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NONLINEAR WAVES IN DOPED MATERIAL WITH UNIAXIAL ELECTRICAL-MAGNETIC COUPLING

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Abstract—Helix particle exhibits uniaxial electrical-magnetic coupling and doped material with helix particles has the nonlinearity properties of electromagnetic waves. Based on the small nonlinearity assumption, nonlinear electromagnetic waves propagating in doped materials with transversely and longitudinally uniaxial electrical-magnetic coupling are analytically formulated, respectively. It is shown that this class of nonlinear material can simultaneously support right- and left-handed elliptically-polarized nonlinear waves. In the case of transversely uniaxial electrical-magnetic coupling, the two nonlinear waves propagate with different phase velocities (sub- and super-luminously, respectively) and spatial profiles. For the case of longitudinally uniaxial electrical-magnetic coupling, the two nonlinear waves exhibit different spatial profiles but propagate with the same phase velocity. It is also found that complex nonlinear waves, which propagate with complex phase factor, could exist for certain constitutive parameters of this class of nonlinear material.

1. INTRODUCTION

With the use of powerful laser that produce light beams of high intensity, nonlinear waves in composite materials have received considerable attention in connection with applications to optical signal processing and integrated optics [1, 2]. The typical methodology for investigating the nonlinear phenomenon has been to solve the nonlinear equations analytically or numerically. With only few exception [3–5], almost all of the formulations have constrained themselves on the Kerr-type nonlinearity, where the nonlinear part of the permittivity of the material is proportional to the square of the electric field strength [6–11]. Therefore, it is necessary to extensively study the phenomena associated with the non-Kerr-type nonlinearity, which is frequently encountered in the realistic situations.

Due to the experimental observation of the nonlinear characteristics of helix particles [12], it becomes interesting to study the nonlinear phenomena in the composite nonlinear materials which are composed of the elements of helix. On the other hand, with advances in polymer synthesis techniques, it has become realizable to fabricate new types of composite materials with helix particles [13]. Due to the introduction of uniaxial electrical-magnetic coupling by the helix particles, doped material with helix particles would exhibit complicated electromagnetic properties. Among the new composite materials consisting of helix particles, one should mention the two basic types: chiral material [13–15] and uniaxial chiral material [16–18]. Self-action of electromagnetic waves and unique spatial soliton phenomena in nonlinear chiral material have been reported [19, 20]. Nevertheless, continuing effort is still required in order to achieve a throughout understanding and to exploit the possible novel phenomena in this class of helix-particle composite nonlinear materials.

In the present study, nonlinear electromagnetic waves in the doped material with transversely and longitudinally uniaxial electrical-magnetic coupling are analytically investigated, respectively. In practice, the doped nonlinear material with uniaxial electrical-magnetic coupling could be created by immersing small helix particles in a nonlinear host medium in such a way that the axes of all helices are oriented with a preferred direction, but distributed in random locations. Here, the nonlinear part of the permittivity of the host nonlinear medium is supposed to be proportional to an arbitrary but specified real power of the electric field strength, recovering the case of Kerr- as well as non-Kerr-type nonlinear material. Basically, there are two methods to make the helices have a preferred direction: the helix elements can be such arranged that the axes of all the helices are parallel or perpendicular to a fixed direction.

This manuscript is organized as follows. In Sections 2 and 3, formulations for the nonlinear electromagnetic waves in doped material with transversely and longitudinally uniaxial electrical-magnetic coupling are analytically presented, respectively. Typical numerical results for the two cases are given in Section 4, separatively. Section 5 concludes the present investigation with emphasis on the three novel characteristics of the nonlinear waves.

In what follows, the harmonic $\exp(i\omega t)$ time dependence is assumed and suppressed.

2. FORMULATIONS FOR THE DOPED NONLINEAR MATERIAL WITH TRANSVERSELY UNIAXIAL ELECTRICAL-MAGNETIC COUPLING

Without any loss of generality, suppose the nonlinear waves propagate along the z axis. Then, the nonlinear material with transversely uniaxial electrical-magnetic coupling can be characterised by the constitutive relations

$$\boldsymbol{D} = \varepsilon\boldsymbol{E} - i(\varepsilon_0\mu_0)^{1/2}Y_t\overline{\boldsymbol{I}}_t\cdot\boldsymbol{H} \tag{1a}$$

$$\boldsymbol{B} = \mu\boldsymbol{H} + i(\varepsilon_0\mu_0)^{1/2}Y_t\overline{\boldsymbol{I}}_t\cdot\boldsymbol{E} \tag{1b}$$

where $\overline{\boldsymbol{I}}_t = \boldsymbol{e}_x\boldsymbol{e}_x + \boldsymbol{e}_y\boldsymbol{e}_y$ is the transverse idem factor, $\varepsilon = \varepsilon_L + \varepsilon_N|\boldsymbol{E}|^\delta$ is the intensity-dependent permittivity representing the nonlinearity of this material, and Y_t stands for the degree of electrical-magnetic coupling. Here, ε_N is a specified positive number and δ can be an arbitrary real number. ε_0 and μ_0 denote the permittivity and permeability of free space, respectively.

To have an idea on the microstructure of the composite material, we consider two sets of helix elements, $5N$ right-handed (left-handed) and N left-handed (right-handed), distributed in a host medium with random locations. Let $2N$ of the right-handed (left-handed) helices have their axes parallel to the x axis, $2N$ of the right-handed (left-handed) parallel to the y axis and the remaining $2N$ helices (N right-handed and N left-handed) parallel to the z axis. Then, the permittivity and permeability of the composite material are isotropic and the electrical-magnetic coupling must have no z component and the constitutive relations posses the forms of Equation (1).

For the sake of simplicity, we consider the canonical (1+1)-dimensional nonlinear waves, where the field quantities depend only on one lateral coordinate. In this case, $\partial/\partial y = 0$ and $\boldsymbol{F}(x, y, z) = \boldsymbol{F}(x)\exp(-i\beta z)$ for $\boldsymbol{F} = \boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{D}$, $\boldsymbol{B}$. Here, for simplicity, only the case corresponding to the positive real part of the phase factor β is concerned. (The case of positive real part β corresponds to wave propagating in the $+z$ direction, while the negative real part of β stands for that travelling in the $-z$ direction.)

2.1. Coupled Differential Equations

Substituting the constitutive relations (1a) and (1b) into the source-free Maxwell equations and taking a reasonable assumption that ε_N is much smaller than ε_L such that the terms containing the derivatives of ε with respect to x are negligible, the field components of the nonlinear wave can be simply represented in terms of E_x and H_x:

$$H_y = \frac{1}{\beta}(\omega\varepsilon E_x - ik_0 Y_t H_x) \tag{2a}$$

$$E_y = -\frac{1}{\beta}(\omega\mu H_x + ik_0 Y_t E_x) \tag{2b}$$

$$H_z = \frac{1}{\beta}\left(\frac{k_0 Y_t}{\omega\mu}\cdot\frac{dE_x}{dx} - i\frac{dH_x}{dx}\right) \tag{2c}$$

$$E_z = -\frac{1}{\beta}\left(\frac{k_0 Y_t}{\omega\varepsilon}\cdot\frac{dH_x}{dx} + i\frac{dE_x}{dx}\right) \tag{2d}$$

A set of coupled differential equations of E_x and H_x can also be obtained after appropriate algebraic manipulation:

$$\omega\varepsilon\frac{d^2E_x}{dx^2} - ik_0 Y_t\frac{d^2H_x}{dx^2} + \omega\varepsilon\left[\left(\omega^2\varepsilon\mu + k_0^2Y_t^2 - \beta^2\right)E_x - 2i\omega\mu k_0 Y_t H_x\right] = 0 \tag{3a}$$

$$i\omega\mu\frac{d^2H_x}{dx^2} - k_0 Y_t\frac{d^2E_x}{dx^2} + \omega\mu\left[i\left(\omega^2\varepsilon\mu + k_0^2Y_t^2 - \beta^2\right)H_x - 2\omega\varepsilon k_0 Y_t H_x\right] = 0 \tag{3b}$$

where $k_0 = \omega(\varepsilon_0\mu_0)^{1/2}$. Let $H_x = -i\eta E_x$ to decouple the differential equations, where η is a constant to be determined. Comparing (3a) with (3b), it is easily found that

$$\eta_\pm = \pm\sqrt{\varepsilon/\mu} \simeq \pm\sqrt{\varepsilon_L/\mu} \tag{4}$$

In Equation (4) and the following formulations, small nonlinearity approximation is assumed for the sake of simplicity such that the condition $\varepsilon_L \gg \varepsilon_N|\boldsymbol{E}|^\delta$ is satisfied. By substituting (4) into (3b), a differential equation of E_x is obtained

$$\frac{d^2E_x}{dx^2} + \left(\omega^2\varepsilon\mu f_\pm + k_0^2 Y_t^2 g_\pm - g_\pm\beta^2\right) E_x = 0 \tag{5}$$

where

$$f_\pm = \frac{\omega\mu\eta_\pm - 2k_0Y_t}{\omega\mu\eta_\pm - k_0Y_t} \tag{6a}$$

$$g_\pm = \frac{\omega\mu\eta_\pm}{\omega\mu\eta_\pm - k_0Y_t} \tag{6b}$$

corresponding to $\eta_\pm$, respectively.

Once solutions to Equation (5) are obtained, the other field components and henceafter the properties of the nonlinear waves can be straightforwardly obtained from Equations (2) and (4).

It should be noted that the positive definite energy function of linear uniaxial chiral material [21] would require a constraint condition on the constitutive parameters: $\omega\sqrt{\varepsilon_L\mu} - k_0|Y_t| > 0$. Here, we assume that this constraint condition on the constitutive parameters still holds in the present investigation, by considering $\varepsilon_N \ll \varepsilon_L$.

2.2. Nonlinear Wave Solutions

Let the nonlinear wave solutions of Equation (5) be of the form:

$$E_x = A_0 \operatorname{sech}^{\frac{2}{\delta}}(\alpha x) \tag{7}$$

Noting the permittivity ε is intensity-dependent and substituting (7) and (2b), (2d) into Equation (5), we have

$$\frac{d^2E_x}{dx^2} + \left(p_\pm - g_\pm\beta^2 + q_\pm|E_x|^\delta\right) E_x = 0 \tag{8}$$

where a higher order term that depends on $|E_x|^{\delta+2}$ has been dropped as we focus in this study solely on the lowest-order nonlinearity. Here,

$$p_\pm = \omega^2\varepsilon_L\mu f_\pm + k_0^2 Y_t^2 g_\pm \tag{9a}$$

$$q_\pm = \omega^2\varepsilon_N\mu f_\pm \left[1 + \frac{1}{\beta^\delta}(\omega\mu\eta_\pm - k_0Y_t)^\delta\right] \tag{9b}$$

After straightforward algebraic manipulation of Equation (8) in conjunction with (9), we obtain

$$\alpha^2 = \frac{\delta^2 q_\pm A_0^\delta}{2(\delta+2)} \tag{10a}$$

$$\beta^2 = \frac{P_\pm + \frac{4\alpha^2}{\delta^2}}{g_\pm} \tag{10b}$$

Generally, for a certain specified δ, numerical method (such as Muller root-searching technique) must be employed to obtain the quantitative information about the properties of the nonlinear waves propagating in the doped nonlinear material with transversely uniaxial electrical-magnetic coupling.

The ellipticity of the polarized nonlinear wave is given by $|E_y/E_x| = (\omega\sqrt{\varepsilon\mu} \mp k_0 Y_t)/\beta_\pm$, which is apparently intensity-dependent. Further straightforward analysis reveals that the phase factor β_+ and β_- correspond to the right- and left-handed elliptically-polarized nonlinear waves, respectively.

It can be easily checked that the present nonlinear wave solutions for Equation (5) unify those of the Kerr-type nonlinear isotropic medium [6, 7] when Y_t and $\delta = 2$. As $\varepsilon_N = 0$, the propagation phase factor $\beta = \omega\sqrt{\varepsilon_L\mu} \pm Y_t k_0$, which recovers those of linear material with transversely uniaxial electrical-magnetic coupling [16–18].

3. FORMULATIONS FOR THE DOPED NONLINEAR MATERIAL WITH LONGITUDINALLY UNIAXIAL ELECTRICAL-MAGNETIC COUPLING

The nonlinear material with longitudinal electrical-magnetic coupling, where the nonlinear waves are supposed to propagate in the $+z$ direction, can be characterized by the constitutive relations

$$\boldsymbol{D} = \varepsilon\boldsymbol{E} - i(\varepsilon_0\mu_0)^{1/2} Y_1 \boldsymbol{e}_z\boldsymbol{e}_z \cdot \boldsymbol{H} \tag{11a}$$

$$\boldsymbol{B} = \mu\boldsymbol{H} + i(\varepsilon_0\mu_0)^{1/2} Y_1 \boldsymbol{e}_z\boldsymbol{e}_z \cdot \boldsymbol{E} \tag{11b}$$

where $\varepsilon = \varepsilon_L + \varepsilon_N|\boldsymbol{E}|^\delta$ is intensity-dependent permittivity and represents the nonlinearity of this material, ε_N is a specified positive number, and δ can be an arbitrary real number. The electrical-magnetic coupling parameter $Y_1 > 0$ ($Y_1 < 0$) corresponds to the right-handed (left-handed) helix elements of the composite nonlinear material, respectively. The positive definite energy function [21] of this nonlinear material requires $\sqrt{\varepsilon\mu} > Y_1\sqrt{\varepsilon_0\mu_0}$. In this material, all the axes of the helix elements are arranged parallel to the z axis but distributed in random distributions.

For the sake of simplicity, we also consider the traditional (1+1)-dimensional nonlinear waves, where the field quantities depend only on one lateral coordinate. In this situation, we have $\partial/\partial y = 0$ and $\boldsymbol{F}(x, y, z) = \boldsymbol{F}(x)\exp(-i\beta z)$ for $\boldsymbol{F} = \boldsymbol{E}, \boldsymbol{H}, \boldsymbol{D}, \boldsymbol{B}$. Here, β characterizes the phase properties of the transmitting wave, and $\boldsymbol{F}(x)$ determines its spatial profile.

Substituting the constitutive relations (11a) and (11b) into the source-free Maxwell equations and taking a reasonable assumption that ε_N is much smaller than ε_L such that the terms containing the derivatives of ε with respect to x are negligible, appropriate mathematical manipulation leads to a set of expressions where the components of the nonlinear wave H_y, E_y, H_z, and E_z are represented in terms of E_x and H_x:

$$H_y = \frac{\omega\varepsilon}{\beta} E_x \tag{12a}$$

$$E_y = -\frac{\omega\mu}{\beta} H_x \tag{12b}$$

$$H_z = \frac{-\frac{\omega\varepsilon}{\beta}}{\omega^2\varepsilon\mu - k_0^2 Y_1^2}\left(k_0 Y_1 \frac{dE_x}{dx} + i\omega\mu \frac{dH_x}{dx}\right) \tag{12c}$$

$$E_z = \frac{\frac{\omega\mu}{\beta}}{\omega^2\varepsilon\mu - k_0^2 Y_1^2}\left(k_0 Y_1 \frac{dH_x}{dx} - i\omega\varepsilon \frac{dE_x}{dx}\right) \tag{12d}$$

And a set of coupled differential equations of E_x and H_x are obtained:

$$\omega\varepsilon k_0 Y_1 \frac{d^2 E_x}{dx^2} + i\omega^2\varepsilon\mu \frac{d^2 H_x}{dx^2} + i\left(\omega^2\varepsilon\mu - k_0^2 Y_1^2\right)\left(\omega^2\varepsilon\mu - \beta\right) H_x = 0 \tag{13a}$$

$$\omega^2\varepsilon\mu \frac{d^2 E_x}{dx^2} + i\omega\mu k_0 Y_1 \frac{d^2 H_x}{dx^2} + \left(\omega^2\varepsilon\mu - k_0^2 Y_1^2\right)\left(\omega^2\varepsilon\mu - \beta^2\right) E_x = 0 \tag{13b}$$

Let $H_x = -i\eta E_x$ to decouple the differential Equations (13a) and (13b), we can rigorously find $\eta_\pm = \pm\sqrt{\varepsilon/\mu}$. Simultaneously, a differential equation of E_x can be obtained within the limitation of small nonlinearity assumption $\varepsilon_N|\boldsymbol{E}|^\delta \ll \varepsilon_L$

$$(1 \pm u_1)\frac{d^2 E_x}{dx^2} + (1 - u_1^2)\left(\omega^2\varepsilon_L\mu - \beta^2 + \omega^2\varepsilon_N\mu\left[1 + \left(\frac{\omega\mu}{\beta}\eta_\pm\right)^\delta\right]|E_x|^\delta\right)E_x = 0 \tag{14}$$

where $u_1 = Y_1\sqrt{\varepsilon_0\mu_0}/\sqrt{\varepsilon_L\mu}$ is the normalized longitudinal electrical-magnetic coupling parameter.

The nonlinear wave solutions of Equation (14) are supposed to be of the form

$$E_x = A_0 \operatorname{sech}^{\frac{2}{\delta}}(\alpha x) \tag{15}$$

where α determines the shape of the profile of the propagating wave. Substituting Equation (15) into (14) and after lengthy algebraic manipulations, we end up with

$$\alpha^2 = \frac{q_\pm A_0^\delta \delta^2}{2(\delta+2)(1\pm u_1)} \tag{16a}$$

$$\beta^2 = \omega^2 \varepsilon_L \mu + \frac{4\alpha^2(1\pm u_1)}{\delta^2(1-u_1^2)} \tag{16b}$$

where

$$q_\pm = \omega^2 \varepsilon_N \mu \left[1 + \left(\frac{\omega\mu}{\beta}\eta_\pm\right)^\delta\right] \tag{17}$$

Once again, for the general case it is often difficult to derive a set of analytical solutions of the nonlinear Equations (16a) and (16b). Usually, numerical implementation must be applied to get the quantitative results of the nonlinear waves propagating in the doped nonlinear material with longitudinally uniaxial electrical-magnetic coupling.

The ellipticity of the polarized nonlinear wave is given by $|E_y/E_x| = \omega\sqrt{\varepsilon\mu}/\beta_\pm$, which is also intensity dependent. Further analysis reveals that the nonlinear wave are right- and left-handed elliptically-polarized, corresponding to β_+ and β_-, respectively.

It can be easily shown that the present nonlinear wave solutions of the nonlinear wave equation for the nonlinear material with longitudinal electrical-magnetic coupling recover that of the conventional nonlinear dielectric [6, 7] with $Y_1 = 0$ and $\delta = 2$.

4. NUMERICAL EXAMPLES

To have an idea about the propagation properties of the nonlinear waves in the doped nonlinear material with uniaxial electrical-magnetic coupling, we have to resort to the Muller root-seeking technique to solve the characteristic Equations (10) and (17), respectively.

Figure 1 typically illustrates the variation of the phase factor against the electrical-magnetic coupling parameter for the case of the transversely uniaxial electrical-magnetic coupling. For convenience, the phase factor β is normalized with respect to $k_L = \omega\sqrt{\varepsilon_L\mu}$, and the normalized transverse electrical-magnetic coupling parameter is defined as $u_t = Y_t\sqrt{\varepsilon_0\mu_0}/\sqrt{\varepsilon_L\mu}$. It should be pointed out that due to the small quantities, the image parts of the normalized phase factors are not plotted in this figure, which exist in the range of $0.5 < |u_t| < 1$. From Figure 1, it is seen that the nonlinear material with transversely

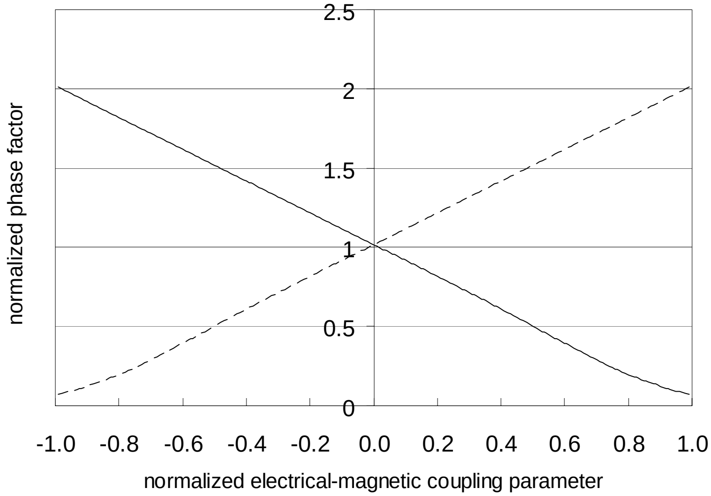

Figure 1. Variation of the normalized phase factor β/k_L against the normalized transverse electrical-magnetic coupling parameter k_0Y_t/k_L for the nonlinear waves traveling in a nonlinear material with transversely uniaxial electrical-magnetic coupling. Due to the small quantities, the image parts of the normalized phase factors are not plotted in this figure, which exist in the range of $0.5 < |u_t| < 1$. The solid and dashed lines correspond to right- and left-handed elliptically-polarized nonlinear waves, respectively. Here, $A_0^2 = 0.2$, $\delta = 1.6$, and $\varepsilon_N/\varepsilon_L = 0.3$.

uniaxial electrical-magnetic coupling can simultaneously support right- and left-handed elliptically-polarized nonlinear waves, which propagate with different phase velocities $\omega/\beta_\pm$. Moreover, for $0.5 < |u_t| < 1$, the multiple nonlinear waves would propagate with attenuation (i.e., the phase factor is complex). Another significant feature of the nonlinear waves is that one of the two waves (right- or left-handed elliptically polarized waves) can propagate with a phase velocity faster than that of light traveling in the corresponding linear medium (ε_L, μ), while the phase velocity of the other wave is slower than the light speed.

In Figure 2, variation of the normalized phase factor of the nonlinear waves against the normalized uniaxial electrical-magnetic coupling parameter u_1 is typically illustrated for nonlinear material with longitudinally uniaxial electrical-magnetic coupling. Once again, due to the small quantities, the image parts of the normalized phase factors are also not illustrated. It is seen that the nonlinear material with longitudinally uniaxial electrical-magnetic coupling can also

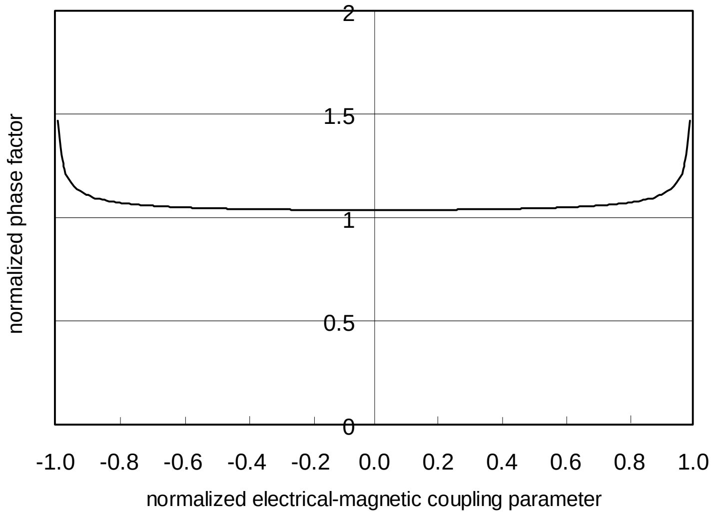

Figure 2. Variation of the normalized phase factor β/k_L against the normalized longitudinal electrical-magnetic coupling parameter k_0Y_1/k_L for the nonlinear waves traveling in a nonlinear material with longitudinally uniaxial electrical-magnetic coupling. Due to the small quantities, the image parts of the normalized phase factors are not plotted in this figure. Here, $A_0^2 = 0.2$, $\delta = 1.6$, and $\varepsilon_N/\varepsilon_L = 0.3$.

simultaneously support right- and left-handed elliptically-polarized nonlinear waves, which propagate with the same phase velocity but different spatial profiles. More interestingly, both of the two nonlinear waves would propagate with attenuation.

5. CONCLUDING REMARKS

In the present study, nonlinear wave solutions for coupled differential wave equations in doped nonlinear material with transversely and longitudinally uniaxial electrical-magnetic coupling are presented respectively. Three important features of the nonlinear waves should be highlighted: (1) Multiple nonlinear waves can simultaneously exist, which are right- and left-handed elliptically-polarized, respectively. (2) In the case of transversely uniaxial electrical-magnetic coupling, the nonlinear waves would propagate with different phase velocities (sub- and super-luminously, respectively) and spatial profiles. For the longitudinally uniaxial electrical-magnetic coupling case, the nonlinear waves would propagate with the same phase velocity but different profiles. (3) For some typical constitutive parameters of this class nonlinear material, spatial complex nonlinear waves can exist, respectively.

It is worthwhile to note that different from the isotropic linear chiral medium [8, 9] where right- and left-handed circularly-polarized electromagnetic waves could coexist, the nonlinear material with uniaxial electrical-magnetic coupling can simultaneously support the right- and left-handed elliptically-polarized nonlinear waves. Compared with the conventional Kerr- and non-Kerr-type nonlinear dielectric where the phase velocity of the nonlinear wave is super-luminous, it is found that the phase velocities of the two electromagnetic waves propagating in the nonlinear material with transversely uniaxial electrical-magnetic coupling are sub- and super-luminous, respectively. Different from the isotropic linear chiral medium [13, 14] and the conventional Kerr-and non-Kerr-type nonlinear dielectric, complex nonlinear waves would exist in the doped nonlinear material with either transversely or longitudinally uniaxial electrical-magnetic coupling.

In the frame works of the classical waveguide theory [22, 23], electromagnetic wave propagating in a lossless waveguide can be classified into three catalogues: oscillatory, evanescent, and complex modes, which are characterized by real, purely imaginary, and complex phase factors, respectively. Since the profiles of the electromagnetic waves in a nonlinear material are bounded, similar to the case as they are in a linear waveguide, one may expect the nonlinear waves to have the types of oscillatory, evanescent, and complex modes. However, to our best knowledge, except the well-known oscillatory nonlinear waves [1–11], the evanescent and complex nonlinear waves have not been reported. In the present study, it is indicated that for certain constitutive parameters, complex nonlinear waves would exist in the doped nonlinear material with either transversely or longitudinally uniaxial electrical-magnetic coupling.

In the conventional nonlinear dielectric, electromagnetic waves can be decomposed into TE and TM waves, which facilitates the analytical analysis. But in the present nonlinear material with uniaxial electrical-magnetic coupling, TE and TM waves are coupled together, and they can not exist separately. This coupling mechanism is noted to be introduced by the uniaxial electrical-magnetic coupling in the constitutive relations. Compared with the previous analytical formulations [1–11], which separately treated TE and TM nonlinear waves, the present study results in the analytical formulations for nonlinear electromagnetic waves in nonlinear material with uniaxial electrical-magnetic coupling, within the approximation of small nonlinearity. The formulations recover the well-known cases of nonlinear isotropic medium and linear medium with uniaxial electrical-magnetic coupling.

Since the two nonlinear waves propagate with different polarization states, the nonlinear material with uniaxial electrical-magnetic coupling may probably find applications in constructing novel error-correction device and message-verification device if the two waves are modulated with the same signal.

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