quad -\zeta\mu_z\xi\,\Big)+\epsilon_t\left(\zeta\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\mu_z\xi\,\right)\big]\omega^2\Big]\Big\}, && \text{(C1c)}\\
\varphi_4 &= -\frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{k_{\lambda_j}\Big[h^2\mu_a\left(-\epsilon_z\mu_z+\zeta\xi\right)\omega\\
&\quad +\lambda_j^2\left(\epsilon_a\mu_t\mu_z+\mu_a\xi^2\right)\omega-\left(\epsilon_z\mu_z-\zeta\xi\right)\Big[\epsilon_a\Big(-\mu_a^2\\
&\quad +\,\mu_t^2\Big)+\zeta\mu_a\xi\Big]\omega^3+ih\Big[\lambda_j^2\left(\mu_t-\mu_z\right)\xi+\mu_t\left(\zeta\right.\\
&\quad \left.-\xi\,\right)\left(-\left(\epsilon_z\mu_z\right)+\zeta\xi\right)\omega^2\Big]\Big]\Big\}, && \text{(C1d)}\\
\varphi_5 &= \frac{1}{(\epsilon_t\mu_t-\zeta\xi)\omega}\Big\{k_{\lambda_j}\Big[ih^2\left(\mu_t-\mu_z\right)\xi+h\left[\epsilon_t\mu_a\mu_z\right.\\
&\quad \left.+\epsilon_a\mu_t\mu_z+\mu_a\xi\left(-\zeta+\xi\right)\right]\omega+i\Big[\epsilon_a\zeta\mu_a\mu_z\\
&\quad +\,\zeta\xi\left(-\zeta\mu_z+\mu_t\xi\right)+\epsilon_t\Big(\zeta\mu_t\mu_z+\mu_a^2\xi-\mu_t^2\xi\Big)\Big]\omega^2\Big]\Big\}, && \text{(C1e)}\\
\varphi_6 &= \frac{1}{h^2(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_{\lambda_j}^2\Big[-\lambda_j^4\mu_t+\left(\epsilon_z\mu_z\right.\\
&\quad \left.-\,\zeta\xi\right)\omega^2\Big[h^2\mu_t+ih\mu_a\left(\zeta-\xi\right)\omega+\left(\epsilon_t\left(\mu_a-\mu_t\right)\left(\mu_a\right.\right.\\
&\quad \left.\left.+\mu_t\,\right)+\zeta\mu_t\xi\,\right)\omega^2\Big]+\lambda_j^2\Big[-h^2\mu_z+ih\mu_a\left(\zeta-\xi\right)\omega\\
&\quad -\Big(\epsilon_z\left(\mu_a-\mu_t\right)\left(\mu_a+\mu_t\right)+\mu_t\Big(\zeta^2-\epsilon_t\mu_z+\xi^2\Big)\Big)\omega^2\Big]\Big]\Big\}, && \text{(C1f)}
\end{aligned}$$

$$\begin{aligned}\varphi_7 &= \frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_{\lambda_j}^2\Big[h\lambda_j^2\mu_t+h^3\mu_z+i\mu_a\Big(\zeta\lambda_j^2\\&\quad+h^2\xi\Big)\omega+h\Big[\zeta^2\mu_z-(\epsilon_a\mu_a+\epsilon_t\mu_t)\mu_z-\zeta\mu_t\xi\\&\quad+\mu_t\xi^2\Big]\omega^2-i\Big[\zeta(\epsilon_t\mu_a+\epsilon_a\mu_t)\mu_z+\epsilon_a(\mu_a\\&\quad-\mu_t)(\mu_a+\mu_t)\xi-\zeta\mu_a\xi^2\Big]\omega^3\Big]\Big\},\end{aligned}\tag{C1g}$$

$$\begin{aligned}\varphi_8 &= \frac{1}{h(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_{\lambda_j}^2\Big[h\lambda_j^2\mu_t+h^3\mu_z-i\mu_a\Big(h^2\zeta\\&\quad+\lambda_j^2\xi\Big)\omega+h\Big[\zeta^2\mu_t-\zeta\mu_t\xi+\mu_z(-\epsilon_a\mu_a-\epsilon_t\mu_t\\&\quad+\xi^2)\Big]\omega^2+i\Big[\mu_a\Big(-\zeta^2+\epsilon_t\mu_z\Big)\xi+\epsilon_a(\zeta(\mu_a\\&\quad-\mu_t)(\mu_a+\mu_t)+\mu_t\mu_z\xi)\Big]\omega^3\Big]\Big\},\end{aligned}\tag{C1h}$$

$$\begin{aligned}\varphi_9 &= \frac{1}{\epsilon_z\lambda_j^2(\epsilon_t\mu_t-\zeta\xi)\omega^2}\Big\{k_{\lambda_j}^2\Big[\epsilon_t\lambda_j^4\mu_t+h^2\epsilon_t\lambda_j^2\mu_z\\&\quad-h^4\zeta\xi-2h^2\zeta\lambda_j^2\xi-\zeta\lambda_j^4\xi-ih\lambda_j^2(\epsilon_t\mu_a+\epsilon_a(\mu_t\\&\quad-\mu_z))(\zeta-\xi)\omega+\Big[\lambda_j^2\Big(\zeta^2(\epsilon_a\mu_a+\epsilon_t\mu_t)\\&\quad+(\epsilon_a-\epsilon_t)(\epsilon_a+\epsilon_t)\mu_t\mu_z)-\zeta\Big[h^2\Big[\zeta^2-2(\epsilon_a\mu_a\\&\quad+\epsilon_t\mu_t))+\lambda_j^2\Big(\zeta^2-\epsilon_t\mu_z\Big)\Big]\xi+\lambda_j^2(\epsilon_a\mu_a+\epsilon_t\mu_t)\xi^2\\&\quad-\zeta k_{\lambda_j}^2\xi^3\Big]\omega^2+2ih\zeta(\epsilon_t\mu_a+\epsilon_a\mu_t)(\zeta-\xi)\xi\omega^3\\&\quad-\zeta\xi((\epsilon_a-\epsilon_t)(\mu_a-\mu_t)-\zeta\xi)((\epsilon_a+\epsilon_t)(\mu_a+\mu_t)-\zeta\xi)\omega^4\Big]\Big\}.\end{aligned}\tag{C1i}$$

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