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# DIFFRACTION BY A TERMINATED SEMI-INFINITE PARALLEL PLATE WAVEGUIDE WITH TWO-LAYER MATERIAL LOADING AND IMPEDANCE BOUNDARIES

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**Abstract**—The plane wave diffraction by a terminated semi infinite parallel-plate waveguide with two-layer material loading and impedance boundaries is rigorously analyzed for $E$ polarization using the Wiener-Hopf technique. Introducing the Fourier transform for the scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the simultaneous Wiener-Hopf equations, which are solved via the factorization and decomposition procedure. The scattered field is evaluated by taking the inverse Fourier transform and applying the saddle point method. The numerical examples of the radar cross section (RCS) are represented for various physical parameters and backscattering characteristics, of considered geometry for open ended cavity and discussed in detail.

# 1. INTRODUCTION

The problem of electromagnetic wave scattering by open-ended parallel plates waveguide cavities is an important topic in radar cross-section (RCS) prediction and reduction studies. This analysis is also one of the important canonical models for duct structures such as jet engine intakes of aircrafts and cracks occurring on surfaces of general complicated bodies. The scattering problem by cavities of various shapes has been investigated by using different analytical and numerical methods in literatures. The commonly used methods were high frequency ray techniques [1–4] and low frequency numerical methods [5, 6]. A hybrid ray-numerical approach has been also used [7]. We should note that the solutions obtained by these methods are not valid uniformly for arbitrary cavity dimensions.

A rigorous RCS analysis of two dimensional cavities with and without material loading having a finite, perfectly conducting parallel plate waveguide is carried out by Kobayashi and his co-workers by using the Wiener-Hopf technique [8–10]. As a related cavity geometry a terminated, semi-infinite parallel plate waveguide with three-layer material loading with perfectly conducting walls has been analyzed in [12] through the Wiener-Hopf technique. This problem has later on generalized for empty cavities having imperfectly conducting walls [11].

The aim of this work is to treat the diffraction of electromagnetic waves by a terminated semi-infinite parallel plate waveguide with two layer material loading where the cavity walls are impedance boundaries. This generalization of impedance boundaries is not openly given and merits to be investigated. The geometry of the problem is given in Figure 1. The original problem can be split up into two simpler one corresponding to even and odd excitations by using the image bisection method. Each sub problem is then analyzed by using the Wiener-Hopf technique in conjunction with the mode-matching method. Later this method of formulation, which is based on expansion of the diffracted field into the series of normal modes in the waveguide region and using the Fourier transform technique elsewhere, gives rise to a scalar modified Wiener-Hopf equation. After application of decomposition and factorization procedures, solution is obtained as a form of infinite number of constants satisfying an infinite system of linear algebraic equations. The numerical solution of this system is obtained for various values of the plate impedances, the distance between the plates, the cavity depth and the parameters of the dielectric loading.

A time factor $e^{-i\omega t}$ with $\omega$ being the angular frequency is assumed and suppressed throughout the paper.

## 2. FORMULATION OF THE PROBLEM

We consider the diffraction of a time harmonic, $E_z$-polarized plane electromagnetic wave by a parallel plates waveguide cavity formed by two impedance parallel half planes defined by; $S_1 = \{(x, y, z),\ x < 0,\ y = d,\ z \in (-\infty, \infty)\}$, $S_2 = \{(x, y, z),\ x < 0,\ y = -d,\ \ z \in (-\infty, \infty)\}$, respectively and a planar interior termination located at; $S_3 = \{(x, y, z),\ x = -\ell_2,\ y \in (-d, d),\ z \in (-\infty, \infty)\}$ as depicted in Fig. 1.

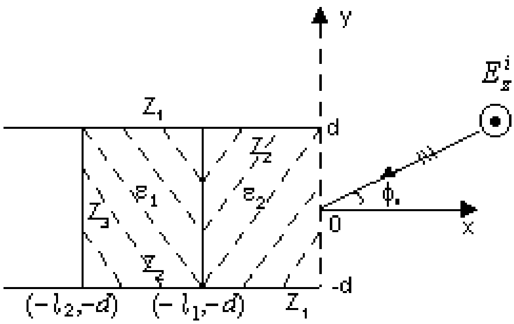


**Figure 1.** Parallel-plate waveguide with impedance walls and two layers impedance loading.

The surfaces $y = \pm d,\ x < 0$ of the horizontal walls of the cavity is assumed to be characterized by constant surface impedance $Z_1 = \eta_1 Z_0$ while the impedance of the inner surfaces $y = \pm d,\ x \in (-\ell_2, 0)$ is $Z_2 = \eta_2 Z_0$. The impedance of the planar interior termination $x = -\ell_2,\ y \in (-d, d)$ is denoted by $Z_3 = \eta_3 Z_0$. Here $Z_0$ is the characteristic impedance of the free space. We will further assume that the cavity is filled by a two layered non magnetic dielectric. Material the dielectric primitivities of the regions along $x \in (-\ell_2, -\ell_1)$ and $x \in (-\ell_1, 0)$ are $\varepsilon_1$ and $\varepsilon_2$, respectively.

In order to determine the scattered field, one can proceed by decomposing the incident in Fig. 2(a) and (b). Relying upon the image bisection principle, it can be shown that the configurations shown in Fig 2(a) and (b) are equivalent to presented figures in Fig. 2(c) and (d), respectively. In what follows, the even and odd excitations will be treated separately.

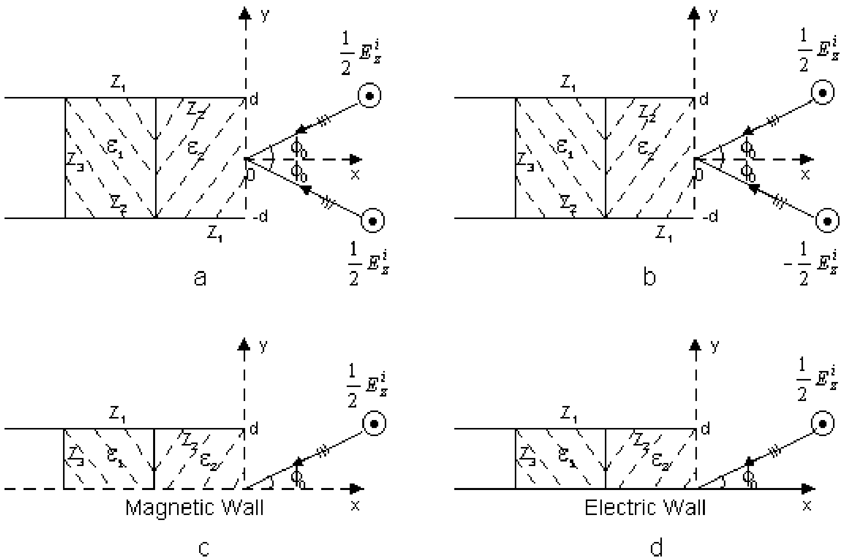


**Figure 2.** Equivalent problems. (a) Symmetric (even) excitation. (b) Asymmetric (odd) excitation. (c) Equivalence to (a). (d) equivalence to (b).

## 2.1. Symmetric (Even) Excitation

Let us consider first the configuration shown in Fig. 2(c), which is equivalent to the even excitation case. Since in this case the field is symmetrical about the plane $y = 0$, the normal derivative of the total electric field must vanish for $y = 0$, $x \in (-\infty, +\infty)$ (magnetic wall).

For analysis purposes, it is convenient to express the total field as follows:

$$u_T^{(e)}(x,y) = \begin{cases} u^i + u^\sigma + u_1^{(e)}; & y > d \\ u_2^{(1e)}[H(-x-\ell_1) - H(-x-\ell_2)] + & \\ u_2^{(2e)}[H(-x) - H(-x-\ell_1)] + u_2^{(3e)}[H(x)], & 0 < y < d \end{cases} \tag{1a}$$

Here, $u^i$ is the incident field given by,

$$E_z^i = u^i(x,y) = \exp\{-ik[x\cos\phi_0 + y\sin\phi_0]\} \tag{1b}$$

while $u^\sigma$ denotes the field reflected from the plane $y = d$, namely

$$u^\sigma = -\frac{1-\eta_1\sin\phi_0}{1+\eta_1\sin\phi_0}\exp\{-ik[x\cos\phi_0 - (y-2d)\sin\phi_0]\} \tag{1c}$$

with $k$ being the free-space wave number. $u^{(e)}$ and $u_2^{(ne)}$, $n = 1, 2, 3$, which satisfy the Helmholtz equation, are to be determined with the following boundary and continuity relations:

$$\left(1 + \frac{\eta_1}{ik}\frac{\partial}{\partial y}\right) u_1^{(e)}(x, d) = 0, \qquad y = d,\ x < 0 \tag{2a}$$

$$\frac{\partial}{\partial y} u_2^{(1e)} u(x, 0) = 0, \qquad y = 0, (-\ell_2 < x < -\ell_1) \tag{2b}$$

$$\frac{\partial}{\partial y} u_2^{(2e)}(x, 0) = 0, \qquad y = 0, (-\ell_1 < x < 0) \tag{2c}$$

$$\left(1 + \frac{\eta_3}{ik_1}\frac{\partial}{\partial x}\right) u_2^{(1e)}(-\ell_2, y) = 0, \qquad x = -\ell_2, (0 < y < d) \tag{2d}$$

$$\left(1 - \frac{\eta_2}{ik_1}\frac{\partial}{\partial y}\right) u_2^{(1e)}(x, d) = 0, \qquad y = d, (-\ell_2 < x < -\ell_1) \tag{2e}$$

$$\left(1 - \frac{\eta_2}{ik_2}\frac{\partial}{\partial y}\right) u_2^{(2e)}(x, d) = 0, \qquad y = d, (-\ell_1 < x < 0) \tag{2f}$$

$$\frac{\partial}{\partial y} u_2^{(3e)}(x, 0) = 0, \qquad y = 0,\ x > 0 \tag{2g}$$

$$u_2^{(1e)}(-\ell_1, y) = u_2^{(1e)}(-\ell_1, y), \qquad \{x = -\ell_1, (0 < y < d)\} \tag{2h}$$

$$\frac{\partial}{\partial x} u_2^{(1e)}(-\ell_1, y) = \frac{\partial}{\partial x} u_2^{(2e)}(-\ell_1, y), \qquad \{x = -\ell_1, (0 < y < d)\} \tag{2i}$$

$$u_2^{(2e)}(0, y) = u_2^{(3e)}(0, y), \qquad \{x = 0, (0 < y < d)\} \tag{2j}$$

$$\frac{\partial}{\partial x} u_2^{(2e)}(0, y) = \frac{\partial}{\partial x} u_2^{(3e)}(0, y), \qquad \{x = 0, (0 < y < d)\} \tag{2k}$$

$$u_1^{(e)}(x, d) - u_2^{(3e)}(x, d) = -\frac{2\eta_1 \sin\phi_0}{1 + \eta_1 \sin\phi_0} e^{-ikd\sin\phi_0} e^{-ikx\cos\phi_0}, \qquad \{y = d, x > 0\} \tag{2l}$$

$$\frac{\partial}{\partial y} u_1^{(e)}(x, d) - \frac{\partial}{\partial y} u_2^{(3e)}(x, d) = \frac{2ik \sin\phi_0}{1 + \eta_1 \sin\phi_0} e^{-ikd\sin\phi_0} e^{-ikx\cos\phi_0}, \qquad \{y = d, x > 0\} \tag{2m}$$

Since $u_1^{(e)}(x, y)$ satisfies the Helmholtz equation in the range of $x \in (-\infty, +\infty)$, its Fourier transform with respect to $x$ gives

$$\left[\frac{d^2}{dy^2} + (k^2 - \alpha^2)\right] F^{(e)}(\alpha, y) = 0 \tag{3a}$$

with

$$F^{(e)}(\alpha, y) = F_+^{(e)}(\alpha, y) + F_-^{(e)}(\alpha, y) \tag{3b}$$

where

$$F_\pm^{(e)}(\alpha, y) = \pm\frac{1}{2\pi}\int_0^{\pm\infty} u_1^{(e)}(x, y)e^{i\alpha x}dx \tag{3c}$$

By taking into account the following asymptotic behaviors of $u_1^{(e)}$ for $x \to \pm\infty$

$$u_1^{(e)}(x, y) = \begin{cases} O(e^{-ikx}); & x \to -\infty \\ O(e^{-ikx\cos\phi_0}); & x \to \infty \end{cases} \tag{4}$$

one can show that $F_+^{(e)}(\alpha, y)$ and $F_-^{(e)}(\alpha, y)$ are regular functions of $\alpha$ in the half-planes $\Im m(\alpha) > \Im m(k\cos\phi_0)$ and $\Im m(\alpha) < \Im m(k)$ respectively. The general solution of (3a), satisfying the radiation condition for $y \to \infty$, reads

$$F_+^{(e)}(\alpha, y) + F_-^{(e)}(\alpha, y) = A^{(e)}(\alpha)e^{iK(\alpha)(y-d)} \tag{5a}$$

with

$$K(\alpha) = \sqrt{k^2 - \alpha^2}; \quad K(0) = k \tag{5b}$$

The square-root function is defined in the complex $\alpha$ plane cut along $\alpha = k$ to $\alpha = k + i\infty$ and $\alpha = -k$ to $\alpha = -k - i\infty$, such that $K(0) = k$. See Fig. 3.

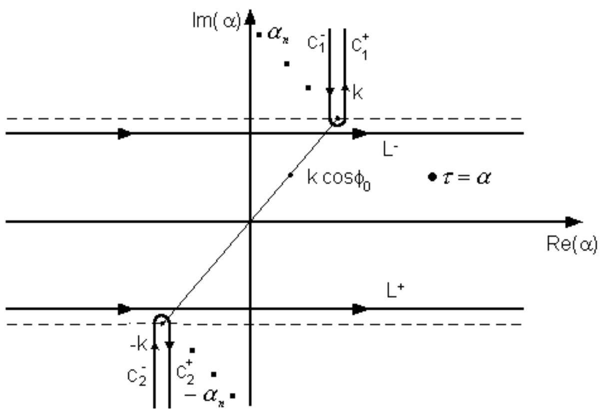


**Figure 3.** Complex $\alpha$ plane.

In the Fourier transform domain, (2a) takes the form

$$F_-^{(e)}(\alpha, d) + \frac{\eta_1}{ik}\dot{F}_-^{(e)}(\alpha, d) = 0 \tag{6}$$

where the dot specifies the derivative with respect to $y$. By using derivative of (5a) with respect to $y$ and (6), one obtains

$$R_+^{(e)}(\alpha) = A^{(e)}(\alpha)\left[1 + \eta_1 \frac{K(\alpha)}{k}\right] \tag{7a}$$

where

$$R_+^{(e)}(\alpha) = F_+^{(e)}(\alpha, d) + \frac{\eta_1}{ik} \cdot \dot{F}_+^{(e)}(\alpha, d) \tag{7b}$$

In the region for $0 < y < d$, $u_2^{(3e)}(x, y)$ satisfies the Helmholtz equation

$$\left[\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + k\right] u_2^{(3e)}(x, y) = 0 \tag{8}$$

in the range of $x > 0$. The half-range Fourier transform of (8) yields

$$\left[\frac{d^2}{dy^2} + K^2(\alpha)\right] G_+^{(e)}(\alpha, y) = \left[f^{(e)}(y) - i\alpha g^{(e)}(y)\right] \tag{9a}$$

with

$$f^{(e)}(y) = \frac{1}{2\pi}\frac{\partial}{\partial x} u_2^{(3e)}(0, y) \tag{9b}$$

$$g^{(e)}(y) = \frac{1}{2\pi} u_2^{(3e)}(0, y) \tag{9c}$$

$G+^{(e)}(\alpha, y)$ which is defined by

$$G+^{(e)}(\alpha, y) = \frac{1}{2\pi}\int_0^\infty u_2^{(3e)}(x, y) e^{i\alpha x} dx \tag{10}$$

is also a function regular in the half plane $\Im m(\alpha) > \Im m(-k)$.

The general solution of (9a) satisfying the Neumann boundary condition at $y = 0$, reads

$$G_+^{(e)}(\alpha, y) = C^{(e)}(\alpha)\cos[Ky] + \frac{1}{K(\alpha)}\int_0^y \left[f^{(e)}(t) - i\alpha g^{(e)}(t)\right] \times \sin[K(y - t)]dt \tag{11}$$

If one obtains the derivativation of (11) with respect to $y$:

$$\dot{G}_+^{(e)}(\alpha, y) = -KC^{(e)}(\alpha)\sin[Ky] + \int_0^y \left[f^{(e)}(t) - i\alpha g^{(e)}(t)\right] \times \cos[K(y - t)]dt \tag{12}$$

Combining (2l) and (2m), one get

$$R_{+}^{(e)}(\alpha, d) = G_{+}^{(e)}(\alpha, d) + \frac{\eta_1}{ik}\dot{G}_{+}^{(e)}(\alpha, d) \tag{13}$$

and if equations (11), (12) and (13) are used together, $C^{(e)}(\alpha)$ can be solved uniquely

$$\begin{aligned} C^{(e)}(\alpha) \quad = \quad & \frac{R_{+}^{(e)}(\alpha)}{M^{(e)}(\alpha)} - \frac{1}{M^{(e)}(\alpha)} \int\limits_{0}^{d} \left[ f^{(e)}(t) - i\alpha g^{(e)}(t) \right] \\ & \times \left[ \frac{\sin[K(d-t)]}{K} + \frac{\eta_1}{ik} \cos[K(d-t)] \right] dt \end{aligned} \tag{14a}$$

with

$$M^{(e)}(\alpha) = \cos[Kd] - \frac{\eta_1}{ik} K \sin[Kd] \tag{14b}$$

Replacing (14a) into (11) we get

$$\begin{aligned} G_{+}^{(e)}(\alpha, y) \quad = \quad & \frac{\cos[Ky]}{M^{(e)}(\alpha)} \left\{ R_{+}^{(e)}(\alpha) - \int\limits_{0}^{d} \left[ f^{(e)}(t) - i\alpha g^{(e)}(t) \right] \right. \\ & \times \left[ \frac{\sin[K(d-t)}{K} + \frac{\eta_1}{ik} \cos[K(d-t)] \right] dt \\ & + \frac{1}{K} \int\limits_{0}^{y} \left[ f^{(e)}(t) - i\alpha g^{(e)}(t) \right] \sin[K(y-t)] dt \end{aligned} \tag{15}$$

Although the left-hand side of (15) is regular in the upper half-plane $\Im m(\alpha) > \Im m(-k)$, the regularity of the right-hand side is violated by the presence of simple poles occurring at the zeros of $M^{(e)}(\alpha)$, namely at $\alpha = \alpha_m^e$ satisfying

$$M^{(e)}\left(\alpha_m^{(e)}\right) = 0, \quad \Im m\left(\alpha_m^{(e)}\right) > \Im m(k); \quad m = 1, 2, \ldots \tag{16}$$

These poles can be eliminated by imposing their residues are zero. This gives

$$R_{+}^{(e)}(\alpha_m^e) = \frac{\sin[K_m^e d]}{2K_m^e} \left[ 1 - \frac{\eta_1^2}{k^2}(K_m^e)^2 \right] d\left[ f_m^e - i g_m^e \right] \tag{17a}$$

where $K^e_m$, $f^e_m$ and $g^e_m$ specify

$$K^e_m = K(\alpha^e_m) \tag{17b}$$

$$\begin{bmatrix} f^e_m \\ g^e_m \end{bmatrix} = \frac{2}{d}\int\limits_0^d \begin{bmatrix} f^e(t) \\ g^e(t) \end{bmatrix} \cos[K^e_m t]dt \tag{17c}$$

Owing to (17c), $f^e(y)$ and $g^{(e)}(y)$ can be expanded into Fourier cosine series as follows [13]

$$\begin{bmatrix} f^{(e)}(y) \\ g^{(e)}(y) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} f^e_m \\ g^e_m \end{bmatrix} \cos\left[K^e_m y\right] \tag{17d}$$

Consider the continuity relation (2l) which reads, in the Fourier transform domain

$$F^{(e)}_+(\alpha, d) - G^{(e)}_+(\alpha, d) = \left(\frac{2\eta_1 \sin\phi_0}{1 + \eta_1 \sin\phi_0}\right)\left(\frac{e^{-ikd\sin\phi_0}}{i(\alpha - k\cos\phi_0)}\right) \tag{18a}$$

where $\Im m(\alpha) > \Im m(\cos\phi_0)$

Replacing of (15) at $y = d$ expression into (18a) obtains

$$F^{(e)}_+(\alpha, d) - \frac{R^{(e)}_+(\alpha)}{M^{(e)}(\alpha)} \cos\left[K^{(e)} d\right] + \int\limits_0^d \left[f^{(e)}(t) - i\alpha g^{(e)}(t)\right]$$
$$x \left\{ \frac{\sin\left[K^{(e)}(d-t)\right]/K^{(e)} + \dfrac{\eta_1}{ik}\cos\left[K^{(e)}(d-t)\right]}{M^{(e)}(\alpha)} \right\}$$
$$- \sin\frac{\left[K^{(e)}(d-t)\right]}{K^{(e)}} dt$$
$$= \frac{2\eta_1 \sin\phi_0}{i\left(1 + \eta_1 \sin\phi_0\right)} \frac{e^{-ikd\sin\phi_0}}{(\alpha - k\cos\phi_0)} \tag{18b}$$

Expression of (5a) at $y = d$

$$F_+(\alpha, d) + F_-(\alpha, d) = A(\alpha) \tag{18c}$$

Combining (7a) and (18c), we get

$$F_+(\alpha, d) = \frac{R_+(\alpha)}{1 + \eta_1 \dfrac{K(\alpha)}{k}} - F_-(\alpha, d) \tag{18d}$$

Replacing (14b) and (18d) into (18b), it is obtained

$$\begin{aligned}&\frac{R_+^{(e)}(\alpha)K^e}{N^{(e)}(\alpha)\left[1+\eta_1\dfrac{K^e(\alpha)}{k}\right]}+\frac{k}{\eta_1}F_-^{(e)}(\alpha,d)\\ =\ &\frac{2ik\sin\phi_0}{(1+\eta_1\sin\phi_0)}\frac{e^{-ikd\sin\phi_0}}{(\alpha-k\cos\phi_0)}\\ &-\frac{i}{M^{(e)}(\alpha)}\int_0^d\left[f^{(e)}(t)-i\alpha g^{(e)}(t)\right]\cos[K^e t]dt \qquad (19a)\end{aligned}$$

where

$$N^{(e)}(\alpha)=M^{(e)}(\alpha)e^{iK(\alpha)d} \qquad (19b)$$

Substitution (17d) in (19a) and evaluating the resultant integral, one obtains the following modified Wiener-Hopf equation of the second kind valid in the strip $\Im m(k\cos\phi_0)<\Im m(\alpha)<\Im m(k)$

$$\begin{aligned}\frac{R_+^{(e)}k\chi(\eta_1,\alpha)}{N^{(e)}(\alpha)}+\frac{k}{\eta_1}F_-^{(e)}(\alpha,d)=&\ \frac{2ik\sin\phi_0e^{-ikd\sin\phi_0}}{1+\eta_1\sin\phi_0(\alpha-k\cos\phi_0)}\\ &-\sum_{m=1}^{\infty}\frac{iK_m^{(e)}\sin[K_m^e d]}{\alpha^2-\alpha_m^2}\left(f_m^{(e)}-i\alpha g_m^{(e)}\right)\end{aligned} \qquad (20a)$$

with

$$\chi(\eta_1,\alpha)=\frac{1}{\eta_1+\dfrac{k}{K^e(\alpha)}} \qquad (20b)$$

(20a) is a function regular in the strip $\Im m(k\cos\phi_0)<\Im m(\alpha)<\Im m(k)$.

The formal solution of (20a) can be obtained through the classical Wiener-Hopf procedure. The result is

$$\begin{aligned}\frac{k\chi_+(\alpha)}{N_+^{(e)}}R_+^{(e)}(\alpha)\ =&\ \frac{2ik\sin\phi_0}{(1+\eta_1\sin\phi_0)}\frac{e^{-ikd\sin\phi_0}}{(k\cos\phi-\alpha_0)}\frac{N_-^{(e)}(k\cos\phi_0)}{\chi_-(k\cos\phi_0)}\\ &-\sum_{m=1}^{\infty}\frac{iK_m^{(e)}\sin[K_m^e d]}{2\alpha_m^e(\alpha^2+\alpha_m^2)}\frac{N_+^{(e)}(\alpha_m^e)}{\chi_+(\alpha)}\left(f_m^e+i\alpha_m^e g_m^e\right)\end{aligned} \qquad (21)$$

where $N_+^{(e)}(\alpha)$, $\chi_+(\alpha)$ and $N_-^{(e)}(\alpha)$, $x_-(\alpha)$ are the split functions, regular and free of zeros in the half-planes $\Im m(\alpha) > \Im m(-k)$ and $\Im m(\alpha) < \Im m(k)$, respectively, resulting from the Wiener-Hopf factorization of the kernel function $\chi(\alpha)/N^{(e)}(\alpha)$ as

$$\frac{\chi(\alpha)}{N^{(e)}(\alpha)} = \frac{\chi_+(\alpha)}{N_+^{(e)}(\alpha)}\frac{\chi_-(\alpha)}{N_-^{(e)}(\alpha)} \tag{22}$$

The explicit expression of $N_+^{(e)}(\alpha)$ can be obtained by following the procedure outlined in [14]:

$$\begin{aligned} N_+^{(e)} &= \left[\cos[kb] - \frac{\eta_1}{i}\sin[kb]\right]^{\frac{1}{2}} \exp\left\{\frac{Kb}{\pi}\ell n\left(\frac{\alpha + iK}{k}\right)\right\} \\ &\quad \times \exp\left\{\frac{i\alpha d}{\pi}\left(1 - C + \ell n\left(\frac{2\pi}{kd}\right) + i\frac{\pi}{2}\right)\right\} \\ &\quad \times \prod_{m=1}^{\infty}\left(1 + \frac{\alpha}{\alpha_m^e}\right)\exp\left(\frac{i\alpha d}{m\pi}\right) \qquad \text{(23a)} \\ N_-^{(e)}(\alpha) &= N_+^{(e)}(-\alpha) \qquad \text{(23b)} \end{aligned}$$

In (23a) $C$ the Euler's constant given by $C = 0,57721\ldots$. As to the split functions $\chi_\pm(\alpha)$, they can be expressed explicitly in terms of Maluizhinets function as follows: [15]

$$\chi_+(k\cos\phi) = 2^{\frac{3}{2}}\sqrt{\frac{2}{\eta_1}}\sin\frac{\phi}{2}\left\{\frac{M_\pi(3\pi/2 - \phi - \theta)M_\pi(\pi/2 - \phi + \theta)}{M_\pi^2(\pi/2)}\right\}^2$$
$$\times\left\{\left[1 + \sqrt{2}\cos\left(\frac{\pi/2 - \phi - \theta}{2}\right)\right]\left[1 + \sqrt{2}\cos\left(\frac{3\pi/2 - \phi - \theta}{2}\right)\right]\right\}^{-1} \tag{24a}$$

with

$$\sin\theta = \frac{1}{\eta_1} \tag{24b}$$

with

$$M_\pi(z) = \exp\left\{-\frac{1}{8\pi}\int_0^z \frac{\pi\sin u - 2\sqrt{2}\sin(u/2) + 2u}{\cos u}du\right\} \tag{24c}$$

Consider now the waveguide $\{(-\ell_2 < x < -\ell_1); (0 < y < d)\}$ and $\{(-\ell_1 < x < 0); (0 < y < d)\}$, where the total fields can be expressed in terms of Fourier transform domain as:

$$u_2^{(1e)}(x,y) = \sum_{n=1}^{\infty} a_n \left( e^{i\beta_{1n}x} - \frac{1+\eta_3\beta_{1n}/k_1}{1-\eta_3\beta_{1n}/k_1} e^{-2i\beta_{1n}\ell_2 - i\beta_{1n}x} \right) \cos\gamma_{1n}y \tag{25a}$$

$$U_2^{(2e)}(x,y) = \sum_{n=1}^{\infty} \left( c_n e^{i\beta_{2n}x} + d_n e^{-i\beta_{2n}x} \right) \cos\gamma_{2n}y \tag{25b}$$

where

$$\beta_{1n} = \sqrt{k_1^2 - \gamma_{1n}^2}; \quad \cos\gamma_{1n}d + \frac{\eta_2}{ik_1}\gamma_{1n}\sin\gamma_{1n}d = 0 \tag{25c}$$

and

$$\beta_{2n} = \sqrt{k_2^2 - \gamma_{2n}^2}; \quad \cos\gamma_{2n}d + \frac{\eta_2}{ik_2}\gamma_{2n}\sin\gamma_{2n}d = 0 \tag{25d}$$

Consider the continuity relations (2h) and (2i) in the Fourier transform domain which reads,

$$\sum_{n=1}^{\infty} a_n \left( e^{-i\beta_{1n}\ell_1} - Ae^{i\beta_{1n}\ell_1} \right) \cos\gamma_{1n}y = \sum_{n=1}^{\infty} \left( c_n e^{-i\beta_{2n}\ell_1} + d_n e^{i\beta_{2n}\ell_1} \right) \cos\gamma_{2n}y \tag{26a}$$

$$\sum_{n=1}^{\infty} i\beta_{1n}a_n \left( e^{-i\beta_{1n}\ell_1} + Ae^{i\beta_{1n}\ell_1} \right) \cos\gamma_{1n}y = \sum_{n=1}^{\infty} i\beta_{2n} \left( c_n e^{-i\beta_{2n}\ell_1} - d_n e^{i\beta_{2n}\ell_1} \right) \cos\gamma_{2n}y \tag{26b}$$

where

$$A = \frac{1+\eta_3\beta_{1n}/k_1}{1-\eta_3\beta_{1n}/k_1} e^{-2i\beta_{1n}\ell_2} \tag{26c}$$

Let us consider (26a) and (26b). Hence obtains

$$c_n = \frac{[x_3(1+T) + x_4(T-1)]}{2x_1} \frac{\cos\gamma_{1n}y}{\cos\gamma_{2n}y} a_n \tag{26d}$$

$$d_n = \frac{[x_3(1-T) - x_4(T-1)]}{2x_2} \frac{\cos\gamma_{1n}y}{\cos\gamma_{2n}y} a_n \tag{26e}$$

and

$$\begin{aligned} P_n &= \frac{c_n + d_n}{c_n - d_n} \\ &= \frac{x_3[(1+T)x_2 + (1-T)x_1] + Ax_4[(T-1)x_2 - (1+T)x_1]}{x_3[(1+T)x_2 - (1-T)x_1] + Ax_4[(T-1)x_2 + (1+T)x_1]} \end{aligned} \tag{26f}$$

where

$$x_1 = e^{-i\beta_{2n}\ell_1},\ x_2 = e^{i\beta_{2n}\ell_1},\ x_3 = e^{-i\beta_{1n}\ell_1},\ x_4 = e^{i\beta_{1n}\ell_1} \tag{26g}$$

$$T = \frac{\beta_{1n}}{\beta_{2n}} \tag{26h}$$

From the continuity relations (2j), (2k) and (9b, c), it is found

$$u_2^{(2e)}(0,y) = g^{(e)}(y); \quad (0 < y < d) \tag{27a}$$

$$\frac{\partial}{\partial x} u_2^{(2e)}(0,y) = f^{(e)}(y); \quad (0 < y < d) \tag{27b}$$

Using (25b) one may write

$$\sum_{n=1}^{\infty} (c_n + d_n) \cos \gamma_{2n} y = g^{(e)}(y) \tag{27c}$$

$$\sum_{n=1}^{\infty} i\beta_{2n}(c_n - d_n) \cos \gamma_{2n} y = f^{(e)}(y) \tag{27d}$$

Now using (17d) it can be written

$$\sum_{n=1}^{\infty} (c_n + d_n) \cos \gamma_{2n} y = \sum_{m=1}^{\infty} g_m^e \cos[K_m^e y] \tag{27e}$$

$$\sum_{n=1}^{\infty} i\beta_{2n}(c_n - d_n) \cos \gamma_{2n} y = \sum_{m=1}^{\infty} f_m^e \cos[K_m^e y] \tag{27f}$$

Let us multiply both sides of the equation (27f) by $\cos[\gamma_{2\ell} y]$ and integrate obtained result in the interval, $(0 < y < d)$. If we consider the orthogonal properties of the trigonometric functions at the solution of the integration step;

$$i\frac{d}{2}\beta_{2\ell}^e(c_\ell - d_\ell) = -\sum_{m=1}^{\infty} \frac{\gamma_{2\ell}^e \sin(\gamma_{2\ell}^e d)}{(K_m^e)^2 - (\gamma_{2\ell}^e)^2} \left( \frac{\eta_2}{ik_2} K_m^e \sin[K_m^e d] + \cos[K_m^e d] \right) f_m \tag{28a}$$

For $(c_\ell - d_\ell)$;

$$(c_\ell - d_\ell) = \frac{2i\gamma_{2\ell}^e}{\beta_{2\ell}^e d}\sin(\gamma_{2\ell}^e d)\sum_{m=1}^{\infty}\frac{\Omega_m}{\vartheta_{m\ell}}f_m \tag{28b}$$

where

$$\Omega_m = \cos[K_m^e d] + \frac{\eta_2}{ik_2}K_m^e\sin(K_m^e d) \tag{28c}$$

$$\vartheta_{m\ell} = (K_m^e)^2 - (\gamma_{2\ell}^e)^2 \tag{28d}$$

By using the same approach above, let us multiply both sides of the equation (27e) by the term $\cos(K_\ell^e y)$ then integrate obtained result in the interval, $(0 < y < d)$. Again, if we consider the orthogonal properties of the trigonometric functions at the solution of the integration step;

$$g_\ell^e\frac{d}{2} = -\sum_{n=1}^{\infty}(c_n + d_n)\frac{\Omega_\ell^e}{\vartheta_{\ell n}^e}\gamma_{2n}^e\sin(\gamma_{2n}^e d) \tag{29a}$$

For $g_\ell^e$ in (29a);

$$g_\ell^e = -\frac{2}{d}\Omega_\ell^e\sum_{n=1}^{\infty}(c_n + d_n)\frac{\gamma_{2n}^e}{\vartheta_{\ell n}^e}\sin(\gamma_{2n}^e d) \tag{29b}$$

$$\vartheta_{\ell n}^e = (K_\ell^e)^2 - (\gamma_{2n}^e)^2 \tag{29c}$$

The term $(c_n + d_n)$ in (26f) can be defined by the terms $(c_n - d_n)$ and replacing in (29b), we get;

$$g_\ell^e = -\frac{2}{d}\Omega_\ell^e\sum_{n=1}^{\infty}P_n(c_n - d_n)\frac{\gamma_{2n}^e}{\vartheta_{\ell n}^e}\sin(\gamma_{2n}^e d) \tag{30a}$$

Instead of $(c_n - d_n)$ in the (30a), if equation (29a) is written, then;

$$g_\ell^e = -\frac{4i}{d^2}\Omega_\ell^e\sum_{n=1}^{\infty}P_n\frac{(\gamma_{2n}^e)^2}{\beta_{2n}^e\vartheta_{\ell n}^e}\sin(\gamma_{2n}^e d)\sum_{m=1}^{\infty}\frac{\Omega_m^e}{\vartheta_{m\ell}^e}f_m^e \tag{30b}$$

Replacing the terms $\alpha = \alpha_1^{(e)}, \alpha_2^{(e)} \ldots$ in (24a), then writing the right side of the equation (17a) for the term $R_+^{(e)}(\alpha_m^{(e)})$:

$$\frac{kd}{2}\left[1 - \frac{\eta_1}{k^2}\left[K_r^{(e)}\right]^2\right]\frac{\sin\left[K_r^{(e)}d\right]}{K_r^{(e)}}\frac{\chi_+(\alpha)}{N_+\left(\alpha_r^{(e)}\right)}\left[f^{(e)} - i\alpha r g_\tau^{(e)}\right]$$

$$= \frac{2ik\sin\phi_0}{(1+\eta_1\sin\phi_0)}\frac{N_-^{(e)}(k\cos\phi_0)}{x-(k\cos\phi_0)}\frac{e^{-ikd\sin\phi_0}}{\left(k\cos\phi_0-\alpha_r^{(e)}\right)}$$
$$-\sum_{m=1}^{\infty}\frac{K_m^{(e)} i\sin\left(K_m^{(e)}d\right)}{2\alpha_m^{(e)}}\frac{N_+^{(e)}(\alpha_m)}{\chi_+(\alpha)}\frac{\left(f_m^{(e)}+i\alpha_m g_m^{(e)}\right)}{\left(\alpha_r^{(e)}+\alpha_m^{(e)}\right)} \quad (31a)$$

Substituting (30b) in (31a), we get infinite number of equations with infinite number of unknowns that yield the constants $f_r^{(e)}$ as follows

$$\frac{kd}{2}\left[1-\frac{\eta_1^2}{k^2}\left[K_r^{(e)}\right]^2\right]\frac{\sin\left[K_r^{(e)}d\right]}{K_r^{(e)}}\frac{\chi_+(\alpha)}{N_+(\alpha_r)}f_r^e$$
$$+i\alpha_r^e\left(\frac{4i}{d^2}\Omega_r\sum_{m=1}^{\infty}\frac{(\gamma_{2n})^2}{\beta_{2n}\vartheta_{\ell n}}\sin^2\gamma P_n\sum_{m=1}^{\infty}\frac{\Omega_m^e}{V_{mr}^e}f_m^e\right)$$
$$+\sum_{m=1}^{\infty}\frac{K_m^{(e)} i\sin\left[K_m^{(e)}d\right]}{2\alpha_m^e(\alpha_r^e+\alpha_m^e)}\frac{N_+(\alpha_m^e)}{\chi_+(\alpha^e)}$$
$$\left[f_m^e-i\alpha_m^e\left(\frac{4i}{d^2}\Omega_m\sum_{n=1}^{\infty}\frac{(\gamma_{2n})^2}{\beta_{2n}\vartheta_{mn}}\sin^2[\gamma_{2n}d]P_n\sum_{s=1}^{\infty}\frac{\Omega_s^e}{\vartheta_{sm}^e}f_s^{(e)}\right)\right]$$
$$= \frac{2ik\sin\phi_0}{1+\eta_1\sin\phi_0}\frac{N_-(k\cos\phi_0)}{x_-(k\cos\phi_0)}\frac{e^{-ikd\sin\phi_0}}{(k\cos\phi_0-\alpha_r)} \quad (31b)$$

## 2.2. Asymmetric (Odd) Excitation

The solution of odd excitation is similar to that of even excitation Indeed, by assuming a representation similar to (1a) with to superscript $(e)$ being replaced by $(o)$; it can be seen that all the boundary and continuity relations in (2a)–(2m) remain valid for the odd excitation case also, except (2b) and (2c), which are to be changed as

$$u_2^{(1o)}(x,0) = 0, \quad y=0, \quad (-\ell_2 < x < -\ell_1) \quad (32a)$$
$$u_2^{(2o)}(x,0) = 0, \quad y=0, \quad (-\ell_1 < x < 0) \quad (32b)$$

In this case, the Wiener-Hopf equation reads

$$\frac{k\chi(\alpha)}{N^{(o)}(\alpha)}R_+^{(o)}(\alpha)-\frac{ik}{\eta_1}F_-^{(o)}(\alpha,d) = \frac{2k\sin\phi_0}{(1+\eta_1\sin\phi_0)}\frac{e^{-ikd\sin\phi_0}}{(\alpha-k\cos\phi_0)}$$
$$+\sum_{m=1}^{\infty}\frac{K_m^o\cos\left[K_m^o d\right]}{\left[\alpha^2-(\alpha_m^o)^2\right]}\left(f_m^o-i\alpha g_m^o\right) \quad (33a)$$

with

$$N^{(o)}(\alpha) = e^{iK(\alpha)d}\left\{[\sin K(\alpha)d] + \frac{\eta_1}{ik}K(\alpha)\cos[K(\alpha)d]\right\} \quad (33b)$$

$$K_m^o = \sqrt{k^2 - (\alpha_m^o)^2} \quad (33c)$$

$$\begin{matrix} f_m^o \\ g_m^o \end{matrix} = \frac{2}{d}\int_0^d \begin{bmatrix} f^o(t) \\ g^o(t) \end{bmatrix} \sin[K_m^o t]\, dt \quad (33d)$$

Owing to (33d) $f^o(y)$ and $g^o(y)$ can be expanded into Fourier series as follows [13]:

$$\begin{bmatrix} f^o(y) \\ g^o(y) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} f_m^o \\ g_m^o \end{bmatrix} \sin[K_m^o y] \quad (33e)$$

where $\alpha_m^o$ are the roots of

$$\sin[K(\alpha)d] + \frac{\eta_1}{ik}K(\alpha)\cos[K(\alpha)d] = 0, \;\; \alpha = \alpha_m^o; \;\; \Im m(\alpha_m^o) > \Im m(k) \quad (34)$$

The application of the Wiener-Hopf procedure to (33a) yields

$$\frac{k\chi_+(\alpha)}{N_+^{(o)}(\alpha)} R_+^{(o)}(\alpha) = \frac{2k\sin\phi_0}{(1+\eta_1\sin\phi_0)} \frac{e^{-ikd\sin\phi_0}}{(k\cos\phi_0 - \alpha)} \frac{N_-^o(k\cos\phi_0)}{\chi_-(k\cos\phi_0)} + \sum_{m=1}^{\infty} \frac{K_m^o\cos[K_m^o d]}{2\alpha_m^o} \frac{N_+^{(o)}(\alpha_m^o)}{\chi_+(\alpha_m^o)} \frac{(f_m^o + i\alpha g_m^o)}{(\alpha + \alpha_m^o)} \quad (35)$$

$N_+^{(o)}(\alpha)$ and $N_-^{(o)}(\alpha)$ are the split functions resulting from the Wiener-Hopf factorization of (33b) as

$$N^{(o)}(\alpha) = N_+^{(o)}(\alpha)\, N_-^{(o)}(\alpha) \quad (36a)$$

The explicit expression of $N_\pm^{(o)}(\alpha)$ are [14].

$$N_+^{(o)}(\alpha) = \sqrt{k+\alpha}\left[\frac{\sin[kd]}{k} + \frac{\eta_1}{ik}\cos[kd]\right]^{1/2} \exp\left\{\frac{K(\alpha)d}{\pi}\ln\left(\frac{\alpha + iK(\alpha)}{k}\right)\right\} \exp\left\{\frac{i\alpha d}{\pi}\left(1 - C + \ln\left(\frac{2\pi}{kd}\right) + i\frac{\pi}{2}\right)\right\} \sum_{m=1}^{\infty}\left(1 + \frac{\alpha}{\alpha_m^o}\right)\exp\left(\frac{i\alpha d}{m\pi}\right) \quad (36b)$$

$$N_{-}^{(o)}(\alpha) = N_{+}^{(o)}(-\alpha) \tag{36c}$$

By using the continuity relations at the aperture $\{(0 < y < d);\ x = 0\}$, the infinite number of equations are obtained with infinite number of unknowns which yields the constants $f_r^o$ as follows:

$$\begin{aligned}
&-\frac{kd}{2}\left[1 - \frac{\eta_1}{k^2}(K_r^o)^2\right]\frac{\cos(K_r^o d)}{K_r^o}\frac{\chi_+(\alpha_r^o)}{N_+^{(o)}(\alpha_r^o)} \times \\
&\left\{f_r^o + i\alpha_r^o\left[\frac{4i}{d^2}\Omega_l\sum_{n=1}^{\infty}\frac{\gamma_{2n}^2\cos^2\gamma_{2n}d}{\beta_{2n\ell n}}P_n\sum_{m=1}^{\infty}\frac{\Omega_m}{m\ell}f_m^o\right]\right\} \\
&-\sum_{\substack{m=1\\M\neq r}}^{\infty}\frac{K_m^o\cos(K_m^o d)}{2\alpha_m^o}\frac{N_+^{(o)}(\alpha_m^o)}{\chi_+(\alpha_m^o)}\frac{1}{(\alpha_r+\alpha_m^o)} \times \\
&\left\{f_m^o - i\alpha_m^o\left[\frac{4i}{d^2}\Omega_l\sum_{n=1}^{\infty}\frac{\gamma_{2n}^2\cos^2\gamma_{2n}d}{\beta_{2n}mn}P_n\sum_{s=1}^{\infty}\frac{\Omega_s}{sm}f_s^o\right]\right\} \\
=\;&\frac{2ik\sin\phi_0}{1+\eta_1\sin\phi_0}\frac{N_-^o(k\cos\phi_0)}{x_-(k\cos\phi_0)}\frac{e^{-ikd\sin\phi_0}}{(k\cos\phi_0-\alpha_r^o)}
\end{aligned} \tag{37}$$

Here $\Omega_m^o$, $\vartheta_{m\ell}$, $\beta_{2n}$ and $P_n$

$$\Omega_m^o = \cos[K_m^o d] + \frac{\eta_2}{ik_2}K_m^o\sin(K_m^o d) \tag{38a}$$

$$\vartheta_{m\ell} = (K_m^o)^2 - (\gamma_{2\ell}^o)^2 \tag{38b}$$

$$U_2^{(10)}(x,y) = \sum_{n=1}^{\infty}\left\{a_n\left[e^{i\beta_{1n}x} - \left(\frac{1+\eta_3\beta_{1n}/k_1}{1-\eta_3\beta_{1n}/k_1}\right) e^{-(2i\beta_{1n}l_2+i\beta_{1n}x)}\right]\right\}\sin\gamma_{1n}y \tag{38c}$$

$$U_2^{(20)}(x,y) = \sum_{n=1}^{\infty}\left[c_n e^{i\beta_{2n}x} + d_n\cdot e^{-i\beta_{2n}x}\right]\cdot\sin\gamma_{2n}y \tag{38d}$$

$$\beta_{1n} = \sqrt{k_1^2-\gamma_{1n}^2};\quad \sin(\gamma_{1n}^o d) - \frac{\eta_2}{ik_1}\gamma_{1n}^o\cos(\gamma_{1n}^o d) = 0 \tag{38e}$$

$$\beta_{2n} = \sqrt{k_2^2-\gamma_{2n}^2};\quad \sin(\gamma_{2n}^o d) - \frac{\eta_2}{ik_2}\gamma_{2n}^o\cos\left(\gamma_{2n}^0 d\right) = 0,\ n = 1,2,\ldots \tag{38f}$$

$$P_n = \frac{c_n+d_n}{c_n-d_n} = \frac{x_3[(1+T)x_2+(1-T)x_1]+Ax_4[(T-1)x_2-(1+T)x_1]}{x_3[(1+T)x_2-(1-T)x_1]+Ax_4[(T-1)x_2+(1+T)x_1]} \tag{38g}$$

with $\gamma_{2n}^o$ being the roots of (38f); $g_r^o$ can be expressed in terms of $f_r^o$ as

$$g_r^o = -\frac{4i}{d^2}\Omega_r^o \sum_{n=1}^{\infty} \frac{\gamma_{2n}^2 \cos^2 \gamma_{2n} d}{\beta_{2n} r n} P_n \sum_{m=1}^{\infty} \frac{\Omega_m}{m\ell} f_m^0 \tag{38h}$$

## 3. ANALYSIS OF THE DIFFRACTED FIELD

The scattered field in the region $y > d$ for even and odd excitations can be obtained by taking the inverse Fourier transform of $F^{(e)}(\alpha, y)$ and $F^{(o)}(\alpha, y)$ respect to $\pounds$

$$u_1^{(e)}(x, y) = \frac{1}{2\pi} \int_{\pounds} F^{(e)}(\alpha, y)\, e^{-i\alpha x} d\alpha \tag{39}$$

$$u_1^{(o)}(x, y) = \frac{1}{2\pi} \int_{\pounds} F^{(o)}(\alpha, y)\, e^{-i\alpha x} d\alpha \tag{40}$$

where $\pounds$ is a straight line parallel to the real $x$ axis laying in the strip $\Im m(k \cos \phi_0) < \Im m(\alpha) < \Im m(k)$. The asymptotic evaluation of the integrals in (39) and (40) through the saddle-point technique enables us to write the diffracted field as,

$$u_1(\rho, \psi) = \frac{u_1^{(e)}(\rho, \psi) + u_1^{(o)}(\rho, \psi)}{2} \tag{41a}$$

with

$$\begin{aligned} u_1^{(e)}(\rho, \psi) \approx & \sqrt{\frac{2\pi}{k\rho}} \frac{N_+^{(e)}(-k\cos\psi)}{\chi_+(-k\cos\psi)} \frac{N_-^{(e)}(k\cos\phi_0)}{\chi_-(k\cos\phi_0)} \\ & \frac{2i \sin\phi_0 e^{-ikd\sin\phi_0} \sin\psi}{(1+\eta_1\sin\phi_0)(1+\eta_1\sin\psi)} \frac{e^{ik\rho}e^{i\pi/4}}{(\cos\phi_0+\cos\psi)} \\ & -2\pi i \frac{N^{(e)}(k\cos\phi_0)}{\chi(k\cos\phi_0)} \frac{2i\sin^2\phi_0 e^{-ikd\sin\phi_0}}{(1+\eta_1\sin\phi_0)^2} \\ & e^{-ik\rho\cos(\psi+\phi_0)} H(\pi-(\psi+\phi_0)) \\ & -\sqrt{\frac{2\pi}{k\rho}} \frac{N_+^{(e)}(-k\cos\psi)}{\chi_+(-k\cos\psi)} \sum_{m=1}^{\infty} \left\{ \frac{iK_m^e \sin[K_m^e d]}{2\alpha_m^e(\alpha_m^e - k\cos\psi)} \right. \\ & \left. \frac{N_+^{(e)}(\alpha_m^e)}{\chi_+(\alpha_m^e)} (f_m + i\alpha_m^e g_m^e) \frac{\sin\psi}{(1+\eta_1\sin\psi)} e^{ik\rho} e^{i\pi/4} \right\} \end{aligned} \tag{41b}$$

$$
\begin{aligned}
u_1^{(o)}(\rho,\psi) \approx & \sqrt{\frac{2\pi}{k\rho}}\frac{N_+^{(o)}(-k\cos\psi)}{\chi_+(-k\cos\psi)}\frac{N_-^{(o)}(k\cos\phi_0)}{\chi_-(k\cos\phi_0)} \\
& \frac{2\sin\phi_0 e^{-ikd\sin\phi_0}\sin\psi}{(1+\eta_1\sin\phi_0)(1+\eta_1\sin\psi)}\frac{e^{ik\rho}e^{i\pi/4}}{(\cos\phi_0+\cos\psi)} \\
& -2\pi i\frac{N^{(o)}(k\cos\phi_0)}{\chi(k\cos\phi_0)}\frac{2\sin^2\phi_0 e^{-ikd\sin\phi_0}}{(1+\eta_1\sin\phi_0)^2} \\
& e^{-ik\rho\cos(\psi+\phi_0)}H(\pi-(\psi+\phi_0)) \\
& +\sqrt{\frac{2\pi}{k\rho}}\frac{N_+^{(o)}(-k\cos\psi)}{\chi_+(-k\cos\psi)}\sum_{m=1}^{\infty}\left\{\frac{K_m^o\cos[K_m^o d]}{2\alpha_m^o(\alpha_m^o-k\cos\psi)}\frac{N_+^{(o)}(\alpha_m^o)}{\chi_+(\alpha_m^o)}\right. \\
& \left.\times(f_m+i\alpha_m^o g_m^o)\frac{\sin\psi}{(1+\eta_1\sin\psi)}e^{ik\rho}e^{i\pi/4}\right\} \qquad (41c)
\end{aligned}
$$

where $(\rho,\psi)$ are the cylindrical polar coordinates defined by

$$x = x = \rho\cos\psi, \qquad y = \rho\sin\psi, \qquad \alpha = -k\cos t$$

In order to show the influence of the values of plate dimensions, surface impedances, distance between the plates and variation of dielectric constants of two different dielectric loading at the region between the parallel plates on the diffraction phenomenon, the numerical results showing the variation of the diffracted field $(20\log|u_d\times\sqrt{k\rho}|)$ with the observation angle are presented. Firstly, the results are checked with empty cavities having same impedance values for all walls by considering $\varepsilon_1$ and $\varepsilon_2$ as $\varepsilon_0$. The good agreement is found with previous result in literature [11]. In Fig. 4, we show the variation of the diffracted field with the truncation number $N$ at a fixed point. It is seen that the diffracted field becomes insensitive to the truncation number $N$ for $N \geq 10$, $\varepsilon_1 = \varepsilon_2 = 1$ , $\ell_1 = \lambda$ , $\ell_2 = 2\lambda$ , $\eta_1 = \eta_2 = \eta_3 = -0.5i$. Fig. 5 shows the variation of the diffracted field with observation angle, for different values distance between the plates. The diffracted field decreases with the increasing values of the distance between the plates. Fig. 6, 7 and 8 show the variation of the diffracted field with observation angle for surface impedance $\eta_1$. The diffracted field decreases with the increasing values of all surface impedances $\eta_1, \eta_2$ and $\eta_3$, respectively. Figs. 9 and 10 show the variation of the diffracted field with observation angle for dielectric loading $\varepsilon_1$ and $\varepsilon_2$, respectively. The diffracted field decreases with the increasing values of the dielectric loading $\varepsilon_1$ and $\varepsilon_2$. Figs. 11 and 12 show the diffracted field with observation angle for the length of $\ell_1$ and $\ell_2$, respectively. The diffracted field decreases with the increasing length of walls $\ell_1$ and $\ell_2$.

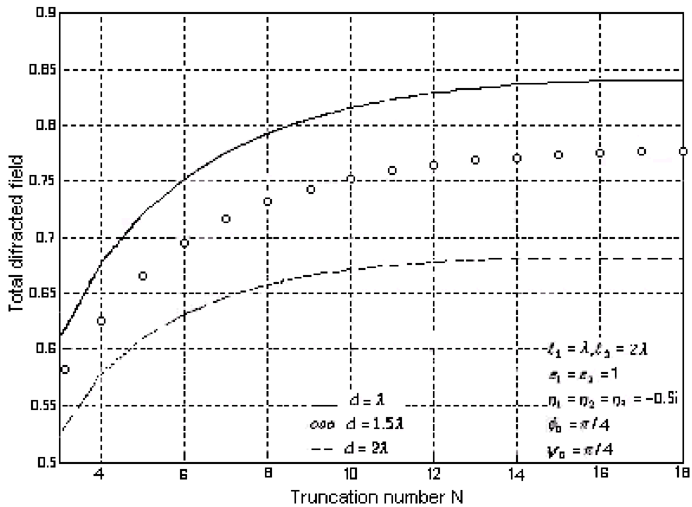


**Figure 4.** The diffracted field versus the truncation number $N$.

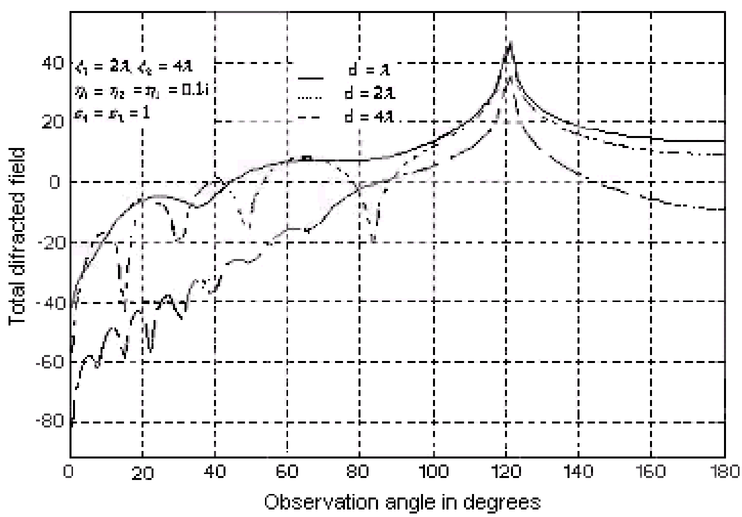


**Figure 5.** The diffracted field versus the observation angle $\psi_0$ for different values of $d$.

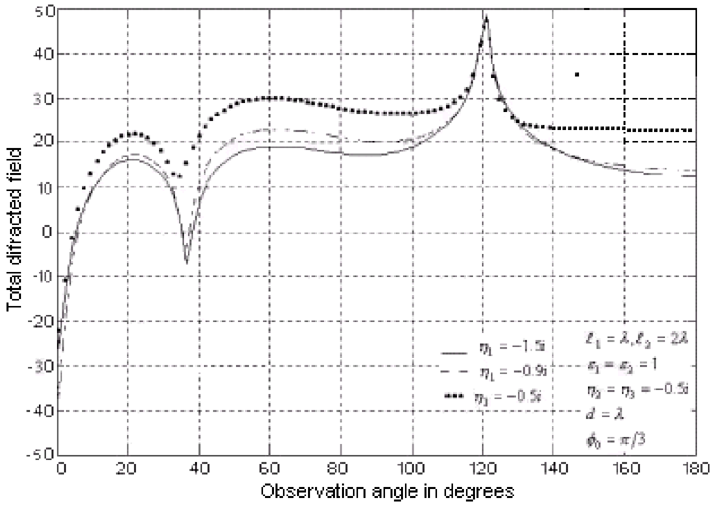


**Figure 6.** The diffracted field versus the observation angle $\psi_0$ for different values of $\eta_1$.

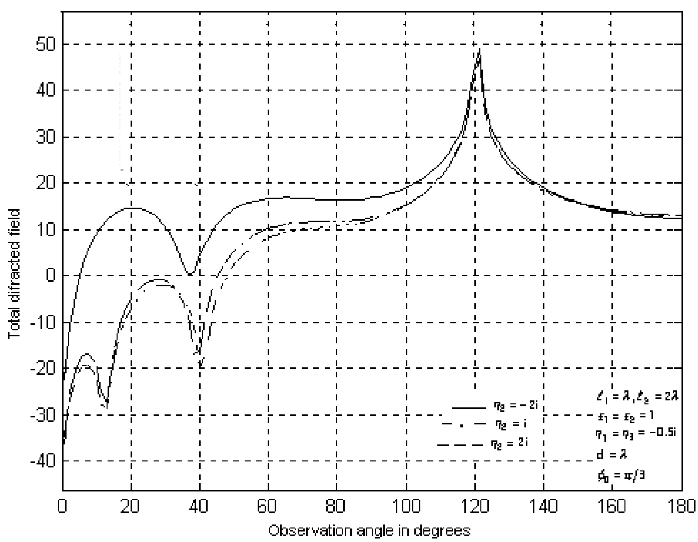


**Figure 7.** The diffracted field versus the observation angle $\psi_0$ for different values of $\eta_2$.

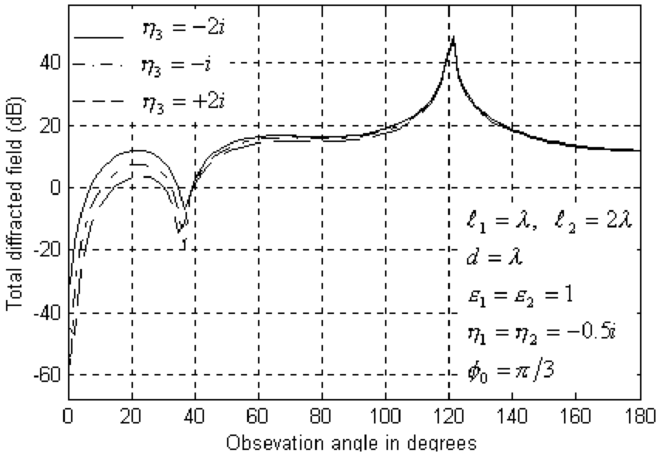


**Figure 8.** The diffracted field versus the observation angle $\psi_0$ for different values of $\eta_3$.

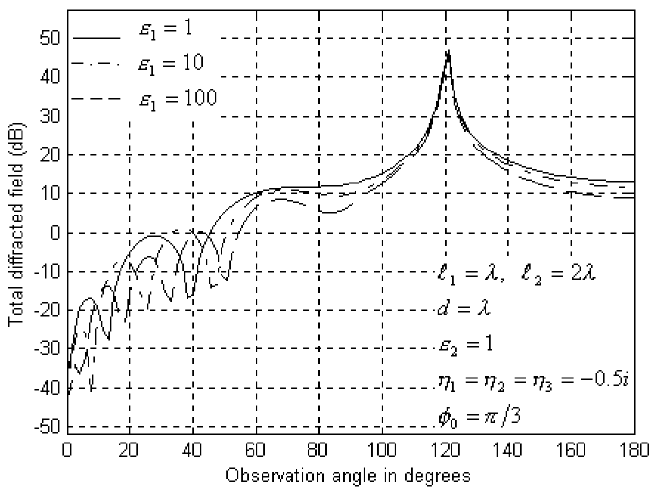


**Figure 9.** The diffracted field versus the observation angle $\psi_0$ for different values of $\varepsilon_1$.

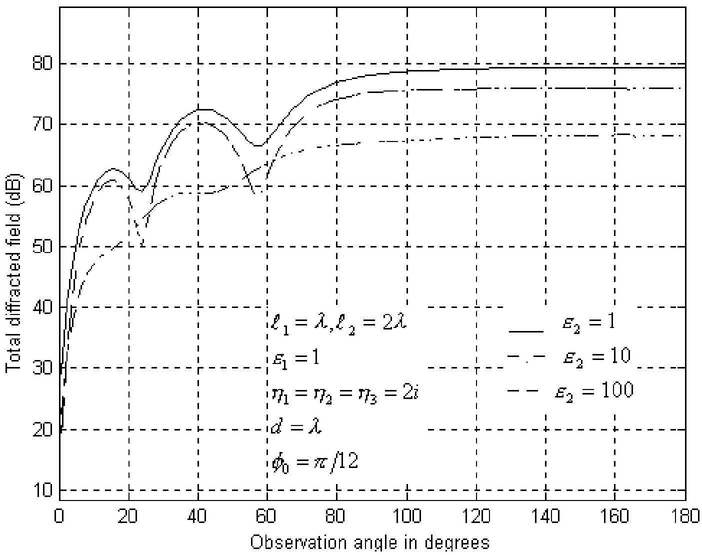


**Figure 10.** The diffracted field versus the observation angle $\psi_0$ for different values of $\varepsilon_2$.

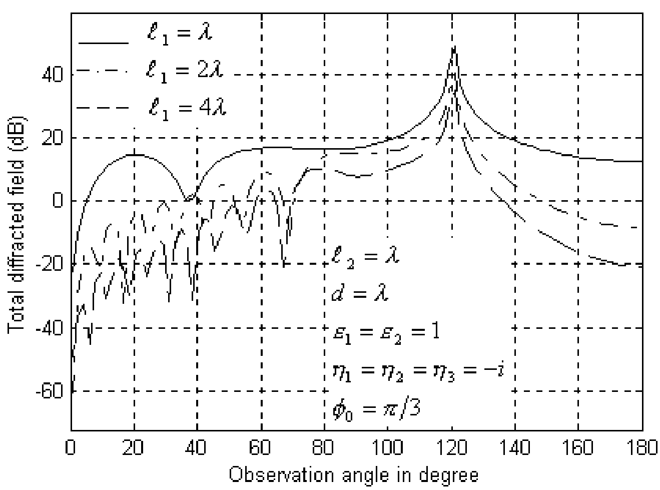


**Figure 11.** The diffracted field versus the observation angle $\psi_0$ for different values of $\ell_1$.

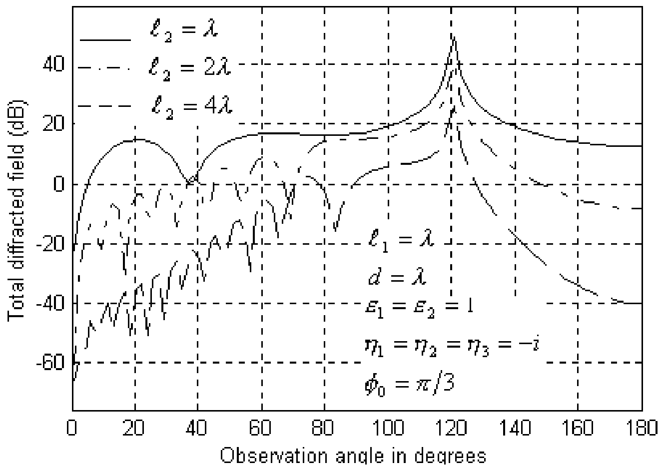


**Figure 12.** The diffracted field versus the observation angle $\psi_0$ for different values of $\ell_2$.

## 4. CONCLUSIONS

In this paper, we have solved the $E$-polarized plane wave diffraction by a terminated, semi-infinite parallel plate waveguide with two-layer material loading and impedance boundaries using the Wiener-Hopf technique. It is to be noted that the final solution obtained here is rigorous and uniformly valid for arbitrary cavity dimensions and impedance boundaries. The numerical examples of the RCS related to considered geometry for various physical parameters are shown that the total diffracted field can be reduced by coating the cavity walls with thin films and refilling cavity with dielectric materials having higher dielectric permittivity of $\varepsilon_1$ and $\varepsilon_2$. This problem is important in several engineering applications. The result obtained in this paper can be used as a reference solution and canonical models for duct structures such as jet engine intakes of aircrafts and cracks occurring on surfaces of general complicated bodies base on approximation methods.

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