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SURFACE WAVE CHARACTER ON A SLAB OF METAMATERIAL WITH NEGATIVE PERMITTIVITY AND PERMEABILITY

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Abstract—Characteristics of surface wave modes on a grounded slab of negative permittivity and negative permeability parameters are investigated. It is shown that, unlike a slab with positive parameters, the dominant mode can have evanescent fields on both sides of the interface between the slab and the surrounding air. Detailed characteristics of such mode for various combinations of the slab parameters are given.

1 Introduction

2 Surface Wave Modes

3 Modes with Evanescent Fields on both Sides of the Interface

4 Concluding Remarks

References

1. INTRODUCTION

Recently, there has been a growing interest in metamaterials having negative permittivity and negative permeability in some frequency range for their potential of novel applications in microwave circuits. Veselago [1] was the first to show that the Poynting vector is antiparallel to the phase progression in such material. Lindell et al. [2, 3] have studied media with negative values of permittivity and permeability and labeled them as backward wave (BW) media or media capable of supporting backward waves. In the presence of an interface between a BW medium and a regular medium (one with positive permittivity and permeability), the power flowing laterally in the BW medium is in opposite direction to that in the regular medium. Other authors have used other labels to such materials, which include negative refractive index [4], left-handed media [5, 6] or double negative parameters media [7–9]. In this work we shall adopt the labeling "backward wave" for materials with negative permittivity and permeability. Several theoretical and experimental studies have been conducted on wave reflection and refraction at an interface between a BW medium and a positive parameter medium [7, 10] and on radiation from a traveling wave source at such interface [8]. One interesting example is the concept of a perfect lens proposed in [4]. Characteristics of surface waves on a slab of BW material have been investigated in [3, 11]. Dispersion relations are determined and mode cutoff effects are studied. It is found that a band-pass region appears for the first mode. Guided modes in metallic waveguides partially loaded by a slab of BW material have been studied [12].

Although surface wave modes on a BW slab are studied in [3, 11], we reconsider them in this research to study the nature of power carried by these modes. We also investigate the possibility of supporting a mode with evanescent fields on both sides of the interface between the slab and the surrounding (regular) medium. Conditions for the absence of surface waves in a certain frequency band are sought. To this end, we first review the main features of surface waves on a grounded slab of BW material, concentrating on the power carried by each mode in Section 2. Detailed study of the dominant TE or TM modes is carried out for various combinations of the permittivity and permeability parameters of the slab in Section 3. Concluding remarks are given in Section 4.

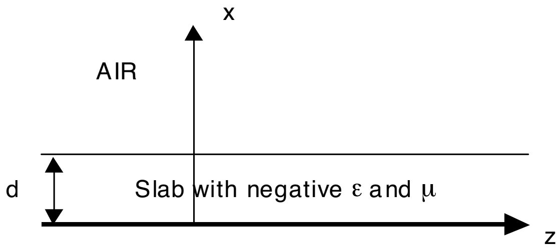

Figure 1. A grounded BW slab.

2. SURFACE WAVE MODES

Surface wave modes on a BW slab have been studied in [3–11]. In this section we review the salient features of surface waves on a grounded slab and then derive the power carried by any surface wave mode. To this end, let us consider a grounded slab of negative relative permittivity ε_r and negative relative permeability μ_r impeded in air as depicted in Fig. 1. We consider TM to z modes and the TE to z modes.

TM modes:

Considering TM modes with the magnetic field totally along the y-direction and independent of y, h_y takes the form:

$$\begin{aligned} h_y &= \cos k_x x \exp[-j\beta z] & 0 \leq x \leq d \\ &= \cos k_x x d \exp[-\alpha(x-d)] & d \leq x < \infty \end{aligned} \tag{1}$$

where: $k_x = (k_0^2\mu_r\varepsilon_r - \beta^2)^{1/2}$, $\alpha = (\beta^2 - k_0^2)^{1/2}$, $k_0 = \omega(\mu_0\varepsilon_0)^{1/2}$ and a time harmonic variation $\exp(j\omega t)$ has been assumed. Using Maxwell equation to get the electric field component e_z as proportional to $\varepsilon_r^{-1}\partial h_y/\partial x$ and enforcing the continuity of e_z at $x = d$, we get the modal equation:

$$[k_x d \tan(k_x d)]/\varepsilon_r = \alpha d = \sqrt{V^2 - k_x^2 d^2} \tag{2}$$

where $V = k_0 d\sqrt{\varepsilon_r\mu_r - 1}$ is a normalized frequency parameter. Equation (2) is the same modal equation as that of a regular slab with positive ε_r and μ_r. However the negative value of ε_r has a drastic effect on the solution for the modal phase constant β as will be seen. As an example, the relative phase constant β/k_0 for the first two TM_n modes is plotted versus V in Fig. 2 for a slab having $\varepsilon_r = -2$ and $\mu_r = -1.5$. The first mode TM_1 of the BW slab starts at some minimum value

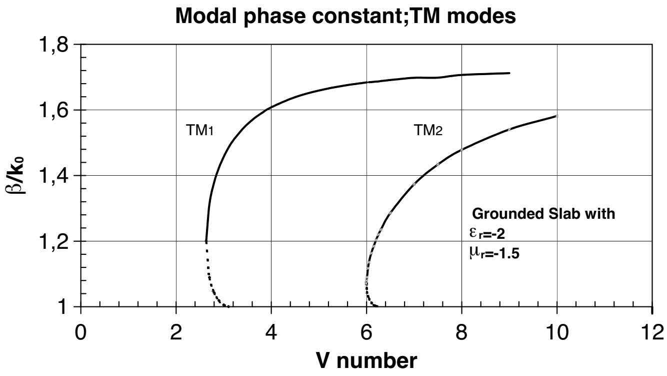

Figure 2. Relative phase constant of TM modes versus normalized frequency V.

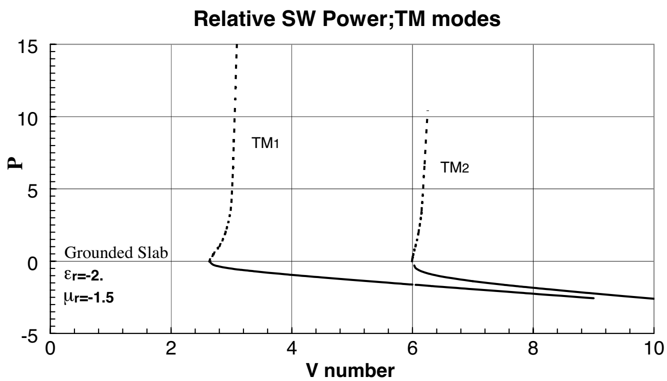

Figure 3. Relative Power of TM modes on a grounded backward slab versus V.

of V; say $V = V_{\min}$. For $V_{\min} < V < \pi$, there are two solutions for the TM_1 mode. One of these has a cutoff at $V = \pi$ and negative group velocity. The other solution exists at any value of $V > V_{\min}$ and represents a tightly bounded mode for large V. Similar behavior is displayed by the second mode. So, in general the TM_n mode has two branches, the one with the lower β has a negative group velocity and suffers cutoff ($\beta = k_0$) at $V = n\pi$. The other branch continues to exist at higher frequencies and has a positive group velocity.

It is interesting at this point to investigate the net modal power of each branch of the TM_n modes; $n > 0$. This is shown in Fig. 3 where it is seen that the modal power is zero at $V = V_{\min}$ where the mode starts to emerge. The power of the mode branch with the lower β is positive (forward) and the power of the mode branch with the higher β is negative (backward). It should be noted here that the net power of a mode is the result of positive power that flows in the air along the direction of phase change and negative power, or backward power, flowing in the BW slab in the opposite direction.

TE modes:

Using similar analysis, we get the modal equation of TE surface wave modes on a BW slab as:

$$[-k_x d Cot(k_x d)]/\mu_r = \alpha d = \sqrt{V^2 - k_x^2 d^2} \tag{3}$$

Again this is the same equation as that for a regular slab, but here the negative value of μ_r results in a drastic change in the solution for the phase constant β. Considering the lowest order mode; TE_0 mode, we notice that when $k_x = 0$ in (3), the LHS tends to $(1/|\mu_r|)$ which is positive and must be equal to V. Actually it can be shown that there is no solution to (3) with sinusoidal field distribution in the slab for $V < (1/|\mu_r|)$. So the TE_0 mode starts at a frequency corresponding to $V = (1/|\mu_r|)$ whence $\alpha d = V$. As $k_x d$ approaches $\pi/2$, αd tends to zero and the mode cutoffs at $V = k_x d = \pi/2$. Thus the TE_0 mode exists alone in the frequency range given by $1/|\mu_r| < V < \pi/2$ (provided that $|\mu_r| > 2/\pi$) and therefore can be considered the dominant mode of the slab. The relative phase constant β/k_0 of the TE_0 mode and the next mode TE_1 mode is plotted against V in Fig. 4 for $\varepsilon_r = -2$ and $\mu_r = -1.5$. It is seen that the TE_1 mode (and other higher order modes) has two branches with opposite group velocities just as the

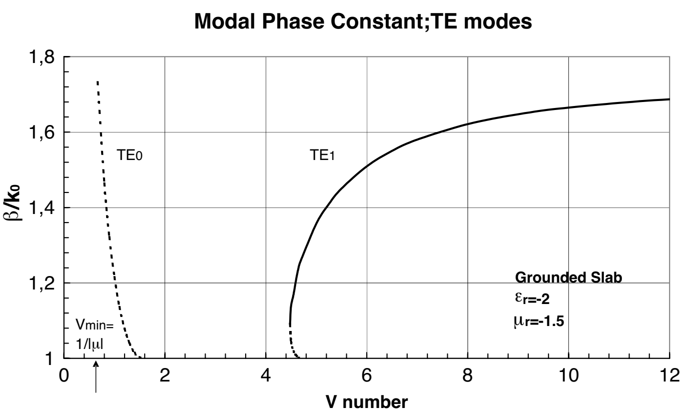

Figure 4. TE modal phase constant.

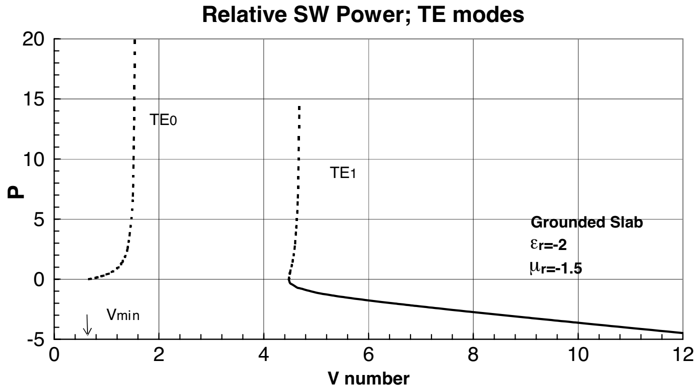

Figure 5. Relative powers of TE mode.

TM modes. In general, a higher order TE_n mode; $n > 0$ starts at a $V_{\min,n}$ such that $n\pi < V_{\min,n} < (n + 1/2)\pi$. The mode has two solutions for β in the band $V_{\min,n} < V < (n + 1/2)\pi$ and one solution for $V > (n + 1/2)\pi$. The corresponding relative power carried by each mode is plotted in Fig. 5. The TE_0 mode carries positive or forward power which increases indefinitely towards cutoff at $V = \pi/2$. Each of the TE_n modes with $n > 0$ has two branches; the one with the lower β carries a net positive or forward power and ends in cutoff at $V = (n + 1/2)\pi$. The other branch carries a net negative power, or backward power, and continues at higher frequencies getting more bounded to the slab.

3. MODES WITH EVANESCENT FIELDS ON BOTH SIDES OF THE INTERFACE

So far we have considered modes with oscillating fields inside the slab and decaying fields in the air. The question that rises now is whether modes with decaying fields on both sides of the interface can exist. It is well known that a slab with positive parameters will not support modes with decaying fields on both sides of the interface. For a backward wave slab, however, it turns out that this is not the case. To study this statement, let us investigate the modal Equations (2) and (3) where we assume that k_x is pure imaginary; that is $k_x = jp$, where p is real. Equations (2) and (3) reduce then to:

$$-[pd\tanh(pd)]/\varepsilon_r = \alpha d = \sqrt{V^2 + p^2 d^2} \quad \text{(TM modes)} \qquad (2')$$

$$[-pdCoth(pd)]/\mu_r = \alpha d = \sqrt{V^2 + p^2 d^2} \quad \text{(TE modes)} \qquad (3')$$

We note that αd must be positive for a proper surface wave mode. Now if ε_r (or μ_r) is positive, the LHS of (2) (or (3)) is negative and there is no solution for a valid surface wave. However, when ε_r (or μ_r) is negative, it is possible to have a surface wave solution for which the fields are evanescent away from both sides of the interface. Note also that for such a mode V^2 does not have to be positive, that is, $\varepsilon_r\mu_r$ can be greater or less than one. It is worth noting here that the same phenomenon has been reported in metamaterial resonators with cascaded BW slab and a regular slab where a mode with evanescent fields away from the interface is detected [13]. Now let us consider the evanescent TE and TM surface wave mode separately.

TE_0 mode; $\varepsilon_r\mu_r > 1$:

For the TE_0 mode, Eq. (3') tells us that $p = 0$ when $V = 1/|\mu_r|$ whence $\beta/k_0 = \sqrt{\varepsilon_r\mu_r}$. At this point, we distinguish between two cases according to whether $|\mu_r|$ is greater or less than one. If $|\mu_r| > 1$, then for $V < 1/|\mu_r|$, 'p' increases and β/k_0 becomes $> \sqrt{\varepsilon_r\mu_r}$. This goes on until V tends to zero and both 'p' and 'α' tend to infinity. To illustrate this behavior, β/k_0 of the TE_0 mode is plotted for a slab having $\varepsilon_r = -2$ and $\mu_r = -1.5$ in Fig. 6 (see solid curve). It is seen that the mode exists in the range $0 < V < \pi/2$. The mode is evanescent, i.e., β/k_0 is greater than $\sqrt{\varepsilon_r\mu_r}$, in the range $0 < V < 1/|\mu_r|$. An explanation for the field decay on both sides of the interface can be given based on the boundary condition on h_z, which requires the

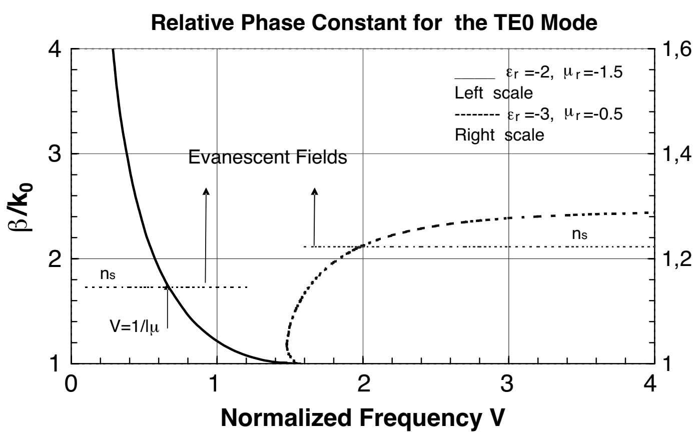

Figure 6. The phrase constant of the dominant TE_0 mode on a BW slab with $n_s \equiv \sqrt{\varepsilon_r\mu_r} > 1$ and $|\mu_r| > 1$ (solid curve) and $n_s \equiv \sqrt{\varepsilon_r\mu_r} > 1$ and $|\mu_r| < 1$ (dotted curve).

continuity of $\mu^{-1}\partial e_y/\partial x$ across the interface. Since μ is changing sign across the interface, so will be $\partial e_y/\partial x$. This allows the field to decay on both sides of the interface. It is thus clear that a surface wave with very high propagation factor is achieved at the interface of a BW slab and a regular medium even in a low frequency range. This phenomenon can be used in the design of superdirective antenna structures. Next, if $|\mu_r| < 1$, it can be shown from (3') that the TE_0 mode is evanescent for all values of $V > 1/|\mu_r|$. This is illustrated by the dotted curve in Fig. 6 where β/k_0 is plotted for a slab with $\varepsilon_r = -3$ and $\mu_r = -0.5$.

TE modes; $\varepsilon_r\mu_r < 1$:

It is interesting to investigate the surface wave modes when the refractive index of the slab is less than that in the surrounding medium; that is $\varepsilon_r\mu_r < 1$. In this case the only possible guided mode is one with hyperbolic fields inside the slab, or evanescent modes. Noting that V^2 is negative, it is appropriate to define $W = k_0 d\sqrt{1-\varepsilon_r\mu_r}$, so that we should use $W^2 = -V^2$ in Equation (3'). In this case consideration of Equation (3') shows that there is no mode when $|\mu_r| < 1$. So we conclude that the slab does not support any guided TE mode when its refractive index is less than that of the air and $|\mu_r| < 1$. On the other hand, an evanescent TE mode does exist at all frequencies when $|\mu_r| > 1$.

TM modes: $\varepsilon_r\mu_r > 1$:

Consideration of (2') shows that when $\varepsilon_r\mu_r > 1$ and $|\varepsilon_r| > 1$, no evanescent TM guided mode can be supported on the slab. To have an evanescent TM mode, one has to have $|\varepsilon_r| < 1$.

TM modes: $\varepsilon_r\mu_r < 1$:

Consideration of (2') shows that when $\varepsilon_r\mu_r < 1$ and $|\varepsilon_r| > 1$, the slab can support a TM guided mode at all frequencies. On the other hand when $|\varepsilon_r| < 1$, a guide TM mode is supported in a finite low frequency range.

4. CONCLUDING REMARKS

Surface wave modes on a grounded slab with negative permittivity and negative permeability (referred to as BW slab), which is embedded in a regular medium have been studied. It is shown that in general each surface wave mode has two branches having opposite group velocities. The net power of a mode is the algebraic sum of forward power in air and backward power in the slab. It is shown that, unlike a regular slab, TM modes do not exist in a low frequency range when $|\varepsilon_r| > 1$.

As an application, a BW slab can thus be used as a substrate in microstrip circuits to avoid the excitation of unwanted surface waves. It has been proved that modes with evanescent fields on both sides of the interface between the slab and air (referred to as evanescent modes) can exist. Namely, the TE_0 mode becomes evanescent in the low frequency band $0 < V < 1/|\mu_r|$ when $n_s \equiv \sqrt{\varepsilon_r \mu_r} > 1$ and $|\mu_r| > 1$. In this case the longitudinal phase constant β attains high values that makes the structure attractive in building superdirective antennas. It is interesting to find out that one or more evanescent surface wave mode can exist on the BW slab even when its refractive index is less than the surrounding medium. Namely when n_s is < 1, a TM evanescent mode can propagate at all frequencies if $|\varepsilon_r| > 1$ and in a finite low frequency if $|\varepsilon_r| < 1$. On the other hand, when n_s is < 1, a TE evanescent mode can propagate at all frequencies if $|\mu_r| > 1$ and ceases to exist if $|\mu_r| < 1$.

For future it will be interesting to study the problem of two layer slab, one layer is of positive permittivity and permeability and the other is of negative permittivity and permeability. In such structure bounded by metal plates a memory device behaviour for evanescent fields is observed [13].

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