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ELECTROMAGNETIC FIELD IN THE PRESENCE OF A THREE-LAYERED SPHERICAL REGION

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Abstract—In this paper, the region of interest consists of the spherical dielectric earth, coated with a dielectric layer under the air. The simple explicit formulas have been derived for the electromagnetic fields of a vertical electric dipole and vertical magnetic dipole in the presence of three-layered spherical region, respectively. Next, basing on the above results, the formulas for the six components of the field in the air generated by a horizontal electric dipole are derived by using reciprocity. The computations show that the trapped surface wave of electric type can be excited efficiently and the trapped surface wave of magnetic type can not be excited when the thickness l of the dielectric layer is somewhat and satisfies the condition $0 < \sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$. When the thickness l of the dielectric layer is somewhat and satisfies the condition $\frac{\pi}{2} < \sqrt{k_1^2 - k_0^2} \cdot l < \pi$, the trapped surface wave of magnetic type can be excited. These formulas and the computations can be applied to the surface communications at the lower frequencies.

1 Introduction

2 The Field in the Air Generated by a Vertical Electric Dipole over the Surface of the Dielectric-Coated Spherical Electrically Earth

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1. INTRODUCTION

The old problem of the electromagnetic fields generated by a vertical electric dipole and horizontal electric dipole in a three-layered planar region has been studied and debated by several investigators because of its useful applications in several cases [1–5]. It is necessary to re-analyze the old problem again. Only several years ago, the complete, simple and accurate formulas were derived for the electromagnetic field generated by a vertical electric dipole in a three-layered planar region. In [6], it has been drawn a conclusion that the trapped surface wave can be induced by a vertical electric dipole in a three-layered planar region. The wave number of the trapped surface wave is between that of the wave number k_0 in the air and the wave number k_1 in the dielectric layer, which is different with that presented in [4]. The amplitude of the trapped wave attenuates as $r^{-1/2}$ in the r direction near the surface and attenuates exponentially as e^{-gz} in the z direction. This development, naturally, rekindled the interest in the spherical problem.

Recently, Pan and Zhang [8] and Li *et al.* [9] had derived the accurate and simple formulas for the electromagnetic fields of a vertical electric dipole and horizontal electric dipole over the surface of a spherical conducting earth coated with a dielectric layer, respectively. When applied to practical situations such as the wave propagation over the dielectric-coated earth, the earth should be regarded as an electrically dielectric sphere. It is necessary to study the problem furthermore.

In the present study, the simple analytical formulas for the electromagnetic fields in the air are outlined when a dipole source (vertical electric dipole, vertical magnetic dipole, or horizontal electric dipole) is located near the surface of the dielectric-coated spherical electrically earth. First, along such research lines of [8] and [9], we will present the formulas for the fields of vertical electric dipole and vertical magnetic dipole over the surface of a dielectric-coated spherical electrically earth. Next, basing on these results, we will derive the formulas for the six components of the electromagnetic field generated by a horizontal electric dipole over the dielectric-coated spherical electrically earth. It should be noted that the conditions of the trapped surface wave of electric type and those of the trapped surface wave of magnetic type are presented clearly in this paper. These formulas and the computations can be applied to the surface communications at the lower frequencies.

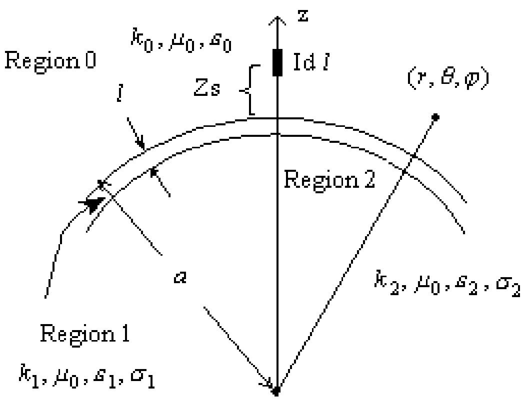


Figure 1. Geometry of a vertical electric dipole in the presence of a three-layered spherical region.

2. THE FIELD IN THE AIR GENERATED BY A VERTICAL ELECTRIC DIPOLE OVER THE SURFACE OF THE DIELECTRIC-COATED SPHERICAL ELECTRICALLY EARTH

First, we will analyze the field of a vertical electric dipole in the presence of three-layered spherical region. In the present study, the geometry of the problem consists of the air (region 0, $r > a$), the dielectric layer with its thickness l (region 1, $a - l < r < a$), and the earth (region 2, $r < a-l$), as illustrated in Figure 1. The wave numbers in the three regions are

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0},$$
$$k_1 = \omega\sqrt{\mu_0\left(\varepsilon_0\varepsilon_{r1} + i\sigma_1/\omega\right)},$$

$$k_2 = \omega\sqrt{\mu_0\left(\varepsilon_0\varepsilon_{r2} + i\sigma_2/\omega\right)} \tag{1}$$

where ε_r is the relative permittivity, σ is the conductivity.

Assuming that the dipole is represented by the current density $\hat{z}Idl\delta(x)\delta(y)\delta(z-b)$, where $b = z_s + a$, and $z_s > 0$ denotes the height of the dipole above the surface of the dielectric-coated earth, and use is made of the time dependence $e^{(-i\omega t)}$, the nonzero components of the field E_r, E_θ, and H_φ, which the field is defined as the field of electric type, can be expressed as follows,

$$E_r^{(i)} = \left(\frac{\partial^2}{\partial r^2} + k_i^2\right)(U_i r) \tag{2}$$

$$E_\theta^{(i)} = \frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}(U_i r) \tag{3}$$

$$H_\varphi^{(i)} = -\frac{i\omega\varepsilon_i}{r}\frac{\partial}{\partial\theta}(U_i r). \tag{4}$$

Here the potential function U_i is the solution of in the scalar Helmholtz equation

$$(\nabla^2 + k_i^2)U_i = 0 \tag{5}$$

where $i = 0, 1, 2$. In spherical coordinates, because of symmetry, the expanded form (5) is written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{U_i}{r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U_i}{\partial\theta}\right) + k_i^2 U_i = 0. \tag{6}$$

Using the separation of variables method, we assume that a solution for U_i can be written in the form of

$$U_i = \frac{1}{r}Z_i(r)\Phi(\theta) \tag{7}$$

then

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta\frac{\Phi(\theta)}{\partial\theta}\right] + \nu(\nu+1)\Phi(\theta) = 0 \tag{8}$$

$$\frac{d^2 Z_i(r)}{dr^2} + k_i^2\left[1 - \frac{\nu(\nu+1)}{k_i^2 r^2}\right]Z_i(r) = 0. \tag{9}$$

The equation (8) is closely related to the well-known Legendre differential equation whose solution is taken as the Legendre functions of first kind $P_\nu(\cos(\pi-\theta))$ [8]. In Region 0, the solution of (9) has been addressed in [8].

$$Z_0(r) = AW_2(t-y) \tag{10}$$

where $W_2(t)$ is the Airy function of the second kind, and t and y are defined as

$$t = \left[\frac{a^3}{2\nu(\nu+1)}\right]^{2/3} \cdot \left[\frac{\nu(\nu+1)}{a^2} - k_0^2\right] \tag{11}$$

$$y = \left[\frac{2\nu(\nu+1)}{a^3}\right]^{1/3} z \approx \left(\frac{2}{k_0 a}\right)^{1/3} k_0 z. \tag{12}$$

In (10), $r = a + z$ and $z \ll a$, $z > 0$ denotes the height above the surface of the dielectric-coated earth. We assume that both the dipole and the point of observation close to the spherical interface between the air and the dielectric layer. In the whole text we take $z_s \ll a$ and $z_r \ll a$.

In Region 1, $a - l \le r \le a$, taking into account $l \ll a$, we take $r \approx a$. Then (9) is simplified as

$$\frac{d^2 Z_1(r)}{dr^2} + (k_1^2 - k_0^2) Z_1(r) = 0 \tag{13}$$

The solution of (13) is

$$Z_1(r) = Be^{i\sqrt{k_1^2-k_0^2}\cdot[r-(a-l)]} + Ce^{-i\sqrt{k_1^2-k_0^2}\cdot[r-(a-l)]}. \tag{14}$$

According to the impedance boundary condition at $r = a - l$, we get

$$\left.\frac{E_\theta^{(1)}}{\eta_1 H_\varphi^{(1)}}\right|_{r=a-l} = -\triangle_g \tag{15}$$

At lower frequencies, the surface impedance $\triangle_g$ can be approximated as,

$$\triangle_g \approx \frac{k_1}{k_2}\sqrt{1 - \left(\frac{k_1}{k_2}\right)^2} \tag{16}$$

Then,

$$\triangle_e = \frac{C}{B} = \frac{\sqrt{k_1^2 - k_0^2} - k_1 \triangle_g}{\sqrt{k_1^2 - k_0^2} + k_1 \triangle_g}. \tag{17}$$

At the boundary $r = a$, with the conditions $E_\theta^{(0)}|_{z=a} = E_\theta^{(1)}|_{z=a}$ and $H_\varphi^{(0)}|_{z=a} = H_\varphi^{(1)}|_{z=a}$, we get

$$-AW_2' \cdot \left(\frac{2}{k_0 a}\right)^{\frac{1}{3}} \cdot k_0 = iB\sqrt{k_1^2 - k_0^2}\left(e^{i\sqrt{k_1^2-k_0^2}\cdot l} - \triangle_e \cdot e^{-i\sqrt{k_1^2-k_0^2}\cdot l}\right) \tag{18}$$

$$A \cdot k_0^2 W_2 = B \cdot k_1^2\left(e^{i\sqrt{k_1^2-k_0^2}\cdot l} + \triangle_e \cdot e^{-i\sqrt{k_1^2-k_0^2}\cdot l}\right). \tag{19}$$

Then, the differential equation is obtained as follows

$$W_2'(t) - qW_2(t) = 0 \tag{20}$$

where

$$q = \frac{-ik_0\sqrt{k_1^2 - k_0^2}}{k_1^2} \cdot \left(\frac{k_0 a}{2}\right)^{\frac{1}{3}} \cdot \left(\frac{1 - \triangle_e \cdot e^{-i2\sqrt{k_1^2 - k_0^2}\cdot l}}{1 + \triangle_e \cdot e^{-i2\sqrt{k_1^2 - k_0^2}\cdot l}}\right). \tag{21}$$

In the spherical conducting case $\triangle_g \longrightarrow 0$, then $\triangle_e = 1$, the above formula reduces to

$$q = \frac{k_0\sqrt{k_1^2 - k_0^2}}{k_1^2} \cdot \left(\frac{k_0 a}{2}\right)^{\frac{1}{3}} \cdot \tan(\sqrt{k_1^2 - k_0^2} \cdot l). \tag{22}$$

The expression of (22) coincides with that given in the literature [8].

If t_s are the roots of the differential equation (37). Then the potential function U_0 in the air is expressed as

$$U_0 = \frac{1}{r}\sum_s A_s F_s(z_r) P_\nu(\cos(\pi - \theta)) \tag{23}$$

where the "height-gain" function $F_s(z_r)$ is

$$F_s(z_r) = \frac{W_2(t_s - y)}{W_2(t_s)}. \tag{24}$$

With the similar method in [8], the coefficients A_s in (23) will be determined.

$$A_s = -\left(\frac{2}{ka}\right)^{1/3} \cdot \frac{k_0 I dl}{2\omega\varepsilon_0} \cdot \frac{F_s(z_s)\cdot e^{i\nu\pi}}{t_s - q^2}. \tag{25}$$

When ν is very large and θ is not close to 0 and π [8].

$$P_\nu(\cos(\pi - \theta)) = \sqrt{\frac{1}{2\pi k_0 a \sin\theta}} \cdot \exp\left[i(k_0 a\theta + t_s x)\right] \cdot \exp\left[i(\nu + \frac{1}{4})\pi\right] \tag{26}$$

where $x = (\frac{k_0 a}{2})^{1/3}\theta$.

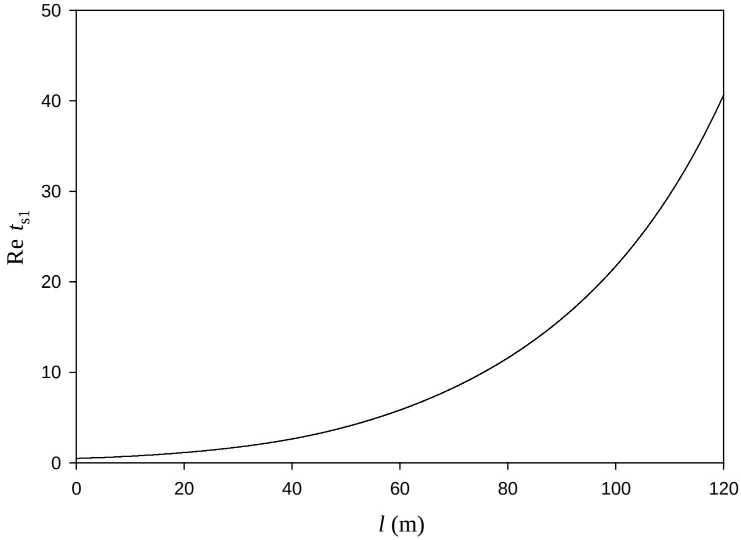
Finally, the accurate and complete formulas for the components E_r, E_θ and ηH_φ of the electromagnetic field generated by a vertical electric dipole at $(a+z_s, 0, 0)$ over the dielectric-coated spherical earth are expressed as follows.

$$\begin{bmatrix} E_r(a+z_r,\theta,\varphi) \\ E_\theta(a+z_r,\theta,\varphi) \\ \eta_0 H_\varphi(a+z_r,\theta,\varphi) \end{bmatrix} = \frac{iIdl\cdot\eta}{\lambda a} \cdot \frac{e^{i(ka\theta+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}} \cdot \sqrt{\pi x} \cdot \begin{bmatrix} \sum_s \frac{F_s(z_s)\cdot F_s(z_r)}{t_s - q^2} \cdot e^{it_s x} \\ \sum_s \frac{iF_s(z_s)\cdot \frac{\partial F_s(z)}{k_0\partial z}\big|_{z=z_r}}{k_0(t_s-q^2)\left[1+\frac{t_s}{2}\left(\frac{2}{ka}\right)^{2/3}\right]} \cdot e^{it_s x} \\ -\sum_s \frac{F_s(z_s)\cdot F_s(z_r)}{(t_s-q^2)\left[1+\frac{t_s}{2}(\frac{2}{ka})^{2/3}\right]} \cdot e^{it_s x} \end{bmatrix} \tag{27}$$

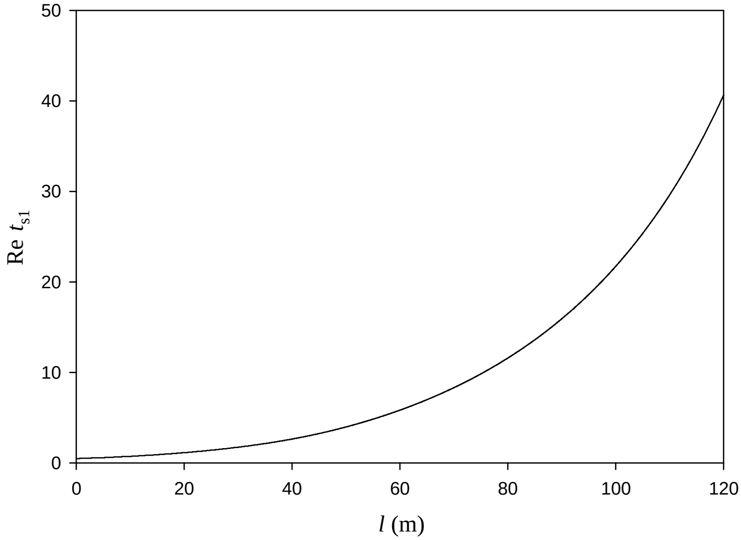
where $r = a + z_r$.

To the asphalt- and cement-coated spherical earth or ice-coated spherical seawater at the lower frequencies, it is easily to compute the components of the electromagnetic field versus the propagating distance at different thicknesses of the dielectric layer.

If the thickness of the dielectric layer is not zero, the parameters t_s can be computed directly by using the method presented in [8]. Assuming that the radius of the earth is taken as $a = 6370\,\text{km}$, the dielectric layer $a-l < r < a$ (Region 1) is characterized by the relative permittivity $\varepsilon_{r1} = 15$, and conductivity $\sigma_1 = 10^{-5}\,\text{S/m}$, the earth in the region $r < a - l$ (Region 2) is characterized by the relative permittivity $\varepsilon_{r2} = 100$, and conductivity $\sigma_2 = 4\,\text{S/m}$, t_s can be calculated easily. Figs. 2 and 3 show both the real parts and the imaginary parts of t_{s1} and t_{s2}–t_{s5}, respectively. It should be noted that q is always imaginary and will increase with the thickness l from zero when the thickness l of the dielectric layer satisfy the condition $0 < \sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$. From Fig. 2, it can be seen that the real part of t_{s1} increase significantly when $l > 40\,\text{m}$ and the imaginary part of t_{s1} approaches zero when the thickness of the dielectric layer l is between 40 m and 50 m. It means that the first propagation mode will attenuate very slowly along the surface of the spherical ground when the thickness of the dielectric layer. Therefore, the first term may be considered as the trapped surface wave [8]. In this paper, we will define the trapped surface wave generated by a vertical electric dipole as the trapped wave of electric type. Taking into account the conductivity σ_1 of the dielectric layer, the imaginary part of t_{s1} increase significantly when $l > 50\,\text{m}$.



(a) The real part of t_{s1}



(b) The imaginary part of t_{s1}

Figure 2. The value of t_{s1} versus the thickness of the dielectric layer l.

In contrast with the effects on $|E_r|$ and $|H_\varphi|$ by the thickness l, we will compute $|E_r|$ and $|H_\varphi|$ at different thicknesses l. Assuming $a = 6370\,\text{km}$, $f = 100\,\text{kHz}$, $\varepsilon_{r1} = 15$, $\varepsilon_{r2} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 4\,\text{S/m}$, and $z_r = z_s = 0$, $|E_r|$ in V/m and $|H_\varphi|$ in A/m due to unit vertical electric dipole at $l = 0\,\text{m}$, $l = 50\,\text{m}$, and $l = 100\,\text{m}$ are plotted in Figs. 4 and 5, respectively. From Figs. 2–5, it can be seen that the trapped surface wave of electric magnetic can be excited efficiently when the thickness l of the dielectric layer is between 40 m and 50 m and satisfies the condition $0 < \sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$.

Compared with the computations presented in [8], it is concluded that the electromagnetic field is affected largely by the conductivity of the earth and the electromagnetic field attenuates rapidly for the dielectric-coated earth with high conductivity. In the case of the perfect conducting earth, the attenuation of the electromagnetic field over the surface of the dielectric-coated spherical conducting earth will be largest.

3. THE FIELD IN THE AIR GENERATED BY A MAGNETIC DIPOLE OVER THE SURFACE OF THE DIELECTRIC-COATED SPHERICAL ELECTRICALLY EARTH

If the excitation source is replaced by a vertical magnetic dipole with its moment $M = Ida$, and da is the area of the loop, the nonzero components of the electromagnetic field in the air are E_φ, H_r, and H_θ (TE modes), which the field is defined as the field of electric type. $E_\varphi^{(i)}$, $H_r^{(i)}$, and $H_\theta^{(i)}$ can be expressed as follows,

$$E_\varphi^{(i)} = \frac{-i\omega\mu_0}{r}\frac{\partial}{\partial\theta}(V_i r) \tag{28}$$

$$H_r^{(i)} = \left(\frac{\partial^2}{\partial r^2} + k^2\right)(V_i r) \tag{29}$$

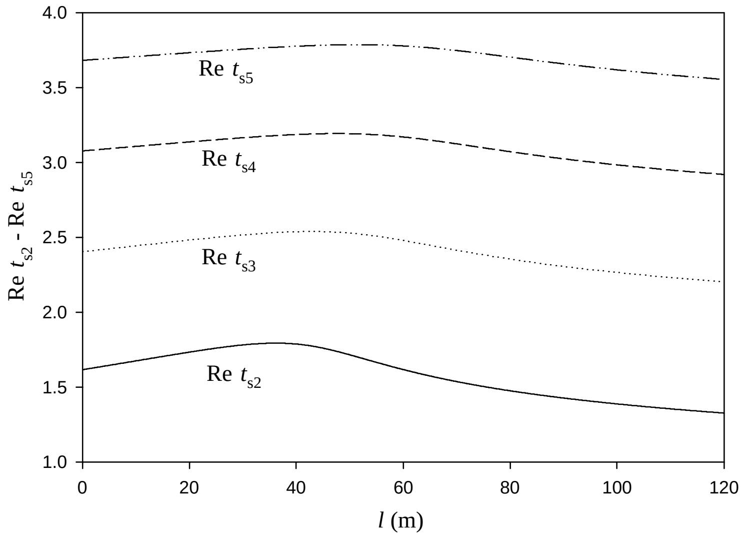
$$H_\theta^{(i)} = \frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}(V_i r). \tag{30}$$

where the potential function V_i is the solution of in the scalar Helmholtz equation

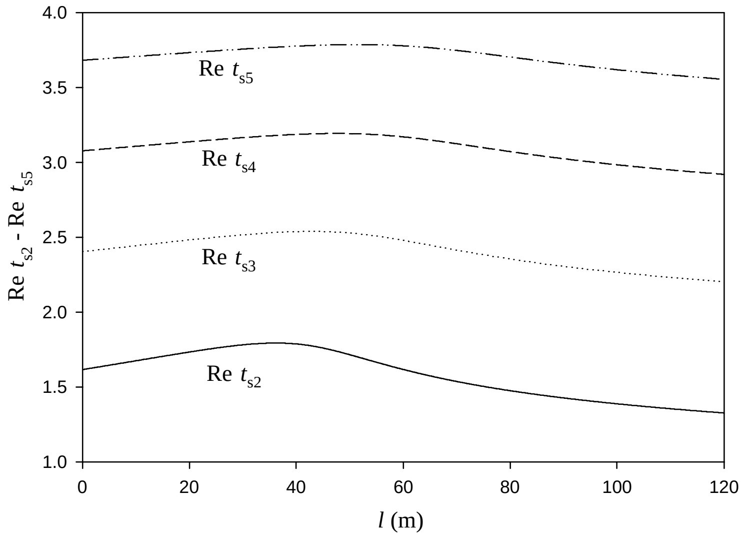
$$(\nabla^2 + k_i^2)V_i = 0, \quad i = 0, 1. \tag{31}$$

It is noted that the surface impedance of magnetic type is

$$\left.\frac{E_\varphi^{(1)}}{\eta_1 H_\theta^{(1)}}\right|_{r=a-l} = \triangle_g \tag{32}$$



(a) The real parts of t_{s2}–t_{s5}



(b) The imaginary parts of t_{s2}–t_{s5}

Figure 3. The values of t_{s2}–t_{s5} versus the thickness of the dielectric layer l.

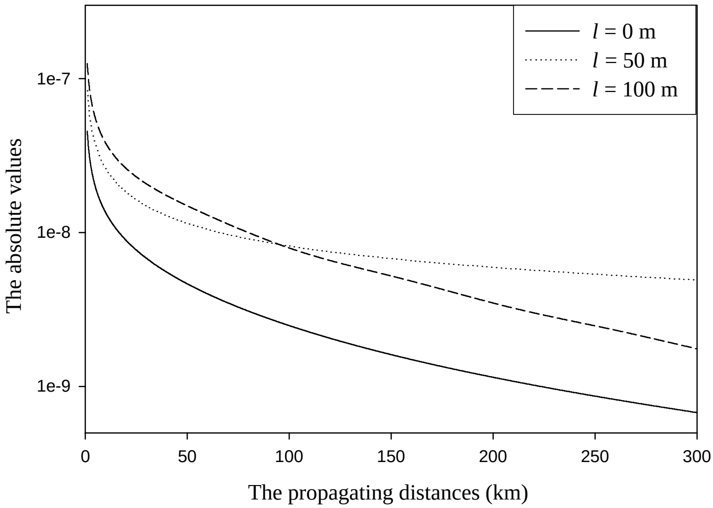


Figure 4. $|E_r|$ in V/m due to unit vertical electric dipole versus the propagating distance ρ at $l = 0\,\text{m}$, $l = 50\,\text{m}$, and $l = 100\,\text{m}$, respectively: $f = 100\,\text{kHz}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 15$, $\varepsilon_{r2} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 4\,\text{S/m}$, and $z_r = z_s = 0$.

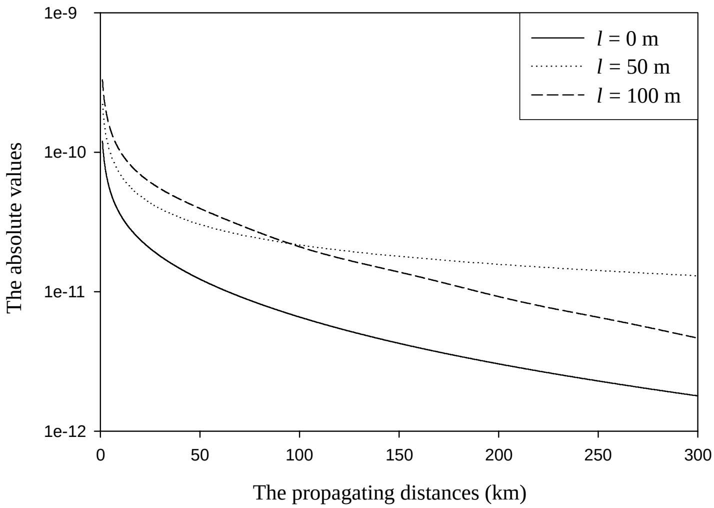


Figure 5. $|H_\varphi|$ in A/m due to unit vertical electric dipole versus the propagating distance ρ at $l = 0\,\text{m}$, $l = 50\,\text{m}$, and $l = 100\,\text{m}$, respectively: $f = 100\,\text{kHz}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 15$, $\varepsilon_{r2} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 4\,\text{S/m}$, and $z_r = z_s = 0$.

Follow the same step in Section 2, we will obtain the formulas of the components E_φ, H_r, and H_θ in the air generated by a vertical magnetic dipole over the dielectric earth.

$$\begin{bmatrix} E_\varphi \\ \eta_0 H_r \\ \eta_0 H_\theta \end{bmatrix} = \frac{\omega\mu_0 I da}{\lambda a} \frac{e^{i(ka\theta+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\sqrt{\pi x}\cdot \begin{bmatrix} \sum_s \frac{G_s(z_s)G_s(z_r)}{t_s-(q^h)^2} e^{it'_s x} \\ \sum_s \frac{G_s(z_s)G_s(z_r)}{t_s-(q^h)^2} e^{it'_s x} \\ i\sum_s \frac{G_s(z_s)\frac{\partial G_s(z)}{k_0\partial z}|_{z=z_r}}{t_s-(q^h)^2} e^{it'_s x} \end{bmatrix}. \tag{33}$$

where

$$q^h = \frac{-i\sqrt{k_1^2-k_0^2}}{k_0}\cdot\left(\frac{k_0 a}{2}\right)^{\frac{1}{3}}\cdot\left(\frac{1-\triangle_m\cdot e^{-i2\sqrt{k_1^2-k_0^2}\cdot l}}{1+\triangle_m\cdot e^{-i2\sqrt{k_1^2-k_0^2}\cdot l}}\right). \tag{34}$$

$$\triangle_m = \frac{\sqrt{k_1^2-k_0^2}\cdot\triangle_g + k_1}{\sqrt{k_1^2-k_0^2}\cdot\triangle_g - k_1}. \tag{35}$$

In the spherical perfectly conducting case $\triangle_g \longrightarrow 0$, then $\triangle_m = -1$, the above formula reduces to

$$q^h = -\left(\frac{k_0 a}{2}\right)^{\frac{1}{3}}\cdot\frac{\sqrt{k_1^2-k_0^2}}{k_0\cdot\tan\left[\sqrt{k_1^2-k_0^2}\cdot l\right]}. \tag{36}$$

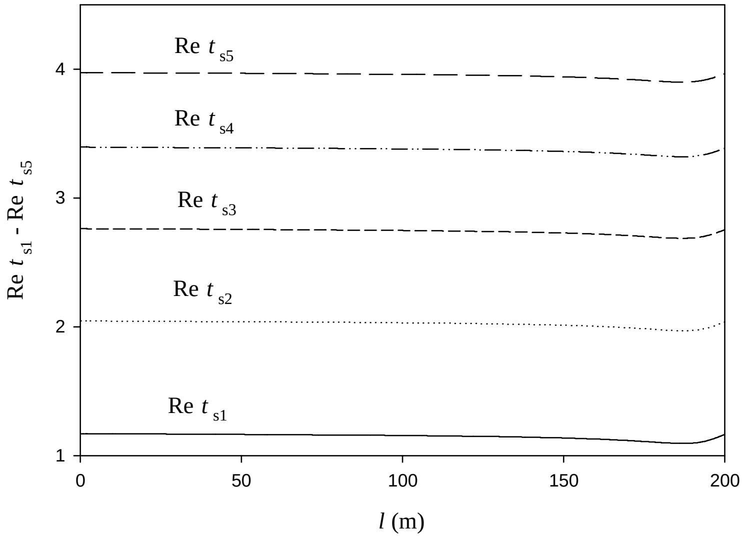
t'_s are the roots of the following differential equation

$$W'_2(t'_s) - q^h W_2(t'_s) = 0 \tag{37}$$

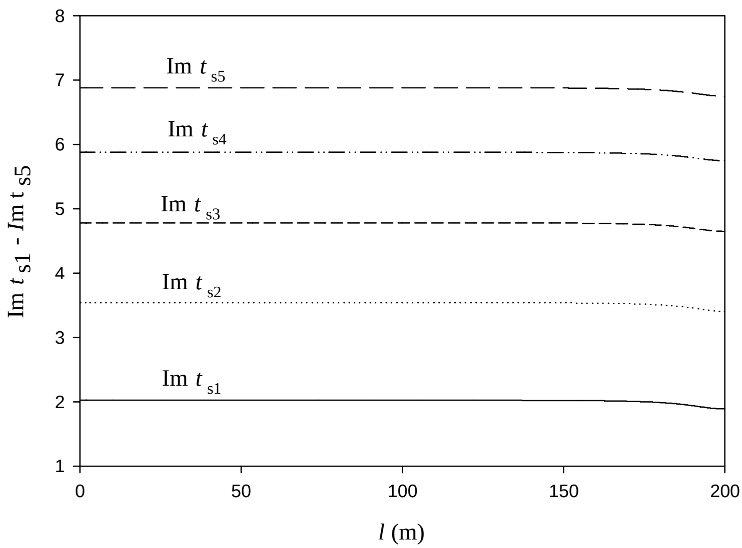
Here, $W_2(t'_s)$ is the Airy function of the second kind. The "height-gain" function $G_s(z_r)$ is

$$G_s(z_r) = \frac{W_2(t'_s - y)}{W_2(t'_s)} \tag{38}$$

Assuming that all the parameters of the dielectric layer $a-l < r < a$ (Region 1) and the earth in the region $r < a-l$ (Region 2) are same with those in Section 2. Both the real parts and the imaginary parts of t'_{s1}–t'_{s5} are calculated at $f = 100\,\text{kHz}$ and plotted in Fig. 6, respectively. Fig. 6 shows both the real part and the imaginary part of



(a) The real parts of t'_{s1}–t'_{s5}



(b) The imaginary parts of t'_{s1}–t'_{s5}

Figure 6. The values of t'_{s1}–t'_{s5} versus the thickness of the dielectric layer l.

t'_{s1}–t'_{s5}. It should be noted that q is always negative and will increase from negative infinite to zero with the thickness l when the thickness l of the dielectric layer satisfy the condition $0 < \sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$. From Fig. 6, it is seen that the surface wave cannot be excited by a vertical magnetic dipole when the thickness l satisfies the condition $0 < \sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$.

When the thickness l of the dielectric layer satisfies the condition $\frac{\pi}{2} < \sqrt{k_1^2 - k_0^2} \cdot l < \pi$, q is always positive and will increase from zero to infinite with the thickness l. When the thickness of the layer is larger than a certain value, the real part of t'_{s1} increase significantly and the imaginary part of t'_{s1} approaches to zero. From the discussions in Section 2, it is seen that the surface wave can be excited by a vertical magnetic dipole when the thickness l of the dielectric layer is larger than a certain value and satisfies the condition $\frac{\pi}{2} < \sqrt{k_1^2 - k_0^2} \cdot l < \pi$. In this paper, the trapped surface wave generated by a vertical electric dipole is defined as the trapped surface wave of magnetic type. In fact, the thickness l of the electric layer such as ice or permafrost is less than 200 m, the above condition cannot be satisfied.

In contrast with the effects on $|E_\varphi|$ and $|H_r|$ by the thickness l, we will compute $|E_\varphi|$ and $|H_r|$ at different thicknesses l. Assuming $a = 6370\,\text{km}$, $f = 100\,\text{kHz}$, $\varepsilon_{r1} = 15$, $\varepsilon_{r2} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 4\,\text{S/m}$, and $z_r = z_s = 0$, $|E_\varphi|$ in V/m and $|H_r|$ in A/m due to unit vertical electric dipole at $l = 0\,\text{m}$, $l = 50\,\text{m}$, and $l = 100\,\text{m}$ are plotted in Figs. 7 and 8, respectively.

4. THE FIELD IN THE AIR GENERATED BY A HORIZONTAL ELECTRIC DIPOLE OVER THE SURFACE OF THE DIELECTRIC-COATED SPHERICAL ELECTRICALLY EARTH

From the above expressions of the electromagnetic field of the vertical electric dipole and those of the vertical magnetic dipole in the air, using reciprocity theorem, the accurate and complete formulas of the vertical components E_r^{he} and H_r^{he} for the electromagnetic field generated by a horizontal electric dipole upon the surface of the dielectric-coated spherical conducting earth in the air can be developed easily.

$$E_r^{he} = -\frac{i\eta I ds^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \cos\varphi \cdot \sum_s \frac{iF_s(z_r) \cdot \frac{\partial F_s(z)}{k_0 \partial z}\big|_{z=z_s}}{(t_s - q^2)[1 + \frac{t_s}{2}(\frac{2}{k_0 a})^{2/3}]} \cdot e^{it_s x} \tag{39}$$

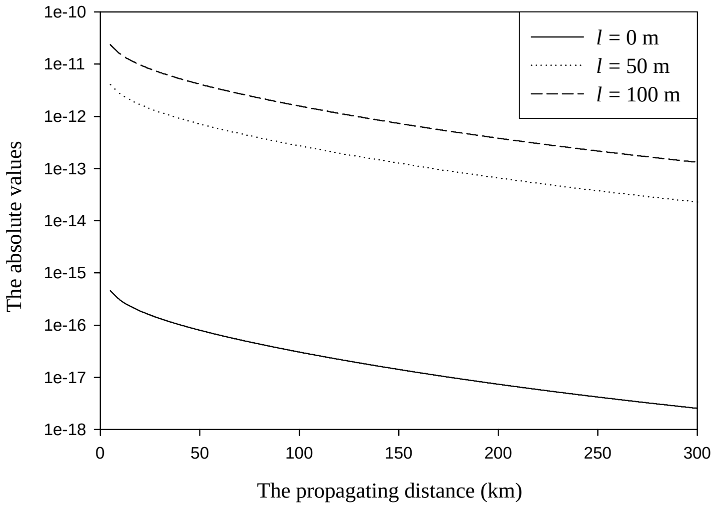


Figure 7. $|E_\varphi|$ in V/m due to unit vertical electric dipole versus the propagating distance ρ at $l = 0\,\text{m}$, $l = 50\,\text{m}$, and $l = 100\,\text{m}$, respectively: $f = 100\,\text{kHz}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 15$, $\varepsilon_{r2} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 4\,\text{S/m}$, and $z_r = z_s = 0$.

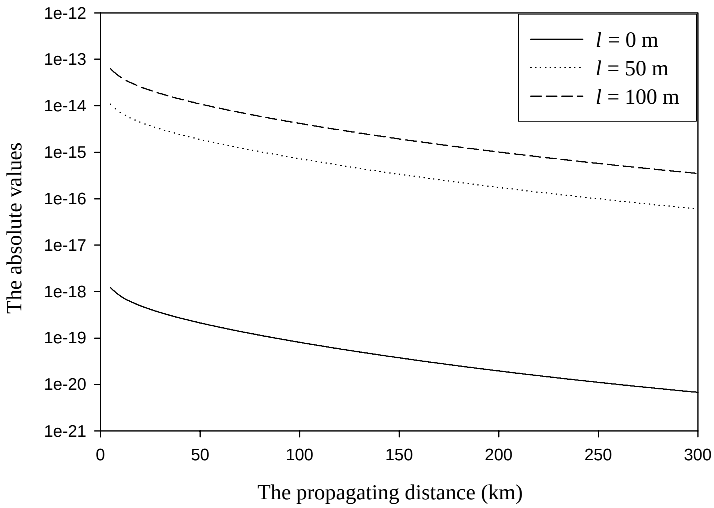


Figure 8. $|H_r|$ in A/m due to unit vertical electric dipole versus the propagating distance ρ at $l = 0\,\text{m}$, $l = 50\,\text{m}$, and $l = 100\,\text{m}$, respectively: $f = 100\,\text{kHz}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 15$, $\varepsilon_{r2} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 4\,\text{S/m}$, and $z_r = z_s = 0$.

$$H_r^{he} = -\frac{iIds^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \sin\varphi \cdot \sum_s \frac{G_s(z_s)G_s(z_r)}{t_s' - (q^h)^2} \cdot e^{it_s' x} \quad (40)$$

In spherical coordinates, from Maxwell's equations, it is easily to express the rest four components E_θ^{he}, E_φ^{he}, H_θ^{he} and H_φ^{he} in terms of E_r^{he} and H_r^{he}

$$\left(\frac{\partial^2}{\partial r^2} + k^2\right)(rH_\varphi^{he}) = i\omega\varepsilon_0 \frac{\partial E_r^{he}}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial^2 H_r^{he}}{\partial r \partial\varphi} \quad (41)$$

$$\left(\frac{\partial^2}{\partial r^2} + k^2\right)(rE_\theta^{he}) = \frac{\partial^2 E_r^{he}}{\partial r \partial\theta} + \frac{i\omega\mu_0}{\sin\theta}\frac{\partial H_r^{he}}{\partial\varphi} \quad (42)$$

$$\left(\frac{\partial^2}{\partial r^2} + k^2\right)(rH_\theta^{he}) = -\frac{i\omega\varepsilon_0}{\sin\theta}\frac{\partial E_r^{he}}{\partial\varphi} + \frac{\partial^2 H_r^{he}}{\partial r \partial\theta} \quad (43)$$

$$\left(\frac{\partial^2}{\partial r^2} + k^2\right)(rE_\varphi^{he}) = \frac{1}{\sin\theta}\frac{\partial^2 E_r^{he}}{\partial r \partial\varphi} - i\omega\mu_0 \frac{\partial H_r^{he}}{\partial\theta} \quad (44)$$

From (41), we get

$$\frac{\nu(\nu+1)}{r}(H_\varphi^{he}) = i\omega\varepsilon_0 \frac{\partial E_r^{he}}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial^2 H_r^{he}}{\partial r \partial\varphi} \quad (45)$$

Taking into account the relation $\nu(\nu+1)/r \approx k^2 a$, the component H_φ^{he} is formulated as follows.

$$\begin{aligned} H_\varphi^{he} = & -\frac{iIds^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \cos\varphi \\ & \times \left[\sum_s \frac{iF_s(z_r) \cdot \frac{\partial F_s(z)}{k_0 \partial z} \mid_{z=z_s}}{t_s - q^2} \cdot e^{it_s x} \right. \\ & \left. + \frac{1}{k_0 a \sin\theta} \cdot \sum_s \frac{\frac{\partial G_s(z)}{k_0 \partial z} \mid_{z=z_r} \cdot G_s(z_s)}{t_s' - (q^h)^2} \cdot e^{it_s' x} \right] \end{aligned} \quad (46)$$

Similarly, the rest three components E_θ^{he}, H_θ^{he} and E_φ^{he} are written as

$$\begin{aligned} H_\theta^{he} = & \frac{Ids^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \sin\varphi \\ & \times \left[\frac{i}{k_0 a \sin\theta} \sum_s \frac{F_s(z_r) \cdot \frac{\partial F_s(z)}{k_0 \partial z} \mid_{z=z_s}}{(t_s - q^2)[1 + \frac{t_s}{2}(\frac{2}{k_0 a})^{2/3}]} \cdot e^{it_s x} \right. \end{aligned}$$

$$-\sum_s \frac{\frac{\partial G_s(z)}{k_0 \partial z}\Big|_{z=z_r} \cdot G_s(z_s)}{t'_s-(q^h)^2}\cdot e^{it'_s x}\Bigg] \tag{47}$$

$$\begin{aligned}E_\theta^{he} &= \frac{\eta I ds^{he}}{\lambda a}\cdot\frac{e^{i(k_0 a\theta+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x}\cdot\cos\varphi \\ &\times\Bigg[-i\sum_s \frac{\frac{\partial F_s(z)}{k_0\partial z}\Big|_{z=z_r}\cdot\frac{\partial F_s(z)}{k_0\partial z}\Big|_{z=z_s}}{t_s-q^2}\cdot e^{it_s x} \\ &+\frac{1}{k_0 a\sin\theta}\cdot\sum_s\frac{G_s(z_r)\cdot G_s(z_s)}{t'_s-(q^h)^2}\cdot e^{it'_s x}\Bigg]\end{aligned} \tag{48}$$

$$\begin{aligned}E_\varphi^{he} &= \frac{i\eta I ds^{he}}{\lambda a}\cdot\frac{e^{i(k_0 a\theta+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x}\cdot\sin\varphi \\ &\times\Bigg[\frac{i}{k_0 a\sin\theta}\cdot\sum_s\frac{\frac{\partial F_s(z)}{k_0\partial z}\Big|_{z=z_r}\cdot\frac{\partial F_s(z)}{k_0\partial z}\Big|_{z=z_s}}{(t_s-q^2)[1+\frac{t_s}{2}(\frac{2}{k_0 a})^{2/3}]}\cdot e^{it_s x} \\ &+\sum_s\frac{G_s(z_r)\cdot G_s(z_s)}{t'_s-(q^h)^2}\cdot e^{it'_s x}\Bigg]\end{aligned} \tag{49}$$

where superscripts *he* designate the horizontal electric dipole source.

From the above derivations, it can be seen that the electromagnetic field generated by a horizontal electric dipole includes the electromagnetic field of electric type and the field of magnetic type. Obviously, for the electromagnetic field of a horizontal electric dipole, the trapped surface wave of electric type can be excited efficiently when the thickness l of the dielectric layer is somewhat and satisfies the condition $0 < \sqrt{k_1^2 - k_0^2}\cdot l < \frac{\pi}{2}$ and the trapped surface wave of magnetic type can be excited when the thickness l of the dielectric layer is somewhat and satisfies the condition $\frac{\pi}{2} < \sqrt{k_1^2 - k_0^2}\cdot l < \pi$.

5. CONCLUSION

In the above analysis, the simple explicit formulas have been derived for the electromagnetic field over the surface of a dielectric-coated spherical electrically earth when a dipole source (a vertical electric dipole, vertical magnetic dipole or horizontal electric dipole) is in the air. The computations show that both the trapped surface wave of electric type and that of magnetic type can be exited efficiently under the different conditions. Also, the electromagnetic field is affected largely by the conductivity of the earth.

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