

## **RAPID SOLUTIONS OF SCATTERING FROM MICROSTRIP ANTENNAS USING WELL-CONDITIONED ASYMPTOTIC WAVEFORM EVALUATION**

**J. X. Wan and C. H. Liang**

The Department of Electromagnetic Field Engineering  
Xidian University  
Xi'an 710071, P. R. China

**Abstract**—The well-conditioned asymptotic waveform evaluation (WCAWE) is applied to the MoM solution of scattering from microstrip antennas so that the reduced order model is obtained to efficiently evaluate the frequency response over a broadband in this paper. In the traditional asymptotic waveform evaluation (AWE) method, the ill conditioning usually leads to stagnation in the moment-matching process. The WCAWE eliminates this difficult. At the same time, to cover the entire bandwidth, a multipoint automatic WCAWE method is also proposed. Numerical examples are given to illustrate the accuracy and robustness of this method.

### **1 Introduction**

### **2 The Mixed Potential Integral Equation**

### **3 The Well-Conditioned Asymptotic Waveform Evaluation**

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## 1. INTRODUCTION

To obtain frequency responses over a band of interest, we have to repeat the calculation at each discrete frequency. This can be computationally intensive for electromagnetic devices with complicated frequency responses. The asymptotic waveform evaluation (AWE) [1] has been proposed to efficiently solve this problem. AWE was originally developed in the circuit community, and then extended for electromagnetic analysis [2, 3]. It is known that the conventional AWE has the problem of instability in the computation of the Pade approximation due to the ill-conditioned moment-matching process [4]. Hence Well-Conditioned asymptotic waveform evaluation (WCAWE) [3] was proposed to perform a fast frequency sweep, and provided good results with the finite element method (FEM). In this paper, we extend WCAWE to handle scattering from microstrip antennas with the method of moments (MoM). As to the authors knowledge, until now no one has successfully applied the conventional AWE to the MoM solution of scattering from microstrip antennas. There are two reasons for this. The first one is that for scattering from microstrip antennas, the derivative of the excitation vector can not be obtained directly. The other one is that the conventional AWE is ill-conditioned and is invalid for microstrip structures. In the following, the derivative of the excitation vector is given firstly. Then the WCAWE is applied to the MoM solution of scattering from microstrip antennas so that the reduced order model is obtained to efficiently evaluate the frequency response over a broadband. On the other hand, in many practical problems with broadband response, one expansion point is not sufficient to cover the entire bandwidth. In such cases, multiple expansion points are necessary. Here, a simple binary search algorithm, as described in [2], is employed to automatically choose the expansion points. The resulting algorithm reduces the computational cost. Several typical examples demonstrate the efficiency and accuracy of the proposed technique.

## 2. THE MIXED POTENTIAL INTEGRAL EQUATION

The mixed potential integral equation formulation is chosen in the present analysis because it provides a less singular kernel as compared with the electric field integral equation (EFIE) method. Suppose the microstrip patch is illuminated by a plane wave  $\vec{E}^i$ , enforcing the boundary condition that the tangential electric field on the perfectly

conducting surface should be zero, we obtain [5]

$$\hat{n} \times [j\omega\vec{A}(\vec{r}) + \nabla\phi(\vec{r})] = \hat{n} \times [\vec{E}^i(\vec{r}) + \vec{E}^r(\vec{r})] \quad (1)$$

Where

$$\begin{cases} \vec{A}(\vec{r}) = \int_S \overline{G}_A(\vec{r}, \vec{r}') \cdot \vec{J}(\vec{r}') ds' \\ \phi(\vec{r}) = \int_S G_q(\vec{r}, \vec{r}') \nabla \cdot \vec{J}(\vec{r}') ds' \end{cases} \quad (2)$$

in which  $\vec{J}(\vec{r}')$  is the unknown currents on the patch surface.  $\overline{G}_A$  and  $G_q$  denote the dyadic and scalar Green's functions for the vector and scalar potentials in the spatial domain respectively. Using the discrete complex image method (DCIM) [6], we can get

$$\overline{G}_A = \sum_{i=1}^{N_A} a_i^A \frac{e^{-jkR_i^A}}{4\pi R_i^A}, \quad G_q = \sum_{i=1}^{N_q} a_i^q \frac{e^{-jkR_i^q}}{4\pi R_i^q} \quad (3)$$

where  $R_i^A = |r - r' + \hat{z}b_i^A|$  and  $R_i^q = |r - r' + \hat{z}b_i^q|$ .  $a_i^A$ ,  $b_i^A$ ,  $a_i^q$  and  $b_i^q$  are the complex coefficients obtained from the DCIM.

For the solution of the mixed potential integral Equation (1), the method of moments with triangular discretization and RWG basis functions are used. The Galerkin's method is employed to yield the following matrix equation

$$ZI = V \quad (4)$$

in which the impedance matrix  $Z$  has the elements given by

$$\begin{aligned} Z_{ij} = & \frac{jk\eta_0}{4\pi} \int_{T_i} \int_{T_j} \vec{f}_i(\vec{r}) \cdot \vec{f}_j(\vec{r}') \cdot \sum_{i=1}^{N_A} a_i^A \frac{e^{-jkR_i^A}}{R_i^A} dr' dr \\ & - \frac{j\eta_0}{4\pi k} \int_{T_i} \int_{T_j} \nabla \cdot \vec{f}_i(\vec{r}) \nabla' \cdot \vec{f}_j(\vec{r}') \sum_{i=1}^{N_q} a_i^q \frac{e^{-jkR_i^q}}{R_i^q} dr' dr \end{aligned} \quad (5)$$

Where  $\vec{f}_i$  and  $\vec{f}_j$  represent the testing and basis functions, and  $T_i$  and  $T_j$  denote their supports respectively [7]. The elements of  $V$  can be computed by

$$V_i = \int_{T_i} \vec{f}_i(\vec{r}) \cdot [\vec{E}^i(\vec{r}) + \vec{E}^r(\vec{r})] dr \quad (6)$$

where the incident field is  $\vec{E}^i = (\hat{\theta}^i E_\theta + \hat{\phi}^i E_\phi) e^{-jk^i \cdot \vec{r}}$  and the reflect field is  $\vec{E}^r = (\hat{\theta}^r R^{TM} E_\theta + \hat{\phi}^r R^{TE} E_\phi) e^{-jk^r \cdot \vec{r}}$ .

Suppose the substrate has the thickness  $d$  and the relative permittivity  $\varepsilon_r$ , we get

$$R^h = \frac{R_0^h - e^{-j2k_2 \cos(\theta_t)d}}{1 - R_0^h e^{-j2k_2 \cos(\theta_t)d}} \quad (7)$$

where  $h = TE$  or  $h = TM$  and

$$R_0^{TE} = \frac{\cos(\theta_i) - \sqrt{\varepsilon_r} \cos(\theta_t)}{\cos(\theta_i) + \sqrt{\varepsilon_r} \cos(\theta_t)}, \quad R_0^{TM} = \frac{\cos(\theta_t) - \sqrt{\varepsilon_r} \cos(\theta_i)}{\cos(\theta_t) + \sqrt{\varepsilon_r} \cos(\theta_i)} \quad (8)$$

in which  $\theta_i$  denotes the angle between the incident wave and  $z$ -axis,  $\theta_t = \sin^{-1} \left( \frac{\sin(\theta_i)}{\sqrt{\varepsilon_r}} \right)$  denotes the angle between the transmission wave and  $z$ -axis.

### 3. THE WELL-CONDITIONED ASYMPTOTIC WAVEFORM EVALUATION

To obtain the frequency responses over a band of interest, we have to repeat the calculation at each discrete frequency. This will be very time consuming for the electromagnetic devices with complicated frequency responses. To alleviate this problem, asymptotic waveform evaluation [1] has been proposed. To apply the AWE technique into the method of moments, select an expansion point  $k_0$  and write the Taylor series for (1) about this point [3]

$$\sum_{n=0}^{a_1} \left( \sigma^n Z^{(n)} \right) I(k) = \sum_{m=0}^{a_2} \sigma^m V^{(m)} \quad (9)$$

where  $\sigma = k - k_0$  and  $a_1$  and  $a_2$  are chosen large enough so no significant higher order  $Z^{(n)}$  and  $V^{(m)}$  term is truncated.  $Z^{(n)}$  is the  $n$ th derivative with respect to  $k$ , of  $Z$  given in Equation (5) and evaluated at  $k_0$ . Utilizing close-form Green's functions, we can obtain [8]

$$\begin{aligned} Z_{ij}^{(n)}(k_0) = & \frac{1}{n!} \left[ \frac{jk\eta_0}{4\pi} \int_{T_i} \int_{T_j} \vec{f}_i(\vec{r}) \cdot \vec{f}_j(\vec{r}') \right. \\ & \cdot \sum_{i=1}^{N_A} a_i^A (-jR_i^A)^n \left( 1 - \frac{n}{jk_0 R_i^A} \right) \frac{e^{-jkR_i^A}}{R_i^A} dr' dr \\ & - \frac{j\eta_0}{4\pi k} \int_{T_i} \int_{T_j} \nabla \cdot \vec{f}_i(\vec{r}) \nabla' \cdot \vec{f}_j(\vec{r}') \\ & \left. \cdot \sum_{i=1}^{N_q} a_i^q (-jR_i^q)^n \sum_{m=0}^n \frac{P(n, m)}{(jk_0 R_i^q)^m} \frac{e^{-jkR_i^q}}{R_i^q} dr' dr \right] \quad (10) \end{aligned}$$

where the permutation function  $P(q, p)$  is defined as  $P(q, p) = \frac{q!}{(q-p)!}$ . Similarly  $V^{(m)}$  is the  $m$ th derivative with respect to  $k$ , of  $V$  given in Equation (6). As to the authors' knowledge, until now no one has successfully applied the conventional AWE to the MoM solution of scattering from microstrip antennas. One reason is that for scattering from microstrip antennas, the derivative of the excitation vector can not be obtained directly. However, from (8) we can see clearly that  $R_0^h$  is independent of  $k$ , so we can obtain  $V^{(m)}$  easily. Then using (6), we can obtain

$$V_i^{(m)} = \int_{T_i} \vec{f}_i(\vec{r}) \cdot \left[ (\vec{E}^i)^{(m)} + (\vec{E}^r)^{(m)} \right] dr \quad (11)$$

and

$$(\vec{E}^i)^{(n)} = \left( \hat{\theta}^i E_\theta + \hat{\varphi}^i E_\varphi \right) \left( -j\vec{k}^i \cdot \vec{r} \right)^{(n)} e^{-j\vec{k}^i \cdot \vec{r}} \quad (12)$$

$$\begin{aligned} (\vec{E}^r)^{(1)} &= \hat{\theta}^r E_\theta \left[ R^{TM(1)} + R^{TM} \left( -j\hat{k}^r \cdot \vec{r} \right) \right] e^{-j\vec{k}^i \cdot \vec{r}} \\ &+ \hat{\varphi}^r E_\varphi \left[ R^{TE(1)} + R^{TE} \left( -j\hat{k}^r \cdot \vec{r} \right) \right] e^{-j\vec{k}^i \cdot \vec{r}} \end{aligned} \quad (13)$$

where  $R^{h(1)} = \frac{jxe^{-jk_1x}[1-(R_0^h)^2]}{(1-R_0^he^{-jk_1x})^2}$ ,  $x = 2\sqrt{\varepsilon_r} \cos(\theta_t)d$ .

Similarly, we can compute  $(\vec{E}^r)^{(n)} (n > 1)$  and further get  $V^{(m)}$ . Then the Asymptotic Waveform Evaluation can be applied to the MoM solution of scattering from microstrip antennas so that the reduced order model is obtained to efficiently evaluate the frequency response over a broadband. The moment-matching AWE subspace for (9) is  $W_q \in C^{N \times q}$  where  $W_q = [w_1, w_2, \dots, w_q]$  with [3]

$$\begin{aligned} w_1 &= Z^{-1}V \\ w_2 &= Z^{-1} \left( V^{(1)} - Z^{(1)}w_1 \right) \\ &\vdots \\ w_q &= Z^{-1} \left( V^{(q-1)} - \sum_{m=1}^{\min(a_1, q-1)} Z^{(m)}w_{q-m} \right) \end{aligned} \quad (14)$$

and  $V^{(k)} = 0$  for  $k > a_2$ .

However, it is known that this traditional AWE has the problem of instability in the computation of the Pade approximation due to the ill-conditioned moment-matching process [4]. Hence the Well-Conditioned asymptotic waveform evaluation [3] was proposed and shows better results with FEM. In this paper, we extend WCAWE to

handle scattering from microstrip antennas With MoM. The moment-matching WCAWE subspace for (9) is  $V_q \in C^{N \times q}$  where  $V_q = [v_1, v_2, \dots, v_q]$ .  $\hat{v}_q$  and  $v_q$  are related by an  $q \times q$  upper-triangular, nonsingular matrix  $U$  (which can be, but does not have to, chosen as the coefficients in a modified Gram-Schmidt process to orthonormalize  $V_q$ ) by the equation.

$$V_q = \hat{V}_q U^{-1} \quad (15)$$

where  $\hat{V}_q = [\hat{v}_1, \hat{v}_2, \dots, \hat{v}_q]$  with

$$\begin{aligned} \hat{v}_1 &= Z^{-1}V \\ \hat{v}_2 &= Z^{-1} \left( V^{(1)} e_1^T P_{U1}(2, 1) e_1 - Z^{(1)} v_1 \right) \\ &\vdots \\ \hat{v}_q &= Z^{-1} \left( \sum_{m=1}^{\min(a_2, q-1)} \left( V^{(m)} e_1^T P_{U1}(q, m) e_{q-m} \right) - Z^{(1)} v_{q-1} \right. \\ &\quad \left. - \sum_{m=2}^{\min(a_1, q-1)} \left( Z^{(m)} V_{q-m} P_{U2}(q, m) e_{q-m} \right) \right) \end{aligned} \quad (16)$$

where  $e_r$  is the vector with all entries equal to zero except the  $r$  th entry which is equal to one.

Furthermore, the correction terms are [3]

$$P_{Uw}(n, m) = \prod_{t=w}^m U(t : n - m + t - 1, t : n - m + t - 1)^{-1} \quad (17)$$

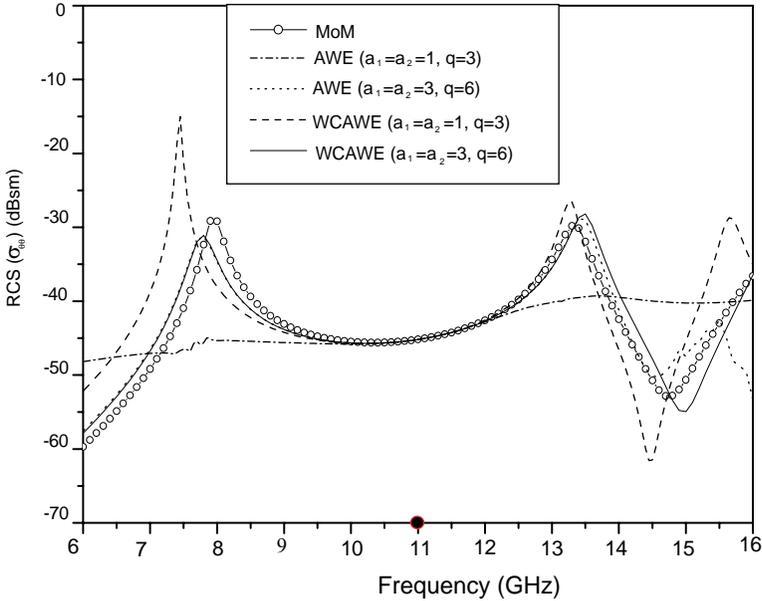
where  $w = 1$  or  $2$ , and the order of the product is  $\prod_{t=1}^2 U_t^{-1} = U_1^{-1} U_2^{-1}$ .

The proof that the vectors in (16) match moments can be found in [3]. The WCAWE algorithm is given in the Appendix A.

In many practical problems with broadband response, one expansion point is not sufficient to cover the entire bandwidth. In such cases, multiple expansion points are necessary. Here, a simple binary search algorithm, as described in [2], is employed to automatically choose the expansion points. In this paper, the first two expansion points are located closer to the region's endpoints. Then multipoint WCAWE (MWCAWE) can be automated using this search algorithm.

#### 4. NUMERAL RESULTS

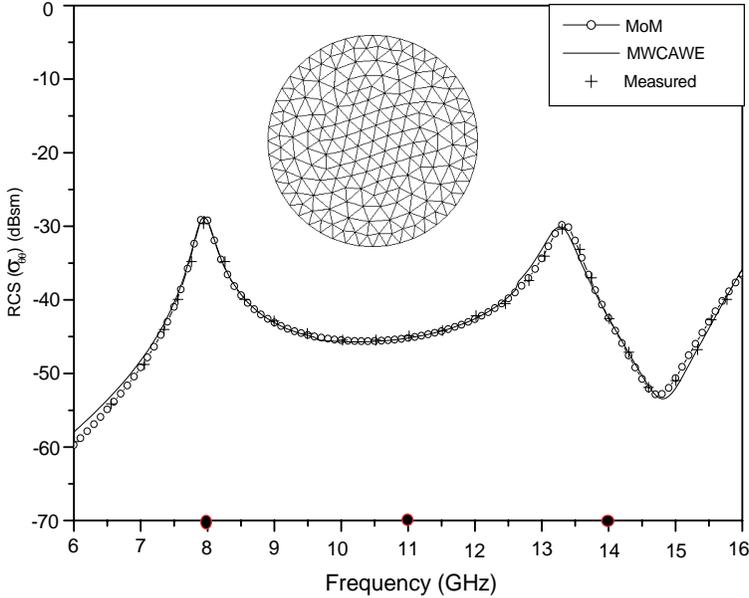
To illustrate the validity and accuracy of the method described above, we present three numerical examples. As the first example, we consider



**Figure 1.** RCS versus frequency for a circle patch using AWE, WCAWE and MoM.

the scattering from a circle patch microstrip antenna with the radius of 7.1 mm. The number of unknowns is 492. The substrate parameters are  $h = 0.7874$  mm,  $\epsilon_r = 2.2$ . The patch is illuminated by an  $\theta$ -polarized incident plane wave traveling along the direction of  $\theta^i = 63^\circ$  and  $\varphi^i = 0^\circ$ .

We choose only one expansion point  $f_0 = 11.0$  GHz and consider two cases:  $a_1 = a_2 = 1$ ,  $q = 3$  and  $a_1 = a_2 = 3$ ,  $q = 6$ . Figure 1 shows the radar cross section (RCS) versus frequency. Three conclusions can be drawn clearly: 1) Using higher order Pade approximation, the AWE and WCAWE solutions agree better with the MoM calculation; 2) the WCAWE result is better than the AWE result, which shows that this novel WCAWE technique is more accurate and robust. 3) the lower order WCAWE gives almost same accurate bandwidth, though the response over the entire bandwidth is worse than the higher order WCAWE does. Hence, to obtain more accurate results, we can use multipoints lower order WCAWE to cover the entire bandwidth, which also can reduce the computer storage requirement. The process can be automated using a simple binary search algorithm, as described in [2]. For this example, we choose  $a_1 = a_2 = 1$ ,  $q = 3$ . Only three expansion

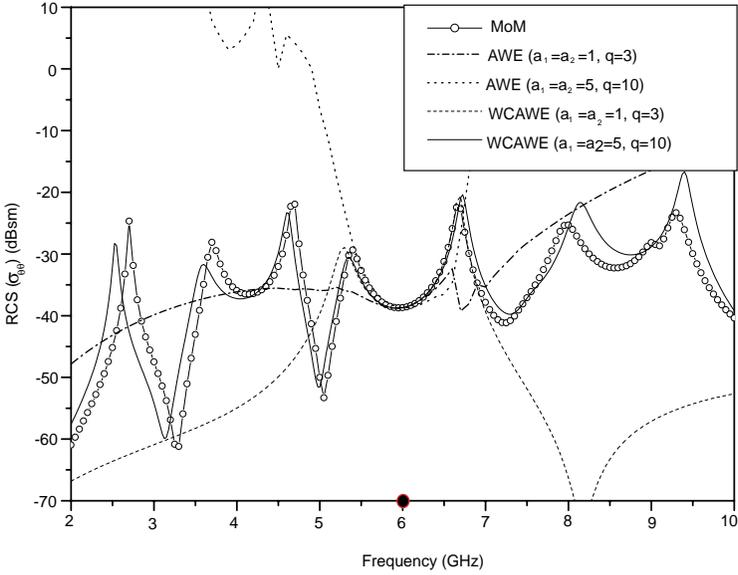


**Figure 2.** RCS versus frequency for a circle patch using MWCAWE, MoM and measurement.

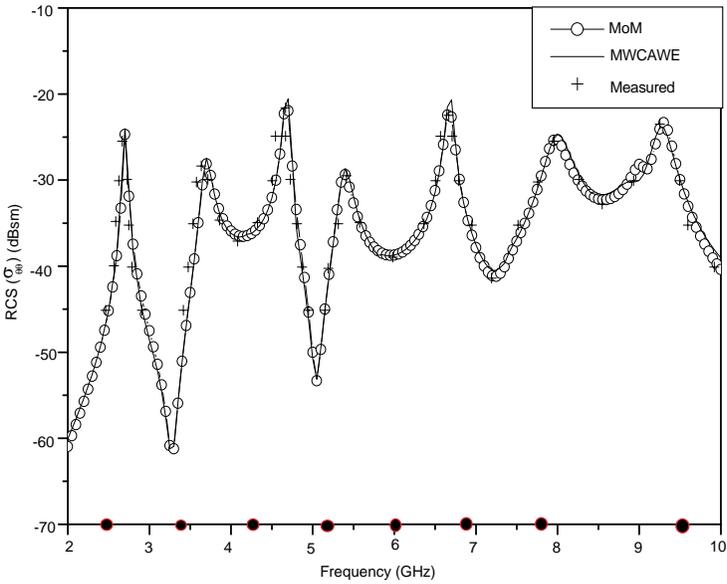
points are needed. As shown in Figure 2, the multipoint WCAWE solutions agree well with the MoM calculation and the measured data [9]. The CPU time using WCAWE and MWCAWE is compared with the direct method of moments calculation to demonstrate their efficiency, as shown in Table 1. It can be seen that the method with MWCAWE is approximately 14.2 times faster than the direct method of moments calculation. It also can be seen that Rao-Wilton-Glisson (RWG) basis function displays a good flexibility to model arbitrarily shape structures.

We next consider the scattering from a rectangular patch microstrip antenna with the length of 36.6 mm and the width of 26 mm. The number of unknowns is 772. The substrate parameters are  $h = 1.58$  mm,  $\epsilon_r = 2.17$ . The patch is illuminated by an  $\theta$ -polarized incident plane wave traveling along the direction of  $\theta^i = 60^\circ$  and  $\varphi^i = 45^\circ$ .

We choose only one expansion point  $f_0 = 6.0$  GHz and consider two cases:  $a_1 = a_2 = 1$ ,  $q = 3$  and  $a_1 = a_2 = 5$ ,  $q = 10$ . Figure 3 shows RCS versus frequency. The same conclusions as in the first example can be obtained. Figure 4 shows the result of multipoint lower order WCAWE with  $a_1 = a_2 = 1$  and  $q = 3$ . Only eight expansion



**Figure 3.** RCS versus frequency for a rectangular patch using AWE, WCAWE and MoM.



**Figure 4.** RCS versus frequency for a rectangular patch using MWCAWE, MoM and measurement.

**Table 1.** Comparison of the CPU time using the MoM and WCAWE.

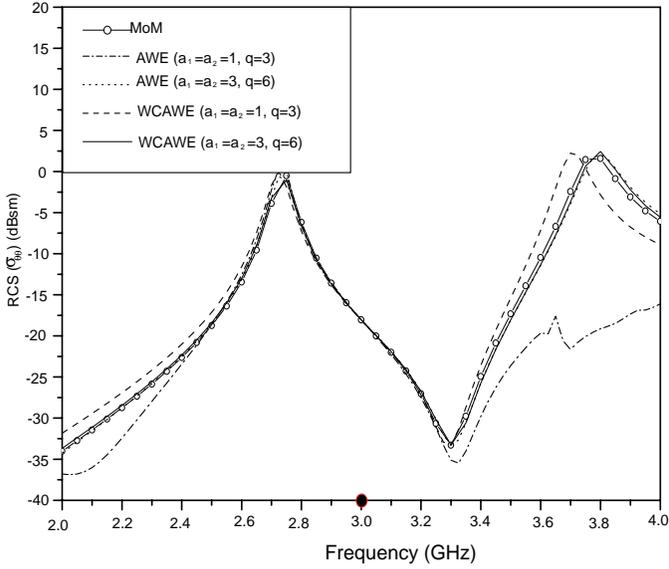
examples	MoM		WCAWE			MWCAWE		
	$N_M$	$T_M$ (S)	$N_W$	$T_W$ (S)	Speed-up	$N_{MW}$	$T_{MW}$ (S)	Speed-up
Ex.1	101	1415.01	201	73.76	19.2	201	99.54	14.2
Ex.2	161	4712.47	321	287.86	16.4	321	605.92	7.8
Ex.3	41	6560.00	81	991.20	6.6	81	1298.76	5.1

$N_M$ : Number of frequencies by MOM;  $N_w$ : Number of frequencies by WCAWE;  $N_{MW}$ : Number of frequencies by MWCAWE;  $T_M$ : Total CPU time by MOM;  $T_W$ : Total CPU time by WCAWE;  $T_{MW}$ : Total CPU time by MWCAWE.

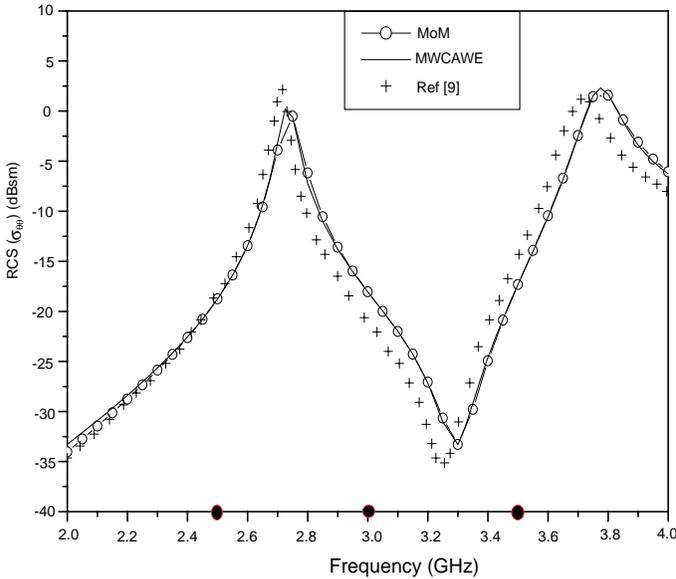
frequencies are chosen automatically by the binary search. As shown in Figure 4, the multipoint WCAWE solutions agree well with the direct MoM calculation and measured data [9]. Table 1 shows the CPU time of the WCAWE, MWCAWE and the direct MoM calculation. It can be seen that the method with MWCAWE is approximately 7.8 times faster than the direct MoM calculation.

At last, we consider the scattering from a  $3 \times 3$  rectangular patch microstrip antenna array. The geometry can be obtained from [10]. The number of unknowns is 1737. The substrate parameters are  $h = 1.58$  mm.  $\epsilon_r = 2.17$ . The array is illuminated by an  $\theta$ -polarized incident plane wave traveling along the direction of  $\theta^i = 0^\circ$  and  $\varphi^i = 45^\circ$ .

We choose only one expansion point  $f_0 = 3.0$  GHz and consider two cases:  $a_1 = a_2 = 1$ ,  $q = 3$  and  $a_1 = a_2 = 3$ ,  $q = 6$ . RCS is shown in Figure 5. It can be seen that the WCAWE result agrees better with the direct MoM calculation and the computed data from [10] than the conventional AWE does. For this example, WCAWE with the higher order Pade approximation provides a very good result comparing with the direct calculation. To reduce the computer storage requirement, we also choose  $a_1 = a_2 = 1$  and  $q = 3$  and use multipoint WCAWE method. Only three expansion frequencies are chosen automatically by the binary search. The CPU time using WCAWE and MWCAWE is compared with the direct MoM calculation to demonstrate the efficiency, as shown in Table 1. It can be seen that using MWCAWE is approximately 5.1 times faster than the direct MoM calculation.



**Figure 5.** RCS versus frequency for a  $3 \times 3$  rectangular patch array using AWE, WCAWE and MoM.



**Figure 6.** RCS versus frequency for a  $3 \times 3$  rectangular patch array using MWCAWE and MoM.

## 5. CONCLUSIONS

In this paper, the well-conditioned AWE is applied to the MoM solution of scattering from microstrip antenna arrays so that the reduced order model is obtained to efficiently evaluate the frequency response over a broadband. Using the higher order Pade approximation, WCAWE provides better estimation of RCS versus frequency than the conventional AWE. If more accurate responses are required, multipoint WCAWE with the lower order Pade approximation can be used, which also can greatly reduce the computer storage requirement. The scattering from several microstrip antennas is analyzed to demonstrate the efficiency of this technique.

## APPENDIX A. THE WCAWE ALGORITHM

$$\hat{v}_1 = Z^{-1}V$$

$$U(1,1) = \|\hat{v}_1\|$$

$$v_1 = \hat{v}_1 U(1,1)^{-1}$$

for  $n = 2, 3, \dots, q$  do

$$\hat{v}_n = Z^{-1} \left( \sum_{m=1}^{\min(a_2, n-1)} (V^{(m)} e_1^T P_{U1}(n, m) e_{n-m}) - Z^{(1)} v_{n-1} \right. \\ \left. - \sum_{m=2}^{\min(a_1, n-1)} (Z^{(m)} V_{n-m} P_{U2}(n, m) e_{n-m}) \right)$$

for  $\alpha = 1, 2, 3, \dots, n-1$  do

$$U(\alpha, n) = v_\alpha^H \hat{v}_n$$

$$\hat{v}_n = \hat{v}_n - U(\alpha, n) v_\alpha$$

end for

$$U(n, n) = \|\hat{v}_n\|$$

$$v_n = \hat{v}_n U(n, n)^{-1}$$

end for

for any desired  $f$  in the range  $f_{\min} < f < f_{\max}$  do

$$\sigma = k - k_0$$

$$g_q(k) = \left( \sum_{i=0}^{a_1} \sigma^i V_q^T Z^{(i)} V_q \right)^{-1} \left( \sum_{j=0}^{a_2} \sigma^j V_q^T b^{(j)} \right)$$

$$I_q(k) = V_q g_q(k)$$

end for

## REFERENCES

1. Pillage, L.T., et al., "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. Comput.-Aided Des. Integrated Circuits and Syst.*, Vol. 9, 352–366, 1990.
2. Kolbehdari, M. A., M. Srinivasan, M. S. Nakhla, Q. J. Zhang, and R. Achar, "Simultaneous time and frequency domain solutions of EM problems using finite element and CFH techniques," *IEEE Microwave Theory Tech.*, Vol. 44, 1526–1534, 1996.
3. Slone, R. D., R. Lee, and J. F. Lee, "Well-conditioned asymptotic waveform evaluation for finite elements," accepted for publication in *IEEE Trans. Antennas and Propagation*.
4. Feldmann, P., et al., "Efficient linear circuit analysis by pade approximation via the lanczos process [J]," *IEEE Trans. Comput.-Aided Des. Integrated Circuits and Syst.*, Vol. 14, 639–649, 1995.
5. Rao, S. M., D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas and Propagation*, Vol. 30, 409–418, 1982.
6. Chow, Y. L., J. J. Yang, D. G. Fang, and G. E. Howard, "A closed-form spatial Green's function for the thick microstrip substrate," *IEEE Microwave Theory Tech.*, Vol. 39, 588–592, 1991.
7. Yuan, N., T. S. Yeo., X.-C. Nie, and L. W. Li, "A fast analysis of scattering and radiation of large microstrip antenna arrays," *IEEE Trans. Antennas and Propagation*, Vol. 51, 2218–2226, 2003.
8. Reddy, C. J., "Application of AWE for RCS frequency response calculations using Method of Moments," NASA Contractor Report 4758, October 1996.
9. Wang, C.-F., F. Ling, and J.-M. Jin, "A fast full-wave analysis of scattering and radiation from large finite arrays of microstrip antennas," *IEEE Trans. Antennas Propagation*, Vol. 46, 1467–1474, 1998.
10. King, A. S. and W. J. Bow, "Scattering from a finite array of microstrip patches," *IEEE Trans. Antennas Propagation*, Vol. 40, 770–774, 1992.

**Wan Jixiang** was born in Henan, China, on May 25, 1978. He received the B.S. and M.S. degrees in electrical engineering from XiDian University, Xi'an, China, in 2000 and 2003, respectively. And he currently is pursuing Ph.D. degree in XiDian University. His research interests include numerical methods for electromagnetic, microstrip antennas, and microwave and millimeter-wave integrated circuits.

**Liang Changhong** was born in Shanghai, China, on December 9, 1943. He graduated in 1965 from the former Xidian University, Xi'an, China, and continued his graduate studies until 1967. From 1980 to 1982, he worked at Syracuse University Syracuse, New York, USA as a visiting scholar. He has been a professor and Ph.D. student advisor of Xidian University since 1986. He was awarded the titles “National Middle-Aged and Young Expert with Distinguished Contribution”, “National Excellent Teacher”, and “One of the 100 National Prominent Professors”, etc. Prof. Liang has wide research interests, which include computational microwave and computational electromagnetic, microwave network theory, microwave measurement method and data processing, lossy variational electromagnetic, electromagnetic inverse scattering, electromagnetic compatibility. Prof. Liang is a Fellow of CIE and a Senior Member of IEEE.