Progress In Electromagnetics Research, PIER 51, 139–152, 2005

GENERALIZED SURFACE PLASMON RESONANCE SENSORS USING METAMATERIALS AND NEGATIVE INDEX MATERIALS

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Abstract—Optical surface plasmon resonance sensors have been known for a long time. In this paper, we discuss the use of metamaterials to construct a surface plasmon sensor which can be used at microwave frequencies. We review the conditions for the existence of surface plasmon and the use of the forward and backward surface waves. A sharp dip in the reflection coefficient occurs when the propagation constant of the incident wave along the surface is nearly equal to the propagation constant of the plasmon surface wave and may be used to probe bulk material characteristics or to determine metamaterial characteristics. Numerical examples are given to illustrate the basic characteristics.

1. INTRODUCTION

The phenomenon of surface plasmon resonance has been known for a long time and has been used for chemical sensors and remote sensing systems [1]. It makes use of a prism and a thin metal layer deposited upon the prism. The p-polarized (TM) reflected light exhibits a sharp dip at the incident angle where the propagation constant along the surface closely matches the propagation constant of the surface plasmon between the metal and the bulk material. This resonance occurs due to the negative dielectric constant of the metal, such as gold or silver, at optical frequencies.

In this paper, we explore the use of the NIM (Negative Index Materials), and more generally metamaterials, to produce the surface plasmon resonance at microwave frequencies. Metamaterials and NIM have attracted considerable attention in recent years [2–6], and the surface plasmon on NIM has also been discussed [7, 8]. We first discuss the surface wave (surface plasmon) modes between metamaterials and the dielectric. This requires the study of all wave types which may exist between the medium with arbitrary ε and μ and the ordinary medium. We discuss the classification of wave types. In particular, we discuss the regimes in the μ-ε diagram where the forward and backward surface waves exist. These regimes give rise to the surface waves, and the reflection coefficient exhibits a sharp dip at this particular angle, similar to the conventional optical surface plasmon resonance sensor.

We clarify the relationships between the p (TM) and s (TE) polarizations. We examine the fields inside NIM, and the interesting behaviors of the Poynting vectors, which point to the opposite direction in the inside and outside of NIM. We discuss the angular and frequency sensitivities of this phenomenon and the effect of the loss and possible difficulties in implementation for practical applications.

2. FORMULATIONS FOR A GENERALIZED SURFACE PLASMON RESONANCE SENSOR

Let us consider the layered structure shown in Fig. 1.

A plane wave is incident from the medium 1 with ε_1 and μ_1 on the layer with ε_2 and μ_2 bounded by the medium 3 with ε_3 and μ_3. Here ε and μ are relative permittivity and relative permeability normalized to free space ε_o and μ_o. The reflection coefficient R is well known [9]

$$R = \frac{A + B/Z_3 - Z_1(C + D/Z_3)}{A + B/Z_3 + Z_1(C + D/Z_3)} \tag{1}$$

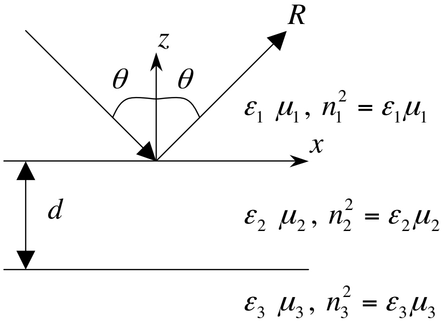

Figure 1. Geometry of layered metamaterials.

where

$$A = D = \cos k_{z2}d,\ \ B = jZ_2 \sin k_{z2}d,\ \ C = \frac{j \sin k_{z2}d}{Z_2}$$

$$k_{zi} = \sqrt{k_i^2 - (k_i \sin\theta_1)^2},\ \ i = 1,2,3,\ \ k_i = k_o n_i$$

$$k_o = \omega/c \text{ is the free space wave number}$$

$$Z_i = \begin{cases} \dfrac{k_{zi}}{\omega\varepsilon_o\varepsilon_i} & \text{for } p\text{-polarization } (E_x,\ E_z,\ H_y) \\ \dfrac{\omega\mu_o\mu_i}{k_{zi}} & \text{for } s\text{-polarization } (H_x,\ H_z,\ E_y) \end{cases}$$

In this formulation, both ε_i and μ_i are complex. However, for a passive medium, we require, using $\exp(j\omega t)$ time dependence,

$$\begin{aligned} \mathrm{Im}(\varepsilon_i) &< 0 \\ \mathrm{Im}(\mu_i) &< 0 \\ \mathrm{Im}(n_i) &< 0 \\ \mathrm{Im}(k_{z_i}) &< 0 \\ \mathrm{Re}\ \sqrt{\mu_i/\varepsilon_i} &> 0 \end{aligned} \tag{2}$$

In the following sections, we examine (1) for metamaterials and discuss its physical meanings.

3. CONVENTIONAL OPTICAL SURFACE PLASMON RESONANCE SENSOR

Before we discuss NIM surface plasmon resonance, we give a brief review of the conventional optical sensor making use of the surface plasmon resonance. A prism with ε_1 and μ_1 has a thin metallic layer with ε_2 and μ_2, and this is inserted into a medium whose characteristics are to be determined. A p-polarized (TM) plane wave is incident from the plasma and the reflected wave is measured as functions of angle and frequency. For the conventional surface plasmon sensor, $\mu_1 = \mu_2 = \mu_3 = 1$.

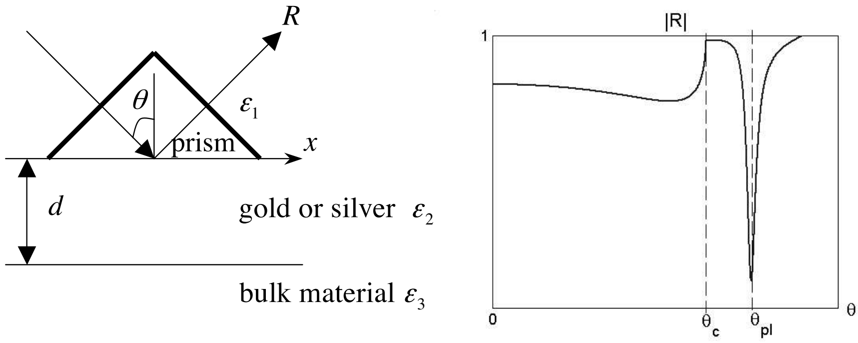

Figure 2. Surface plasmon geometry and the reflection coefficient plot of a conventional optical surface plasmon sensor $\varepsilon_1 = 2.25$, $\varepsilon_2 = -10 - j0.1$, $\varepsilon_3 = 1.75$, $\mu_1 = \mu_2 = \mu_3 = 1$, $d = 0.05\,\mu\text{m}$, $\lambda = 0.6\,\mu\text{m}$.

As an example, Fig. 2 shows a reflection coefficient R as a function of the incident angle. The metal layer, normally gold or silver, has a negative real part of ε_2 at optical frequency. There are two angles θ_c and θ_{pl} in Fig. 1, which are closely related to the material characteristics. The angle θ_c is close to the total reflection angle between the medium 1 and medium 3,

$$n_1 \sin\theta_c = n_3 \tag{3}$$

The angle θ_{pl} is when the propagation constant of the incident wave along the surface is close to the propagation constant of the surface plasmon between medium 2 and 3,

$$k_o n_1 \sin\theta_{pl} = k_{sp} = k_o S \tag{4}$$

The propagation constant of the surface wave is well known [1, 9]

$$S^2 = \frac{\varepsilon_2 \varepsilon_3}{\varepsilon_2 + \varepsilon_3} \tag{5}$$

The sharp dip at θ_{pl} is used to determine ε_3, and there is no surface plasmon for s-polarization.

4. SURFACE PLASMON FOR METAMATERIALS

We now generalize the conventional plasmon sensor in Section 3 to include the metamaterials with arbitrary ε and μ. To do this, we first examine the plasmon which exists between the medium with ε_d and μ_d and the metamaterial with ε_m and μ_m.

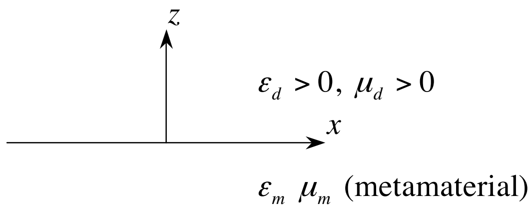

Figure 3. Surface wave along x between two media.

For p-polarization, we have

$$\begin{aligned} H_{yd} &= \exp\{-jk_oC_dz - jk_oSx\} \quad \text{for } z > 0 \\ H_{ym} &= \exp\{+jk_oC_mz - jk_oSx\} \quad \text{for } z < 0 \end{aligned} \tag{6}$$

Satisfying the boundary condition that E_x and H_y be continuous at $z = 0$, we get

$$\frac{C_d}{\varepsilon_d} + \frac{C_m}{\varepsilon_m} = 0 \tag{7}$$

From this, we get the propagation constant S

$$S^2 = \frac{n_d^2\varepsilon_m^2 - n_m^2\varepsilon_d^2}{\varepsilon_m^2 - \varepsilon_d^2} \tag{8}$$

The above equation is for the p-polarized (TM) wave. But it can be shown that for the s-polarized (TE) wave, S is given by switching ε and μ

$$S^2 = \frac{n_d^2\mu_m^2 - n_m^2\mu_d^2}{\mu_m^2 - \mu_d^2} \tag{9}$$

Here, the medium for $z > 0$ is assumed to be an ordinary medium with $\varepsilon_d > 0$ and $\mu_d > 0$, and this may be called the "double positive medium" (DPO) as opposed to the "double negative medium" (DNG) with $\varepsilon < 0$ and $\mu < 0$. Here, we assume that the losses are negligibly small and therefore ε and μ are real with negligible negative imaginary parts.

First, we note that in order to have the surface wave, we require

$$S^2 > n_d^2 \tag{10}$$

Furthermore, the choice of the sign for S needs to be determined by Eq. (7). Also, we need to choose S such that

$$\begin{aligned} \mathrm{Im}(C_d) &< 0 \\ \mathrm{Im}(C_m) &< 0. \end{aligned} \tag{11}$$

In Fig. 4, we show the regions in μ-ε diagram where $S^2 > n_d^2$.

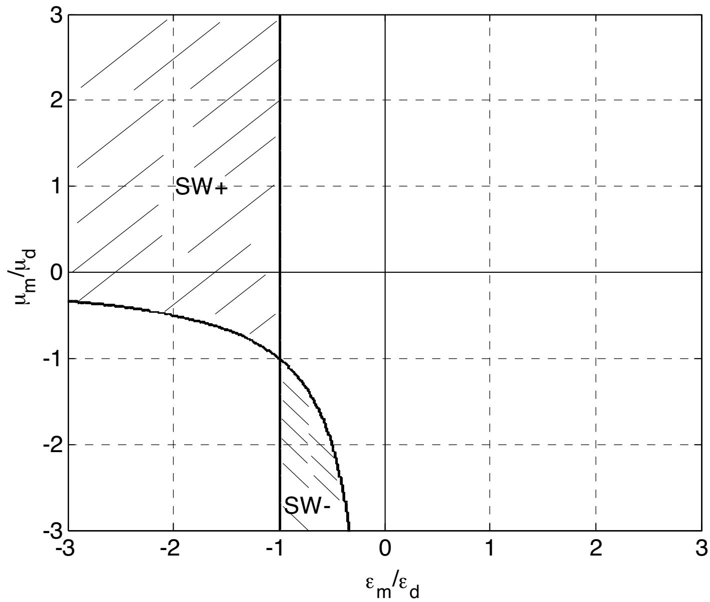

Figure 4. Forward (SW+) and backward (SW−) surface plasmons. The curve in the third quadrant is given by $(\mu_m\varepsilon_m)/(\mu_d\varepsilon_d) = -1$.

Eqs. (7), (8) and the conditions (11) give two regions where the surface wave propagates in the forward ($+x$) direction (SW+), and in the backward ($-x$) direction (SW−). Note that Fig. 4 is for p-polarization. For s-polarization, ε and μ are switched and the horizontal axis is μ_m/μ_d and the vertical axis is $\varepsilon_m/\varepsilon_d$, and all discussion on SW+ and SW− is unchanged. The fields inside and outside the metamaterial show that in the region of SW− in Fig. 4, the phase velocities along the surface in the medium (ε_d, μ_d) and the NIM $(\varepsilon_m < 0,\ \mu_m < 0)$ are in the same direction, but the Poynting vectors and the group velocities along the surface in the medium (ε_d, μ_d) and in NIM are in the opposite direction.

5. SURFACE PLASMON RESONANCE SENSOR

Let us now consider the structure shown in Fig. 5.

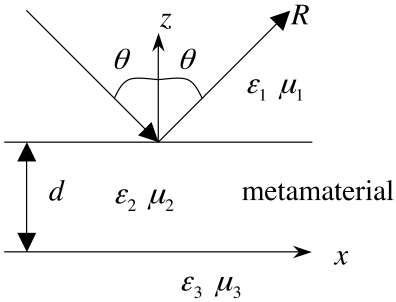

Figure 5. Metamaterial with ε_2 and μ_2 is placed under the prism with ε_1 and $\mu_1 = 1$ and the dielectric material with ε_3 and $\mu_3 = 1$.

The surface plasmon propagation constant S between the medium 2 and 3 is given by Eq. (8) or (9). The dip in the reflection coefficient should occur in the regions of SW+ and SW− in Fig. 4 with $\varepsilon_m = \varepsilon_2$, $\mu_m = \mu_2$, and $\varepsilon_d = \varepsilon_3$, $\mu_d = \mu_3$. Furthermore, in order to have the plasmon resonance sensor, we need to have a total reflection requiring

$$n_1 > n_3 \tag{12}$$

and the dip occurs at the angle θ_{pl} where

$$n_1 \sin\theta_{pl} = S \tag{13}$$

Combining with Eq. (10), we need to have

$$n_3^2 < S^2 < n_1^2 \tag{14}$$

This is shown in Fig. 6.

The reflection coefficients as functions of incident angle θ are shown in Fig. 7. The results are for the following cases:

$$\begin{aligned} &\text{Case A}: \quad \varepsilon_2 = -1.5 - 0.001j \quad \mu_2 = -2 \\ &\text{Case B}: \quad \varepsilon_2 = -4.5 - 0.001j \quad \mu_2 = 1 \\ &\text{Case C}: \quad \varepsilon_2 = -0.66 - 0.001j \quad \mu_2 = -2 \\ &d = 0.3\lambda_o \end{aligned} \tag{15}$$

Note that the plasmon does not exist in Case C, and there is no dip in reflection coefficient.

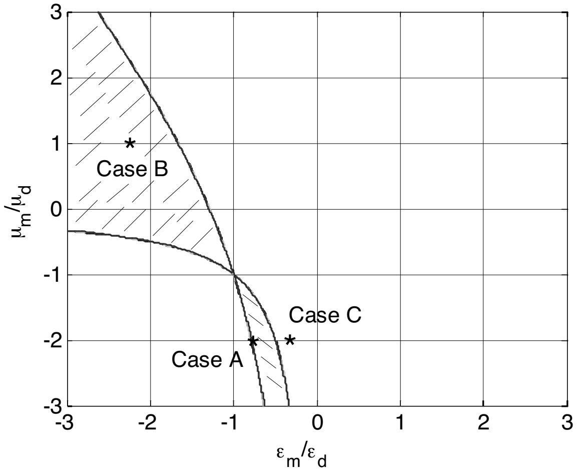

Figure 6. Surface plasmon resonance sensor $\varepsilon_1 = 5$, $\varepsilon_3 = 2$, $\mu_1 = \mu_3 = 1$. Surface waves can exist in the shaded regions.

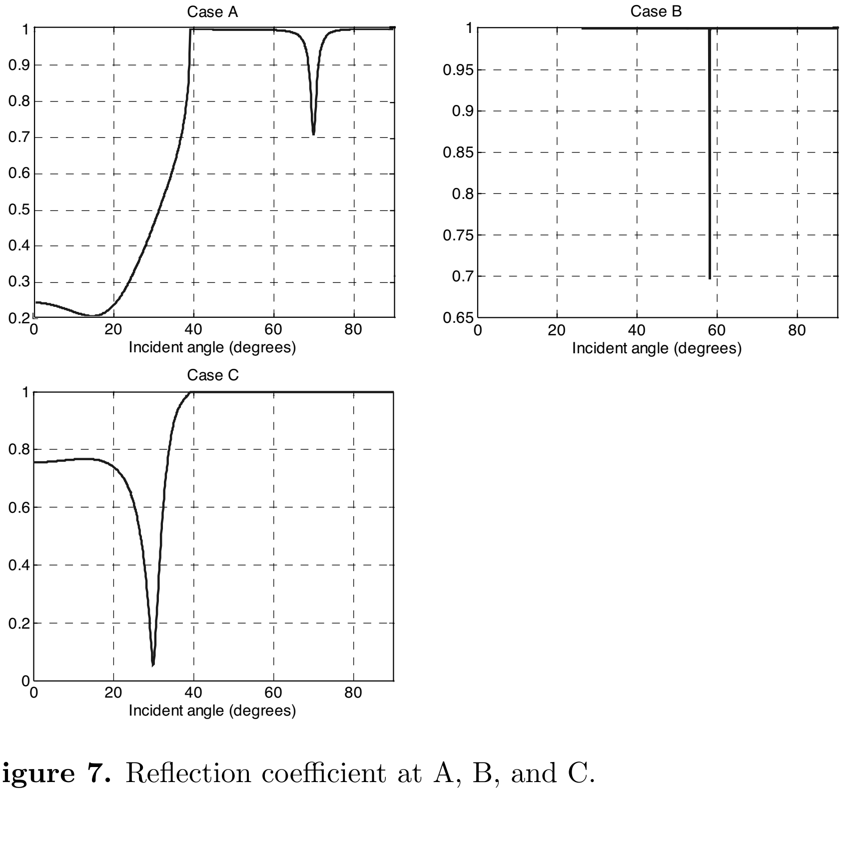

Figure 7. Reflection coefficient at A, B, and C.

6. SURFACE PLASMON SENSOR WITH GAP

Next we consider the structure in Fig. 8.

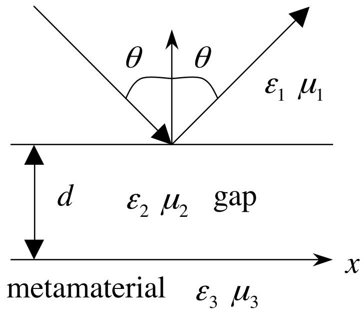

Figure 8. Structure with gap.

Here, we use $\varepsilon_1 = 5$, $\varepsilon_2 = 2$, $\mu_1 = \mu_2 = 1$. Figure 9 shows the cases A, B, and C where ε_3 and μ_3 are metamaterials shown in Eq. (15). Note that there is a sharp dip for Case A and B, but not for Case C. This configuration may be convenient to determine the characteristics of a metamaterial.

7. FREQUENCY DEPENDENCE AND EFFECTS OF LOSS

It is known that metamaterials are often highly dispersive and lossy. Therefore, it is important to examine the frequency dependence of the surface plasmon sensor. However, the frequency characteristics of metamaterials depend on how the material is made. In particular, there appears to be no general formula for $\mu(\omega)$, even though the Lorentz model and Drude model have been used [6]. Here we consider a narrow band approximation where $|\Delta\omega| = |\omega - \omega_o| \ll \omega_o$.

Noting the group refractive index n_g is given by

$$n_g = \frac{\partial}{\partial\omega}(n\omega), \tag{16}$$

we get, in narrow band approximation,

$$n(\omega) = n(\omega_o) + [n_g(\omega_o) - n(\omega_o)]\,\frac{\Delta\omega}{\omega_o} \tag{17}$$

where, in general, both and $n(\omega_o)$ are $n_g(\omega_o)$ are complex.

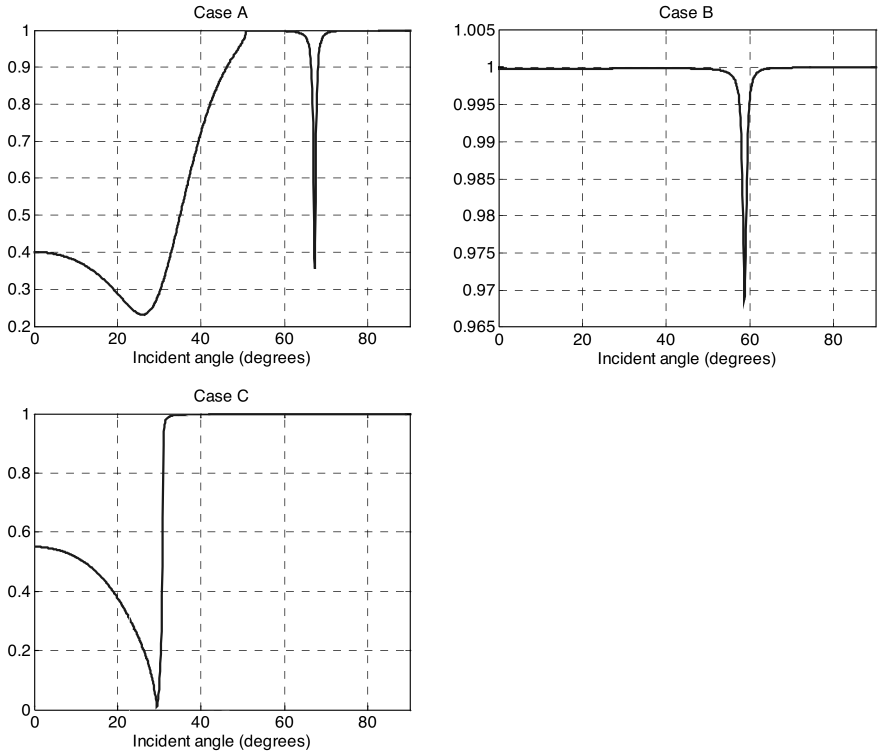

Figure 9. Reflection coefficient as a function of incident angle in gap structure.

Similarly, we can approximate ε and μ as

$$\begin{aligned} \varepsilon(\omega) &= \varepsilon(\omega_o) + \delta_e \frac{\Delta\omega}{\omega_o} \\ \mu(\omega) &= \mu(\omega_o) + \delta_m \frac{\Delta\omega}{\omega_o} \end{aligned} \tag{18}$$

where $\delta_e = \left.\frac{\partial\varepsilon}{\partial\omega}\right|_{\omega_o} \omega_o$ and $\delta_m = \left.\frac{\partial\mu}{\partial\omega}\right|_{\omega_o} \omega_o$. We then get

$$n_g(\omega_o) = n(\omega_o)\left[1 + \frac{1}{2}\left(\frac{\delta_e}{\varepsilon(\omega_o)} + \frac{\delta_m}{\mu(\omega_o)}\right)\right] \tag{19}$$

Note that for NIM, $n(\omega_o)$, $\varepsilon(\omega_o)$, and $\mu(\omega_o)$ are negative, but δ_e, δ_m, and n_g are positive.

As an example, we take Case A and let $\delta_e = \delta_m = \delta$. Figure 10

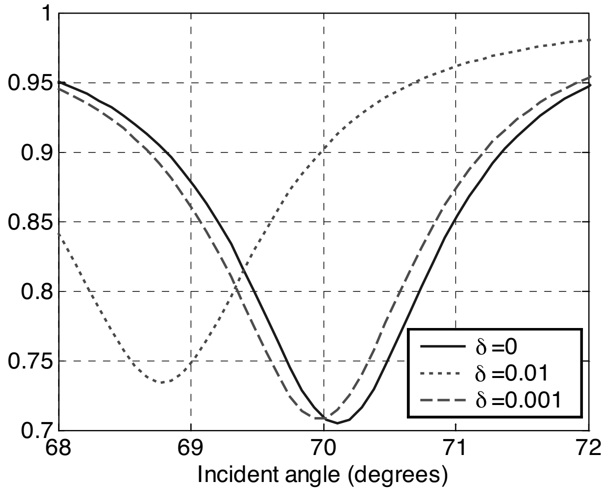

Figure 10. Effect of the frequency dependence to the reflection coefficient.

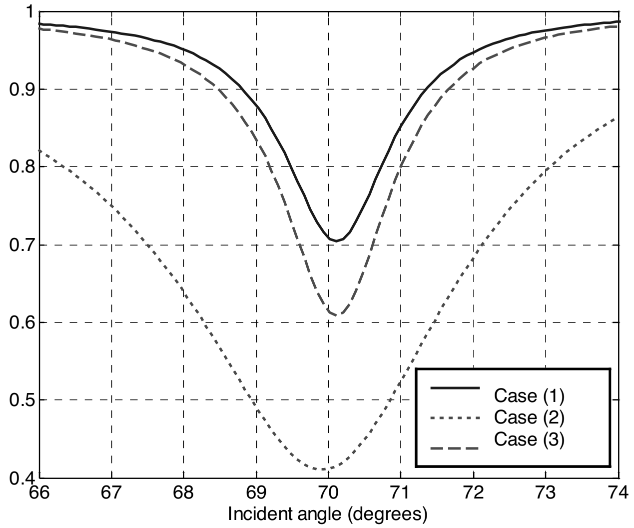

Figure 11. The effect of loss. (1) $\varepsilon_2 = -1.5 - 0.001j$, $\mu_2 = -2$, (2) $\varepsilon_2 = -1.5 - 0.001j$, $\mu_2 = -2 - 0.01j$, (3) $\varepsilon_2 = -1.5 - 0.001j$, $\mu_2 = -2 - 0.001j$.

shows the variation of the dip of the reflection coefficient for a small change of δ.

Next, we consider the effect of loss. For Case A, ε_2 and μ_2 are given in Eq. (15). We calculate and show the effect of loss in Fig. 11. Note that increased loss broadens the dip as expected.

8. CONCLUSIONS

We present the use of metamaterials for plasmon resonance sensors at microwave frequencies. The conditions for the existence of forward and backward surface waves are clarified using the μ-ε diagram. Metamaterial surface plasmon sensors may be useful for remote sensing of material characteristics or for determining metamaterial characteristics. This paper deals with a surface plasmon sensor making use of isotropic and homogeneous metamaterials. However, there are some practical issues, including the questions of how to construct such metamaterials, and what are the effects of anisotropic characteristics. Since most metamaterials are known to be highly dispersive and lossy, the sensitivities need to be carefully studied. It is also important to investigate how to construct practical broadband, low-loss metamaterials.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation (ECS-990 8849).

REFERENCES

1. Jorgenson, R. C. and S. S. Yee, "A fiber-optic chemical sensor based on surface plasmon resonance," *Sensors and Actuators B*, Vol. 12, 213–220, 1993.
2. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Sov. Phy. Usp.*, Vol. 10, No. 4, 509–514, Jan.–Feb. 1968.
3. Pendry, J. B., A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Microwave Theory and Techniques*, Vol. 47, No. 11, 2075–2084, November 1999.
4. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative

permeability and permittivity," *Phys. Rev. Lett.*, Vol. 84, No. 18, 4184–4187, May 2000.
5. Pendry, J. B., "Negative refraction makes a perfect lens," *Phy. Rev. Lett.*, Vol. 85, No. 18, 3966–3969, October 2000.
6. Ziolkowski, R. W. and E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Phy. Rev. E*, Vol. 64, No. 5, 056625, 2001.
7. Ruppin, R., "Surface polaritons of a left-handed medium," *Phy. Lett. A*, Vol. 277, 61–64, 2000.
8. Caloz, C., A. Sanada, and T. Itoh, "Surface plasmons at the interface between right-handed and left-handed 2D metamaterials," *IEEE-APS International Symposium*, Vol. 3, Columbus, OH, June 2003.
9. Ishimaru, A., *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice Hall, 1991.

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