Progress In Electromagnetics Research, PIER 43, 257–271, 2003

AN INVESTIGATION OF MAGNETIC ANTENNAS FOR GROUND PENETRATING RADAR

P. T. Bellett and C. J. Leat

Cooperative Research Centre for Sensor Signal
and Information Processing (CSSIP)
University of Queensland
St Lucia, Brisbane, QLD 4072, Australia

Abstract—For ground penetrating radar (GPR), smaller antennas would provide considerable practical advantages. Some of which are: portability; ease of use; and higher spatial sampling. A theoretical comparison of the fundamental limits of a small electric field antenna and a small magnetic field antenna shows that the minimum Q constraints are identical. Furthermore, it is shown that only the small magnetic loop antenna can be constructed to approach, arbitrarily closely, the fundamental minimum Q limit. This is achieved with the addition of a high permeability material which reduces energy stored in the magnetic fields. This is of special interest to some GPR applications. For example, applications requiring synthetic aperture data collection would benefit from the increased spatial sampling offered by electrically smaller antennas. Low frequency applications may also benefit, in terms of reduced antenna dimensions, by the use of electrically small antennas. Under these circumstances, a magnetic type antenna should be considered in preference to the typical electric field antenna. Numerical modeling data supports this assertion.

1. INTRODUCTION

Given the dominance in GPR antenna design of the requirement for wide bandwidth, it may seem a peculiar choice to consider the loop antenna. This is especially true considering the fact that the penalty incurred in the radiation resistance of an electrically small magnetic loop antenna is more severe than the small electric dipole antenna. The obvious objection is that the fractional bandwidth,

$$B_{frac} \approx \frac{1}{Q} = \frac{R_{rad}}{X}, \tag{1}$$

is much smaller for a typical single turn loop antenna than a typical small electric dipole of length equal to the loop's diameter. The reason for this may be understood by considering the destructive effect upon radiation of the oppositely directed currents on either side of the loop. However, it has been established by Chu [1] that this apparent limitation is not fundamental to magnetic dipole antennas. Indeed, magnetic dipole antennas may be constructed which approach the theoretical best performance more closely than electric dipoles [2]. These matters are considered more closely in Section 4.

The idea of using magnetic loop antennas for GPR is not new. However, very little published record is available in the literature. From the small number of available publications, the loop is typically used only as a receive probe. Yarovoy et al. [3] claims an advantage in making a more compact GPR antenna by using a loop as a receive antenna to obtain a higher spatial sampling by receiving the fields at a local point. Compared with a linear dipole of similar aperture, their loop antenna is reported to have a very small amount of ringing. Yarovoy et al. use the loop below a dielectrically loaded horn antenna with a cross-polarized orientation so the loop is insensitive to the transmitted fields. Chubinsky [4] and de Jongh [5] report the use of a loop probe to assess the performance of GPR antennas. The magnetic loop probe is placed underneath the ground and used to measure the radiation pattern in the lower half-space. Also of special interest, in the context of GPR, is the array of shielded loop antennas used by Sato et al. [6] for a directional borehole radar receive antenna. Each of the loop antennas are fed against a thick conducting cylindrical sleeve surrounding the borehole radar probe. The currents induced on the cylindrical conducting surface of the borehole probe by incoming electromagnetic waves are measured by the small broadband loop probes. In a later publication [7], two different current probe orientations were used to separately measure two components of the magnetic field on the conducting cylinder (i.e., H_z and H_ϕ).

Further support for the use of loop antennas in GPR comes indirectly from complementary antenna structures, such as the complementary bowtie [8] and the slot antenna. As an example, a magnetic dipole slot antenna is used by Sato [9] to provide a cross-polarized receive antenna for borehole radar. A slot antenna is also considered by Druchinin [10] for GPR that has an optimized "front to back" radiation pattern ratio by the use of a resistive material. Both these antenna types can be conceptualized as two adjacent current loops.

In addition, we believe that some benefits may be offered due to the predominantly magnetic near fields of the magnetic antenna being insensitive to changes in permittivity. Such changes may frequently happen in ground-contact GPR as the antenna feed-point traverses ground of varying nature [11] and at varying effective heights.

The structure of this paper will be to expand the argument for GPR loop antennas, drawing upon the literature and analytical methods, and then to examine critically the conclusions using the results from a number of numerical models. Firstly, readers are reminded of the use of quality factor and its relationship to small antenna bandwidth.

2. A MEASURE FOR ANTENNA BANDWIDTH

The *quality factor* (Q) is a dimensionless quantity used as a measure of the bandwidth or frequency selectivity of a resonant circuit and is defined as [12],

$$Q = 2\pi \frac{\text{Time-Avg. Energy Stored at a resonant frequency}}{\text{Energy dissipated in one period of this frequency}}. \tag{2}$$

It can readily be shown that the relative bandwidth is inversely related to Q. The Q-factor is a general concept and is not restricted to resonant electrical circuits. However, for an electric circuit with complex impedance $Z = R + iX = |Z|e^{i\delta}$ tuned to resonance with an equal amount of opposite reactance (i.e., $-X$), the Q of the circuit may be approximated as,

$$Q \approx \frac{X}{R} = \tan\delta. \tag{3}$$

Where X is the reactive component and R is the resistive component of the complex impedance Z. This is a reasonable approximation where one type of energy storage is predominant, such as magnetic energy in the case of the small loop antenna.

3. FUNDAMENTAL LIMITS OF SMALL ANTENNAS APPLY EQUALLY TO MAGNETIC AND ELECTRIC DIPOLES

The problem of an arbitrary antenna geometry supporting an arbitrary but axi-symmetric current distribution is addressed in an elegant mathematical formulation by Chu [1]. Chu's approach concealed all the physical details of the antenna inside the smallest possible enclosing sphere and considered only the fields outside the sphere. In showing the physical limitations of omni-directional antennas, Chu represents the fields outside the sphere as a complete set of orthogonal, spherical waves, propagating radically outward. For a vertically polarized electric dipole, omni-directional antenna, only TM_{m0} circularly symmetric, Hankel-Legendre modes are required. For each of the modes, the apparent impedance implied by the ratio of the electric (E_θ) and magnetic (H_ϕ) fields is constant over the surface of the sphere [13]. Thus:

$$Z = \frac{E_\theta}{H_\phi} = |Z|e^{i\delta} \tag{4}$$

To a good approximation, the Q for each spherical wave mode may be evaluated in accordance with the resonant circuit analogy of Equation (3), the ratio of stored verses radiated energy. Because of the orthogonal nature of the spherical wave modes, the Q is formed from the independent contribution of all the modes.

To deal with the fact that the fields outside the antenna sphere do not uniquely define a current distribution, Chu assumes that an optimal antenna exists with an appropriate source distribution inside the sphere. The antenna (or its matching circuit) provides adequate energy storage inside itself to tune the reactive energy storage of the space outside the antenna sphere. It is also assumed that the only loss mechanism is associated with the radiation resistance of the antenna; implying a radiation efficiency of 100%.

The fields of the magnetic dipole correspond to Chu's TE modes, for which the energy storage and radiation and, hence, Q contributions, are the same as the TM modes of the electric dipole. Thus, for the purposes of this paper, the interesting result of Chu's work is the realization, which is entirely sensible in terms of complementarity, that the minimum Q of a magnetic antenna, corresponding to his TE modes, is the same as that of an equivalently sized electric antenna, corresponding to his TM modes.

4. MAGNETIC ANTENNA ALONE ABLE TO APPROACH CHU LIMIT

In Section 3, it was indicated from Chu's analysis, based on the fields external to the containing volume, that the minimum possible Q for magnetic and electric dipoles is the same. This needs to be reconciled with the earlier statement regarding the generally high Q of loop antennas resulting from a low radiation resistance. Now consider the argument that a magnetic antenna filling the Chu sphere may be constructed to add no extra energy storage inside the sphere using available materials, but that the equivalent cannot be said for the electric antenna.

The fundamental limitation on bandwidth and performance of small antennas was first introduced by Wheeler [14] in 1947 in terms of a "radiation power factor," described as proportional to the "effective volume" of a small antenna. Wheeler describes a small antenna as equivalent to either a capacitor or an inductor, consequently requiring an additional reactor of the opposite kind to tune to a resonance. This is in line with the lumped circuit model, inspired by the theoretical argument of dominant fields, from the analytic expressions for an electric and magnetic dipole. The "power factor" Wheeler speaks of is now more preferably referred to in terms of the radiation Q of an antenna. The "power factor" can be interpreted as the fractional bandwidth ($B_{frac} = \Delta f / f_0$) or the inverse of the antenna Q.

Before continuing, it is important to re-examine the important difference between the two types of small radiators. The radiation from an electric dipole is a direct consequence of the completion of the current path along the antenna conductor in the capacitance of the space surrounding the antenna. Maxwell explains this with the addition of a *displacement current density* term to Ampere's Law. This is distinct from the magnetic dipole that radiates because of the completion of a magnetic flux path in the space surrounding the antenna. The continuous magnetic flux paths that must necessarily exist around the current path of the equivalent small conducting loop antenna helps to visualize this. By thinking of the electric dipole as a parallel plate capacitor and the magnetic dipole as an inductor, it is easier to visualize that the flux tubes inside and outside the capacitor are in the same direction; thus, in effect, in parallel. However, they are antiparallel and thus, in series, for the inductor. Wheeler [2] states that, "decreasing the effective length of the internal flux path by inserting some material has the effect of increasing the stored energy in the capacitor but decreasing it in the inductor." This can be explained in terms of the boundary conditions imposed by Maxwell's equations. In the case of the capacitor, the tangential component of the electric field must be the same on either side of the boundary. If a higher permittivity ($\varepsilon_r > 1$) material is placed inside the capacitor, the energy density inside will increase according to Equation (6). The energy stored in the magnetic field of an inductor and the electric field of a capacitor are given by Cheng [12] as,

$$\tilde{W}_m = \iiint_{\nu} \frac{1}{2}\mu|\mathbf{H}|^2 d\nu = \text{Time-Average Magnetic Energy } (J) \tag{5}$$

and

$$\tilde{W}_e = \iiint_{\nu} \frac{1}{2}\varepsilon|\mathbf{E}|^2 d\nu = \text{Time-Average Electric Energy } (J) \tag{6}$$

For the magnetic loop, or the inductor, the normal component of the magnetic flux density must be equal across the boundary; imposed by the solenoid nature of $\mathbf{B}$ (i.e., $\nabla \cdot \mathbf{B} = 0$) . Consequently, Equation (5) needs to be re-written in terms of the magnetic flux density ($\mathbf{B}$) instead of the magnetic intensity vector ($\mathbf{H}$),

$$\tilde{w}_m = \frac{|\mathbf{B}|^2}{2\mu} \quad (J/m^3). \tag{7}$$

Tangential $\mathbf{H}$ is not continuous within and without the inductor, due to the surface currents imposed by the exciting loop. Equation (7) now reveals that the insertion of a material with higher permeability ($\mu_r > 1$) inside the inductor reduces the energy density within. Conversely, the effective radioactive strength of the magnetic dipole is dependent on the $\mathbf{B}$ flux emerging from the poles, which has already been shown to be unaffected due to the solenoid principle. Such a reduction in stored energy inside the radian sphere has the effect of reducing the Q of the small loop antenna. No readily available materials have an equivalent effect for the small electric field antenna. This would require a relative dielectric constant less than unity.

Wheeler [15] described a small spherical inductor which is "conceptually filled with a perfect magnetic material, so there is no stored energy inside the sphere." This provides the optimal (minimum Q) lossless antenna within a spherical volume of radius a, since any reactive fields at $r < a$ can only add to the amount of stored energy and increase the Q. It is assumed that no loss mechanism exists in the permeable material or the conducting wire, with the only loss attributed to the radiation resistance. In other words, how close the bandwidth of an electrically small antenna gets to the fundamental limit is determined by how effective the antenna geometry is in making use of the volume within the antenna sphere to avoid energy storage. Energy storage outside the antenna sphere is completely unavoidable. The minimum Q is formulated by Wheeler [2] in terms of effective volume as,

$$Q \approx \frac{1}{(ka)^3} = \frac{\text{Volume of Radiansphere}}{\text{Volume of Antenna Sphere}}, \tag{8}$$

where k is the wave number and a is the antenna sphere radius. The effect of an imperfect magnetic (i.e., $\mu_r \neq \infty$) material is predicted by Wheeler by an additional term to Equation (8),

$$Q = \frac{1}{(ka)^3}\left[1 + \frac{1}{\mu_r}\right]. \tag{9}$$

Equation (9) predicts that, provided a material can be manufactured with a large enough μ_r, the effect of using a finite permeability material produces only a marginal increase in the minimum obtainable Q. For example, $\mu_r = 50$ gives only a 2% increase in Q, which is a tolerable compromise.

We arrive at an important result; the loop antenna is capable of achieving the theoretical limit of maximum bandwidth for a small antenna, while an electric antenna apparently cannot. There have been numerous attempts to approach the ideal for electric dipoles. As is clear from above, dielectric material is counterproductive in this aim. Practical approaches use conducting material to eliminate energy storage from parts of the antenna sphere, and provide large capacitive hats to avoid electric field concentrations. An example of a small antenna designed to reduce energy storage within the antenna sphere is Goubau's [16] multi-element monopole antenna.

In the remainder of the paper we use numerical models to verify these assertions by comparing the Q of equivalently sized dielectrically loaded electric dipoles antennas with permeability loaded magnetic dipoles antennas.

5. METHOD

Agilent's High Frequency Structure Simulator (HFSS) [17] was used to produce finite element models of equivalently sized electric dipole

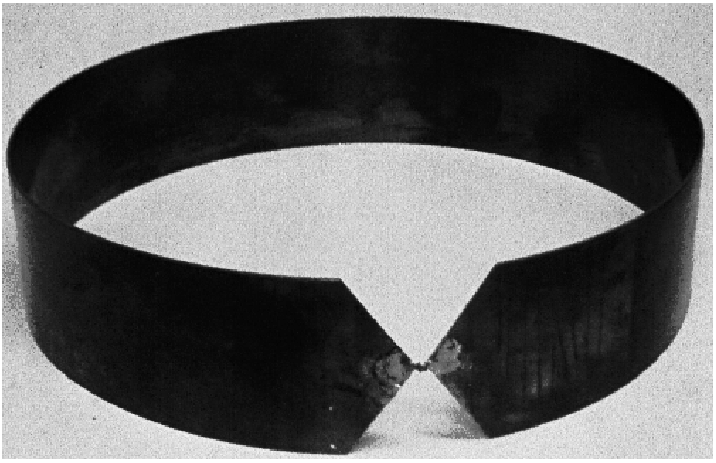

Figure 1. The experimental magnetic loop antenna used for the Q measurements above a "real ground" half-space. The low ESR capacitor used to tune the loop is shown in the foreground at the feed-point of the loop antenna.

antennas and magnetic loop antennas. The dimensions of the modeled loop antenna are taken from the experimental loop antenna shown in Figure 1. The copper structure is modeled as a perfect conductor with a height of 70 mm and an outer radius of 146 mm. The thickness of the loop conductor is 2 mm. The copper losses are very small and estimated to be an order of magnitude smaller then the radiation resistance of loop in free-space. The antenna feed-point is the tapered section shown in Figure 1. The HFSS model is shown in Figure 2. To simplify and reduce the problem space required for meshing and to help facilitate simulation convergence, the HFSS model incorporates the use of symmetry planes. Geometric planes of symmetry corresponding to the electromagnetic E and H-planes of symmetry are shown in Figure 2. The symmetric E-plane is defined by the centre of the feed-point and the loop axis. The loop is fed with a voltage source that is now placed between the E-plane and one side of the loop feed-point. It also lies along the H-plane that bisects the antenna in the plane of the loop. The results presented in this paper were obtained at 50 MHz, ensuring the electrically small criterion ($D/\lambda \approx 1/20$).

The HFSS model for the equivalently sized bi-conical electric dipole, also exploiting symmetry planes is shown in Figure 3. The electric dipole exhibits an additional geometric symmetry that

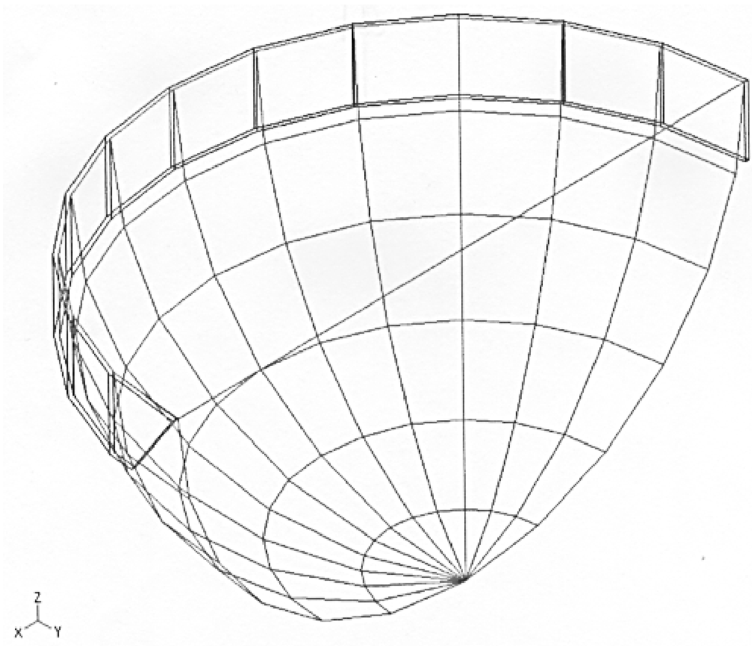

Figure 2. HFSS model of the experimental magnetic loop antenna. A symmetric H-plane and E-plane was used to simplify the geometry to help facilitate simulation convergence. The use of symmetry planes reduces the problem space to one quarter of the original size.

corresponds to an additional symmetric H-plane. The dipole element is also modeled as a perfect conductor, with the voltage source placed between the symmetric E-plane and the apex feed-point. The voltage source also lies along both the H-planes.

Both models show a spherical volume for which the material properties are adjusted to perform the appropriate simulation. These spherical volumes are equal, and therefore can provide an equivalently fair comparison relative to the fundament Wheeler-Chu limit of small antennas.

Simulation convergence was validated by checking that the power radiated through the HFSS radiation boundary balanced the power supplied by the applied voltage source. The complex antenna input impedance, obtained from the S_{11} parameter, was used to calculate the antenna Q in accordance with Equation (3).

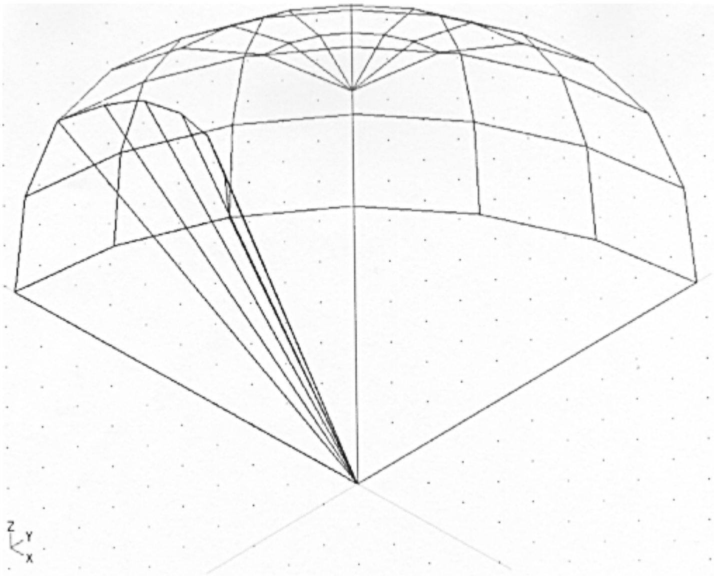

Figure 3. HFSS model for the bi-conical electric dipole. Two symmetric H-planes and one symmetric E-plane are used. The problem space is reduced to one eighth of the original size.

6. DISCUSSION

One conceivable way of reducing the Q of a loop antenna, as discussed in Section 4, is to fill the volume inside the antenna sphere around the antenna with a highly permeable material ($\mu_r > 1$). While the inductance of the loop is increased by the addition of the permeable material, the radiation resistance is increased significantly more — effectively lowering the Q and making the antenna more broadband. The inductance is proportional to the relative permeability of the sphere, $L \propto \mu_r$ and the radiation resistance is proportional to the square of the relative permeability, $R_r \propto \mu_r^2$ [18]; this implies the radiation resistance is increased by much large factor, and the Q of the antenna should consequently decrease. The simulation results, shown in Figure 4, support this argument. The Wheeler-Chu limit for minimum Q for an antenna of this size in free-space, calculated from Equation (8), is approximately 280. When the permeability of the material within the loop antenna is increased to only $\mu_r = 16$, the Q is reduced to approximately 324. This is reasonable, considering Wheeler's "perfect permeable material" or $\mu_r = \infty$ produces a Q of only 280.

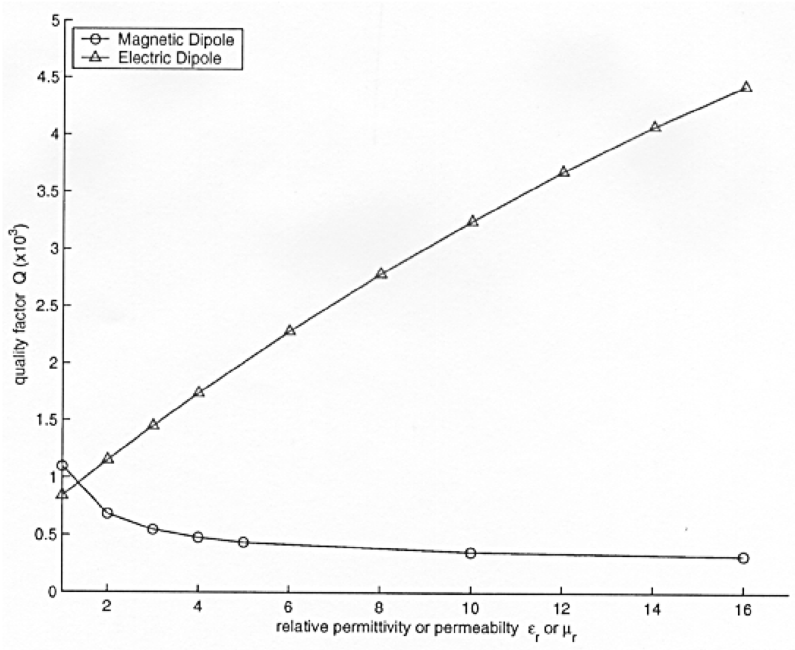

Figure 4. A modeled (HFSS) comparison of the effect of material loading on the free-space Q of a small magnetic and electric field antenna. An improvement in the Q of the small magnetic loop antenna is shown as a function of the relative permeability ($\mu_r > 1$) of the material in the antenna sphere. A similar reduction is not observable with the equivalently sized electric dipole surrounded by a higher permittivity ($\varepsilon_r > 1$) material.

It should also be noted that the modeled materials used to load the antennas in this paper have lossless properties (i.e., magnetic and electric loss-tangents are both zero) and, as a consequence, are not directly coupling additional dissipative losses to unfairly improve the antenna Q.

The simulated Q results shown in Figure 4 also provide supporting evidence that a small electric dipole antenna surrounded by a spherical volume of dielectric material with permittivity greater than air ($\varepsilon_r > 1$), adversely affects the antenna bandwidth. The addition of the dielectric in the antenna sphere does reduce the capacitive reactance of the antenna impedance, but, unfortunately, decreases the radiation resistance by a larger amount.

These loaded electric dipole results are in general supported by a recent paper by Ida [19]. However, for the cylindrical dielectric geometry chosen by Ida, a local minima in Q is reported to exist for $\varepsilon_r \approx 4$. These results may be particular to the geometry employed, and were obtained for a monopole over a ground plane with an electrical size approximately twice that of the electric dipole reported in this paper. Also, the dielectric volume was cylindrical rather than circular.

Notably the most practical approach for improving the Q of a small electric dipole, as discussed in Section 4, is to increase the amount of conducting material used to construct the elements of the dipole. For example, instead of uniformly sized thin wire, a bi-conical shaped dipole can be used. This is a useful argument for explaining why the fractional bandwidth of the small electric antenna is generally thought of as being much larger than an equivalently sized small magnetic antenna. This is consistent with the general use of bi-conical or pyramidal shaped electric dipoles in GPR. It is also generally accepted that such shapes offer greater frequency independence. However, a distinction should be made in the case of the electrically small antenna. These shapes are partially frequency independent in the sense of being mainly defined by angle, but the improved bandwidth in the case of electrically small antennas is a result of the larger surface over which charges can be spread to reduce the electric field strength.

The bi-conical electric dipole used in these simulations (Figure 3) has an inclusive angle of 60 degrees. With this increase in the amount of conductive material within the antenna sphere, the Q is seen in Figure 4 to be almost a factor of 1.5 better than the equivalent sized loop antenna (Figure 2). Additional simulation results of the bi-conical antenna with inclusive angles of 10 and 30 degrees support the concept of energy storage reduction within the antenna sphere with the use of highly conducting materials. The capacitive reactance of the dipole is seen to increase with a decrease in the inclusive angle, while not significantly affecting the radiation resistance. This results in an improved or lower Q in accordance with Equation (3), as the inclusive angle is increased.

Reducing the inner radius of the loop annulus achieves a similar reduction in the amount of reactive stored energy by increasing the amount of conducting material within the loop antenna sphere. However, unlike the electric dipole, the increase in conducting material also has a deleterious effect on the loop radiation resistance, consequently counteracting any gain in bandwidth. This is because of the very strong relationship between the loop radius and the radiation resistance ($R_{rad} \propto r^4$) [18], and the fact that the inner loop radius has been reduced.

These results show that if antennas are required to be electrically small, magnetic loop antennas can be constructed to closely achieve the fundamental bandwidth limit of small antennas. It should also be recognized that while the achieved Q is still high is terms of the bandwidth required for GPR, this value is indicative of the only loss being attributed to radiation. It is common practice to resistively load GPR antennas to compromise radiation efficiency for bandwidth. Furthermore, these results are for antennas in free-space. The proximity to a typical ground half-space may also provide additional benefit for small GPR antennas [13].

7. CONCLUSIONS

It has been shown that for GPR applications that require electrically smaller antennas, they are best constructed as loop antennas. Such magnetic antennas may achieve the lowest theoretical ringing for a given antenna size constraint. The bandwidth of a small magnetic antenna can be improved by the inclusion of a higher permeability material within the near-field surrounding the antenna. A similar improvement is not easily achievable with readily available dielectric materials, with a higher permittivity material having an adverse effect on the bandwidth of a small electric antenna.

ACKNOWLEDGMENT

The authors would like to acknowledge Glen Stickley for commenting on the manuscript.

REFERENCES

1. Chu, L. J., "Physical limitations of omni-directional antennas," *Journal of Applied Physics,* Vol. 19, 1163–1175, 1948.
2. Wheeler, H. A., "The radiansphere around a small antenna," *Proceedings of the I.R.E.,* Vol. 47, 1325–1331, 1959.
3. Yarovoy, A. G., P. V. Genderen, and L. P. Ligthart, "Ground penetrating impulse radar for landmine detection," *Eighth International Conference on Ground Penetrating Radar, GPR 2000*, 856–860, Gold Coast, Australia, May 23–26, 2000.
4. Chubinsky, N. and A. Krampuls, "Probe of pulse amplitude of electromagnetic field for investigation of GPR antenna performance," *Eighth International Conference on Ground Penetrating Radar, GPR 2000*, 75–79, Gold Coast, Queensland, Australia, May 23–29, 2000.

5. De Jongh, R. V., A. G. Yarovoy, I. V. Kaploun, and A. D. Schukin, "Design and analysis of new GPR antenna concepts," *Seventh International Conference on Ground Penetrating Radar, GPR'98*, 81–68, Lawrence, Kansas, USA, May 27–30, 1998.
6. Sato, M. and T. Tanimoto, "A shielded loop array antenna for a directional borehole radar," *Fourth International Conference on Ground Penetrating Radar, Geological Survey of Finland,* 323–327, Rovaniemi, Finland, June 8–13, 1992.
7. Ebihara, S., M. Sato, and H. Niitsuma, "Estimation of wave polarization in directional borehole radar measurements with the music algorithm," *The Third Well Logging Symposium of Japan,* 1–8, Chiba, Japan, September 25–25, 1997.
8. Chong, A. A., "Complementary GPR antennas and watertank testing," M.Eng.Sc. (Research) Thesis in Department of Computer Science and Electrical Engineering, University of Queensland, Australia, 2001.
9. Sato, M., T. Ohkubo, and H. Niitsuma, "Cross-polarization borehole radar measurements with a slot antenna," *Journal of Applied Geophysics,* Vol. 33, 53–61, 1995.
10. Druchinin, S. V., "Analysis of characteristics of a slot antenna used in georadar," *Seventh International Conference on Ground Penetrating Radar, GPR'98,* 643–648, Lawrence, Kansas, USA, May 27–30, 1998.
11. Noon, D. A. and R. M. Narayanan, "Subsurface remote sensing," *Review of Radio Science 1999–2002,* W. Ross Stone (Ed.), 535–552, IEEE Press and John Wiley & Sons, 2002.
12. Cheng, D. K., *Field and Wave Electromagnetics,* Second Edition, Addison Wesley, 1989.
13. Leat, C. J., "The fundamental limit of small antenna bandwidth: How it affects small GPR antennas," *Workshop on Applications of Radio Science, WARS'02,* Leura, Australia, February 2002.
14. Wheeler, H. A., "Fundamental limitations on small antennas," *Proceedings of the I.R.E.,* Vol. 35, 1479–1484, 1947.
15. Wheeler, H. A., "Small antennas," *IEEE Transactions on Antennas and Propagation,* Vol. AP-23, No. 4, 462–469, 1975.
16. Goubau, G., "Multi-element monopole antenna," *Proc. Workshop on Electrically Small Antennas ECOM,* 63–67, Ft Monmouth, N.J., May, 1976.
17. Agilent, Agilent High-Frequency Structure Simulator, Version 5.6, Agilent Technologies, 2000.
18. Balanis, C. A., *Antenna Theory Analysis and Design,* Second

Edition, John Wiley & Sons, 1997.

19. Ida, I., J. Sato, T. Sekizawa, H. Yoshimura, and K. Ito, "Dependence of the efficiency-bandwidth product on electrical volume of small dielectric loaded antennas," *IEEE Transactions on Antennas and Propagation,* Vol. 50, No. 6, 821–826, 2002.