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THE SURFACE IMPEDANCE OF A HOMOGENEOUS TM-TYPE PLANE WAVE AT SKEW INCIDENCE UPON AN INCLINED ANISOTROPIC HALF-SPACE

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Abstract—General expressions for the electromagnetic fields of homogeneous TM-type plane waves at a skew angle of incidence upon an inclined anisotropic half-space are derived. Previous analyses have only considered fields of homogeneous plane waves in the problems of a laterally anisotropic half-space, and not the problem of an inclined anisotropic half-space. Previous analyses also have assumed that the linear polarization of the incident magnetic field is maintained, regardless of the anisotropy present. The results presented in this paper have shown that while this assumption is valid only for the magnetic field, the electric field is elliptically polarised in the anisotropic half-space. This is demonstrated through a model study and experimental verification at VLF. The solutions obtained converge on the expected values for the special cases presented by other authors.

1. INTRODUCTION

The physical basis for the surface impedance of a homogeneous plane wave polarised parallel or perpendicular to the strike of a two-dimensional half-space is that the electric and magnetic fields generated are orthogonal. In any other situation, the electric and magnetic field vectors are not orthogonal. This can occur when the source field is scattered by a half-space that is three-dimensional and inhomogeneous, or when the source field is not polarised parallel or perpendicular to the strike of the half-space [1]. At frequencies less than 1 kHz, the source fields are elliptically polarised [2] and this can lead to some difficulties in surface impedance data reduction and interpretation [18]. At higher frequencies, the principle source field for surface impedance measurements is the radiation from lightning discharges or artificial signals from navigation beacons and radio transmitters [3]. These very low frequency (VLF) waves propagate with very little loss in the earth-ionosphere waveguide as a series of waveguide modes. Cloud-to-ground lightning discharges and VLF antennas are effectively vertical electric dipoles, and launch linearly polarised TM-type waves. At imperfect ionospheric and terrestrial boundaries, the TM-type waves can be partly reflected as TE-type waves, which propagate independently with higher attenuation than the lower order TM-type waves that dominate VLF propagation over very large distances in the earth-ionosphere waveguide [4]. However, the extent of this mechanism for elliptical polarization has been calculated to be less than 1% [5] and the minor/major axis ratio of the magnetic field polarization ellipse has been measured to less than 1% [6].

The one-dimensional surface impedance of VLF radio waves is commonly measured using portable surface impedance meters [7, 8] that measure the horizontal electric and magnetic field magnitudes, and the phase difference between them. In the interpretation of the VLF surface impedance measurements, the earth is usually assumed to be isotropic. However, some authors have presented methods of interpreting one-dimensional VLF surface impedance data above a laterally anisotropic half-space [9–11]. In these formulations, it has been assumed that the horizontal magnetic field component maintains its linear polarization. This is not exactly true in the presence of an arbitrary anisotropic half-space. Further, no analyses have been presented for the surface impedance of an inclined anisotropic half-space at arbitrary skew angles. In this paper, we will develop the method of auxiliary potentials proposed by Chetaev and Belen'kaya [12–14] to solve for the fields of a linearly polarised, homogeneous TM-type plane wave incident at a skew angle upon an inclined anisotropic half-space. In the anisotropic half-pace, all field components are coupled and hence a TM-type wave incident upon the half-space will generate a reflected TE-type wave in addition to the reflected TM-type wave. This influences the measured electric and magnetic field components in one-dimensional surface impedance measurements. Expressions for all elements of the impedance tensor are derived, and their applications to VLF surface impedance measurements are discussed.

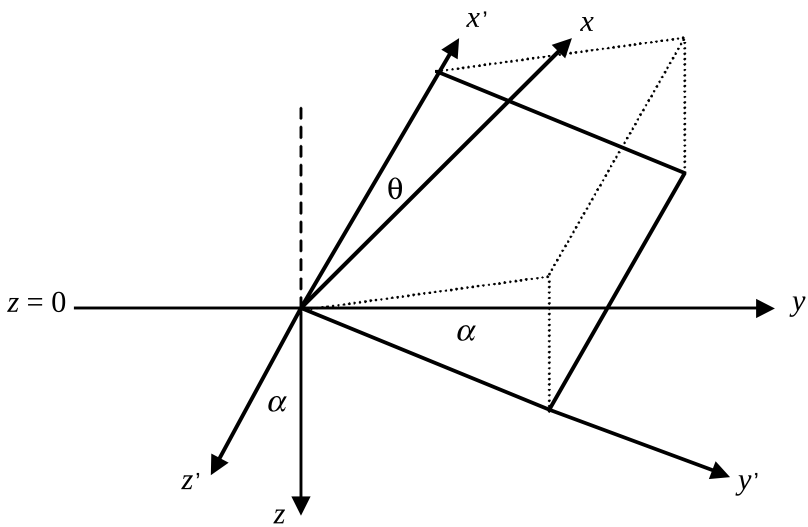

Figure 1. Geometry for the homogeneous plane wave incident at a skew angle θ to the anisotropic half space inclined at angle α about the x-axis.

2. GENERAL SOLUTIONS FOR AN ANISOTROPIC MEDIUM

The surface impedance of homogeneous, monochromatic TM-type plane waves incident at a skew angle upon a homogeneous half-space with inclined uniaxial anisotropic conductivity (see Figure 1) is considered using Chetaev's method of auxiliary potentials [12, 13]. Geologically, this is equivalent to the propagation of VLF radio waves in a sedimentary environment, where the bedding planes are inclined with respect to the surface of the earth. In the $\{x', y', z'\}$ co-ordinate system, the half-space is characterized by the uniaxial conductivity tensor:

$$\hat{\sigma} = \begin{bmatrix} \sigma_t & \sigma & 0 \\ 0 & \sigma_t & 0 \\ 0 & 0 & \sigma_n \end{bmatrix}, \tag{1}$$

where σ_t is the conductivity parallel to the bedding plane, and σ_n is the conductivity normal to the bedding plane, and both σ_t and σ_n can be complex. Given Maxwell's equations for monochromatic fields with time variance of $e^{j\omega t}$ in the inclined co-ordinate system of a uniaxial anisotropic medium with no free charges or extraneous currents:

$$\nabla \times \mathbf{H}' = \hat{\sigma}\mathbf{E}', \tag{2}$$
$$\nabla \times \mathbf{E}' = -j\omega\mathbf{B}', \tag{3}$$
$$\nabla \cdot \mathbf{B}' = 0, \tag{4}$$
$$\nabla \cdot \mathbf{J}' = 0. \tag{5}$$

The electromagnetic potentials can be written as:

$$\mathbf{B}' = \nabla \times \mathbf{A}', \tag{6}$$
$$\mathbf{E}' = -j\omega\mathbf{A}' - \nabla\Phi. \tag{7}$$

Substituting equations (6) and (7) into Maxwell's equations (3) and (4) results in:

$$\nabla^2\mathbf{A}' - \nabla(\nabla \cdot \mathbf{A}') - j\omega\mu\hat{\sigma}\mathbf{A}' - \mu\hat{\sigma}\nabla\Phi = 0,$$

and substituting equations (6) and (7) into (2) and (5) results in:

$$\nabla \cdot \hat{\sigma}\nabla\Phi + j\omega\nabla \cdot \hat{\sigma}\mathbf{A}' = 0, \tag{8}$$

where $\nabla \cdot \hat{\sigma}$ can not be abbreviated if $\hat{\sigma}$ is anisotropic [15]. Using the optimal Lorentz gauge condition [16]:

$$\Phi = -\frac{1}{\mu\sigma_t}\nabla \cdot \mathbf{A}', \tag{9}$$

and substituting equation (9) into equation (4), then:

$$\nabla^2\mathbf{A}' - j\omega\mu\hat{\sigma}\mathbf{A} + \left(\frac{\hat{\sigma}}{\sigma_t} - 1\right)\nabla(\nabla \cdot \mathbf{A}') = 0. \tag{10}$$

for the vector potential $\mathbf{A}$ in $\{x', y', z'\}$ co-ordinates. For $A_{x'}$ and $A_{y'}$ which are associated with σ_t, equation (10) reduces to the homogeneous Helmholtz equations:

$$\nabla^2 \left\{ \begin{array}{c} A_{x'} \\ A_{y'} \end{array} \right\} - j\omega\mu\sigma_t \left\{ \begin{array}{c} A_{x'} \\ A_{y'} \end{array} \right\} = 0, \tag{11}$$

From the Lorentz gauge condition, the full condition on the vector potential is stated as:

$$\frac{-1}{\mu\sigma_t}\frac{\partial A_{z'}}{\partial z'} = \Phi, \tag{12a}$$

$$\frac{\partial A_{x'}}{\partial x'}\frac{\partial A_{y'}}{\partial y'} = 0. \tag{12b}$$

For $A_{z'}$, which is associated with σ_n equation (10) reduces to:

$$\nabla^2 A_{z'} - j\omega\mu\sigma_n + \left(\Lambda^2 - 1\right)\frac{\partial A_{z'}}{\partial z'^2} = 0, \tag{13}$$

where $\Lambda = \sqrt{\sigma_n/\sigma_t} = 1/\lambda$, the reciprocal of the coefficient of anisotropy, λ. The co-ordinate rotation matrix for rotations through angles α and θ about the x- and z-axes respectively can be written as:

$$\mathbf{R}(\theta,\alpha) = \mathbf{R}(\theta)\mathbf{R}(\alpha) = \begin{bmatrix} \cos\theta & \sin\theta\cos\alpha & -\sin\theta\sin\alpha \\ -\sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}. \tag{14}$$

$\mathbf{R}(\theta,\alpha)$ rotates the co-ordinates from the fundamental co-ordinate system to the inclined co-ordinate system. Equation (11) is invariant, while equation (12) takes the form of:

$$-\frac{\partial A_{x'}}{\partial z}\sin\theta\sin\alpha + \frac{\partial A_{y'}}{\partial z}\cos\theta\sin\alpha = 0, \tag{15}$$

and equation (13) takes the form of:

$$\Delta A_{z'} - j\omega\mu\sigma_n + \left(\Lambda^2 - 1\right)\frac{\partial A_{z'}}{\partial z^2}\cos^2\alpha = 0, \tag{16}$$

since all partial derivatives of the homogeneous plane wave field with respect to x and y are equal to zero. When one considers equation (12) with the spatial variance $\exp(-k_1, z)$, then $A_{x'}$ and $A_{y'}$ satisfy the wave number:

$$k_1^2 = j\omega\mu\sigma_t, \tag{17}$$

which is the wave number for an ordinary wave, provided $\mathrm{Re}k_1 > 0$ to prevent exponentially divergent solutions in $A_{x'}$ and $A_{y'}$. As a result of the spatial variances, from equation (15), the relation between the vector potential components of the ordinary wave:

$$A_{y'} = A_{x'}\tan\theta, \tag{18}$$

is obtained. From co-ordinate rotations, the vector potential components for the ordinary wave can be written as:

$$\begin{aligned} A_x &= A_{x'}\cos\theta + A_{y'}\sin\theta\cos\alpha, && (19)\\ A_y &= -A_{x'}\sin\theta + A_{y'}\cos\theta\cos\alpha, && (20)\\ A_z &= A_{y'}\sin\alpha. && (21) \end{aligned}$$

By substituting equation (18) into equations (19) to (21), the vector potential components in fundamental co-ordinates can be written as:

$$\begin{aligned} A_x &= A_{x'}\left(\frac{\cos^2\theta + \sin^2\theta\cos\alpha}{\cos\theta}\right), && (22)\\ A_y &= A_{x'}\sin\theta(\cos\alpha - 1), && (23)\\ A_z &= A_{x'}\tan\theta\sin\alpha. && (24) \end{aligned}$$

As a consequence of equation (12), it is observed for the ordinary wave that $\Phi = 0$, which implies:

$$\mathbf{E} = -j\omega\mathbf{A}. \tag{25}$$

As a result, the components of the electric fields of the ordinary wave in fundamental coordinates can be written as:

$$\begin{aligned} E_x &= -j\omega A_{x'}\left(\frac{\cos^2\theta + \sin^2\theta\cos\alpha}{\cos\theta}\right), && (26)\\ E_y &= -j\omega A_{x'}\sin\theta(\cos\alpha - 1), && (27)\\ E_z &= -j\omega A_{x'}\tan\theta\sin\alpha. && (28) \end{aligned}$$

From equation (6), the components of the magnetic flux density of the ordinary wave in fundamental co-ordinates can be written as:

$$\begin{aligned} B_x &= jk_1 A_{x'}\sin\theta(\cos\alpha - 1), && (29)\\ B_y &= -jk_1 A_{x'}\left(\frac{\cos^2\theta + \sin^2\theta\cos\alpha}{\cos\theta}\right), && (30)\\ B_z &= 0. && (31) \end{aligned}$$

After considering equation (16) with the spatial variance $\exp(-k_2 z)$, then it is observed that $A_{z'}$ satisfies the wave number:

$$k_2^2 = \frac{j\omega\mu\sigma_t}{1 + (\lambda^2 - 1)\sin^2\alpha}, \tag{32}$$

which is the wave number for an extraordinary wave, provided $\mathrm{Re}k_2 > 0$ to prevent exponentially divergent solutions in $A_{z'}$. For the extraordinary wave, it is observed that the scalar potential is not zero but is given by:

$$\Phi = \frac{jk_2}{\mu\sigma_t} A_{z'} \cos\alpha. \tag{33}$$

From equation (33), it follows that:

$$\nabla\Phi = \left(\frac{k_2^2}{\mu\sigma_t} A_{z'} \cos\alpha\right)\hat{\mathbf{k}}. \tag{34}$$

From co-ordinate rotations, the vector potential components for the ordinary wave can be written as:

$$A_x = -A_{z'} \sin\theta \sin\alpha, \tag{35}$$
$$A_y = -A_{z'} \cos\theta \sin\alpha, \tag{36}$$
$$A_z = A_{z'} \cos\alpha. \tag{37}$$

The components of the electric field for the extraordinary wave are obtained by substituting equations (34) to (37) into equation (7):

$$E_x = j\omega A_{z'} \sin\theta \sin\alpha, \tag{38}$$
$$E_y = j\omega A_{z'} \cos\theta \sin\alpha, \tag{39}$$
$$E_z = \left(\frac{k_1^2 - k_2^2}{\mu\sigma_t}\right) A_{z'} \cos\alpha. \tag{40}$$

From equation (6), the components of the magnetic flux density of the ordinary wave in fundamental co-ordinates can be written as:

$$B_x = -jk^2 A_{z'} \cos\theta \sin\alpha, \tag{41}$$
$$B_y = jk^2 A_{z'} \sin\theta \sin\alpha, \tag{42}$$
$$B_z = 0. \tag{43}$$

The observed fields will be the sum of the ordinary and extraordinary fields:

$$\mathbf{E} = \mathbf{E}_{\text{ordinary}} + \mathbf{E}_{\text{extraordinary}}, \tag{44a}$$
$$\mathbf{H} = \mathbf{H}_{\text{ordinary}} + \mathbf{H}_{\text{extraordinary}}. \tag{44b}$$

From equations (26) to (31) and (38) to (43), the components of the total electric and magnetic fields in fundamental co-ordinates in the inclined anisotropic half-space become:

$$E_x = -j\omega A_{x'} \left(\frac{\cos^2\theta + \sin^2\theta\cos\alpha}{\cos\theta}\right) + j\omega A_{z'} \sin\theta \sin\alpha, \tag{45}$$

$$\begin{aligned}
E_y &= -j\omega A_{x'} \sin\theta(\cos\alpha - 1) + j\omega A_{z'} \cos\theta \sin\alpha, && (46)\\
E_z &= -j\omega A_{x'} \tan\theta \sin\alpha + \left(\frac{k_1^2 - k_2^2}{\mu\sigma_t}\right) A_{z'} \cos\alpha, && (47)\\
B_x &= jk_1 A_{x'} \sin\theta(\cos\alpha - 1) - jk_2 A_{z'} \cos\theta \sin\alpha, && (48)\\
B_y &= -jk_1 A_{x'} \left(\frac{\cos^2\theta + \sin^2 \cos\alpha}{\cos\theta}\right) + jk_2 A_{z'} \sin\theta \sin\alpha, && (49)\\
B_z &= 0. && (50)
\end{aligned}$$

Equation (50) is expected for a homogeneous plane wave.

3. GENERAL SOLUTIONS FOR PROPAGATION IN AIR

In the upper half-space, the general solution for the electromagnetic fields are the superposition of the TE- and TM-type waves, which propagate independently but satisfy the same wave number:

$$k_0^2 = \omega^2 \varepsilon\mu. \tag{51}$$

From (2) and (3) in free space, the components of the homogeneous TM-type waves will be:

$$\begin{aligned}
\frac{\partial B_y}{\partial t} &= -\frac{\partial E_x}{\partial z}, && (52)\\
\varepsilon_0 \frac{\partial E_x}{\partial t} &= -\frac{1}{\mu}\frac{\partial B_y}{\partial z} && (53)
\end{aligned}$$

The general solutions for the magnetic field component of the homogeneous TM-type waves take the form:

$$B_y = M\exp(-jk_0 z) + N\exp(jk_0 z), \tag{54}$$

so the corresponding electric field component can be written as:

$$E_x = \frac{k_0}{\omega\mu\varepsilon}\left\{M\exp(-jk_0 z) - N\exp(jk_0 z)\right\}, \tag{55}$$

where M and N are (complex) constants independent of $\{x, y, z, t\}$, representing the coefficients for the down- and up-going homogeneous plane waves. From equations (2) and (3) in free space, the components of the homogeneous TE-type waves can be written

$$\begin{aligned}
\varepsilon_0 \frac{\partial E_y}{\partial t} &= \frac{-1}{\mu}\frac{\partial B_x}{\partial z}, && (56)\\
\frac{\partial B_x}{\partial t} &= -\frac{\partial E_y}{\partial z}. && (57)
\end{aligned}$$

The general solutions for the electric field component of the homogeneous TE-type waves then can take the form:

$$E_y = P\exp(-jk_0 z) + Q\exp(jk_0 z), \tag{58}$$

so the corresponding magnetic field component can be written as:

$$B_x = \frac{k_0}{\omega}\{P\exp(-jk_0 z) - Q\exp(jk_0 z)\}, \tag{59}$$

where P and Q are (complex) constants independent of $\{x, y, z, t\}$, representing the coefficients for the down- and up-going homogeneous plane waves.

4. TM-TYPE WAVE INCIDENCE

Consider a TM-type wave incident upon the anisotropic half-space at a skew angle of incidence θ. Since the ordinary and extraordinary waves in the anisotropic half-space couple all field components, then a TE-type wave must also be reflected, in addition to the reflected TM-type wave. The problem is to determine the constants P, N, $A_{x'}$, and $A_{z'}$. in terms of the amplitude of the incident TM-type wave, M. There are four continuity conditions for the horizontal field components. First, $A_{z'}$ must be expressed in terms of $A_{x'}$ from continuity of the horizontal components of the TE-type wave that is generated and reflected. At the boundary $z = 0$, setting $P = 0$ in equation (58), only an up-going TE-type wave in air is considered. Equating the fields of the reflected TE-type wave with the corresponding field components in the anisotropic half-space, using equations (58) and (59), then

$$E_y\Big|_{z=0} = Q, \tag{60}$$

$$B_x\Big|_{z=0} = -\frac{k_0}{\omega}Q. \tag{61}$$

Equating equations (60) and (61) with equations (46) and (48) respectively leads to the relation

$$A_{x'} = \frac{(k_0 - k_2)\cos\theta\sin\alpha}{(k_0 - k_1)\sin\theta(\cos\alpha - 1)}A_{z'}. \tag{62}$$

From equations (49) and (62), then:

$$B_y = F_{TE}A_{x'}, \tag{63}$$

Where:

$$F_{TE} = jk_1 \sin\theta(\cos\alpha - 1) + j\left(\frac{k_1^2 - k_2^2}{\mu\sigma_t}\right)\frac{(k_0 - k_1)\tan\theta(\cos\alpha - 1)}{(k_0 - k_2)\tan\alpha}. \tag{64}$$

Similarly:

$$B_y = G_{TE}A_{z'}, \tag{65}$$

where:

$$G_{TE} = jk_1(k_0 - k_2)\cos\theta\sin\alpha + j\left(\frac{k_1^2 - k_2^2}{\mu\sigma_t}\right)\cos\alpha. \tag{66}$$

Now, all field components for the electromagnetic field at the surface of the half-space can be expressed in terms of B_x:

$$E_x = \left[-j\omega\frac{1}{F_{TE}}\left(\frac{\cos^2\theta + \sin^2\theta\cos\alpha}{\cos\theta}\right) + j\omega\frac{1}{G_{TE}}\sin\theta\sin\alpha\right]B_x, \tag{67}$$

$$E_x = \left[-j\omega\frac{1}{F_{TE}}\sin\theta(\cos\alpha - 1) + j\omega\frac{1}{G_{TE}}\cos\theta\sin\alpha\right]B_x, \tag{68}$$

$$E_{z'} = \left[-j\omega\frac{1}{F_{TE}}\tan\theta\sin\alpha + \left(\frac{k_1^2 - k_2^2}{\mu\sigma_t}\right)\frac{1}{G_{TE}}\cos\alpha\right]B_x, \tag{69}$$

$$B_y = \left[-jk_1\frac{1}{F_{TE}}\left(\frac{\cos^2\theta + \sin^2\theta\cos\alpha}{\cos\theta}\right) + jk_2\frac{1}{G_{TE}}\sin\theta\sin\alpha\right]B_x, \tag{70}$$

$$B_z = 0. \tag{71}$$

Note that when $\theta = 0°$ or $180°$:

$$Z_{xy} = \sqrt{\frac{j\omega\mu}{\sigma_t}\{1 + (\lambda^2 - 1)\sin^2\alpha\}}, \tag{72}$$

$$Z_{xy} = \sqrt{\frac{j\omega\mu}{\sigma_t}}. \tag{73}$$

When $\theta = 90°$ or $270°$:

$$Z_{xy} = 0, \tag{74}$$

$$Z_{yx} = \sqrt{\frac{j\omega\mu}{\sigma_t}\{1 + (\lambda^2 - 1)\sin^2\alpha\}}. \tag{75}$$

Equations (72) to (75) correlate to the expected values of the surface impedance when the source fields are parallel and perpendicular to the strike of the anisotropic half-space [17].

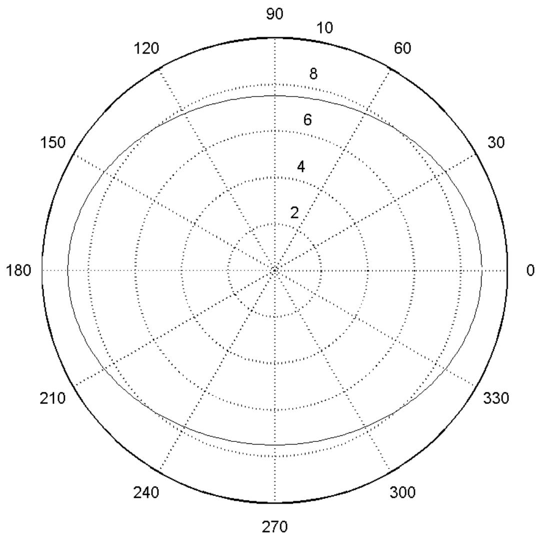

Figure 2. $|Z_{xy}|$ (in ohms) of a homogeneous TM-type plane wave as a function of θ for a half space with $\sigma_t = 0.001\,\mathrm{S/m}$, $\sigma_n = 0.01\,\mathrm{S/m}$, and $\alpha = 45°$.

5. DISCUSSION

From equations (67) to (71), the elements of the impedance tensor can be calculated:

$$Z_{mn} = \frac{\mu E_m}{B_n}. \tag{76}$$

The principal interest of this work is for TM-type propagation problems related to VLF propagation and surface impedance measurements. Polar diagrams for one of the principle components and one of the additional components of the surface impedance tensor for a 10 kHz homogeneous plane wave at all angles of skew incidence upon a homogeneous half-space with $\sigma_t = 0.001\,\mathrm{S/m}$, $\sigma_n = 0.01\,\mathrm{S/m}$, and $\alpha = 60°$ are presented in Figures 2 and 3. Note that the values converge to the expected impedances for $\theta = 0°$, $90°$, $180°$ and $270°$. It should be observed that Figures 2 and 3 correlate to the additional impedance polar diagrams expected from a two-dimensional structure [18]. The principle impedance polar diagram exhibits the phase of a one-dimensional half- space, but has the magnitude characteristics of a two-dimensional half-space [18]. This reinforces the well-known principle of the existence of ambiguity in surface impedance measurements, where

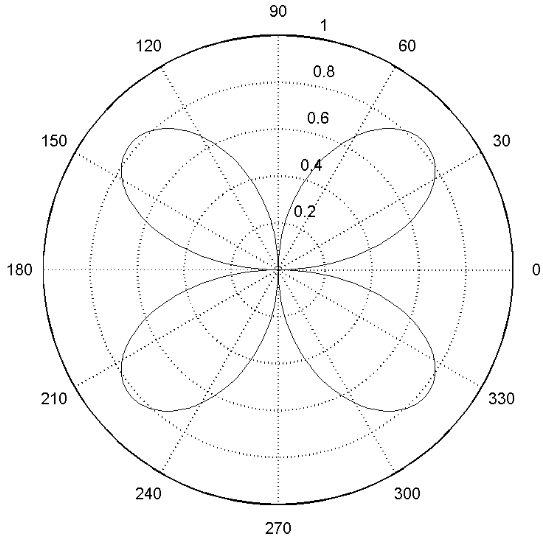

Figure 3. $|Z_{yy}|$ (in ohms) of a homogeneous TM-type plane wave as a function of θ for a half space with $\sigma_t = 0.001\,\mathrm{S/m}$, $\sigma_n = 0.01\,\mathrm{S/m}$, and $\alpha = 45°$.

it is difficult to differentiate between the surface impedance response of a one-dimensional homogeneous and anisotropic half-space, and a two-dimensional homogeneous (and anisotropic) half-space.

As a further case, we examine the effect of an anisotropic half-space on the measured electric and magnetic fields in an ideal one-dimensional surface impedance measurement. Writing ψ as the angle between the orientation of the surface impedance meter and the direction of propagation, i.e., the angle of the instrument with respect to θ. The fields induced in the horizontal electric and magnetic dipoles of the surface impedance meter will be proportional to the electric and magnetic field components of the total fields in the direction of the dipole axes. Ignoring the influences of the radiation patterns of the electric and magnetic dipoles, for any ψ, components of the observed electric and magnetic fields are:

$$E^{obs} = E_x \cos\psi + E_y \sin\psi, \tag{77}$$

$$H^{obs} = -H_x \sin\psi + H_y \cos\psi. \tag{78}$$

If the homogeneous half-space was isotropic, then equations (77) and

(78) would simply reduce to:

$$E^{obs} = E_x \sin\psi, \tag{79}$$

$$H^{obs} = H_y \cos\psi, \tag{80}$$

and the observed surface impedance Z_s, at any ψ would be given by:

$$Z_s = \frac{E_x}{H_y}\tan\psi, \tag{81}$$

which exhibits asymptotes when $\theta = 90°$ or $180°$, as expected for linearly polarized fields. As an example, consider a 10 kHz homogeneous TM-type plane wave incident upon a homogeneous half-space with $\sigma_t = 0.001\,\mathrm{S/m}$, $\sigma_n = 0.01\,\mathrm{S/m}$, and $\alpha = 60°$, where $\theta = 45°$, $90°$ and $135°$. The fields were calculated used the expressions derived in Section 4. From equations (77) and (78), the one-dimensional surface impedance can be calculated, and the corresponding polar diagrams for the normalized magnitude of the magnetic and electric fields are shown in Figures 4 and 5 respectively. It is observed in Figure 4 that at VLF frequencies, the polarization of the horizontal magnetic field of a homogeneous TM-type plane wave is independent of the anisotropy of the half-space. This verifies the assumptions of previous authors [5, 9–11] that the magnetic field maintained linear polarization above an anisotropic half-space. In Figure 5, it is observed that at VLF frequencies, the polarization of the horizontal electric fields of a homogeneous TM-type plane wave are elliptically polarized and are dependent upon the anisotropy of the half-space. For the model presented here, the major axis of the electric field ellipse is observed to be rotated approximately 12° about the horizontal plane with respect to the magnetic field, maximizing for values of $\theta = 45°$ and $135°$. It is suggested that by measuring the polar radiation fields of the electric field and magnetic fields in one-dimensional surface impedance measurements, information about the presence of anisotropy in the earth can be determined. It is emphasized here that the inverse problem of solving for the direction of anisotropic strike has not been considered in this paper.

To demonstrate this effect of anisotropy in a practical VLF survey, the surface impedance of a VLF radio wave was measured at 10° instrument orientations with respect to the direction of propagation of the VLF fields [19]. Measuring the 19.8 kHz fields of the North West Cape (Western Australia) VLF transmitter, the surveys were conducted at a Central Queensland coal mine in Queensland, Australia in July 1997 at different times of day (5:15 am, 6:15 am, 7:15 am, 8:15 am, 9:15 am, 4:25 pm). The local site consisted of quartz

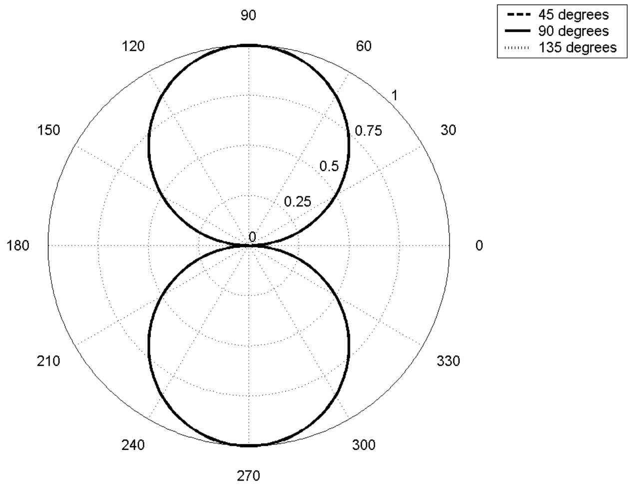

Figure 4. Normalized observed (measured) magnetic field of a homogeneous TM-type plane wave as a function of ψ for a half space with $\sigma_t = 0.001\,\mathrm{S/m}$, $\sigma_n = 0.01\,\mathrm{S/m}$ and $\alpha = 45°$ for $\theta = 45°$, $90°$ and $135°$. Note that the fields overlap, indicating that there is no variation in the magnetic field polarization due to the anisotropy of the half space.

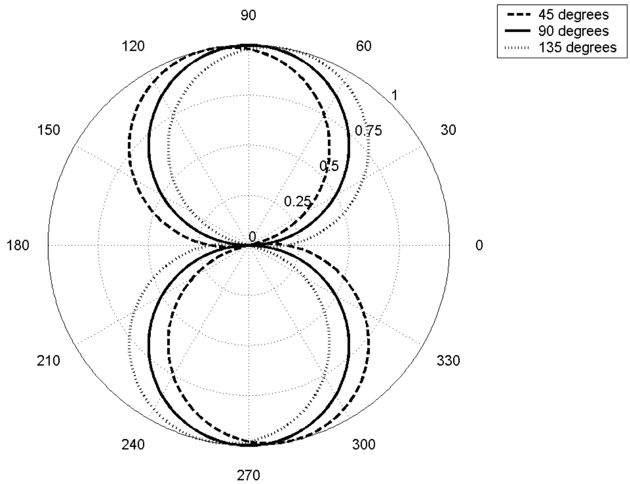

Figure 5. Normalized observed (measured) electric field of a homogeneous TM-type plane wave as a function of ψ for a half space with $\sigma_t = 0.001\,\mathrm{S/m}$, $\sigma_n = 0.01\,\mathrm{S/m}$ and $\alpha = 45°$ for $\theta = 45°$, $90°$ and $135°$.

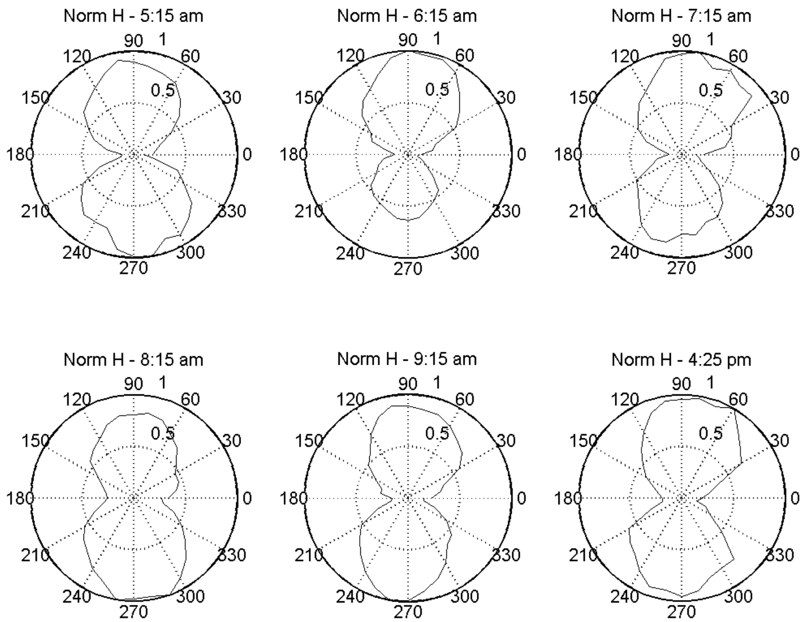

Figure 6. Observed (measured) magnetic field of a 19.8 kHz VLF wave as a function of ψ measured at different times of day in July 1997 (data from [19]). Data is normalized for a maximum of one at every angle.

sandstones and shales with shallow dips, at a strike approximately 30° to the direction of propagation. The magnitudes of the electric and magnetic fields, and the phase difference between them were measured using a meter similar to Thiel [7]. The magnetic field was measured using a ferrite cored multi-turn loop antenna. The electric field was measured using an electrically short, insulated dipole antenna, which has been demonstrated to be an effective antenna for measuring the horizontal electric field [20]. Figures 6 and 7 respectively present the normalized magnetic and electric fields from these surveys.

It is noted that during the measurements between 6:15 am and 8:15 am relate to the sunrise period along the 3.5 Mm propagation path between the transmitter and receiver sites, explaining rapidly varying degradation in the magnetic and electric field polarizations at those times. This is a result of the destructive interference that occurs between TM_{01} and TM_{02} modes during periods of sunrise and sunset [21]. However is observed in Figure 6 that the magnetic fields demonstrate a linear polarization for stable waveguide propagation

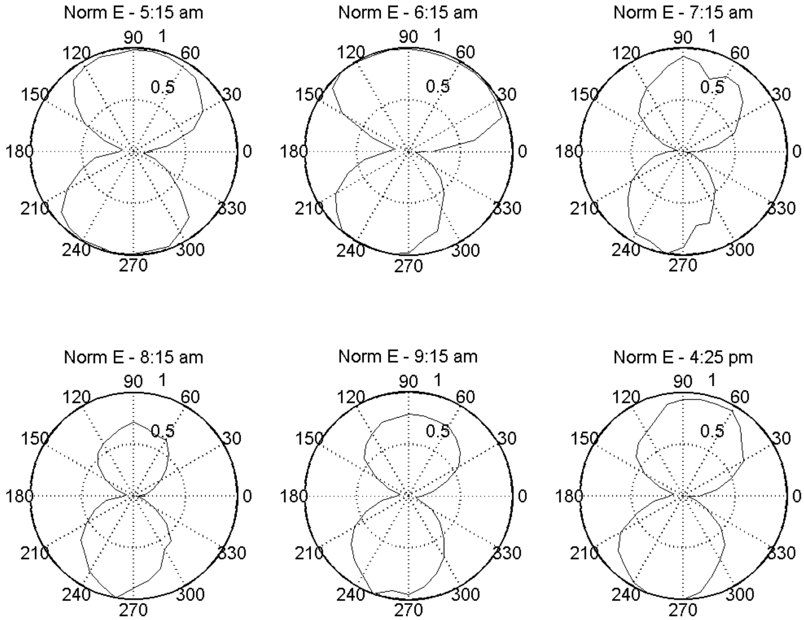

Figure 7. Observed (measured) electric field of a 19.8 kHz VLF wave as a function of ψ measured at different times of day in July 1997 (data from [19]). Data is normalized for a maximum of one at every angle.

conditions (5:15 am, 9:15 am, 4:25 pm). Comparing Figure 7 to Figure 6, it is observed that the horizontal electric field is elliptically polarised, with the major axis of the electric field polar pattern rotated approximately 10° from the magnetic field polar pattern. As discussed earlier in the model study, this suggests the presence of anisotropy in the local earth. However, from the surface impedance data available, it is not possible to determine whether the anisotropy is due to inclined or lateral anisotropy in the local earth.

6. CONCLUSION

In this paper, the general expressions for the fields of TM-type homogeneous plane waves at a skew angle of incidence upon an inclined anisotropic half-space have been derived. Previous analyses have only considered fields of homogeneous plane waves in the problems of a laterally anisotropic half-space (i.e., $\alpha = 90°$), and have not considered the problem of an inclined anisotropic half-space. Further, previous analyses have not considered the effect of mode conversions at the air-half-space boundary, and in the case of TM-type wave, have assumed linear polarization of the magnetic field is maintained. The results presented in this paper have shown that the assumption that the linear polarization of the magnetic field is maintained and further the solutions obtained have been shown to converge on the expected values for the special cases presented by Wilson and Thiel [17] and can be considered as general solutions for the homogeneous plane wave incident upon an inclined anisotropic half-space.

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