ELECTROMAGNETIC FIELD FOR A HORIZONTAL ELECTRIC DIPOLE BURIED INSIDE A DIELECTRIC LAYER COATED HIGH LOSSY HALF SPACE

H.-Q. Zhang †
National Time Service Center of Chinese Academy of Science
Xi’an, 710600, P. R. of China

W.-Y. Pan
China Research Institute of Radiowave Propagation, Qingdao Branch
19 Qingda 1 Road, Qindao, Shandong, 266071, P. R. China

K. Li
School of Electrical and Electric Engineering
Nanyang Technological University
Singapore 639798

K.-X. Shen
National Time Service Center of Chinese Academy of Science
Xi’an, 710600, P. R. China

Abstract—In this paper, analytical formulas have been derived for the electromagnetic field generated by a horizontal electric dipole inside high lossy half-space coated with a dielectric layer. This problem is corresponding to the electromagnetic field generated by a horizontal antenna in a submarine under an ice layer, or the measurement of the conductivity of the oceanic lithosphere with a horizontal antenna as the source, and a layer of sediment on the sea floor. These formulas obtained for the electromagnetic field can be employed to calculated the total field including the lateral-wave term and the trapped-surface-wave term. Because the wave number of the trapped-surface-wave term is different from that of the lateral-wave term, the interference appears in the total field. Additionally, this paper has presented the approximative formulas for a thin dielectric layer, which can be used for the communication in low frequencies region.

† Also with China Research Institute of Radiowave Propagation, Qingdao Branch, 19 Qingda 1 Road, Qindao, Shandong, 266071, P. R. China
1. INTRODUCTION

The electromagnetic wave propagation along the surface of a dielectric-layer coated high lossy half space is an interesting research topic for various practical applications. For example, a dielectric layer is frequently applied to superstrate to cover conducting strip in the integrated microstrip circuits and printed antennas in the frequency range of 1 GHz through 50 GHz, asphalt or cement to coat the ground for VHF communications in the frequency range of 100 MHz through 1000 MHz, and a thick ice layer to coat sea water for communications between two submarines in the frequency range of 30 kHz through 3 MHz at high-latitude regions.

From 1991 to 1994, King and Sandler [1–3] presented a set of analytical formulas about electromagnetic fields generated by a vertical or horizontal electric dipole over a conductor coated with a dielectric layer. In 1998, Wait [4] commented the 1994 King’s paper and claimed that the paper had overlooked a trapped-surface-wave term, which varies as $\rho^{-1/2}$ in the far region without detail description. The debates between King and Wait rekindled the interest in the study on the electromagnetic field generated by a vertical electric dipole placed on a planar conductor covered with a dielectric layer coating [6, 8]. In previous works by the authors, it has been found that the electromagnetic field on the surface of the dielectric layer does include both the trapped-surface-wave term and the lateral-wave term, and the trapped-surface-wave varies as $\rho^{-1/2}$ in the horizontal direction and attenuates exponentially in the $z$ direction. Later, further studies have been investigated on the electromagnetic field generated by a horizontal dipole over a dielectric-coated lossy material [9].
In 1986, J. M. Dunn [12] had already discussed the electromagnetic field generated by a horizontal dipole buried inside a high lossy material coated with a dielectric layer, and the problem had been embodied in King’s monograph [13].

In this paper, the author analyzes the problem same as Dunn’s paper, and finds Dunn had overlooked the trapped surface wave. The author has obtained the completed expressions of the electromagnetic field generated by an unit horizontal electric dipole buried inside a high lossy half-space coated with a dielectric layer. These expressions contain both the trapped surface wave and the lateral wave. And these formulas and computations can be applied to the communications in lower frequencies region.

2. THE INTEGRATED FORMULAS OF THE ELECTROMAGNETIC FIELD

The relevant geometry is illustrated in Fig. 1, where an unit horizontal electric dipole in the x-direction is located at (0, 0, −d). Use is made of the time dependence $e^{-i\omega t}$, and comparing Fig. 1 in this paper with the corresponding one in [1], exchanging $k_0$ and $k_2$ and taking

\[ \lambda \]

**Figure 1.** The poles and branches of the integrand in the $\lambda$ complex plane.
are obtained readily.

\[ E_{2\rho}(\rho, \varphi', z') = -\frac{\omega\mu_0}{4\pi k_2^2} \cdot \cos \varphi' [F_{\rho\rho}(\rho, z' - d') - F_{\rho\rho}(\rho, z' + d')] + F_{\rho\rho}(\rho, z' + d') \] (1)

\[ E_{2\varphi}(\rho, \varphi', z') = \frac{\omega\mu_0}{4\pi k_2^2} \sin \varphi' [F_{\varphi\rho}(\rho, z' - d') - F_{\varphi\rho}(\rho, z' + d')] + F_{\varphi\rho}(\rho, z' + d') \] (2)

\[ E_{2z}(\rho, \varphi', z') = \frac{i\omega\mu_0}{4\pi k_2^2} \cos \varphi' [F_{\varphi\rho}(\rho, z' - d') - F_{\varphi\rho}(\rho, z' + d')] + F_{\varphi\rho}(\rho, z' + d') \] (3)

\[ B_{2\rho}(\rho, \varphi', z') = -\frac{\mu_0}{4\pi} \sin \varphi' [G_{\rho\rho}(\rho, z' - d') - G_{\rho\rho}(\rho, z' + d')] + G_{\rho\rho}(\rho, z' + d') \] (4)

\[ B_{2\varphi}(\rho, \varphi', z') = -\frac{\mu_0}{4\pi} \cos \varphi' [G_{\varphi\rho}(\rho, z' - d') - G_{\varphi\rho}(\rho, z' + d')] + G_{\varphi\rho}(\rho, z' + d') \] (5)

\[ B_{2z}(\rho, \varphi', z') = \frac{i\mu_0}{4\pi} \sin \varphi' [G_{\varphi\rho}(\rho, z' - d') - G_{\varphi\rho}(\rho, z' + d')] + G_{\varphi\rho}(\rho, z' + d') \] (6)

The first and the second terms in (1) through (6) can be expressed as follows:

\[ F_{\rho\rho}(\rho, z' \pm d') = -e^{ikz_2r} \left[ 2k_2 \frac{2i}{r^2} + \left( \frac{z' \pm d'}{r} \right) \left( \frac{ik_2^2}{r} - \frac{3k_2}{r^2} - \frac{3i}{r^3} \right) \right] \] (7)

\[ F_{\varphi\rho}(\rho, z' \pm d') = -e^{ikz_2r} \left( \frac{ik_2^2}{r} - \frac{k_2}{r^2} - \frac{i}{r^3} \right) \] (8)

\[ F_{\varphi\varphi}(\rho, z' \pm d') = -e^{ikz_2r} \left( \frac{\rho}{r} \right) \left( \frac{z' \pm d'}{r} \right) \left( \frac{k_2^2}{r} + \frac{3ik_2}{r^2} - \frac{3}{r^3} \right) \] (9)

\[ G_{\rho\rho}(\rho, z' \pm d') = -e^{ikz_2r} \left( \frac{z' \pm d'}{r} \right) \left( \frac{ik_2}{r} - \frac{1}{r^2} \right) \] (10)

\[ G_{\varphi\rho}(\rho, z' \pm d') = -e^{ikz_2r} \left( \frac{\rho}{r} \right) \left( \frac{k_2}{r} + \frac{i}{r^2} \right) \] (11)

In the above formulas, when using “−”, \( r = r_1 = \sqrt{\rho^2 + (z' - d')^2} \), and when using “+”, \( r = r_2 = \sqrt{\rho^2 + (z' + d')^2} \). Here, \( r_1 \) is the path length of the direct wave, \( r_2 \) is the path length of the reflected wave. At
EM field for a horizontal electric dipole 167

large distances between the source and the observation point, because of the high attenuation, both the direct and the reflected waves in region 2 are negligible.

The third terms in (1) and (2) can be expressed as

\[ F_\rho_1'(\rho, z' + d') = F_\rho_2'(\rho, z' + d') + F_\rho_3'(\rho, z' + d') \]  
(12)

\[ F_{\varphi_1}'(\rho, z' + d') = F_{\varphi_2}'(\rho, z' + d') + F_{\varphi_3}'(\rho, z' + d') \]  
(13)

where

\[ F_{\rho_2}'(\rho, z' + d') = \frac{1}{2} \int_0^\infty \gamma_2(Q_3 + 1)[J_0(\lambda \rho) \mp J_2(\lambda \rho)]e^{i\gamma_2(z' + d')} \cdot \lambda d\lambda \]  
(14)

\[ F_{\varphi_2}'(\rho, z' + d') = -\frac{k_2^2}{2} \int_0^\infty \gamma_2(Q_3 + 1)[J_0(\lambda \rho) \mp J_2(\lambda \rho)]e^{i\gamma_2(z' + d')} \cdot \lambda d\lambda \]  
(15)

The third term in (3) can be expressed as

\[ F_{z_1}'(\rho, z' + d') = \int_0^\infty (Q_3 + 1)J_1(\lambda \rho)e^{i\gamma_2(z' + d')} \cdot \lambda^2 d\lambda \]  
(16)

Similarly,

\[ G_{\rho_1}(\rho, z' + d') = G_{\rho_2}(\rho, z' + d') + G_{\rho_3}(\rho, z' + d') \]  
(17)

\[ G_{\varphi_1}(\rho, z' + d') = G_{\varphi_2}(\rho, z' + d') + G_{\varphi_3}(\rho, z' + d') \]  
(18)

where

\[ G_{\rho_2}(\rho, z' + d') = \frac{1}{2} \int_0^\infty (Q_3 + 1)[J_0(\lambda \rho) \pm J_2(\lambda \rho)]e^{i\gamma_2(z' + d')} \cdot \lambda d\lambda \]  
(19)

\[ G_{\varphi_2}(\rho, z' + d') = -\frac{k_2^2}{2} \int_0^\infty (P_3 - 1)[J_0(\lambda \rho) \pm J_2(\lambda \rho)]e^{i\gamma_2(z' + d')} \cdot \lambda d\lambda \]  
(20)

\[ G_{z_1}(\rho, z' + d') = -\int_0^\infty (P_3 - 1)\gamma_2^{-1}J_1(\lambda \rho)e^{i\gamma_2(z' + d')} \cdot \lambda^2 d\lambda \]  
(21)

where \(Q_3\) and \(P_3\) are the reflected coefficients of the electric-type wave and the magnetic-type wave, respectively. They are
\[ \frac{\gamma_2^2}{2}(Q_3 + 1) = \frac{k_2^2 \gamma_2}{k_1^2} \left( \frac{k_4^2 \gamma_0}{k_0^2} - i \gamma_1 \tan \gamma_1 l \right) \frac{\gamma_2}{\gamma_2 + \frac{k_2^2 \gamma_0}{k_0^2} - i \left( \frac{k_2^2 \gamma_1}{k_1^2} + \frac{k_4^2 \gamma_0}{k_0^2} \right) \tan \gamma_1 l} \]  

(22)

\[ \frac{k_2^2}{2 \gamma_2}(P_3 - 1) = \frac{k_2^2}{\gamma_1 + \frac{\gamma_1 \gamma_2}{\gamma_0} - i \left( \frac{\gamma_1}{k_1^2} + \frac{\gamma_2}{k_0^2} \right) \gamma_1 \tan \gamma_1 l} \]  

(23)

where

\[ \gamma_m = \sqrt{k_m^2 - \lambda^2}, \quad m = 0, 1, 2 \]  

(24)

In the next step, it is necessary to evaluate the above Sommerfeld integrals.

3. EVALUATION FOR THE ELECTRIC-TYPE TERMS OF THE ELECTROMAGNETIC FIELD

The terms in (14), (16) and (19) including the factor \((Q_3 + 1)\), which depends on the coefficient of the electric-type wave, are defined by the electric-type terms. Similarly, the terms in (15), (20) and (21) including the factor \((P_3 - 1)\), which depends on the coefficient of the magnetic-type wave, are defined by the magnetic-type terms. Substituting (22) into (14), the following expressions are obtained readily.

\[ F_{\rho_2}(\rho, z' + d') = \int_0^\infty \frac{\left( \frac{\gamma_0}{k_0^2} - \frac{\gamma_1}{k_1^2} \tan \gamma_1 l \right) \gamma_2}{\gamma_2 + \frac{\gamma_0}{k_0^2} - i \left( \frac{\gamma_1}{k_1^2} + \frac{k_4^2 \gamma_0}{k_0^2} \right) \tan \gamma_1 l} \cdot e^{i \gamma_2(z' + d')} \left[ J_0(\lambda \rho) \pm J_2(\lambda \rho) \right] \lambda d \lambda \]  

(25)

Taking into account the following relations

\[ J_n(\lambda \rho) = \frac{1}{2} \left[ H_n^{(1)}(\lambda \rho) + H_n^{(2)}(\lambda \rho) \right] \]  

(26)

\[ H_n^{(1)}(-\lambda \rho) = (-1)^{n+1} H_n^{(2)}(\lambda \rho) \]  

(27)
\[
\begin{align*}
F_{\rho_2}(\rho, z' + d') - F_{\phi_2}(\rho, z' + d') &= \frac{1}{2} \int_{-\infty}^{\infty} (\gamma_0 - i\gamma_1 \tan \gamma_1 l) \gamma_2 \\
&\quad \cdot e^{i\gamma_2(z' + d')} \left[ H_0^{(1)}(\lambda \rho) \mp H_2^{(1)}(\lambda \rho) \right] \lambda d\lambda \quad (28)
\end{align*}
\]

where
\[
q(\lambda) = \frac{\gamma_2}{k_0^2} + \frac{\gamma_0}{k_0^2} - i \left( \frac{\gamma_1}{k_1^2} + \frac{k_1^2 \gamma_0 \gamma_2}{k_0^2 k_2^2 \gamma_1} \right) \tan \gamma_1 l \quad (29)
\]

In order to evaluate the Sommerfeld integrals, the poles of the integrand should be considered. The integrand of (28) has the poles \(\lambda_j\) to satisfy the equation \(q(\lambda) = 0\). When \(k_2\) is much larger than \(k_1\) and \(k_0\), using the limit of \(k_2 \to \infty\), we can convert the equation \(q(\lambda) = 0\) to
\[
q^*(\lambda) = \frac{\gamma_0}{k_0^2} - i \frac{\gamma_1}{k_1^2} \tan \gamma_1 l = 0 \quad (30)
\]

Equation (30) has been discussed in [6], when the thickness \(l\) of the dielectric layer satisfy \((n - 1)\pi < \sqrt{k_2^2 - k_0^2 l} < n\pi\), equation (30) has \(n\) roots on the up-half space of the plane of \(\lambda\). If \(k_2 \neq \infty\), and is much larger than \(k_1\) and \(k_0\), following the manner addressed in [6, 8], under the same condition, (29) still has \(n\) roots and may be solved by Newton method numerically.

In addition, the integrand has two branch points at \(\lambda = k_0\) and \(\lambda = k_2\), the branch cuts and the poles are taken on the \(\lambda\) complex plane as shown in Fig. 2.

On both sides of the branch cut \(\Gamma_0\), the phase of \(\Gamma_0\) will change to 180°, but \(\Gamma_2\) remains the same value, and vice versa for the branch cut \(\Gamma_2\). Under these conditions, (28) may be re-written as
\[
\begin{align*}
F_{\rho_2}(\rho, z' + d') - F_{\phi_2}(\rho, z' + d') &= \pi i \sum_j \gamma_2^* \left( \frac{\gamma_0}{k_0^2} - i \frac{\gamma_1^*}{k_1^2} \tan \gamma_1^* l \right) \\
&\quad \cdot e^{i\gamma_2(z' + d')} \lambda_j \left[ H_0^{(1)}(\lambda_j \rho) \mp H_2^{(1)}(\lambda_j \rho) \right] \\
&\quad + \frac{1}{2} \int_{\Gamma_0 + \Gamma_2} \gamma_2 \left( \frac{\gamma_0}{k_0^2} - i \frac{\gamma_1}{k_1^2} \tan \gamma_1 l \right) \\
&\quad \cdot e^{i\gamma_2(z' + d')} \lambda \left[ H_0^{(1)}(\lambda \rho) \mp H_2^{(1)}(\lambda \rho) \right] d\lambda \quad (31)
\end{align*}
\]
Figure 2. The poles and branches of the integrand in the $\lambda$ complex plane.

where

$$q'(\lambda_j) = -\frac{\lambda_j}{k_2^2 \gamma_2^2} - \frac{\lambda_j}{k_0^2 \gamma_0^2} - i \tan \gamma_1^* l$$

$$- \left[ \frac{\lambda_j}{k_1^2 \gamma_1^2} \frac{k_1^2}{k_0^2 k_2^2} \left( -\frac{\lambda_j \gamma_2^*}{\gamma_1^* \gamma_0^*} - \frac{\lambda_j \gamma_0^*}{\gamma_1^* \gamma_2^*} + \frac{\gamma_0^* \gamma_2^* \lambda_j}{\gamma_1^*} \right) \right]$$

$$+ i \sec^2 \gamma_1^* l \cdot \left( \frac{\gamma_1^*}{k_1^2} + \frac{k_1^2 \gamma_0^* \gamma_2^*}{k_0^2 k_2^2 \gamma_1^*} \right) \frac{\lambda_j l}{\gamma_1^*}$$

$$\gamma_m^* = \sqrt{k_m^2 - \lambda_j^2} \quad m = 0, 1, 2$$

The first term in (31) is the sum of residues at the poles, if the thickness $l$ of the dielectric layer satisfy $(n - 1)\pi < \sqrt{k_1^2 - k_0^2} < n\pi$, the integrand should have $n$ poles. On both sides of the branch cut $\Gamma_0$, let

$$\lambda = k_0 (1 + i\tau^2)$$

thus,

$$H_n^{(1)}(\lambda \rho) \approx \sqrt{\frac{2}{\pi k_0 \rho}} \exp \left[ i \left( k_0 \rho - \frac{n\pi}{2} - \frac{\pi}{4} \right) - k_0 \rho \tau^2 \right]$$

If $k_0 \rho \gg 1$, it may be seen that the dominant contribution of the integral along $\Gamma_0$ comes from the neighborhood of $k_0$, and following
approximation may be taken

\[
\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx k_0 e^{i\frac{\lambda}{2k_0}} \sqrt{2} \cdot \tau
\]  
(36)

\[
\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx \gamma_1' = \sqrt{k_1^2 - k_0^2}
\]  
(37)

\[
\gamma_2 = \sqrt{k_2^2 - \lambda^2} \approx \gamma_2' = \sqrt{k_2^2 - k_0^2}
\]  
(38)

Substituting (36)–(38) into the integral (31) along the branch cut \( \Gamma_0 \), following expressions may be obtained

\[
\frac{1}{2} \int_{\Gamma_0} \frac{\gamma_2 \left( \frac{\gamma_0}{k_0^2} - i \frac{\gamma_1}{k_1^2} \tan \gamma_1' l \right)}{q(\lambda)} \cdot e^{i\gamma_2(z' + d')} \left[ H_0^{(1)}(\lambda \rho) \pm H_2^{(1)}(\lambda \rho) \right] \cdot \lambda \cdot d\lambda
\]

\[
= 2k_0^2 \left[ \frac{2}{\pi k_0 \rho} \exp \left[ i k_0 \rho \mp i \gamma_2'(z' + d') + i \frac{\pi}{4} \right] \cdot \frac{\gamma_2'}{1 - i \frac{k_1^2 \gamma_2'}{k_2^2 \gamma_1'} \tan \gamma_1' l} \right]
\]

\[
\cdot \left( -A + e^{i \pi \Delta} \right) \left( \frac{\pi}{k_0 \rho} + \pi i \sqrt{2} \Delta e^{-iP^*} F(P^*) \right) \left\{ \begin{array}{c} 1 \\ -i \end{array} \right\} \quad (39)
\]

where

\[
\Delta = \sqrt{2} \left( 1 - i \frac{k_1^2 \gamma_2'}{k_2^2 \gamma_1'} \tan \gamma_1' l \right)
\]  
(40)

\[
A = e^{-i \pi \Delta} \frac{k_0 \gamma_1'}{2k_1^2} \tan \gamma_1' l
\]  
(41)

\[
P^* = k_0 \rho \Delta^2
\]  
(42)

\[
F(P^*) = \frac{1}{2} (1 + i) + \int_0^{P^*} \frac{e^{it}}{\sqrt{2\pi t}} dt \quad (43)
\]

\( F(P^*) \) is Fresnel integral.

Since Region 2 is a high lossy medium, so the integral along the branch cut \( \Gamma_2 \) can be neglected. Therefore, (31) may be simplified as

\[
F_{\nu_2'}(\rho, z' + d') = \pi i \sum_j \gamma_2' \left( \frac{\gamma_0^*}{k_0^2} - \frac{i \gamma_1^*}{k_1^2} \tan \gamma_1^* l \right) \frac{q'(\lambda_j)}{q(\lambda_j)} e^{i\gamma_2'(z' + d') \lambda_j}
\]
\[ H_0^{(1)}(\lambda_j \rho) \mp H_2^{(1)}(\lambda_j \rho) - \frac{2k_0^2(A - e^{i\frac{\pi}{4}\Delta})}{1 - i\frac{k_1^2\gamma_2}{k_2^2\gamma_1}} \tan \gamma_1 l \]

\[ \sqrt{\frac{2}{\pi k_0 \rho}} \exp \left[ ik_0 \rho + i\gamma_2'(z' + d') + i\frac{\pi}{4} \right] \left\{ -\frac{1}{k_0 \rho} \right\} \]

\[ \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \sqrt{2} \cdot \pi \cdot i \cdot \Delta \cdot e^{-iP^* F(P^*)} \right] \]

(44)

Apparently, in (44), the first term is the trapped-surface-wave term, and the second one is the lateral-wave term. Similarly, \( F_{z_1}(\rho, z' + d') \) and \( G_{\rho_2}(\rho, z' + d') \) may be re-written as

\[ F_{z_1}(\rho, z' + d') = \int_0^\infty (Q_3 + 1)J_1(\lambda \rho)e^{i\gamma_2(z' + d') \cdot \lambda^2} \cdot d\lambda \]

\[ = 2\pi i \sum_j \frac{\gamma_0^* - i\gamma_1^*}{k_0^2 - k_j^2} \tan \gamma_j \]

\[ \cdot \frac{2k_0^2(A - e^{i\frac{\pi}{4}\Delta})}{1 - i\frac{k_1^2\gamma_2}{k_2^2\gamma_1}} \tan \gamma_1 l \]

\[ \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \sqrt{2} \cdot \pi \cdot i \cdot \Delta \cdot e^{-iP^* F(P^*)} \right] \]

(45)

\[ G_{\rho_2}(\rho, z' + d') = i\pi \sum_j \frac{\gamma_0^* - i\gamma_1^*}{k_0^2 - k_j^2} \tan \gamma_j \]

\[ \cdot e^{i\gamma_2(z' + d') \cdot \lambda_j} - \frac{2k_0^2(A - e^{i\frac{\pi}{4}\Delta})}{1 - i\frac{k_1^2\gamma_2}{k_2^2\gamma_1}} \sqrt{\frac{2}{\pi k_0 \rho}} \]

\[ \cdot \exp \left[ ik_0 \rho + i\gamma_2'(z' + d') + i\frac{\pi}{4} \right] \left\{ -\frac{1}{k_0 \rho} \right\} \]

\[ \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \sqrt{2} \cdot \pi \cdot \Delta \cdot e^{-iP^* F(P^*)} \right] \]

(46)
4. EVALUATION OF THE TERMS FOR THE MAGNETIC-TYPE WAVE

Substituting (23) into (15), the following expressions may be obtained

\[
\frac{F_{\rho_3}(\rho, z' + d')}{F_{\varphi_3}(\rho, z' + d')} = \int_0^{\infty} \frac{k_2^2}{\gamma_2 p(\lambda)} e^{i\gamma_2(z' + d')} [J_0(\lambda \rho) \pm J_2(\lambda \rho)] \lambda d\lambda
\]

where

\[
p(\lambda) = \frac{\gamma_1}{\gamma_0} + \frac{\gamma_1}{\gamma_2} - i \left( 1 + \frac{\gamma_1^2}{\gamma_0 \gamma_2} \right) \tan \gamma_1 l
\]

The electromagnetic field due to a horizontal electric dipole over a lossless half-space coated with a dielectric layer have been addressed in [9]. When \(k_2 \to \infty\), (48) has been simplified as (27) as shown in [9]. It has been pointed out that when the thickness \(l\) of the dielectric layer satisfies the condition \((n - \frac{1}{2})\pi \leq \sqrt{k_1^2 - k_0^2} l \leq (n + \frac{1}{2})\pi\), equation \(p(\lambda) = 0\) has \(n\) roots. If \(k_2 \neq \infty\) and \(k_2\) is much larger than \(k_0\) and \(k_1\), equation \(p(\lambda) = 0\) still has \(n\) roots, and the roots \(\lambda_j (j = 1, 2, \ldots, n)\) can be solved numerically by Newton method.

Therefore, (47) can be re-written as

\[
\frac{F_{\rho_3}(\rho, z' + d')}{F_{\varphi_3}(\rho, z' + d')} = \pi i \sum_j \frac{k_2^2}{\gamma_2 p'(\lambda_j)} e^{i\gamma_2(z' + d')} \lambda_j \cdot \left[ H_0^{(1)}(\lambda_j \rho) \pm H_2^{(1)}(\lambda_j \rho) \right]
\]

\[
+ \frac{1}{2} \int_0^{\infty} \frac{k_2^2}{\gamma_0} \frac{\gamma_1^*}{\gamma_0} - i \tan \gamma_1 l \frac{\gamma_2^* p'(\lambda)}{\gamma_2 p(\lambda)} e^{i\gamma_2(z' + d')}
\]

\[
\cdot \left[ H_0^{(1)}(\lambda \rho) \pm H_2^{(1)}(\lambda \rho) \right] \lambda d\lambda
\]

where

\[
p'(\lambda) = \lambda \gamma_1 \left( \frac{1}{\gamma_0} + \frac{1}{\gamma_2} \right) - \frac{\lambda}{\gamma_1} \left( \frac{1}{\gamma_0} + \frac{1}{\gamma_2} \right) - i \frac{\lambda \sin \gamma_1 l}{\gamma_0 \gamma_2} \left( \frac{\gamma_1^2}{\gamma_2^2} + \frac{\gamma_2^2}{\gamma_0^2} + 2 \right)
\]

\[
+ \frac{i \lambda}{\gamma_1} \sec^2 \gamma_1 l \left( 1 + \frac{\gamma_1^2}{\gamma_0 \gamma_2} \right)
\]

(50)
Since the dominant contribution of the integral along the branch cut $\Gamma_0$ comes from the neighborhood of $k_0$, the second term in (49) may be simplified as

$$
\frac{1}{2} \int_{\Gamma_0} \frac{k_0^2}{\gamma_2 p(\lambda)} \gamma_{12} \left( \gamma_{1} - i \tan \gamma_1 \right) e^{i \gamma_2 (z' + d')} \left[ H_0^{(1)}(\lambda \rho) \pm H_2^{(1)}(\lambda \rho) \right] \lambda d\lambda
= \frac{2k_0^2}{\gamma_2} \gamma_{1} \tan \gamma_1 \left\{ -i \frac{k_0}{\gamma_2} \right\} \cdot \left[ B + e^{i \frac{\pi}{4}} \cdot \Delta' \right] \cdot \left[ \sqrt{\frac{\pi}{k_0 \rho}} + i \sqrt{2} \Delta' e^{-iP_2} F(P_2) \right] (51)
$$

where

$$
\Delta' = \frac{\gamma_1 - \gamma_1}{\gamma_2} \tan \gamma_1
\frac{k_0 \rho \gamma_1}{\frac{\gamma_1}{\gamma_2} \left\{ -i \frac{k_0}{\gamma_2} \right\}} (52)
$$

$$
B = e^{-i \frac{\pi}{4}} \frac{\gamma_1}{k_0 \sqrt{2} \tan \gamma_1} (53)
$$

$$
P_2 = k_0 \rho \Delta' (54)
$$

thus,

$$
F_{\rho_2}(\rho, z' + d') = \frac{\pi i}{2} \sum_j \frac{k_0^2}{\gamma_2 p(\lambda_j)} \gamma_{12} \left( \gamma_{1}^* - i \tan \gamma_1 \right) e^{i \gamma_2 (z' + d')} \lambda_j
\cdot \left[ H_0^{(1)}(\lambda_j \rho) \pm H_2^{(1)}(\lambda_j \rho) \right]
+ \frac{2k_0^2}{\gamma_2} \gamma_{1} \tan \gamma_1 \left\{ -i \frac{k_0}{\gamma_2} \right\} \cdot \left[ B + e^{i \frac{\pi}{4}} \cdot \Delta' \right] \cdot \left[ \sqrt{\frac{\pi}{k_0 \rho}} + i \sqrt{2} \Delta' e^{-iP_2} F(P_2) \right] (55)
$$
5. APPROXIMATION OF THE ELECTROMAGNETIC FIELD FOR THIN DIELECTRIC LAYER

Taking into account that two submarines are buried inside in the seawater covered with a layer of ice, and they communicate the message with the horizontal antenna each other. The relevant geometry of this problem is illustrated in Fig. 3. In this problem, the complex dielectric constants of there three kinds of medium satisfy the condition of $|k_0^2| \ll |k_1^2| \ll |k_2^2|$, and the thickness $l$ of the dielectric layer satisfies the relation $|k_1^2 l^2| \ll 1$, the equation $q(\lambda) = 0$ has only one root $\lambda_1$, and equation $p(\lambda) = 0$ has no root. In this case, (40) and (41) may be simplified as

$$
\Delta \approx \frac{k_0 - ik_0 l}{\sqrt{2}} \ll 1
$$
Figure 3. The geometry for two submarines are buried inside in the seawater covered with a layer of ice.

\[ A \approx e^{-i\frac{\pi}{4}k_0 l} \ll 1 \] (59)

(52) and (53) may be approximated as

\[ \Delta' \approx \frac{k_2}{\sqrt{2}k_0(1 - ik_2l)} \gg 1 \] (60)

\[ B \approx \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2}k_0 l} \gg 1 \] (61)

So that the terms of the electric-type wave may be expressed as

\[
\frac{F_{\rho 2}(\rho, z' + d')}{F_{\phi 2}(\rho, z' + d')} = \pi i \frac{\left( \frac{\gamma_0}{k_0^2} - i \frac{\gamma_1^2 l}{k_1^2} \right) \gamma_2^*}{q(\lambda_1)} e^{i\gamma_2^*(z' + d')} \lambda_1 \\
\cdot \left[ H_0^{(1)}(\lambda_1 \rho) + \tilde{H}_2^{(1)}(\lambda_1 \rho) \right] \\
+ 2k_0^3 \sqrt{\frac{1}{\pi k_0 \rho}} e^{i k_0 \rho + i k_2 (z' + d') + i \frac{\pi}{2}} \left\{ \frac{1}{-i k_0 \rho} \right\} \\
\cdot \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \pi i \left( \frac{k_0}{k_2} - i k_0 l \right) e^{-i P^* F(P^*)} \right]
\] (62)
EM field for a horizontal electric dipole

\[ F_\gamma (\rho, z' + d') = 2\pi i \cdot \frac{\gamma_0^* - \gamma_1^* l_k}{k_0^2 q(\lambda_1)} H^{(1)}_1(\lambda_1 \rho) \lambda_1^2 e^{i \gamma(z' + d')} \]

\[ - 2 \frac{k_0^4}{k_1^2} \sqrt{\frac{1}{\pi k_0 \rho}} e^{i k_0 \rho + i k_2'(z' + d') + i \frac{\pi}{2}} \cdot \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \pi i \left( \frac{k_0}{k_2} - i k_0 l \right) e^{-i P^*} F(P^*) \right] \]

\[ G_{\rho_3} (\rho, z' + d') = \frac{\gamma_0^* - \gamma_1^* l_k}{k_0^2 q(\lambda_1)} e^{i \gamma(z' + d')} \lambda_1 \]

\[ G_{\phi_3} (\rho, z' + d') = \frac{\gamma_0^* - \gamma_1^* l_k}{k_0^2 q(\lambda_1)} e^{i \gamma(z' + d')} \lambda_1 \]

\[ \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \pi i \left( \frac{k_0}{k_2} - i k_0 l \right) e^{-i P^*} F(P^*) \right] \]

Since \( P_2 = k_0 \Delta^2 \gg 1 \), the function \( \text{erfc}(\sqrt{x}) \) may be approximated as

\[ \text{erfc}(\sqrt{x}) = \frac{1}{\pi} e^{-x} \cdot \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma \left( k + \frac{1}{2} \right)}{x^{k+1/2}} + R_n \]

The lateral wave term of (55) may be simplified as

\[ \left[ \sqrt{\frac{\pi}{k_0 \rho}} + \pi i \sqrt{2} \cdot \Delta' \cdot e^{-i P^2} F(P_2) \right] \]

\[ \approx \sqrt{\frac{\pi}{k_0 \rho}} \left[ \frac{-i k_0^2 (1 - i k_2 l)^2}{k_2^2} - \frac{3k_0^4 (1 - i k_2 l)^4}{k_4^2 (k_0 \rho)^2} + \cdots \right] \]

thus,

\[ F_{\rho_3} (\rho, z' + d') = 2k_0^4 \frac{1}{k_1^2} e^{i k_0 \rho + i k_2'(z' + d') + i \frac{\pi}{2}} \cdot \left\{ -\frac{i}{k_0 \rho} \right\} \]

\[ G_{\rho_3} (\rho, z' + d') = 2k_0^4 \frac{1}{k_1 k_2} e^{i k_0 \rho + i k_2'(z' + d') + i \frac{\pi}{2}} \cdot \left\{ -\frac{i}{k_0 \rho} \right\} \]
Substituting (62)–(64) and (67)–(69) into (1)–(6), neglecting the direct wave and the reflected wave in the conductor, the final expressions of the electromagnetic field for the thin dielectric layer are

\[ E_{2\rho}(\rho, \varphi', z') = -\frac{\omega \mu_0}{4\pi k_0^2} \cos \varphi' \left\{ \pi i \left( \frac{\gamma_0^*}{k_0^2} - i \frac{\gamma_1^*}{k_1^2} \right) \frac{\gamma_2^*}{q'(\lambda_1)} e^{i\nu_2^* (z'+d')} \cdot \lambda_1 \right\} \]

\[ + \left\{ \frac{1}{k_0^0} + i \frac{k_0}{k_1^0 k_0^2} \right\} \left( \frac{k_0}{k_2} - i k_0 l \right) \cdot e^{-iP^* F(P^*)} \right\} \]  \hspace{1pt} (70)

\[ E_{2\varphi}(\rho, \varphi', z') = \frac{\omega \mu_0}{4\pi k_0^2} \sin \varphi' \left\{ \pi i \left( \frac{\gamma_0^*}{k_0^2} - i \frac{\gamma_1^*}{k_1^2} \right) \frac{\gamma_2^*}{q'(\lambda_1)} e^{i\nu_2^* (z'+d')} \cdot \lambda_1 \right\} \]

\[ + \left\{ \frac{1}{k_0^0} + i \frac{k_0}{k_1^0 k_0^2} \right\} \left( \frac{k_0}{k_2} - i k_0 l \right) \cdot e^{-iP^* F(P^*)} \right\} \]  \hspace{1pt} (71)

\[ E_{2z}(\rho, \varphi', z') = \frac{i \omega \mu_0}{4\pi k_0^2} \cos \varphi' \left\{ 2\pi i \left( \frac{\gamma_0^*}{k_0^2} - i \frac{\gamma_1^*}{k_1^2} \right) \frac{\gamma_2^*}{q'(\lambda_1)} e^{i\nu_2^* (z'+d')} H_1^{(1)}(\lambda_1 \rho) \lambda_1^2 \right\} \]

\[ - \frac{2k_0^4}{k_2} e^{i k_0 \rho + i k_2 (z'+d')} \left\{ \frac{1}{k_0^0} + i \sqrt{\frac{\pi}{k_0^0}} \left( \frac{k_0}{k_2} - i k_0 l \right) \right\} \]  \hspace{1pt} (70)
\[ -e^{-iP^* F(P^*)} \] \quad (72) \]

\[ B_{2\rho}(\rho, \varphi', z') = -\frac{\mu_0}{4\pi} \sin \varphi' \left\{ \frac{\gamma_0}{k_0^2} - i \frac{\gamma_0^{2l}}{k_1^2} e^{i\gamma_0^2 (z'+d')} \lambda_1 \right. \]
\[ \cdot \left[ H_0^{(1)}(\lambda_1 \rho) + H_2^{(1)}(\lambda_1 \rho) \right] \]
\[ + 2 \frac{k_0^3}{k_2} e^{ik_0 \rho + ik_2 (z'+d')} e^{-i \frac{\pi}{2} \frac{1}{k_0 \rho}} \left[ 1 - \frac{i}{k_0 \rho} + \frac{1}{k_1 \rho} \right] \]
\[ \left. -i\sqrt{\frac{\pi}{k_0 \rho}} \left( \frac{k_0}{k_2} - ik_0 l \right) e^{-iP^* F(P^*)} \right\} \quad (73) \]

\[ B_{2\varphi}(\rho, \varphi', z') = -\frac{\mu_0}{4\pi} \cos \varphi' \left\{ \frac{\gamma_0^*}{k_0^2} - i \frac{\gamma_0^{*2l}}{k_1^2} e^{i\gamma_0^{*2} (z'+d')} \lambda_1 \right. \]
\[ \cdot \left[ H_0^{(1)}(\lambda_1 \rho) - H_2^{(1)}(\lambda_1 \rho) \right] \]
\[ + 2 \frac{k_0^3}{k_2} e^{ik_0 \rho + ik_2 (z'+d')} e^{-i \frac{\pi}{2} \frac{1}{k_0 \rho}} \left[ 1 - \frac{i}{k_0 \rho} + \frac{1}{k_1 \rho^3} \right] \]
\[ +i\sqrt{\frac{\pi}{k_0 \rho}} \left( \frac{k_0}{k_2} - ik_0 l \right) e^{-iP^* F(P^*)} \right\} \quad (74) \]

\[ B_{2z}(\rho, \varphi', z') = \frac{i\mu_0}{4\pi} \sin \varphi' \frac{2k_0^3}{k_1^2 k_2^2 \rho^2} e^{ik_0 \rho + ik_2 (z'+d')} e^{-i \frac{\pi}{2}} \quad (75) \]

6. CALCULATIONS AND DISCUSSIONS

Taking into account that a horizontal antenna of a submarine is buried inside in the sea water covered with a layer of ice, with the operating frequency \( f = 1 \text{MHz} \), the conductivity and the relative dielectric constant of ice being approximately \( \sigma_1 = 1 \times 10^{-5} \text{S/m} \) and \( \varepsilon_{1r} = 3.2 \), respectively, and the conductivity and the relative dielectric constant of sea water being \( \sigma_2 \approx 4 \text{S/m}, \varepsilon_{2r} = 80 \), respectively, the wave number of the electric-type trapped surface wave \( \lambda_1 \) varies as a function of the thickness \( l \) of ice layer. The real part and the imaginary part of \( \lambda_1/k_0 \) are shown in Figs. 4 and 5, respectively. In contrast, when Region 2 is a perfect conductor, the corresponding real part and the imaginary part of \( \lambda_1/k_0 \) are also plotted in the same figures. From the Figs. 4 and 5, it is seen that the real part for perfect conductor is closed.
Figure 4. The real parts of $\lambda_1/k_0$ for electric-type trapped surface wave vary with thickness $l$ of ice layer.

Figure 5. The imaginary parts of $\lambda_1/k_0$ for electric-type trapped surface wave vary with thickness $l$ of ice layer.
to that for the high lossy material, but the imaginary parts of $\lambda_1/k_0$ for both substrates have a little difference. The trapped surface wave for Region 2 being high lossy material attenuates faster than that for Region 2 being perfect conductor. Similarly, the real and imaginary parts of the wave number for the magnetic-type trapped surface wave as a function of the thickness $l$ are shown in Figs. 6 and 7, respectively.

If the transmitting and receiving antennas are located at $d' = z' = 0.5\, \text{m}$ underneath the ice layer, Figs. 8 and 9 show the amplitude of the component $E_{2\rho}$ at $\varphi' = 0^\circ$ as a function of the propagation distances with the thicknesses of the ice layer $l = 2\, \text{m}$ and $8\, \text{m}$, respectively. In Figs. 10 and 11, the amplitude of $E_{2\varphi}$ in the direction $\varphi' = 90^\circ$ as a function of propagation distance are shown with $l=2\, \text{m}$ and $8\, \text{m}$, respectively. From Figs. 9 and 11, it is seen that the interference effect will appear in the total field.

From Fig. 9 to Fig. 11, the curves of lateral wave is the same as Dunn's paper, and Dunn overlooked the trapped surface wave.
Figure 7. The imaginary parts of $\lambda_1/k_0$ for magnetic-type trapped surface wave vary with thickness $l$ of ice layer.

Figure 8. The components of $|E_{2\rho}|$ vary with $\rho$ for $l = 2$ m in direction $\phi = 0^\circ$.
Figure 9. The components of $|E_{2\rho}|$ vary with $\rho$ for $l = 8\,\text{m}$ in direction $\varphi' = 0^\circ$.

Figure 10. The components of $|E_{2\varphi}|$ vary with $\rho$ for $l = 2\,\text{m}$ in direction $\varphi' = 90^\circ$. 
7. CONCLUSIONS

From the above derivations and computations, it is concluded as following: (i) the horizontal antenna underneath the ice layer can efficiently generate both the trapped surface wave and the lateral wave, and the amplitudes of both the waves are at about the same level and can not be neglected; (ii) the wave number of the trapped surface wave is dependent on the thickness $l$ of the dielectric layer as well as the electric parameters of the medium. If the dielectric is lossless, the trapped surface wave will attenuate in the rate of $\rho^{-1/2}$ in the far-zone region, and the trapped surface wave is dominant term; (iii) if the thickness of the dielectric layer is relatively large, the wave number of the trapped surface wave $\lambda_l$ will be significantly different from the wave number of the lateral wave $k_0$.

When the thickness $l$ of the dielectric layer satisfied $\sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$, the electric-type trapped surface wave can be excited efficiently and the magnetic-type trapped surface wave cannot be excited. When the thickness $l$ of the dielectric layer satisfied $\sqrt{k_1^2 - k_0^2} \cdot l > \frac{\pi}{2}$, both the electric-type and magnetic-type trapped surface waves can be excited efficiently. Under the condition of thick dielectric cases, the total field will be more complex. When $l$ satisfies $n\pi < \sqrt{k_1^2 - k_0^2} \cdot l < (n + 1)\pi$, the electric-type trapped surface wave has $n + 1$ modes, and if $l$ is in the condition of $n\pi - \frac{\pi}{2} < \sqrt{k_1^2 - k_0^2} \cdot l < n\pi + \frac{\pi}{2}$, the surface there are $n$ modes of the magnetic-type trapped surface wave.
REFERENCES

Hong-Qi Zhang received his B.Sc. degree in Radio Physics and Ph.D. degree in Astrophysics from Lanzhou University, Lanzhou, China, in 1986, and Shaanxi Astronomical Observatory, the Chinese Academy of Sciences, Shaanxi, China, in 2001, respectively. Since 1986, he has been on the faculty of China Research Institute of Radiowave Propagation (CRIRP). In 1996 he advanced to his present rank of senior engineer at CRIRP. And in 2002 he advanced to his present rank of Professor at CRIRP. His research interests include wave propagation, thunderstorm location technology, and transient electromagnetic field. Dr. Zhang won the Chinese Academy of Sciences awards for his excellent Ph.D. thesis in 2001. (zhqoffice@163.com)

Wei-Yan Pan was born in Changzhou, Jiangsu, China, in November, 1939. He graduate from Chengdu Institute of Radio Engineering (CIRE), Chengdu, China, with a major in Applied Mathematics in 1962. He continued his graduate program in Applied Physics from 1962 to 1965 at CIRE. During the period from June 1983 to August 1985 and June 1993 to December 1993, he was a visiting scholar at Harvard University, USA. Since 1966, he has been with China Research Institute of Radiowave Propagation (CRIRP). In 1987 he advanced to his present rank of Professor at CRIRP. His research interests include LF/VLF wave propagation and applications, thunderstorm location technology, and analytical techniques in electromanetic theory.

Kai Li received his B.Sc. degree in Physics, M.Sc. degree in Radio Physics and Ph.D. degree in Astrophysics from Fuyang Normal University, Anhui, China, in 1990, Xidian University, Xi’an, Shaanxi, China, in 1994 and Shaanxi Astronomical Observatory, the Chinese Academy of Sciences, Shaanxi, China, in 1998, respectively. From Aug. 1990 to Dec. 2000, he had been on the faculty of China Research Institute Radiowave Propagation (CRIRP). From Jan. 2001 to Dec. 2002, he was a postdoctoral fellow at Information and Communications University, Taejon, South Korea. Since Jan. 2003, he has been a research fellow in the School of Electrical and Electronic Engineering, Nanyang Technology University (NTU), Singapore. His current research interests include electromagnetic theory and radio wave propagation. Dr. Li is a senior member of Chinese Institute of Electronics (CIE) and a member of Chinese Institute of Space Science (CISS).

Kai-Xian Shen, Professor, National Time Service Center of Chinese Academy of Science, Xi’an, China, 710600.