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IMPEDANCE BOUNDARY CONDITIONS ON A CHIRAL FILM

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Abstract—Using a technique borrowed from Idemen [1] and requiring the Fourier transform of the x, y-components of the electric and magnetic fields, we obtain the impedance boundary conditions for electromagnetic plane waves with horizontal, vertical and arbitrary polarization incident on a infinite, smooth, chiral film located at $z = 0$ and deposited on a metallic substrate. As an application, we discuss the scattering of harmonic plane waves and of a finite beam on such a film.

1. INTRODUCTION

The simplest and one of the most important boundary value problem in electromagnetism is the scattering of a uniform plane wave incident on a plane boundary between two different media because its solutions can be used to analyse scattering problems on solids large compared with wavelength. To solve problems of this kind, we need the appropriate boundary conditions at the discontinuity surfaces and, in many cases, only the external field components on the surface of the body are interrelated by them so that the determination of the field inside the body becomes avoided which greatly simplifies the task since we have just to introduce proper impedance conditions on the surface of the body.

Idemen [1] has obtained the impedance boundary conditions for harmonic plane waves incident from free space on infinite plane slabs made of different stratified dielectric homogeneous media. And, prompted by the emergence of chiral materials [2] used for instance to reduce target radar cross-sections, we investigate here, with the Idemen technique, the impedance boundary conditions at the surface of a chiral film deposited on a metallic substrate with the ultimate purpose to analyse the scattered fields.

This chiral film is endowed with the constitutive relations [3]

$$\mathbf{D} = \varepsilon\mathbf{E} + i\gamma\mathbf{B}, \qquad \mathbf{H} = \mu^{-1}\mathbf{B} + i\gamma\mathbf{E} \tag{1}$$

that we write

$$\mathbf{D} = \mu(\beta\mathbf{E} + i\gamma\mathbf{H}), \qquad \mathbf{B} = \mu(\mathbf{H} - i\gamma\mathbf{E}), \qquad \beta = \varepsilon\mu^{-1} + \gamma^2 \tag{1a}$$

The parameters ε, μ, γ are permittivity, permeability and chirality respectively and for an harmonic field with time dependence $\exp(ikct)$, the Maxwell equations are

$$\nabla \wedge \mathbf{E} = -ik\mathbf{B}, \quad \nabla \wedge \mathbf{H} = ik\mathbf{D}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{D} = 0 \tag{2}$$

from which we get the wave equations

$$\nabla \wedge \nabla \wedge \{\mathbf{E}, \mathbf{H}\} + 2k\gamma\mu\nabla \wedge \{\mathbf{E}, \mathbf{H}\} - k^2\mu\varepsilon\{\mathbf{E}, \mathbf{H}\} = 0 \tag{2a}$$

On this chiral film located at $z = 0$, impinges an harmonic plane wave with horizontal, vertical or arbitrary polarization and to get the exact form of Maxwell's equations in these situations we introduce the unit vectors $\eta_{H,V}(\mathbf{k})$ in which $\mathbf{k}$ is the wave vector and $\mathbf{k}_z$ the vector with k_z as only nonnull component

$$\eta_H(\mathbf{k}) = (\mathbf{k} \wedge \mathbf{k}_z)|\mathbf{k} \wedge \mathbf{k}_z|^{-1} \quad \eta_V(\mathbf{k}) = [\mathbf{k} \wedge \eta_H(\mathbf{k})]|\mathbf{k} \wedge \eta_H(\mathbf{k})|^{-1} \tag{3}$$

The electromagnetic plane wave with horizontal polarization is defined by the relation in which $\Psi(\mathbf{x}) = \exp(i\mathbf{k}\cdot\mathbf{x})$ with $\mathbf{x} = (x, y, z)$

$$\mathbf{E}(\mathbf{x}) = \eta_H(\mathbf{k})\Psi(\mathbf{x}), \qquad \mathbf{H}(\mathbf{x}) = \eta_V(\mathbf{k})\Psi(\mathbf{x}) \tag{4a}$$

while for vertical polarization and free space impedance unity

$$\mathbf{E}(\mathbf{x}) = \eta_V(\mathbf{k})\Psi(\mathbf{x}), \qquad \mathbf{H}(\mathbf{x}) = -\eta_H(\mathbf{k})\Psi(\mathbf{x}) \tag{4b}$$

These relations become with the spherical polar angles θ, ϕ

$$\Psi(\mathbf{x}) = \exp(ik_x x + ik_y y + ik_z z) \tag{5}$$

$$k_x = -k\sin\theta\cos\phi, \quad k_y = k\sin\theta\sin\phi, \quad k_z = k\cos\theta \tag{5a}$$

and for horizontal polarization

$$\begin{aligned} E_x(\mathbf{x}) = \sin\phi\Psi(\mathbf{x}), \quad E_y(\mathbf{x}) = \cos\phi\Psi(\mathbf{x}), \quad E_z = 0 \\ ikH_x(\mathbf{x}) = \partial_z E_y(\mathbf{x}), \quad ikH_y(\mathbf{x}) = -\partial_z E_x(\mathbf{x}), \\ -ikH_z(\mathbf{x}) = \partial_x E_y(\mathbf{x}) - \partial_y E_x(\mathbf{x}) \end{aligned} \tag{6a}$$

while for vertical polarization

$$\begin{aligned} H_x(\mathbf{x}) = -\sin\phi\Psi(\mathbf{x}), \quad H_y(\mathbf{x}) = -\cos\phi\Psi(\mathbf{x}), \quad H_z = 0 \\ ikE_x(\mathbf{x}) = -\partial_z H_y(\mathbf{x}), \quad ikE_y(\mathbf{x}) = \partial_z H_x(\mathbf{x}), \\ -ikE_z(\mathbf{x}) = \partial_x H_y(\mathbf{x}) - \partial_y H_x(\mathbf{x}) \end{aligned} \tag{6b}$$

These relations show that we only need two components either (E_x, E_y) or (H_x, H_y) to get all the other components of the electromagnetic field so that the scattering problem requires the boundary conditions satisfied by these two couples on the interface $z = 0$.

Then, to get the impedance boundary conditions, Idemen [1] works with the Fourier transforms $\{\mathbf{e}(\rho, z), \mathbf{h}(\rho, z)\}$ of the electromagnetic field in which $\rho = (\xi, \eta)$

$$\{\mathbf{e}(\rho, z), \mathbf{h}(\rho, z)\} = \iint_{-\infty}^{\infty} dx\, dy \{\mathbf{E}(\mathbf{x}), \mathbf{H}(\mathbf{x})\} \exp(i\xi x + i\eta y) \tag{7}$$

and, for a slab of thickness d deposited on a metallic substrate, imposing the continuity of $e_{x,y}$, $h_{x,y}$ at the interface $z = 0$ and the constraint $e_{x,y} = 0$ on the perfectly reflecting substrate at $z = -d$, suffices to get the impedance boundary conditions satisfied by the harmonic plane wave at the film surface. The inverse Fourier transform of these boundary conditions can be performed analytically in some cases and numerically otherwise.

This paper is organized as follows: we discuss in Sec. 2 the nature of electromagnetic waves able to propagate inside an infinite chiral medium with the constitutive relations (1). Sec. 3 is devoted to impedance boundary conditions for an harmonic plane wave with horizontal or vertical polarization incident on a chiral film of thickness d located at $z = 0$ and deposited on a metallic substrate. We also obtain the boundary conditions for TM and TE waves impinging on 1D-chiral films. The same problem is investigated in Sec. 4 for a plane wave with arbitrary polarization. We discuss in Sec. 5 the scattering of harmonic plane waves and of a finite beam from a chiral film. Conclusive comments are given in Sec. 6.

2. ELECTROMAGNETIC WAVE PROPAGATION INSIDE CHIRAL MEDIA

With the Fourier transform (7), the Maxwell equations (2) take the explicit form

$$\begin{aligned} i\eta e_z - \partial_z e_y &= -ik\mu(h_x - i\gamma e_x) \\ \partial_z e_x - i\xi e_z &= -ik\mu(h_y - i\gamma e_y) \\ \xi e_y - \eta e_x &= -k\mu(h_z - i\gamma e_z) \\ i\beta(\xi e_x + \eta e_y) - \gamma(\xi h_x + \eta h_y) + \partial_z e_z &= 0 \end{aligned} \tag{Ia}$$

$$\begin{aligned} i\eta h_z - \partial_z h_y &= ik\mu(\beta e_x + i\gamma h_x) \\ \partial_z h_x - i\xi h_z &= ik\mu(\beta e_y + i\gamma h_y) \\ \xi h_y - \eta h_x &= k\mu(\beta e_z + i\gamma h_z) \\ i(\xi h_x + \eta h_y) + \gamma(\xi e_x + \eta e_y) + \partial_z h_z &= 0 \end{aligned} \tag{Ib}$$

The numerals I, II, III... are reserved to systems of equations and the arguments ρ, z are deleted when there exists no risk of confusion.

We look for solutions of (Ia,b) in the form

$$e_{x,y}(\rho, z) = p_{x,y}(\rho)\exp(i\lambda z), \qquad h_{x,y}(\rho, z) = q_{x,y}(\rho)\exp(i\lambda z) \tag{8}$$

We first get from Eqs. (Ia_3) and (Ib_3):

$$\begin{aligned} k\varepsilon h_z &= \beta(\eta e_x - \xi e_y) - i\gamma(\eta h_x - \xi h_y) \\ k\varepsilon e_z &= \xi h_y - \eta h_x + i\gamma(\xi e_y - \eta e_x) \end{aligned} \tag{9}$$

and using (8)

$$\begin{aligned} k\varepsilon h_z &= [\beta(\eta p_x - \xi p_y) - i\gamma(\eta q_x - \xi q_y)] \exp(i\lambda z) \\ k\varepsilon e_z &= [\xi q_y - \eta q_x + i\gamma(\xi p_y - \eta p_x)] \exp(i\lambda z) \end{aligned} \tag{9a}$$

Substituting (8) and (9a) into Eqs. ($\mathrm{Ia}_{1,2}$), ($\mathrm{Ib}_{1,2}$) gives the homogeneous system of equations satisfied by the amplitudes $p_{x,y}(\rho),\ q_{x,y}(\rho)$

$$\begin{aligned} &p_x(i\lambda - \gamma\xi\eta/\varepsilon k) + \gamma p_y(k\mu - \xi^2/\varepsilon k) + i\xi\eta q_x/\varepsilon k + iq_y(k\mu - \xi^2/\varepsilon k) = 0 \\ &\gamma p_x(k\mu + \eta^2/\varepsilon k) - p_y(i\lambda + \gamma\xi\eta/\varepsilon k) + iq_x(k\mu - \eta^2/\varepsilon k) + i\xi\eta q_y/\varepsilon k = 0 \\ &-i\beta\eta p_x/\varepsilon k - i\beta p_y(k\mu - \xi^2/\varepsilon k) + q_x(i\lambda - \gamma\xi\eta/\varepsilon k) + \gamma q_y(k\mu + \xi^2/\varepsilon k) = 0 \\ &-i\beta p_x(k\mu - \eta^2/\varepsilon k) - i\beta\xi\eta p_y/\varepsilon k + \gamma q_x(k\mu + \eta^2/\varepsilon k) - q_y(i\lambda + \gamma\xi\eta/\varepsilon k) = 0 \end{aligned} \tag{II}$$

This system has nontrivial solutions if its determinant $\Delta = 0$, writing $\Delta = \Delta_1 + \Delta_2$ we get from (II)

$$\Delta_1 = \begin{vmatrix} i\lambda & k\gamma\mu & 0 & ik\mu \\ k\gamma\mu & -i\lambda & ik\mu & 0 \\ 0 & -i\beta k\mu & i\lambda & k\gamma\mu \\ -i\beta k\mu & 0 & k\gamma\mu & -i\lambda \end{vmatrix} = (k\mu)^4 \begin{vmatrix} \nu & -i\gamma & 0 & 1 \\ i\gamma & \nu & 1 & 0 \\ 0 & -\beta & \nu & -i\gamma \\ -\beta & 0 & i\gamma & \nu \end{vmatrix} \tag{10a}$$

in which $\nu = \lambda/\mu k$ and

$$\Delta_2 = \eta^2\xi^2/\varepsilon^2 k^2 \begin{vmatrix} -\gamma & \gamma & i & -i \\ \gamma & -\gamma & -i & i \\ -i\beta & i\beta & -\gamma & \gamma \\ i\beta & -i\beta & \gamma & -\gamma \end{vmatrix} = 0 \tag{10b}$$

so that $\Delta = 0$ implies $\Delta_1 = 0$ and it is easily proved that the roots of Δ_1 are the solutions of the quartic equation

$$\nu^2(\nu^2 - \gamma^2 - 3\beta) + (\beta - \gamma^2)(\nu^2 + \beta - \gamma^2) = 0 \tag{11}$$

using the relation (1a) $\beta - \gamma^2 = \varepsilon\mu^{-1}$ and writing $-\gamma^2 - 3\beta = \beta - \gamma^2 - 4\beta$, this equation becomes

$$\nu^4 + 2\nu^2(\varepsilon\mu^{-1} - 2\beta) + \varepsilon^2\mu^{-2} = 0 \tag{12}$$

with the solutions

$$\begin{aligned} \nu^2 &= 2\beta - \varepsilon\mu^{-1} \pm \left[(2\beta - \varepsilon\mu^{-1})^2 - \varepsilon^2\mu^{-2}\right]^{1/2} \\ &= 2\beta - \varepsilon\mu^{-1} \pm [4\beta - 4\beta\varepsilon\mu^{-1}]^{1/2} \\ &= (\pm\gamma + \beta^{1/2}) \end{aligned} \tag{12a}$$

so that since $\lambda = k\mu\nu$

$$\lambda_{\pm} = k\mu \left[\pm\gamma + (\gamma^2 + \varepsilon\mu^{-1})^{1/2}\right] \tag{13}$$

So, in agreement with a well known result [3, 4], a chiral medium can only support two modes of propagation respectively right and left circularly polarized.

3. HORIZONTALLY AND VERTICALLY POLARIZED FIELDS

The electromagnetic field for $z > 0$ in free space, noted with primed letters, is the sum of the incident and scattered waves, $\varepsilon = \mu = 1$, and according to (7) the Fourier transform of this field is $\{\mathbf{e}'(\rho, z), \mathbf{h}'(\rho, z)\}$.

3.1. Horizontal Polarization

As shown in the introduction, the z-component of the electric field is null, $e'_z = 0$ and only the boundary conditions on $e'_{x,y}$ are needed. Then, the Fourier transform of the Maxwell equations (6a) have the explicit form:

$$\begin{aligned} \partial_z e'_y - ikh'_x &= 0 \\ \partial_z e'_x + ikh'_y &= 0 \\ \xi e'_y - \eta e'_x &= -kh'_z \\ \xi e'_x + \eta e'_y &= 0 \end{aligned} \tag{I'a}$$

$$\begin{aligned} i\eta h'_z - \partial_z h'_y &= ike'_x \\ \partial_z h'_x - i\xi h'_z &= ike'_y \\ \xi h'_y - \eta h'_x &= 0 \\ i\xi h'_x + i\eta h'_y + \partial_z h'_z &= 0 \end{aligned} \tag{I'b}$$

The boundary conditions for the electric and magnetic fields at the interface $z = 0$ between free space and chiral film are (the subscript zero means that these relations are only valid there)

$$\left(e_{x,y} - e'_{x,y}\right)_0 = 0, \qquad \left(h_{x,y} - h'_{x,y}\right)_0 = 0 \tag{14}$$

But these relations are not independent since according to Eqs. (I'a$_4$) and (I'b$_3$):

$$\left(\xi e_x + \eta e_y\right)_0 = 0, \qquad \left(\xi h_y - \eta h_x\right)_0 = 0 \tag{14a}$$

so that they imply only one constraint on e_x or e_y and another one on h_x or h_y.

The normal components of the **B**, **D** fields are continuous on $z = 0$ and according to Eq. (9) written on $z = 0$ together with (14a) and using the relation $\mu(\beta - \gamma^2) = \varepsilon$ we get

$$\left(\varepsilon e_z + i\gamma h'_z\right)_0 = 0, \qquad \left(\varepsilon h_z - \beta h'_z\right)_0 = 0 \tag{15}$$

with in addition

$$(\partial_z e_z)_0 = 0, \qquad \left[\partial_z (h_z - h'_z)\right]_0 = 0 \tag{16}$$

Applying the conditions (14) to the systems (Ia,b), (I'a,b) proves that the relations (15), (16) are fulfilled and they may be ignored.

To sum up, the boundary conditions impose only two constraints on the electromagnetic field $\{\mathbf{e}(\rho, z), \mathbf{h}(\rho, z)\}$, consequently the two circularly polarized modes inside the chiral film have the same amplitude, a necessary condition to get a Faraday rotation of the plane of polarization [5]. So, using the wave numbers (8) we may write $e_x(\rho, z)$

$$\begin{aligned} e_x(\rho, z) &= A_x(\rho)\exp^*(i\lambda z) + B_x(\rho)\exp^*(-i\lambda z), \\ \exp^*(i\lambda z) &= \exp(i\lambda_+ z) + \exp(i\lambda_- z) \end{aligned} \tag{17}$$

in which according to (14)

$$2(A_x + B_x) = \left(e'_x\right)_0 \tag{17a}$$

A second relation to determine A_x, B_x is obtained by using Eq. (Ia$_2$) on $z = 0$

$$k\mu\,(h_y)_0 = i\gamma k\mu\,(e_y)_0 + i\,(\partial_z e_x)_0 + \xi\,(e_z)_0 \tag{18}$$

so that taking into account (14), (14a) and (15), the boundary condition $\left(h_y - h'_y\right)_0 = 0$ becomes

$$\begin{aligned} -i\gamma\xi\eta^{-1}k\mu\left(e'_x\right)_0 + i\,(\partial_z e_x)_0 - i\gamma\xi\varepsilon^{-1}\left(h'_z\right)_0 &= k\mu\left(h'_y\right)_0 \\ &= i\mu(\partial_z e'_x)_0 \end{aligned} \tag{18a}$$

with Eq. (I'a$_2$) used on the right hand side. We write (18a)

$$i\,(\partial_z e_x)_0 = i\left(a'_x\right)_0, \ \left(a'_x\right)_0 = \mu\left(\partial_z e'_x\right)_0 + \gamma\xi\eta^{-1}k\mu\left(e'_x\right)_0 + \gamma\xi\varepsilon^{-1}\left(h'_z\right)_0 \tag{19}$$

in which according to Eqs. (I'a$_{3,4}$):

$$\left(h'_z\right)_0 = (k\eta)^{-1}(\eta^2 + \xi^2)\left(e'_x\right)_0 \tag{19a}$$

Substituting (17) into the left hand side of (19) gives the second relation

$$(\lambda_+ + \lambda_-)(B_x - A_x) = i\left(a'_x\right)_0 \tag{20}$$

and we get from (17a) and (20) with $2\lambda^* = \lambda_+ + \lambda_-$

$$4B_x = \left(e'_x\right)_0 + i\left(a'_x\right)_0/\lambda^*, \quad 4A_x = \left(e'_x\right)_0 - i\left(a'_x\right)_0/\lambda^* \tag{21}$$

But, we have still to impose the proper boundary conditions on the perfectly conducting substrate located at $z = -d$

$$[e_{x,y}(\rho, z)]_{z=-d} = 0 \tag{22}$$

that is using (17) and the corresponding expression for e_y

$$A_{x,y}\exp^*(-i\lambda d) + B_{x,y}\exp^*(i\lambda d) = 0 \tag{22a}$$

Substituting (21) into (22a) gives the impedance boundary condition satisfied by the component $e'_x(\rho, z)$ of the total field incident plus scattered on the chiral film

$$\left[\lambda^* e'_x(\rho, z)\cos^*(\lambda d) - a'_x(\rho, z)\sin^*(\lambda d)\right]_0 = 0 \tag{23}$$

in which $[a'_x(\rho, z)]_0$ is the expression (19), $2\lambda^* = \lambda_+ + \lambda_-$ and

$$\cos^*(\lambda d) = \cos(\lambda_+ d) + \cos(\lambda_- d), \quad \sin^*(\lambda d) = \sin(\lambda_+ d) + \sin(\lambda_- d) \tag{23a}$$

The component $e'_y(\rho, z)$ satisfies a similar boundary condition

$$\left[\lambda^* e'_y(\rho, z)\cos^*(\lambda d) - a'_y(\rho, z)\sin^*(\lambda d)\right]_0 = 0 \tag{24}$$

and, proceeding as for the x-component, we get easily from Eqs. (Ia$_1$), (14), (14a), 15) and using the relation $(h_x - h'_x)_0 = 0$

$$\left[a'_y(\rho, z)\right]_0 = \mu\left(\partial_z e'_y\right)_0 - \gamma\eta\xi^{-1}k\mu\left(e'_y\right)_0 + \gamma\eta\varepsilon^{-1}\left(h'_z\right)_0 \tag{25}$$

in which according to Eqs. (I'a$_{3,4}$)

$$\left(h'_z\right)_0 = -(k\xi)^{-1}(\eta^2 + \xi^2)\left(e'_y\right)_0 \tag{25a}$$

There is no connection between the boundary conditions on the x and y components of the electric field.

3.2. Vertical Polarization

For a vertical polarization $h'_z = 0$ and the Fourier transform of the Maxwell equations (6b) have the explicit form

$$\begin{aligned} i\eta e'_z - \partial_z e'_y &= -ikh'_x \\ \partial_z e'_x - i\xi e'_z &= -ikh'_y \\ \xi e'_y - \eta e'_x &= 0 \\ i\xi e'_x + i\eta e'_y + \partial_z e'_z &= 0 \end{aligned} \tag{II'a}$$

$$\begin{aligned} \partial_z h'_y &= -ike'_x \\ \partial_z h'_x &= ike'_y \\ \xi h'_y - \eta h'_x &= ke'_z \\ \xi h'_x + \eta h'_y &= 0 \end{aligned} \tag{II'b}$$

As proved in the introduction, we only need the boundary conditions on the components $h'_{x,y}$ of the incident magnetic field but to satisfy (22) on the metallic substrate, it is easier to look first for the boundary conditions on $e'_{x,y}$. For the components e'_z and h'_z we have now

$$\left[e'_z - \varepsilon e_z\right]_0 = 0, \qquad \left[i\gamma h'_z - \varepsilon h_z\right]_0 = 0 \tag{26}$$

and we write

$$e_x(\rho, z) = C_x(\rho)\exp^*(i\lambda z) + D_x(\rho)\exp^*(-i\lambda z) \tag{27}$$

in which according to the relation $(e_x - e'_x)_0 = 0$

$$2(C_x + D_x) = \left(e'_x\right)_0 \tag{27a}$$

A second relation to determine C_x, D_x, is still obtained by using Eq. (18) but here, instead of (14a), we get from Eqs. (II'a$_3$), (II'b$_4$)

$$(\xi e_y - \eta e_x)_0 = 0, \qquad (\xi h_x + \eta h_y)_0 = 0 \tag{28}$$

So, still using (14) together with (26) and (28), the relation (18) becomes

$$i\left(\partial_z e_x\right)_0 = k\mu\left(h'_y\right)_0 - i\gamma\eta\xi^{-1}k\mu\left(e'_x\right)_0 - \xi\varepsilon^{-1}\left(e'_z\right)_0 \tag{29}$$

in which according to Eq. (II'a$_2$)

$$k\mu\left(h'_y\right)_0 = i\mu\left(\partial_z e'_x\right)_0 + \mu\xi\left(e'_z\right)_0 \tag{29a}$$

Taking into account (29a) we may write (29)

$$i\left(\partial_z e_x\right)_0 = i\left(b'_x\right)_0,\qquad \left(b'_x\right)_0 = \mu\left(\partial_z e'_x\right)_0 - \gamma\eta\xi^{-1}k\mu\left(e'_x\right)_0 + i\xi\left(\varepsilon^{-1}-\mu\right)\left(e'_z\right)_0 \tag{30}$$

Substituting (27) into (30) gives the second relation satisfied by C_x and D_x

$$(\lambda_+ + \lambda_-)(D_x - C_x) = i\left(b'_x\right)_0 \tag{31}$$

Then, comparing (27a), (31) to (17a), (20) shows that we have just to change a'_x into b'_x in (23) and similarly a'_y into b'_y in (24) to get the boundary conditions satisfied by $e'_{x,y}$

$$\left[\lambda^* e'_{x,y}(\rho,z)\cos^*(\lambda d) - b'_{x,y}(\rho,z)\sin^*(\lambda d)\right]_0 = 0 \tag{32}$$

in which $[b'_x(\rho,z)]_0$ is the expression (30) while a calculation similar to the previous one gives

$$i\left(\partial_z e_y\right)_0 = i\left(b'_y\right)_0,\qquad \left(b'_y\right)_0 \equiv \left[b'_y(\rho,z)\right]_0 = -\mu\left(\partial_z e'_y\right)_0 + \gamma\xi\eta^{-1}k\mu\left(e'_y\right)_0 + i\eta(\varepsilon^{-1}+\mu)\left(e'_z\right)_0 \tag{33}$$

But, we are interested to the boundary conditions for the components $h'_{x,y}$ and taking into account the two equations (II'b$_{1,2}$), we get

$$\begin{aligned}&\left[ik^{-1}\lambda^*\partial_z h'_y(\rho,z)\cos^*(\lambda d) - b'_x(\rho,z)\sin^*(\lambda d)\right]_0 = 0\\ &\left[ik^{-1}\lambda^*\partial_z h'_x(\rho,z)\cos^*(\lambda d) + b'_y(\rho,z)\sin^*(\lambda d)\right]_0 = 0\end{aligned} \tag{34}$$

in which using the equations (II'a$_{1,2}$) and (II'b$_{3,4}$), we may write the expressions (30) and (33) of $b'_{x,y}$

$$\begin{aligned}\left[b'_x(\rho,z)\right]_0 &= -ik\mu\left(h'_y\right)_0 - i\gamma\eta\xi^{-1}k\mu\left(\partial_z h'_y\right)_0 + i(k\varepsilon)^{-1}(\eta^2+\xi^2)\left(h'_y\right)_0\\ \left[b'_y(\rho,z)\right]_0 &= -ik\mu\left(h'_x\right)_0 - i\gamma\xi\eta^{-1}k\mu\left(\partial_z h'_x\right)_0 - i(k\varepsilon)^{-1}(\eta^2+\xi^2)\left(h'_x\right)_0\end{aligned} \tag{34a}$$

so that the boundary conditions for the x, y-components of the magnetic field depend only on h'_x, $\partial_z h'_x$ and h'_y, $\partial_z h'_y$ respectively.

3.3. Electromagnetic Wave Scattering on 1D-Chiral Films

We now assume that the electromagnetic fields outside and inside the chiral film do not depend upon the coordinate y so that $\eta = 0$ in the Fourier transforms of Maxwell's equations. Then, for TM waves only the components e'_y, h'_x, h'_z are nonnull and the Maxwell equations (I'a,b) reduce to

$$\begin{aligned} \partial_z e'_y - ikh'_x &= 0 \\ \xi e'_y + kh'_z &= 0 \end{aligned} \tag{III'a}$$

$$\begin{aligned} \partial_z h'_x - i\xi h'_z &= ike'_y \\ i\xi h'_x + \partial_z h'_z &= 0 \end{aligned} \tag{III'b}$$

The boundary conditions are

$$\left[e_y(\xi, z) - e'_y(\xi, z)\right]_0 = 0, \quad \left[h_x(\xi, z) - h'_x(\xi, z)\right]_0 = 0 \tag{35}$$

while taking into account (35), we get from Eq. (9) written on $z = 0$ with $\eta = 0$

$$k\varepsilon \left[e_z(\xi, z)\right]_0 - i\gamma\xi \left[e'_y(\xi, z)\right]_0 = 0, \quad k\varepsilon \left[h_z(\xi, z)\right]_0 + \beta\xi \left[e'_y(\xi, z)\right]_0 = 0 \tag{36}$$

We now write

$$e_y(\xi, z) = A_y(\xi) \exp^*(i\lambda z) + B_y(\xi) \exp^*(-i\lambda z) \tag{37}$$

in which according to (35)

$$2[A_y(\xi) + B_y(\xi)] = \left[e'_y(\xi, z)\right]_0 \tag{37a}$$

We now get from Eq. (Ia_1) with $\eta = 0$

$$\left[\partial_z e_y(\xi, z)\right]_0 = ik\mu \left[h'_x(\xi, z)\right]_0 \tag{38}$$

and taking into account Eq. ($III'a_1$),

$$\left[\partial_z e_y(\xi, z)\right]_0 = \mu \left[\partial_z e'_y(\xi, z)\right]_0 \tag{38a}$$

Using (37) in the left hand side of (38a) gives

$$i(\lambda_+ + \lambda_-)[A_y(\xi) - B_y(\xi)] = \mu \left[\partial_z e'_y(\xi, z)\right]_0 \tag{39}$$

Substituting A_y, B_y obtained from (37a) and (39) into (37) and using the boundary condition $\left[e_y(\xi, z)\right]_{z=-d} = 0$ on the metallic substrate supplies the impedance boundary condition satisfied by $e'_y(\xi, z)$ on the 1D-chiral film

$$\left[\lambda^* e'_y(\xi, z) \cos^*(\lambda d) + \mu \partial_z e'_y(\xi, z) \sin^*(\lambda d)\right]_0 = 0 \tag{40}$$

Similarly for a TE wave, the nonnull components are h'_y, e'_x, e'_z, so that with $\eta = 0$ the Maxwell equations (II'a) reduce to

$$\begin{aligned} \partial_z e'_x - i\xi e'_z &= -ikh'_y \\ i\xi e'_x + \partial_z e'_z &= 0 \end{aligned} \tag{IV'a}$$

$$\begin{aligned} \partial_z h'_y + ike'_x &= 0 \\ \xi h'_y - ke'_z &= 0 \end{aligned} \tag{IV'b}$$

The boundary conditions are now

$$\left[e_x(\xi, z) - e'_x(\xi, z)\right]_0 = 0, \quad \left[h_y(\xi, z) - h'_y(\xi, z)\right]_0 = 0 \tag{41}$$

and we get from Eqs. (9) and (41)

$$k\varepsilon \left[e_z(\xi, z)\right]_0 - \xi \left[h'_y(\xi, z)\right]_0 = 0, \quad k\varepsilon \left[h_z(\xi, z)\right]_0 - i\gamma\xi \left[h'_y(\xi, z)\right]_0 = 0 \tag{41a}$$

Then, $e_x(\xi, z)$ has the representation

$$e_x(\xi, z) = C_x(\xi) \exp^*(i\lambda z) + D_x(\xi) \exp^*(-i\lambda z) \tag{42}$$

which according to (41) and (IV'b$_1$)

$$2[C_x(\xi) + D_x(\xi)] = \left[e'_x(\xi, z)\right]_0 = ik^{-1} \left[\partial_z h'_y(\xi, z)\right]_0 \tag{42a}$$

Now, we get from Eq. (Ia$_2$) written on $z = 0$ together with (41) and (41a)

$$\begin{aligned} \left[\partial_z e_x(\xi, z)\right]_0 &= i\xi \left[e_z(\xi, z)\right]_0 - ik\mu \left[h'_y(\xi, z)\right]_0 \\ &= i(\xi^2/k\varepsilon - k\mu) \left[h'_y(\xi, z)\right]_0 \end{aligned} \tag{43}$$

which gives taking into account (42)

$$(\lambda_+ + \lambda_-)[C_x(\xi) - D_x(\xi)] = (\xi^2/k\varepsilon - k\mu) \left[h'_y(\xi, z)\right]_0 \tag{43a}$$

Then, substituting C_x and D_x obtained from (42a) and (43a) into (42) and using the boundary condition on the metallic substrate $\left[e_y(\xi, z)\right]_{z=-d} = 0$ gives the impedance boundary condition satisfied by $h'_y(\xi, z)$ on the chiral film

$$\left[\lambda^* k^{-1} \partial_z h'_y(\xi, z) \cos^*(\lambda d) + i(\xi^2/k\varepsilon - k\mu) h'_y(\xi, z) \sin^*(\lambda d)\right]_0 = 0 \tag{44}$$

So, the impedance boundary conditions (40) and (44) do not depend on the chirality factor γ and in addition, the expression (40) has a simple inverse Fourier transform

$$\left[\lambda^* E'_y(x,z)\cos^*(\lambda d) + \mu\partial_z E'_y(x,z)\sin^*(\lambda d)\right]_0 = 0 \tag{45}$$

which is a Robin-Leontovich boundary condition, and calculations are easier with TM than with TE 2D-fields.

4. ARBITRARY POLARIZED INCIDENT FIELD

In this case, we have to work with the full Maxwell equations in free space and their Fourier transform is

$$\begin{aligned} i\eta e'_z - \partial_z e'_y &= -ikh'_x \\ \partial_z e'_x - i\xi e'_z &= -ikh_{y'} \\ \xi e'_y - \eta e'_x &= -kh'_z \\ i\xi e'_x + i\eta e'_y + \partial_z e'_z &= 0 \end{aligned} \tag{V'a}$$

$$\begin{aligned} i\eta h'_z - \partial_z h'_y &= ike'_x \\ \partial_z h'_x - i\xi h'_z &= ike_{y'} \\ \xi h'_y - \eta h'_x &= -ke'_z \\ i\xi h'_x + i\eta h'_y + \partial_z h'_z &= 0 \end{aligned} \tag{V'b}$$

The relations (14) are the boundary conditions for the x, y-components of the electric and magnetic fields and we get from (9) and (14)

$$\begin{aligned} k\varepsilon\,(h_z)_0 &= \beta\left(\eta e'_x - \xi e'_y\right)_0 - i\gamma\left(\eta h'_x - \xi h'_y\right)_0 \\ k\varepsilon\,(e_z)_0 &= \left(\xi h'_y - \eta h'_x\right)_0 + i\gamma\left(\xi e'_y - \eta e'_x\right)_0 \end{aligned} \tag{46}$$

Now, by using the Maxwell equations, we may prove that all the components of the electric and magnetic fields can be expressed in terms of their y-components. Taking into account (46), we get from Eqs. (Ia,b_1)

$$\begin{aligned} i\eta\xi(h_y + i\gamma e_y) - k\varepsilon\partial_z e_y &= -i(k^2\varepsilon\mu - \eta^2)h_x - \gamma(k^2\varepsilon\mu + \eta^2)e_x \\ i\eta\xi(\beta e_y - i\gamma h_y) + k\varepsilon\partial_z h_y &= \gamma(k^2\varepsilon\mu + \eta^2)h_x - i\beta(k^2\varepsilon\mu - \eta^2)e_x \end{aligned} \tag{47}$$

from which we obtain easily e_x and h_x, in particular:

$$e_x = \alpha_1 e_y + \alpha_2 h_y + \alpha_3\partial_z e_y + \alpha_4\partial_z h_y \tag{47a}$$

with the α-coefficients given in Appendix A.

4.1. Impedance Boundary Condition on e'_y

As in Sec. 3 we write $e_y(\rho, z)$

$$e_y(\rho, z) = A_y(\rho) \exp(^* i\lambda z) + B_y(\rho) \exp^*(-i\lambda z) \tag{48}$$

in which according to (14)

$$2(A_y + B_y) = \left(e'_y\right)_0 \tag{48a}$$

Now, from Eq. (Ia$_1$) written on $z = 0$ and from (14) we get

$$(\partial_z e_y)_0 = \left(a'_y\right)_0, \qquad \left(a'_y\right)_0 = i\eta \left(e_z\right)_0 + ik\mu \left(h'_x\right)_0 + k\gamma\mu \left(e'_x\right)_0 \tag{49}$$

where $(e_z)_0$ has the expression (48) and taking into account (48), we obtain a second relation to determine A_y, B_y

$$i(\lambda_+ + \lambda_-)(A_y - B_y) = \left(a'_y\right)_0 \tag{49a}$$

so that from (48a) and (49a)

$$4A_y = \left(e'_y\right)_0 - i\lambda^* \left(a'_y\right)_0, \quad 4B_y = \left(e'_y/2\right)_0 + i\lambda^* \left(a'_y\right)_0 \tag{50}$$

and substituting (50) into (22a) gives the impedance boundary condition on e'_y

$$\left[\lambda^* e'_y(\rho, z) \cos^*(\lambda d) - a'_y(\rho, z) \sin^*(\lambda d)\right]_0 = 0 \tag{51}$$

But, we have still to express a'_y in terms of e'_y, h'_y, $\partial_z e'_y$, $\partial_z h'_y$, and we get at once from Eqs. (V'a$_1$), (V'b$_3$) on one hand and from Eqs.(V'b$_1$), (V'a$_3$) on the other hand

$$(k^2 - \eta^2)h'_x = -\eta\xi h'_y - ik\partial_z e'_y, \quad (k^2 - \eta^2)e'_x = \eta\xi e'_y + ik\partial_z h'_y \tag{52}$$

so that according to (46)

$$k\varepsilon \left(e_z\right)_0 = (k^2 - \eta^2) \left[k^2\xi(h'_y + i\gamma e'_y) - ik\eta(\partial_z e'_y - i\gamma\partial_z h'_y)\right]_0 \tag{52a}$$

Substituting (52) and (52a) into (49) achieves to determine $a'(\rho, z)$ in terms of the y-components of the electric and magnetic fields. So, for an arbitrary polarization, the impedance boundary conditions for e'_y and h'_y are coupled which is not the case for horizontal and vertical polarizations.

4.2. Impedance Boundary Conditions on h'_y

We now write

$$h_y(\rho, z) = C_y(\rho)\exp^*(i\lambda z) + D_y(\rho)\exp^*(-i\lambda z) \tag{53}$$

in which according to (14)

$$2(C_y + D_y) = \left(h'_y\right)_0 \tag{53a}$$

In addition, from Eq. (I'b$_1$) written on $z = 0$ and from (14), we get

$$(\partial_z h_y)_0 = \left(b'_y\right)_0, \qquad \left(b'_y\right)_0 = i\eta\,(h_z)_0 - ik\mu\beta\left(e'_x\right)_0 + k\gamma\mu\left(h'_x\right)_0 \tag{54}$$

Substituting (46) and (52) into (54) gives $\left(b'_y\right)_0$ in terms of e'_y, h'_y, $\partial_z e'_y$, $\partial_z h'_y$ and taking into account (53), we obtain a second relation to determine A_y, B_y

$$i(\lambda_+ + \lambda_-)(C_y - D_y) = \left(b'_y\right)_0 \tag{54a}$$

so that from (53a) and (54a)

$$4C_y = \left(h'_y\right)_0 - i\lambda^*\left(b'_y\right)_0, \qquad 4D_y = \left(h'_y\right)_0 + i\lambda^*\left(a'_y\right)_0 \tag{55}$$

But we need e_x to satisfy the boundary condition (22) on the metallic substrate and, using (48) and (53), we get from (47a)

$$\begin{aligned} e_x(\rho, z) \;=\; & [\alpha_1 A_y(\rho) + \alpha_2 C_y(\rho)]\exp^*(i\lambda z) \\ & + i(\lambda_+ + \lambda_-)[\alpha_3 B_y(r) + \alpha_4 D_y(\rho)]\exp^*(-i\lambda z) \end{aligned} \tag{56}$$

Substituting (56) into (22) gives the impedance boundary condition

$$\begin{aligned} & [\alpha_1 A_y(\rho) + \alpha_2 C_y(\rho)]\exp^*(-i\lambda d) \\ & + i(\lambda_+ + \lambda_-)[\alpha_3 B_y(r) + \alpha_4 D_y(\rho)]\exp^*(i\lambda d) = 0 \end{aligned} \tag{57}$$

in which the α-parameters are given in Appendix A while the amplitudes A_y, B_y and C_y, D_y are supplied respectively by the relations (50) and (55).

5. TE WAVE SCATTERING

Let the TE wave (6a) impinge on a chiral film with thickness d located at $z = 0$. Then, the components $e'_{x,y}$ of the total field incident plus scattered in the half space $z \geq 0$ satisfy the boundary conditions (23) and (24) that we write:

$$\left[\partial_z e'_{x,y}(\xi, \eta, z) + s_{x,y}(\xi, \eta) e'_{x,y}(\xi, \eta, z)\right]_0 = 0 \tag{58}$$

in which with $\rho^2 = \xi^2 + \eta^2$

$$\begin{aligned} s_x(\xi,\eta) &= -\mu\lambda^* \cot^*(\lambda d) + \gamma k\xi\eta^{-1}(1+\rho^2/k^2\varepsilon\mu) \\ s_y(\xi,\eta) &= -\mu\lambda^* \cot^*(\lambda d) - \gamma k\eta\xi^{-1}(1+\rho^2/k^2\varepsilon\mu) \end{aligned} \tag{58a}$$

We note $e^i_{x,y}$, $e^s_{x,y}$, the x, y-components of the incident and scattered electric fields in $z \geq 0$ so that $e'_{x,y} = e^i_{x,y} + e^s_{x,y}$ with according to (6a) and (7)

$$\left\{e^i_x(\xi,\eta,z), e^i_y(\xi,\eta,z)\right\} = 4\pi^2\{\sin\phi, \cos\phi\} \exp(ik_z z)\delta(\xi+k_x)\delta(\eta+k_y) \tag{59}$$

in which δ is the Dirac distribution. So, we may write the total field:

$$\begin{aligned} \left\{e'_x(\xi,\eta,z), e'_y(\xi,\eta,z)\right\} = 8\pi^2\{\sin\phi,\cos\phi\}[A_{x,y}(\xi,\eta)\cos(k_z z) \\ +B_{x,y}(\xi,\eta)\sin(k_z z)]\delta(\xi+k_x)\delta(\eta+k_y) \end{aligned} \tag{60}$$

and the amplitudes $A_{x,y}$, $B_{x,y}$ must fulfill two constraints: the boundary conditions (58) have to be satisfied implying

$$k_z B_{x,y}(\xi,\eta) + s_{x,y}(\xi,\eta)A_{x,y}(\xi,\eta) = 0 \tag{61a}$$

and the $\exp(ik_z z)$ term in (60) is the incident field (59), so:

$$A_{x,y}(\xi,\eta) - iB_{x,y}(\xi,\eta) = 0 \tag{61b}$$

We get from (61a,b)

$$\begin{aligned} A_{x,y}(\xi,\eta) &= k_z\left[k_z + is_{x,y}(\xi,\eta)\right]^{-1}, \\ B_{x,y}(\xi,\eta) &= -s_{x,y}(\xi,\eta)\left[k_z + is_{x,y}(\xi,\eta)\right]^{-1} \end{aligned} \tag{62}$$

Then, the $\exp(-ik_z z)$ term in (60) represents the scattered field

$$\begin{aligned} \left\{e^s_x(\xi,\eta,z), e^s_y(\xi,\eta,z)\right\} &= 4\pi^2\{\sin\phi,\cos\phi\}m_{x,y}(\xi,\eta) \\ &\quad \times \exp(-ik_z z)\delta(\xi+k_x)\delta(\eta+k_y) \end{aligned} \tag{63}$$

with according to (62)

$$m_{x,y}(\xi,\eta) = [k_z - is_{x,y}(\xi,\eta)][k_z + is_{x,y}(\xi,\eta)]^{-1} \tag{63a}$$

and the inverse Fourier transform of (63) is

$$\left\{E^s_x(\mathbf{x}), E^s_y(\mathbf{x})\right\} = \{\sin\phi,\cos\phi\}m_{x,y}(-k_x,-k_y)\exp(ik_x x + ik_y y - ik_z z) \tag{64}$$

the amplitude of the scattered wave depends on the wave length and on the angle of incidence.

We now suppose that instead of the harmonic plane wave (5), the incident field is the finite width beam

$$\begin{aligned}\Psi^i(\mathbf{x}) \;=\; & kb\pi^{-1/2}\int_{-\pi/2}^{\pi/2} d\theta \exp(-k^2b^2\theta^2)\\ & \times\exp[-ik\sin\theta(x\cos\phi - y\sin\phi) + ik\cos\theta z)] \end{aligned} \tag{65}$$

which is a 3D-generalization of a beam used to analyze wave scattering from 2D-random gratings [6] and which represents in the limit of large kb a Gaussian beam whose angle of incidence measured from the normal to the surface $z = 0$ is θ. Now, since the contribution to this integral comes from the values of θ smaller than $1/kb$, small for kb large, we may replace the finite limits of integration by infinite limits and approximate $\sin\theta$, $\cos\theta$ by θ and $1 - \theta^2/2$ respectively. Then, the expression (65) becomes

$$\begin{aligned}\Psi^i(\mathbf{x}) \;\approx\; & kb\pi^{-1/2}\exp(ikz)\int_{-\infty}^{\infty} d\theta \exp[-k^2b^2\theta^2 w(z)]\\ & \times\exp[-ik\theta(x\cos\phi - y\sin\phi)] \end{aligned} \tag{66}$$

$$w(z) = 1 + iz/2kb^2 \tag{66a}$$

and using the well known integral

$$\int_{-\infty}^{\infty} dx \exp(-\sigma^2x^2 + itx) = \pi^{1/2}\sigma^{-1}\exp(-t^2/4\sigma^2) \tag{67}$$

we get

$$\Psi^i(\mathbf{x}) \approx w^{-1/2}(z)\exp(ikz)\exp[-(x\cos\phi - y\sin\phi)^2/4b^2(w(z)] \tag{68}$$

Introducing the variables $X = x\cos\phi - y\sin\phi$, $Y = x\sin\phi + y\cos\phi$ and still using (67), the Fourier transform of (68) is

$$\psi^i(\xi,\eta,z) = 4b\pi^{3/2}\delta(\xi\sin\phi + \eta\cos\phi)\exp[-b^2w(z)(\xi\cos\phi - \eta\sin\phi)^2] \tag{69}$$

and according to (6a), the Fourier transforms of the x, y-components of the incident electric field are

$$\begin{aligned}\left\{e_x^i(\xi,\eta,z), e_y^i(\xi,\eta,z)\right\} = & 4b\pi^{3/2}\{\sin\phi, \cos\phi\}\exp(ikz)\delta(\xi\sin\phi + \eta\cos\phi)\\ & \times\exp[-b^2w(z)(\xi\cos\phi - \eta\sin\phi)^2] \end{aligned} \tag{70}$$

and we write these components for the total field

$$\left\{e'_x(\xi,\eta,z), e'_y(\xi,\eta,z)\right\} = 8b\pi^{3/2}\{\sin\phi, \cos\phi\}[A_{x,y}(\xi,\eta)\cos(kz) + B_{x,y}(\xi,\eta)\sin(kz)]\delta(\xi\sin\phi + \eta\cos\phi) \times \exp[-b^2 w(z)(\xi\cos\phi - \eta\sin\phi)^2] \quad (71)$$

and, the same constraints as previously imposed on (71) give the relation (61a,b) with k_z changed into k so that

$$\begin{aligned} A_{x,y}(\xi,\eta) &= k[k + is_{x,y}(\xi,\eta)]^{-1}, \\ B_{x,y}(\xi,\eta) &= -s_{x,y}(\xi,\eta)[k + is_{x,y}(\xi,\eta)]^{-1} \end{aligned} \quad (72)$$

Then, the components $e^s_{x,y}$ of the scattered electric field are

$$\left\{e^s_x(\xi,\eta,z), e^s_y(\xi,\eta,z)\right\} = 4b\pi^{3/2}\{\sin\phi, \cos\phi\} m_{x,y}(\xi,\eta)\exp(-ikz) \times\delta(\xi\sin\phi + \eta\cos\phi) \times \exp[-b^2 w(z)(\xi\cos\phi - \eta\sin\phi)^2] \quad (73)$$

in which $m_{x,y}(\xi,\eta)$ is the expression (63) with k instead of k_z.

Using the variables $\zeta = \xi\sin\phi + \eta\cos\phi$, $\nu = \xi\cos\phi - \sin\phi$, the inverse Fourier transform of (73) is

$$\begin{aligned} \left\{E^s_x(\mathbf{x}), E^s_y(\mathbf{x})\right\} &= b\pi^{-1/2}\{\sin\phi, \cos\phi\}\exp(-ikz)\iint_{-\infty}^{\infty} d\zeta d\nu \delta(\zeta) \\ &\times \exp[-ix(\zeta\sin\phi + \nu\cos\phi) - iy(\zeta\cos\phi - \nu\sin\phi)] \\ &\times m_{x,y}(\zeta\sin\phi + \nu\cos\phi, \zeta\cos\phi - \nu\sin\phi)\exp[-b^2 w(z)\nu^2] \\ &= b\pi^{-1/2}\{\sin\phi, \cos\phi\}\exp(-ikz) \\ &\int_{-\infty}^{\infty} d\nu \exp[-i\nu(x\cos\phi - y\sin\phi)] \\ &\times m_{x,y}(\nu\cos\phi, -\nu\sin\phi)\exp[-b^2 w(z)\nu^2] \end{aligned} \quad (74)$$

The inverse Fourier transform (74) in which according to (58a) and (62a) $m_{x,y}$ is a function of ν^2 has to be performed numerically and there now exist different powerful techniques to do this job efficiently [7].

We would obtain similar results for TM wave scattering but calculations are more intricate for an arbitrarily polarized electromagnetic field.

6. DISCUSSION

The impedance boundary conditions obtained in this work are exact for plane waves impinging on infinite smooth slabs and their use in wave scattering, for instance from wedges or half-slabs can only be considered as approximations.And, the quality of these approximations has to be checked on some simple problems by comparison with the solutions obtained by other analytical (think of the Raman-Krishnan solution [8] of the Sommerfeld diffraction problem) or purely numerical techniques. Idemen [1] insists strongly on this point which seems to be ignored by some authors.

But, as noticed in the introduction, plane wave scattering from infinite flat surfaces is important because its solutions can be used as first order approximations in scattering problems on convex solids large compared with wavelength. Now, experimental conditions are more closely approximate by rough than by smooth surfaces with a roughness function, periodic, random or fractal of small amplitude also compared with the wavelength. Then, a natural question is: how is it possible to use impedance boundary conditions to investigate wave scattering from infinite rough surfaces such as for instance a dielectric or chiral film deposited on a metallic substrate? Work is in progress to answer this question.

APPENDIX A.

We write (47)

$$\begin{aligned} i\eta\xi h_y - \gamma\eta\xi e_y - k\varepsilon\partial_z e_y &= -i\nu_- h_x - \gamma\nu_+ e_x \\ i\eta\xi\beta e_y + \gamma\eta\xi h_y + k\varepsilon\partial_z h_y &= \gamma\nu_+ h_x - i\beta\nu_- e_x \end{aligned} \tag{A1}$$

with

$$\nu_\pm = k^2\varepsilon\mu \pm \eta^2 \tag{A2}$$

A simple calculation gives

$$i\gamma\eta\xi(\nu_+ + \nu_-)h_y - \eta\xi(\gamma^2\nu_+ + \beta\nu_-)e_y - k\varepsilon\gamma\nu_+\partial_z e_y + ik\varepsilon\nu_-\partial_z h_y = (\beta\nu_-^2 - \gamma^2\nu_+^2)e_x \tag{A3}$$

so that the α-coefficients in (47a) are since $\nu_+ + \nu_- = 2k^2\varepsilon\mu$

$$\begin{aligned} \alpha_1 &= -\eta\xi(\gamma^2\nu_+ + \beta\nu_-)(\beta\nu_-^2 - \gamma^2\nu_+^2)^{-1}, \quad \alpha_2 = 2i\gamma\eta\xi k^2\varepsilon\mu(\beta\nu_-^2 - \gamma^2\nu_+^2)^{-1} \\ \alpha_3 &= -k\varepsilon\gamma\nu_+(\beta\nu_-^2 - \gamma^2\nu_+^2)^{-1}, \quad \alpha_4 = ik\varepsilon\nu_-(\beta\nu_-^2 - \gamma^2\nu_+^2)^{-1} \end{aligned} \tag{A4}$$

REFERENCES

1. Idemen, M., "Universal boundary conditions and Cauchy data for the electromagnetic fields," *Essay on the Formal Aspects of Electromagnetic Theory*, A. Lakhtakia (ed.), World Scientific, Singapore, 1993.
2. Jaggard, D. L. and N. Engheta, "Chirosorb as invisible medium," *Electrn. Lett.*, Vol. 25, 173–174, 1984.
3. Bassiri, S., C. H. Papas, and N. Engheta, "Electromagnetic wave propagation through a dielectric chiral interface and through a chiral slab," *J. Opt. Soc. Am.*, Vol. A.5, 1450–1459, 1988.
4. Bassiri, S., "Electromagnetic wave propagation and radiation in chiral media," Ph.D. Thesis, Cal. Tech., Pasadena, 1987.
5. Kelso, J. M., *Radiowave Propagation in the Ionosphere*, McGraw Hill, New York, 1991.
6. Maradudin, A. A., T. Michel, A. R. McGurn, and E. R. Mendez, "Enhanced back scattering of light from a random grating," *Ann. Phys.*, Vol. 203, 255–307, 1990.
7. Dahlquist, G., "A multigrid extension of FFT," *BIT*, Vol. 33, 85–103, 1993.
8. Raman, C. V. and K. S. Krishnan, "The diffraction of light by metallic screens," *Proc. Roy. Soc A*, Vol. CXVI, 254–265, 1927.