*Progress In Electromagnetics Research*, PIER 46, 165–188, 2004

# ANALYSIS OF RESONANCE AND QUALITY FACTOR OF ANTENNA AND SCATTERING SYSTEMS USING COMPLEX FREQUENCY METHOD COMBINED WITH MODEL-BASED PARAMETER ESTIMATION

**L. Li and C.-H. Liang**

School of Electronic Engineering, Xidian University
Xi'an, Shaanxi 710071, P. R. China

**Abstract**—The generalized system function, $H(s)$, directly associated with the radiated or scattered fields is presented in this paper, which is constructed by applying the model-based parameter estimation (MBPE) technique combined with the complex frequency theory. A complex frequency $\tilde{\omega}$ relating the real resonant frequency with radiated or scattered $Q$ factor is introduced to antenna and scattering systems. By analyzing the characteristics of complex poles and zeros of $H(s)$ in a finite operational frequency band, and combining with adaptability of MBPE, we can determine the resonant frequency and $Q$-value of the antenna and scattering systems effectively. The intensity of resonance can be estimated in terms of $Q$-value and residues at the complex resonant frequencies. Some examples of the practical antenna arrays and scattering systems are given to illustrate the application and validity of the proposed approach in this paper.

## 1. INTRODUCTION

A rigorous analysis of resonant behaviors in the electromagnetic compatibility (EMC) has been an interesting and challenging problem for years [1–6]. With the increasingly complicated electromagnetic environment, the interaction or mutual coupling between antennas and scatterers become more and more severe that sometimes give rise to the strong electromagnetic oscillation phenomena [7–9]. In addition, an important parameter specifying selectivity, and performance in general, of a resonant system is the quality factor, $Q$. The research of antenna or scattering $Q$ has also been paid wide attention to for years [10–14]. In this paper, the generalized system function, $H(s)$, directly associated with the radiated or scattered fields, is presented to analyze the resonance behaviors and quality factor of the antenna and scattering open systems effectively. $H(s)$ is constructed by making use of model-based parameter estimation (MBPE) technique combined with the complex frequency theory. The complex frequency $\tilde{\omega}$ presented in circuit theory [15] is firstly introduced to antenna and scattering systems in Section 2. The model-based parameter estimation (MBPE) [16–18] is a form of "smart" curve fitting, with broad applications to a fast analysis of radiation patterns of antennas or RCS of scatterers in a widely operating bandwidth. In this paper, it is used to construct the generalized system function $H(s)$ of the antenna and scattering systems in complex frequency domain, which is directly associated with the radiated or scattered fields. By analyzing the characteristics of poles and zeros of $H(s)$, we can predict the resonant frequencies of antenna and scattering systems efficiently. Based on MBPE technique, the complex frequency method for calculating the quality factor $Q$ of the antenna and scattering resonance systems is also presented in this paper. It is shown that the intensity of resonance can be estimated effectively by the values of $Q$ and residues at the complex resonant frequencies. Furthermore, time response can be simply obtained by taking inverse Laplace transform of the generalized system function $H(s)$. The response of exponentially damped quasi-period oscillation in time domain also reflects the resonance characteristics profoundly. Some examples and discussion, two parallel dipoles, a five-element circular array for finding direction system, and two conducting objects scattering system, are given in this paper.

## 2. COMPLEX RESONANT FREQUENCY

For an arbitrary lossy resonant system, the complex resonant frequency [15] can be introduced and written as

$$\tilde{\omega} = \omega_0(1 + j\xi) \tag{1}$$

where $\omega_0$ is a real resonant frequency of the system, $\xi$ represents the losses of the resonant system. In general sense, the electric field can be written as

$$\vec{E} = \vec{E}_m e^{j\tilde{\omega}t} = \vec{E}_m e^{-\xi\omega_0 t} e^{j\omega_0 t} \tag{2}$$

With losses present, the energy stored in the resonant system will decay at a rate proportional to the average energy present at any time, so that

$$W = W_0 e^{-2\xi\omega_0 t} \tag{3}$$

where $W_0$ is the average energy present at $t = 0$. But the rate of decrease of $W$ must equal the power loss, so that

$$P_L = -\frac{dW}{dt} = 2\omega_0\xi W \tag{4}$$

In addition, an important parameter specifying selectivity, and performance in general, of a resonant system is the quality factor, $Q$. A general definition of $Q$ that is applicable to all resonant system is

$$Q = \frac{\omega_0 W}{P_L} \tag{5}$$

Substituting (5) into (4), we can easily get

$$\xi = \frac{P_L}{2\omega_0 W} = \frac{1}{2Q} \tag{6}$$

Therefore, the general expression of $\tilde{\omega}$ is

$$\tilde{\omega} = \omega_0\left(1 + j\frac{1}{2Q}\right) \tag{7}$$

It can be seen that the introduction of $\tilde{\omega}$ unifies the resonant frequency and $Q$ of a resonant system. Each complex resonant frequency is corresponding to one resonant mode of the system. It is well known that antenna or scattering system is essentially equivalent to a lossy network. Assume the system media are lossless, the loss $P_L$ represents the radiated or scattered power from the antennas or scattering bodies. The average stored energy $W$ denotes the sum of stored electric field and magnetic field energies around the antennas or scatterers, which is independent of the radiated energies from the antennas or scatterers [14]. Therefore, the complex resonant frequency is applicable to not only the resonant cavity in the closed system, but also the antenna and scattering resonant problems in the open system.

## 3. MODEL-BASED PARAMETER ESTIMATION TECHNIQUE

The model-based parameter estimation (MBPE) is a smart curve fitting technique [16–18], which has been widely applied to fast analysis of radiation patterns of antennas or RCS of scatterers over a wide frequency band. MBPE makes use of low-order analytical formulas as fitting models, while the unknown coefficients for the fitting model are obtained by matching it to multi-point sampled values or fitting it to frequency derivatives of the function at one or two frequency points. In this paper, MBPE is mainly used to construct the generalized system function directly associated with electromagnetic fields in the complex frequency domain. According to the observed objects, one form of a fitting model that is commonly employed in MBPE is represented by Padé rational function as follows

$$H(s) = \frac{Y(s)}{F(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n} \tag{8}$$

where $b_i (i = 0, \cdots, m)$ and $a_j (j = 0, \cdots, n)$ represent the coefficients of numerator and denominator polynomials, respectively. Note that $a_0$ or $a_n$ can be normalized to 1 in denominator coefficients. Thus, (8) has $p = m + n + 1$ unknown complex coefficients. $s$ represents the complex frequency $j\omega$. It is obvious that MBPE utilizes the rational function approximation and extends it into complex frequency domain, which provides an appropriate tool for analyzing the resonance characteristics of antenna and scattering systems from the point of view of complex frequency. The unknown coefficients of the Padé rational function can be obtained by frequencies sampling or frequency-derivative sampling.

**Scheme I** Frequency-derivatives sampling

It is applicable to the case that the frequency derivatives of the approximating function are available. The coefficients of the rational function are obtained by using frequency derivatives of the integro-partial-differential equation formed by the method of moments (MoM) or finite element method (FEM). If the frequency derivatives are available at one frequency $s_0$, the variable in the rational function can be replaced with $(s - s_0)$. Let $a_0 = 1$, we can get $b_0 = H_0(s_0)$ and the following matrix equation

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & -H_0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & -H_1 & -H_0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 & -H_{m-1} & -H_{m-2} & \cdots & 0 \\ 0 & 0 & \cdots & 0 & -H_m & -H_{m-1} & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & -H_{m+n-1} & -H_{m+n-2} & \cdots & -H_m \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_m \\ H_{m+1} \\ \vdots \\ H_{m+n} \end{bmatrix} \tag{9}$$

where $H_m = \frac{H^{(m)}(s_0)}{m!}$, and $H^{(m)}(s_0)$ is the $m^{\text{th}}$ derivatives of $H(s)$ at the complex frequency $s_0$. By solving (9), we can get the coefficients of the rational function. If the frequency derivative information is known for more than one frequency, a rational functional matching all the samples can be obtained which results in a wider frequency response [16].

### Scheme II Multi-frequency sampling

This case is found wider applications. By sampling $H(s)$ at a total of $p = m + n + 1$ frequencies (either calculated or measured), the expression in (8) can be written as a matrix equation of the form, here $a_n = 1$,

$$\begin{bmatrix} 1 & s_1 & \cdots & s_1^m & -H(s_1) & -H(s_1)s_1 & \cdots & -H(s_1)s_1^{n-1} \\ 1 & s_2 & \cdots & s_2^m & -H(s_2) & -H(s_2)s_2 & \cdots & -H(s_2)s_2^{n-1} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 1 & s_m & \cdots & s_m^m & -H(s_m) & -H(s_m)s_m & \cdots & -H(s_m)s_m^{n-1} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 1 & s_p & \cdots & s_p^m & -H(s_p) & -H(s_p)s_p & \cdots & -H(s_p)s_p^{n-1} \end{bmatrix}$$

$$\times \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_m \\ a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{bmatrix} = \begin{bmatrix} H(s_1)s_1^n \\ H(s_2)s_2^n \\ \vdots \\ H(s_m)s_m^n \\ \vdots \\ H(s_p)s_p^n \end{bmatrix} \tag{10}$$

By solving (10), we can also obtain the coefficients of the rational function. In the antenna and scattering systems, the frequency response of radiation patterns, RCS, input impedance, and near electric fields or magnetic fields can be approximated directly by using this scheme. For a more complex system, more sampling points (oversampling) are required to estimate the coefficients of the fitting model more accurately. In this case, a least-squares approach can be implemented in order to solve the resulting matrix equation for the desired coefficients.

In this paper, the generalized system functions directly associated with the radiated or scattered fields need to be constructed to analyze the behaviors of resonances, and thus Scheme II is adopted to calculate the coefficients of the Padé rational function straightforward in the following examples. It is worth pointing out that because of the adaptability of MBPE, only a few of sampling points are enough to get the exact results. According to the uniform approximation theory [19], the error of MBPE interpolation is minimum when $m = n$ or $|m - n| = 1$, and the properties of existence and uniqueness of rational function approximation can be demonstrated [20].

## 4. GENERALIZED SYSTEM FUNCTION AND POLE-ZERO CHARACTERISTICS

Based on the theory of signals and systems, we know a particularly important and useful class of linear time-invariant (LTI) systems is those for which the input and output satisfy a linear constant-coefficient differential equation of the form [21]

$$\begin{aligned} & y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \cdots + a_1 y'(t) + a_0 y(t) \\ = \; & b_m f^{(m)}(t) + b_{m-1} f^{(m-1)}(t) + \cdots + b_1 f'(t) + b_0 f(t) \end{aligned} \tag{11}$$

where $f(t)$ and $y(t)$ represent the input and output time functions, respectively. Taking the Laplace transform of (11), we obtain

$$H(s) = \frac{Y(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{12}$$

$H(s)$ is commonly referred to as the system function or, alternatively, the transfer function. Many properties of LTI systems are closely associated with the characteristics of the system function in the s plane. It is very interesting that (12) is consistent with (8) formed by MBPE in mathematical representation. In physical sense, (12) represents the system function, which is the Laplace transform of impulse response of LTI systems. In the analysis of antenna or scattering electromagnetic systems, the ideal source models [22] of voltage, current, or unit plane wave are commonly utilized as the excitation functions, and the frequency responses of antenna properties, such as the current distribution $I(s)$, input impedance $Z_{in}(s)$, radiation patterns $E_{rad}(s, \theta, \varphi)$, RCS, or near fields $E_{near}(s, x, y, z)$, etc., can be thought of as the output functions. In this case, the output functions are just corresponding to the impulse responses of the antenna or scattering system in time domain. If we make use of MBPE technique to approximate the frequency responses of the output function, (8) is characterized by the system function. Therefore, the generalized system function $H(s)$ directly associated with the radiated and scattered fields can be constructed by MBPE technique in a limited operational bandwidth with a model containing a finite number of suitably chosen complex poles, which describes the intrinsic characteristics of the antenna or scattering systems. (8) can be further factored into the form (Let $a_n = 1$)

$$\begin{aligned} H(s) &= \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \cdots + s^n} \\ &= k\frac{(s - r_1)(s - r_2)\cdots(s - r_m)}{(s - s_1)(s - s_2)\cdots(s - s_n)} = k\frac{\prod_{i=1}^{m}(s - r_i)}{\prod_{j=1}^{n}(s - s_j)} \end{aligned} \tag{13}$$

where $k$ is scale factor. $s_1,\ s_2,\ \cdots s_n$ and $r_1,\ r_2,\ \cdots r_n$ are the complex poles and zeros of the generalized system function respectively. We know the denominator polynomial of $H(s)$ represents the characteristic polynomial of the antenna or scattering systems. The zeros of the denominator polynomial, namely the poles of $H(s)$, define the locations of the natural resonances of antenna or scattering systems. According to the stability of the electromagnetic systems, we know the true poles of the systems should reside in the left half of the complex frequency plane. The validity of the complex poles obtained by (13) will be discussed in the following section. It is assumed that the poles are all simple. This has been substantiated numerically. A partial fraction expansion yields

$$H(s) = \sum_{i=1}^{n} \frac{R_i}{s - s_i} \tag{14}$$

where $s_i$ represents the complex pole and $R_i$ is the corresponding residue. Therefore, many properties of antenna and scattering electromagnetic systems can be characterized by a few pole locations with the corresponding residues.

When the antenna and scattering systems are regarded as the multi-port networks, assumed $m$ input ports and $n$ output ports, the generalized system function matrix can be similarly constructed by MBPE based on the linear superposition principle, which can be expressed as

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \\ \vdots \\ Y_n(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \cdots & H_{1m}(s) \\ H_{21}(s) & H_{22}(s) \cdots & H_{2m}(s) \\ \vdots & & \\ H_{n1}(s) & H_{n2}(s) \cdots & H_{nm}(s) \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \\ \vdots \\ F_m(s) \end{bmatrix} \tag{15}$$

Namely

$$[Y(s)] = [H(s)] \cdot [F(s)] \tag{16}$$

where $[H(s)]$ represents the generalized system function matrix, and $H_{ij}(s)$ is referred to as the sub-system function. All true poles of the generalized system function matrix define the natural resonances of the antenna or scattering systems. $[H(s)]$ is a square matrix when $m = n$, thus the poles are the solutions to the following equation

$$\det[H(s_\alpha)] = 0 \tag{17}$$

where symbol **det**[ ] indicates taking the determinant of matrix.

## 5. RESONANT FREQUENCY, QUALITY FACTOR $Q$, AND RESIDUES OF OPEN SYSTEMS

We know that a finite number of suitably chosen complex poles of $H(s)$ define the natural resonances of the antenna or scattering electromagnetic open systems. Assume the pole $s_\delta = -\alpha + j\beta(\alpha > 0)$, the corresponding partial fraction of the generalized system function can be expressed as

$$H_\delta = \frac{R_\delta}{s - s_\delta} \tag{18}$$

The time response corresponding to the complex pole is

$$h_\delta(t) = R_\delta e^{s_\delta t} = R_\delta e^{-\alpha t} e^{j\beta t} \tag{19}$$

Comparing (19) with (2), we can see that the residue $R_\delta$ represents the complex magnitude of the electric field at $t = 0$, if the complex frequency response of the electric field $\vec{E}(s)$ at one point in space is chosen as the generalized system function. The relationship between the complex pole and the complex resonant frequency $\tilde{\omega}$ presented in the previous section is

$$s_\delta = j\tilde{\omega} = -\frac{\omega_0}{2Q} + j\omega_0 = -\alpha + j\beta \tag{20}$$

Therefore, the resonant frequency and $Q$ of the electromagnetic open system can be easily obtained

$$\begin{cases} \omega_0 = \beta \\ Q = \dfrac{\omega_0}{2\alpha} = \dfrac{\beta}{2\alpha} \end{cases} \tag{21}$$

Obviously, by calculating the complex poles of the generalized system function $H(s)$ based on the physical models, we can directly get the resonant frequency $\omega_0$ and corresponding $Q$. It is well known that the quantitative analysis of electric and magnetic field energies stored in the near-field zone of the antennas or scatterers is very difficult to give, thus the calculation of antenna or scattering $Q$ has been an interesting and challenging problem for years [10–14]. In this paper, the complex frequency method combined with the generalized system function is used to calculate the antenna or scattering $Q$ efficiently, which has been illustrated by the later numerical tests.

By analyzing the characteristics of poles and zeros of the generalized system function $H(s)$ and combining with adaptability of MBPE, we can predict the occurrence of resonance phenomena and determine the resonant frequency of the antenna and scattering systems exactly. It is shown that the intensity of resonance can be estimated effectively by the values of $Q$ and residues at the complex resonant frequencies. Only if both the resonant $Q$ and the residue of the resonant frequency are larger, are the resonance phenomena characterized by the strong peak field in the near region and strong frequency sensitivity in the far field region of the antennas and scattering bodies. If the optimization design of the excitations and radiation pattern of the resonant antenna array were carried out, the superdirective characteristic might be realized [5, 23]. If there exists a fact that a zero counteracts a pole in the generalized system function, it implies the resonant response contributed by the pole will vanish.

It should be pointed out that the complex poles referred above must be the true and stable poles of the antenna or scattering electromagnetic systems. The validity of the poles of the generalized system function constructed by MBPE can be demonstrated from two aspects. On the one hand, according to the stability of the practical antenna and scattering systems, these complex poles must locate in the left half of the s plane. Therefore, the poles occurred in the right half of the s plane must be invalid poles of the system. On the other hand, in MBPE, if the Padé rational functions with different numerator and denominator orders $(m, n)$ are used to construct the generalized system function, we might get some different complex poles. One knows that the true poles are corresponding to the complex natural resonant frequency of systems, which should be independent of the form of fitting function and the orders of the rational function. Thus, the locations of the true poles are stable or invariant. However, the other poles besides those residing in the right half of the s plane will vary with the orders of the rational functions. These poles must also be invalid poles and referred to as "parasitical" poles.

It should be noted that the resonance peak commonly occurs at the resonant frequency in high $Q$ electromagnetic systems. But in low $Q$ resonant systems, the location of the resonance peak is not consistent with the resonant frequency, which should locate at the frequencies satisfied the following equation

$$\frac{d|H(j\omega)|^2}{d\omega^2} = 0 \tag{22}$$

## 6. TIME DOMAIN RESPONSE

Based on the previous analysis, we have seen that the generalized system function of the antenna or scattering electromagnetic systems can be constructed by using MBPE technique in a limited operational bandwidth with a model containing a finite number of suitably chosen complex poles. A partial fraction expansion yields

$$H(s) = \sum_{i=1}^{k} \frac{R_i}{s - s_i} \tag{23}$$

where $k$ is the number of the true and stable poles. Taking the inverse Laplace transform for (23), we easily get

$$h(t) = \sum_{i=1}^{k} R_i e^{s_i t} = \sum_{i=1}^{k} R_i e^{-\frac{\omega_i}{2Q_i} t} e^{j\omega_i t} \tag{24}$$

where $\omega_i$, $Q_i$, and $R_i$ are the resonant frequency, quality factor and the corresponding residue of the $i^{\text{th}}$ natural resonance, respectively. We know that the time response of the antenna or scattering system should be a real time series, thus the true time domain impulse response of the antenna or scattering system can be expressed as

$$h(t) = \mathrm{Re}\left(\sum_{i=1}^{k} R_i e^{s_i t}\right) = \sum_{i=1}^{k} e^{-\frac{\omega_i}{2Q_i}t}\mathrm{Re}(R_i e^{j\omega_i t}) \tag{25}$$

where Re( ) indicates taking the real part. (25) shows that the time domain behavior of the antenna or scattering system is also the exponentially damped wave. When strong resonance behavior occurs, the corresponding $Q$ and $R$ are much larger than those of other resonant modes, and the impulse response decays very slowly. In the later response, the quasi-period decaying oscillation must take place, whose period is

$$T = \frac{1}{f_0} \tag{26}$$

where $f_0$ is the corresponding resonance frequency of the antenna and scattering systems. Therefore, the time domain behavior of the quasi-period decaying oscillation also reflects the characteristics of the resonance behavior profoundly. Nowadays many time-domain methods, such as Finite-Difference Time-Domain (FDTD) and Time-Domain MoM (TDMoM) [24], are extensively used to analyze the antenna and scattering problems. According to the time response waveform and the period of decaying oscillation in the later response, we can determine the resonance frequency from (26) effectively.

## 7. APPLICATIONS AND DISCUSSION

In the following examples, some special resonance behaviors will be analyzed by the generalized system functions associated with the radiated and scattered fields of some local regions, involving far fields and near fields. The results of the following numerical tests show the external resonance phenomena are very remarkable by virtue of the strong interaction and mutual coupling between antennas or scatterers.

**Test 1** Two parallel dipoles system

Consider the two parallel dipoles system shown in Fig. 1. Dipole 1 will be excited by the ideal voltage source, and the terminal of dipole 2 shorted. The length both of them is $L = 7.4\,\mathrm{m}$, with radius $a = 0.01\,\mathrm{m}$. The distance between them is $d = 1.0\,\mathrm{m}$. The frequency response of the radiated electric field of observation point at $(\theta, \varphi) = (90°, 0°)$ in the far zone is chosen as the output function, i.e., the generalized

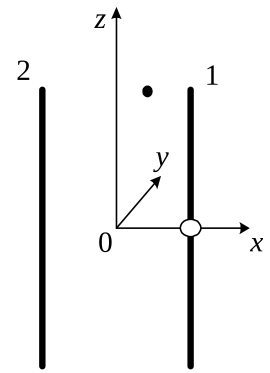



**Figure 1.** Two parallel dipoles system.

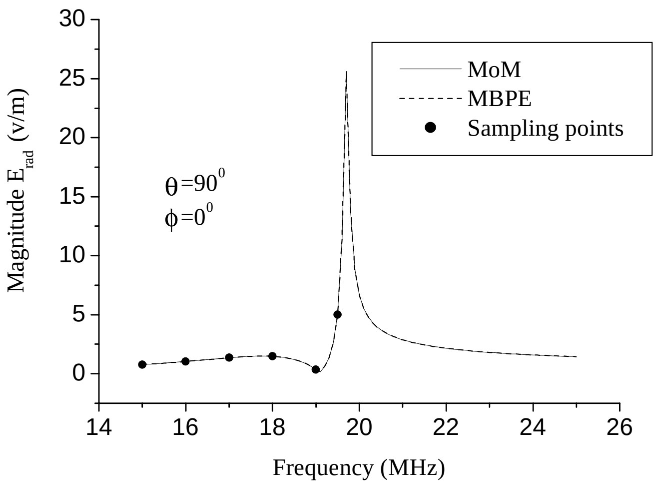



**Figure 2.** Frequency response of system function (far $E$-field) magnitude.

system function $H(s)$. The MBPE technique described in the previous section is applied to the antennas system over a frequency range of 15–25 MHz, using the radiated electric field data obtained from a numerically rigorous method of moments (MoM) computer codes. The Padé rational function is chosen to set the numerator order $m = 2$, the denominator order $n = 3$.

Fig. 2 shows the frequency response of the generalized system function constructed by MBPE, with comparisons being made of the MoM result. As can be seen from Fig. 2, the two curves are nearly graphically indistinguishable. In this case, only six sampling frequencies are required for the MBPE technique. The actual sampling points that were used are indicated by dots on the plots contained in Fig. 2. It is interesting that all of fitting frequencies are sampled before the resonant frequency, but the resonant behavior can be found efficiently according to the adaptability of MBPE technique.

The characteristics of zeros, poles, and corresponding residues of the generalized system function are shown in Table 1. Note that the data in the table have been transformed from $\omega$ to $f$ (MHz).

**Table 1.** Zeros, poles, and residues of the generalized system function.

| $E_{\text{rad}}[2, 3]$ | Zero | Poles | Residues |
|---|---|---|---|
| 1 | $-0.03698 + j19.0868$ | $17.7589 + j47.0234$ | $-16.0997 - j3.4361$ |
| 2 | $-0.7774 + j4.1237$ | $-0.05657 + j19.6921$* | $-0.03243 + j1.4617$ |
| 3 | | -2.1034+j18.4607* | 4.0613+j1.9741 |

The facts show that there are two true and stable complex poles in the antenna system within the finite operation frequency band, which are marked by asterisks in the Table 1. Based on the theory of the complex frequency presented in the previous section, from (21), the resonant frequencies and $Q$ of the two parallel dipoles system are obtained respectively as follows

$$\begin{cases} f_{res1} = 19.6921\,\text{MHz} \\ Q_1 = 174.05 \end{cases}, \qquad \begin{cases} f_{res2} = 18.4607\,\text{MHz} \\ Q_2 = 4.3883 \end{cases}.$$

It can be found the resonance behavior to occur at the frequency 19.6921 MHz with high $Q$, as shown in Fig. 2. It is worth pointing out that the calculation of antenna $Q$ is definite and efficient by using the complex frequency method. To demonstrate the validity of $Q$, a classical formula for finding $Q$ presented in [14] based on the Foster reactance theorem, $Q = \frac{[I^*]^t \left[\omega_0 \frac{\partial [X]}{\partial \omega} \pm [X]\right] [I]}{[I^*]^t [[Z] + [Z^*]^t][I]}$, has been used to calculate the antenna $Q$ of the two parallel dipoles system. Utilizing a first-order accurate difference approximation to the partial frequency derivative of the reactance matrix $[X]$, we obtain $Q = 169.1$ at the resonant frequency 19.6921 MHz, which is very closed to the result of the complex frequency method. It can be seen that the other resonant mode, 18.4607 MHz, makes a little contribution to the resonance behavior in this case for the low $Q$ and residue.

To gain insight into the characteristics of the resonance, the time domain response of the electric field at the observation point in the far

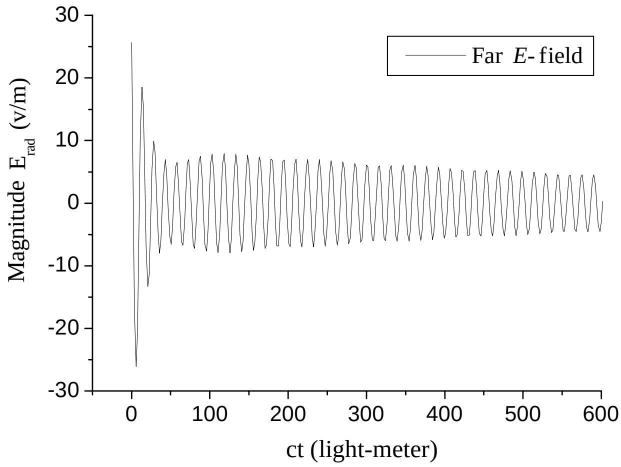



**Figure 3.** Time domain response of the far field.

zone was also calculated by (25) according to two true poles marked by asterisks in the Table 1, as shown in Fig. 3. It is noted that the convergence of the time domain wave is considerably slow, which is characterized by the quasi-period decaying oscillation in the later response, with the period $cT = 15.22$ light-meter. From (26), we can get the corresponding resonant frequency about 19.71 MHz.

To further understand the behavior of the resonance, we calculated the frequency response of the electric field magnitude at the observation point in the vicinity of the dipole 1, as shown in Fig. 4. It can be seen that the behavior of the resonance is also remarkably embodied by the phenomenon of strong peak field in the near zone of the antenna system. The comparison of the electric field magnitude distribution in the $xz$-plane in the near zone of the two parallel dipoles system at resonance with nonresonance is given in the Fig. 5a and (b), respectively. It should be noted that the electric fields are much stronger at resonance than those at nonresonance in the same excitation, but only accumulating in the vicinity of the dipoles. The symmetric distribution of the electric fields magnitude at resonance implies the balance of the electric field energy and magnetic field energy stored in the antenna open system physically.

**Test 2** Five-Element Circular Array for Finding Direction

Consider a practical five-element circular array for finding direction shown in Fig. 6. The antenna system is the circular array with radius 1.2 m, consisting of five same dipoles with length 2 m and

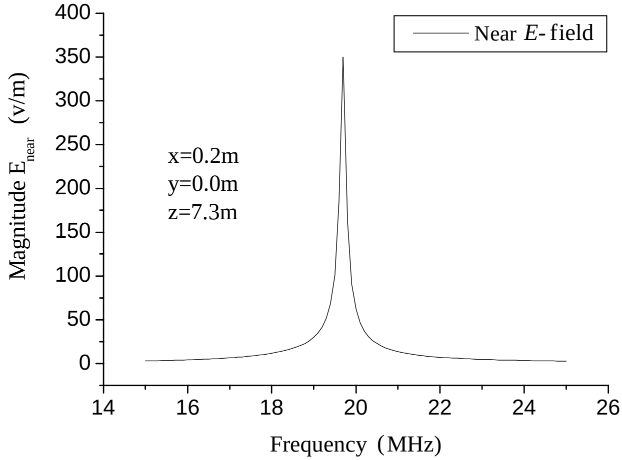
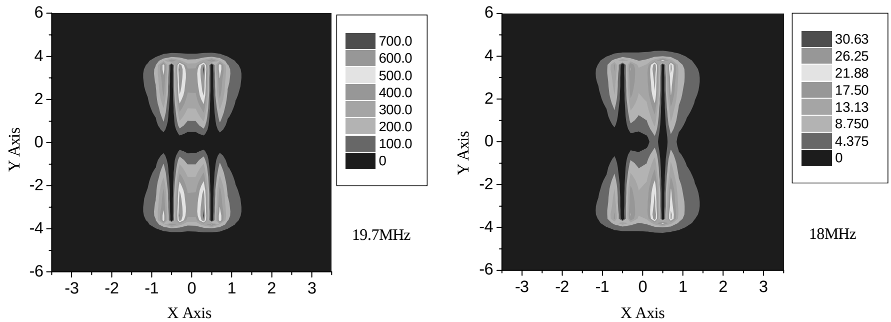



**Figure 4.** Frequency response of the near $E$-field magnitude.




(a) Resonance (b) Nonresonance

**Figure 5.** Comparison of the electric field distribution around dipoles at resonance with nonresonance.

radius of dipoles 0.015 m. The frequency band varies from 30 MHz to 110 MHz, and the feed points locate in the $xoy$ plane. Two kinds of cases are considered here.

**Case 1** Dipole 1 excited by the ideal voltage source and the others shorted

The frequency response of the radiated electric field of observation point at $\theta = 90°$ and $\phi = 0°$ in the far zone is chosen as the generalized system function $H(s)$ by utilizing MBPE technique. The

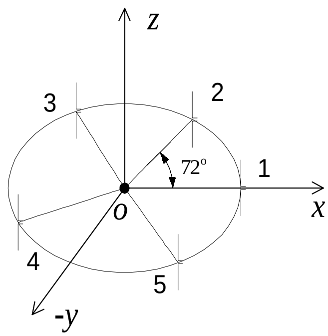



**Figure 6.** Five-element circular array for finding direction.

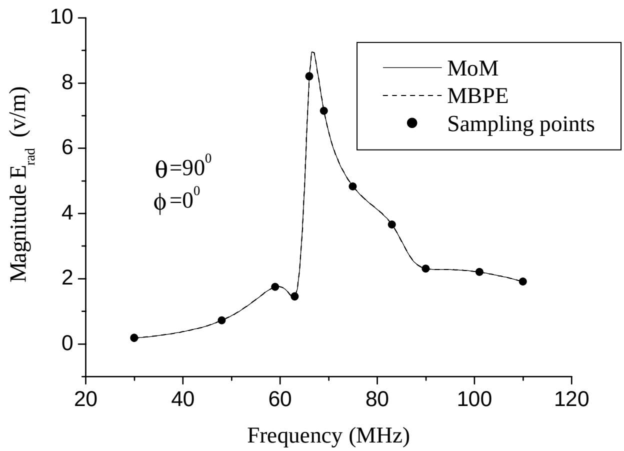



**Figure 7.** Frequency response of system function magnitude.

Padé rational function is chosen to have a numerator order $m = 5$, a denominator order $n = 5$. MoM-generated and MBPE-calculated curves of the radiated electric field magnitude versus frequency for this case are shown in Fig. 7, which shows excellent agreement between them. Table 2 shows the zeros, poles, and corresponding residues of the generalized system function. By analyzing the characteristics of the poles and zeros, we found three true and stable complex poles within the finite operation frequency band for the antenna system, which were marked by asterisks in the Table 2. On the basis of the complex frequency theory, from (21), the resonant frequencies and $Q$ of the five-element circular array for finding direction are obtained as

**Table 2.** Zeros, poles, and residues of the generalized system function.

| $E_{\rm rad}$ [5, 5] | Zeros | Poles | Residues |
|---|---|---|---|
| 1 | $-5.810 + j140.657$ | $98.157 + j110.231$ | $8.639 - j279.965$ |
| 2 | $-4.518 + j87.020$ | $-55.655 + j126.831$ | $-124.406 + j6.919$ |
| 3 | $-1.215 + j63.300$ | $-5.056 + j85.287$* | $4.818 - j3.442$ |
| 4 | $-20.642 + j42.254$ | $-1.525 + j65.951$* | $11.690 - j6.153$ |
| 5 | $3.020 + j16.846$ | $-8.031 + j63.340$* | $16.419 + j20.658$ |

follows

$$\begin{cases} f_{res1} = 63.34\,\text{MHz} \\ Q_1 = 3.9235 \end{cases}, \begin{cases} f_{res2} = 65.951\,\text{MHz} \\ Q_2 = 21.6233 \end{cases}, \begin{cases} f_{res3} = 85.287\,\text{MHz} \\ Q_3 = 8.4342 \end{cases}$$

As can be seen from Fig. 7, the peak field appears at the second resonant frequency (65.951 MHz) in the case of dipole 1 excited. It is worth pointing out that the resonant $Q$-value, approximatively calculated by $Q = \frac{[I^*]^t \left[\omega_0 \frac{\partial [X]}{\partial \omega} \pm [X]\right][I]}{[I^*]^t [[Z]+[Z^*]^t][I]}$, is about 19.2, which is of small error with the result of the complex frequency method.

**Case 2** All dipoles excited by the ideal voltage sources.

In this case, the generalized system function matrix of multi-input and single-output will be constructed by MBPE technique, which can be expressed as follows

$$[Y(s)] = \vec{E}_{\rm rad}(s) = [H_1(s)\ H_2(s)\ H_3(s)\ H_4(s)\ H_5(s)] \begin{bmatrix} F_1(s) \\ F_2(s) \\ F_3(s) \\ F_4(s) \\ F_5(s) \end{bmatrix} \quad (27)$$

where $H_i(s)$ represents the sub-system function of $i^{\rm th}$ antenna excited by the ideal voltage source and the others shorted. The same observation point and radiated electric field function, the same numerator and denominator orders for the Padé rational function with Case 1 were used. According to the symmetries of the antenna structure and observation point, we have $H_2(s) = H_5(s)$ and $H_3(s) = H_4(s)$ Thus, the above expression can be reduced to

$$E_{\rm rad}(s) = H_1(s) + 2H_2(s) + 2H_3(s) \quad (28)$$

**Table 3.** Zeros, poles, and residues of the sub-system function $H_2(s)$.

| $H_2(s)$ [5, 5] | Zeros | Poles | Residues |
|---|---|---|---|
| 1 | $-59.545 + j34.543$ | $-101.862 + j68.364$ | $55.50 - j119.480$ |
| 2 | $-0.8424 + j88.325$ | $68.205 + j48.011$ | $6.677 + j11.423$ |
| 3 | $-5.418 + j67.944$ | $-5.020 + j85.286$* | $4.762 - j3.375$ |
| 4 | $-20.773 + j58.714$ | $-1.524 + j65.954$* | $9.457 + j4.935$ |
| 5 | $0.369 + j16.270$ | $-8.031 + j63.343$* | $4.844 + j6.495$ |

**Table 4.** Zeros, poles, and residues of the sub-system function $H_3(s)$.

| $H_3(s)$ [5, 5] | Zeros | Poles | Residues |
|---|---|---|---|
| 1 | $168.159 + j65.150$ | $-31.535 + j133.755$ | $-28.460 - j38.289$ |
| 2 | $2.764 + j9.550$ | $-35.095 + j51.299$ | $-7.582 + j9.437$ |
| 3 | $-46.263 + j65.335$ | $-5.062 + j85.284$* | $4.701 - j3.266$ |
| 4 | $-2.882 + j82.018$ | $-1.525 + j65.953$* | $-3.614 + j1.891$ |
| 5 | $-2.276 + j67.279$ | $-8.036 + j63.348$* | $-13.703 + j16.591$ |

The frequency responses of three sub-system functions are shown in Fig. 8 calculated by MBPE technique, of which the zeros, poles and residues are shown in Tables 2, 3, and 4, respectively. Note that $H_1(s)$ is the same as the system function of Case 1.

From Table 2 to 4, it is three true complex poles that exist in the antenna system within the frequency band, which are marked by asterisks in the Tables 2, 3, and 4, respectively. However, for the total output response $E_{rad}(s)$, using the linear superposition (28), we found the residues corresponding to two natural resonant frequencies (63.34 MHz and 65.95 MHz) nearly vanish. It implies that these poles make a little contribution to the total resonant behavior in this case. As can be seen from Fig. 9, the resonance peak field does not occurs at the frequency 65.95 MHz like Case 1, but at the frequency 85.286 MHz. The $Q$-value corresponding to the resonant frequency is 8.43, which is calculated by the complex frequency method described in this paper. Compared with the result of 8.05, which is calculated by the Foster reactance theorem formula [14], it is to further illustrate the validity of the complex frequency method.

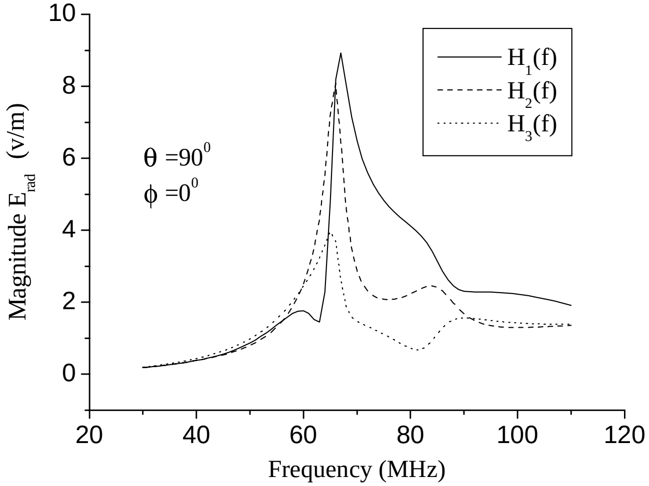



**Figure 8.** Frequency responses of three sub-system functions.

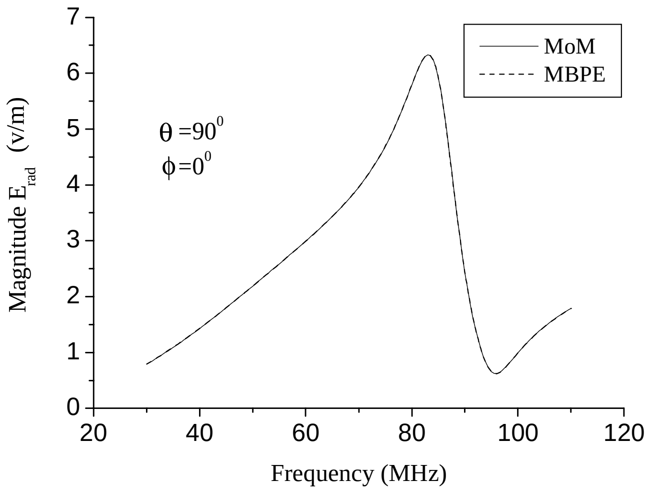



**Figure 9.** Total response of the radiated electric field magnitude.

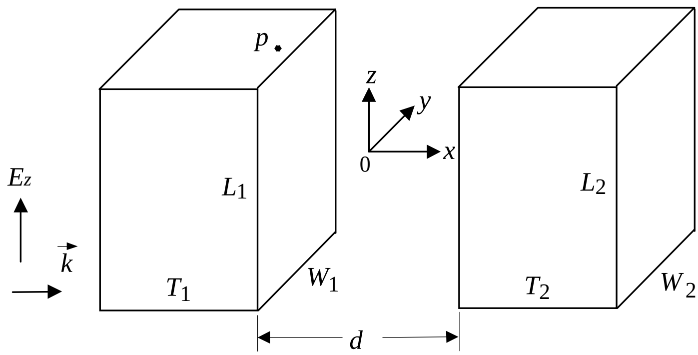



**Figure 10.** Two conducting bodies scattering system.

**Test 3** Two conducting bodies scattering system

Consider the two conducting bodies scattering system shown in Fig. 10, which is excited by the normalized $z$-polar plane wave $\vec{E}_z = e^{-j\vec{k}\cdot\vec{r}}$, and the direction of propagation is $\hat{k} = \hat{x}$. The sizes of two perfect conducting bodies are $T_1 = T_2 = 1.8\,\text{m}$, $W_1 = W_2 = 1.96\,\text{m}$, $L_1 = L_2 = 6.3\,\text{m}$, and the two conducting bodies are $d = 1.2\,\text{m}$ apart. The scattered electric fields of the observation point, $P(-0.6, 0.0, 3.45)$, in the near zone of the scattering bodies are chosen as the generalized system functions in order to analyze the resonance characteristics of near fields. Because the scattered electric field in the near region has two main components $E^x_{sca}$ and $E^z_{sca}$, the MBPE technique are used to approximate the frequency response of the scattered fields in two directions, respectively. The Padé rational function are chosen to have the same numerator order $m = 4$ and denominator order $n = 4$ . From Fig. 11, it can be seen that the solid line calculated by MoM is mostly hidden by the MBPE curve.

The zeros, poles and residues of the generalized system functions of $x$ and $z$ directions are calculated and shown in Tables 5 and 6, respectively. From Tables 5 and 6, we found there exist two true and stable complex poles in the scattering system, which are marked by asterisks respectively. Based on the theory of the complex frequency, we can easily get the resonant frequencies and scattering $Q$ as follows

$$\begin{cases} f_{res1} = 18.169\,\text{MHz} \\ Q_1 = 9.6633 \end{cases}, \qquad \begin{cases} f_{res2} = 13.809\,\text{MHz} \\ Q_2 = 1.318 \end{cases}$$

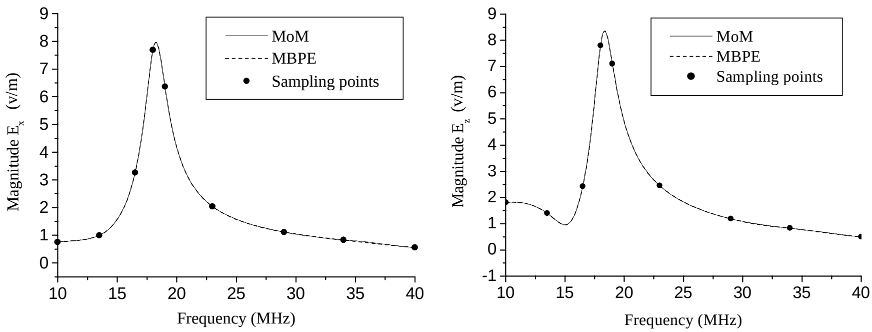
According to the $Q$-value and the corresponding residue, we can estimate that the resonance phenomenon of strong peak near field

**Table 5.** Zeros, poles, and residues of the $x$-direction system function $H_x(s)$.

| $H_x(s)[4,\ 4]$ | Zeros | Poles | Residues |
|---|---|---|---|
| 1 | $33.4845 + j52.0653$ | $-13.3008 + j30.6331$ | $-1.9322 - j8.2252$ |
| 2 | $4.9419 + j30.6398$ | $5.0925 + j30.8850$ | $0.1968 + j0.01556$ |
| 3 | $-20.6723 + j17.7313$ | $-0.9401 + j18.1687$* | $4.4467 - j6.2500$ |
| 4 | $-2.7890 + j13.7039$ | $-5.2580 + j13.8088$* | $3.3030 + j0.6869$ |

**Table 6.** Zeros, poles, and residues of the $z$-direction system function $H_z(s)$.

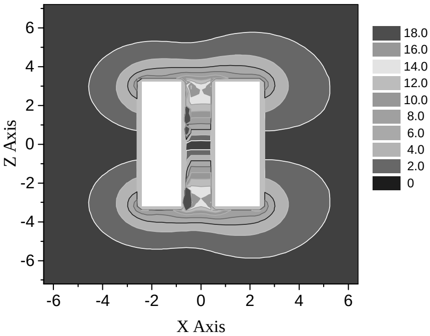
| $H_z(s)[4,\ 4]$ | Zeros | Poles | Residues |
|---|---|---|---|
| 1 | $68.3855 + j43.6232$ | $-13.5184 + j48.2835$ | $-1.1581 - j9.4775$ |
| 2 | $2.1261 + j50.8969$ | $-2.6378 + j33.2229$ | $-0.1892 - j0.08011$ |
| 3 | $-2.6497 + j33.0230$ | $-0.9401 + j18.1683$* | $5.0282 + j6.7202$ |
| 4 | $-0.9475 + j15.2295$ | $-5.2583 + j13.8090$* | $-14.3396 + j3.4025$ |



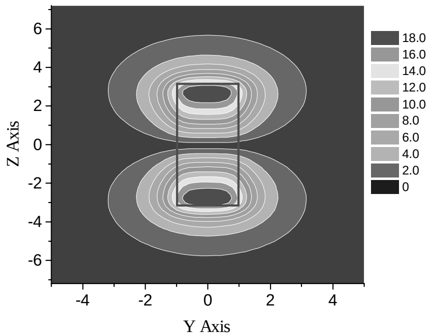

(a) $E^x_{sca}$ component (b) $E^z_{sca}$ component

**Figure 11.** Frequency responses of the scattered near field magnitude.

would appear at the frequency 18.169 MHz, as shown in Fig. 11. The magnitude distributions of the scattered electric field in $XZ$ and $YZ$ plane at resonance (18.169 MHz) are shown in Fig. 12(a) and (b), respectively. It is interesting that the scattered electric fields mainly accumulate in the region between the two conducting bodies, being strong at both sides and weak at center, with stand-wave-like distribution.




(a) $XZ$ plane




(b) $YZ$ plane

**Figure 12.** Scattered electric field magnitude distributions in $XZ$ and $YZ$ plane at resonance (18.169 MHz).

## 8. CONCLUSION

This paper has presented the generalized system function $H(s)$ directly associated with electromagnetic fields to analyze the resonance behavior and quality factor of the antenna and scattering open systems, which is constructed by the MBPE technique in the complex frequency domain. By analyzing the characteristics of poles, zeros, and corresponding residues of $H(s)$, we can predict the occurrence of resonance phenomena and determine the resonant frequencies of the antenna or scattering systems exactly. Based on the theory of the complex frequency, which unifies the resonant frequency and the antenna or scattering $Q$, an effective method for calculating the antenna and scattering $Q$ has been presented in this paper. The intensity of resonance can be estimated effectively in terms of $Q$-value and residues at the complex resonant frequencies, which give a theory basis of the EMC analysis and design. Because of strong interaction and mutual coupling between antennas or scatterers, the resonance behaviors are so remarkable that those may be characterized by the strong peak field in the near region and superdirectivity in the far region of the antenna and scattering systems in the frequency responses, and by the exponentially damped quasi-period oscillation in time domain responses.

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**Li Long** was born in Anshun, Guizhou, P. R. China on January 7, 1977. He received the Bachelor degree in Electromagnetic Field and Microwave Technology from Xidian University in 1998. Since 1999, he has taken a combined master-doctor program and worked for Ph.D. in Xidian University. His present research interests are mainly in electromagnetic slots coupling, hybrid algorithms and electromagnetic compatibility.

**Liang Changhong** was born in Shanghai, China on December 9, 1943. He graduated in 1965 from the former Xidian University, Xi'an, P. R. China, and continued his graduate studies until 1967. From 1980 to 1982, he worked at Syracuse University, New York, USA as a visiting scholar. He has been a professor and Ph.D. student advisor of Xidian University since 1986. Prof. Liang is a Fellow of CIE and a senior member of IEEE.