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COMPLEX GUIDED WAVE SOLUTIONS OF GROUNDED DIELECTRIC SLAB MADE OF METAMATERIALS

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Abstract—This paper focuses on the complex guided wave solutions of grounded slab made of metamaterials. Complex solutions of both TE and TM modes have been analyzed. It is found that they are distributed on the proper Riemann sheet. This property differs dramatically from that of a conventional grounded slab. A number of other distribution properties are studied analytically. Some numerical examples are given to verify the analytical results. It is important to take the peculiar complex solutions into account in the problems of microstrip and surface wave excitation.

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1. INTRODUCTION

Theoretical possibility for metamaterials (MTMs) with simultaneously negative permittivity and permeability was introduced by Vesalago in 1968 [1]. Recently, these materials have been a topic of high interest after the first experimental demonstration of left-handed metamaterials was reported [2]. It has been shown that such materials have unique properties such as backward wave (BW) and negative refraction index (NRI). So far, a lot of attention has been drawn to find the potential applications of such materials. One of the pioneering works was Pendry's suggestion of perfect lens made of MTMs [3]. Recent advances in the theoretical and experimental researches of the composite MTMs [4, 5] make further contributions to the realization of these applications.

The guided surface waves at the interfaces between MTMs and conventional materials can offer new interesting phenomena and may have some important applications [6–9]. So far, a number of unusual properties have been found for the real guided wave solutions of these structures, such as the absence of the fundamental modes and the existence of slow wave mode [9]. In fact, there may exist lots of complex solutions for the eigenvalue equations. These solutions are important because their transition effects should be included in excitation of surface waves and microstrip problems, especially when the contour in the integral of the Green's functions is deformed.

In this paper, we consider a slab made of MTMs located on a perfect ground and place our emphasis on the investigation of the complex guided wave solutions. Based on this analysis, it was found that there may exist an infinite number of complex TE and TM surface wave solutions on the proper Riemann sheet. This property differs dramatically from that of a conventional grounded slab. We also study the distribution properties of the complex solutions and find some of them in the proper Riemann sheet by numerical methods. It is important to take these peculiar solutions into account when considering excitation of surface waves and microstrip problems.

2. EIGENVALUE EQUATIONS FOR GROUNDED SLAB MADE OF MTMS

Surface waves of both the TE and TM type may be guided along a grounded dielectric slab. Let the thickness of the slab be h, the relative permittivity be ε_r, and relative permeability be μ_r, as shown in Figure 1. $\varepsilon_r < -1$ and $\mu_r < -1$ are assumed and suppressed in this paper, which means the refractive index of the slab is higher than that

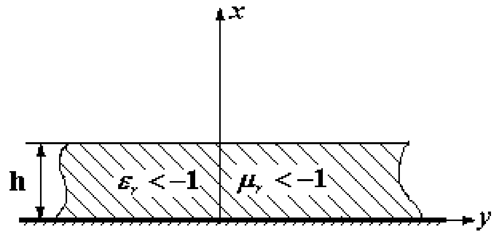


Figure 1. Grounded slab made of MTMs.

of vacuum.

The eigenvalue equations can be obtained by matching the fields tangential to the interface at $x = h$ [10]

$$T_e(k) = \alpha_1 h + \frac{1}{\mu_r} k_2 h \cot(k_2 h) = 0 \tag{1a}$$

$$T_m(k) = \alpha_1 h - \frac{1}{\varepsilon_r} k_2 h \tan(k_2 h) = 0 \tag{1b}$$

(1a) is for TE modes and (1b) is for TM modes, k is in general complex, and

$$\alpha_1^2 = k^2 - k_0^2 \tag{2a}$$

$$k_2^2 = k_0^2 \mu_r \varepsilon_r - k^2 \tag{2b}$$

It is convenient to rewrite (2a) and (2b) as follows:

$$(\alpha_1 h)^2 + (k_2 h)^2 = k_0^2 h^2 (\mu_r \varepsilon_r - 1) = a^2 \tag{3}$$

Solutions satisfying $\mathrm{Re}(\alpha_1) > 0$ lie on the proper Riemann sheet and lead to surface wave modes, while solutions with $\mathrm{Re}(\alpha_1) < 0$ lie on the improper Riemann sheet and give rise to 'leaky wave' modes. Graphical methods to investigate the solutions with real k have been addressed in [8, 9]. In the present work, emphasis is placed on analyzing the solutions with complex k. Complex solutions of TE and TM modes are analyzed in Sections 3 and 4, respectively. Some distribution properties are presented in each section, with numerical examples given to verify the analytical results.

3. COMPLEX GUIDED WAVE SOLUTIONS OF TE MODES

From Section 2, the guided wave solutions of TE modes are determined by a simultaneous roots of (1a) and (3). In order to find the complex solutions, let $\alpha_1 h$ be the complex variable

$$\alpha_1 h = W = u + iv \tag{4}$$

and let $k_2 h$ be the complex variable

$$Z = x + iy \tag{5}$$

Equation (1a) becomes

$$W = -\frac{1}{\mu_r} Z \cot(Z) \tag{6}$$

or in component form

$$u = -\frac{1}{\mu_r} \frac{y \tanh(y)[1 + \tan^2(x)] + x \tan(x)[1 - \tanh^2(y)]}{\tan^2(x) + \tanh^2(y)} \tag{7a}$$

$$v = \frac{1}{\mu_r} \frac{x \tanh(y)[1 + \tan^2(x)] - y \tan(x)[1 - \tanh^2(y)]}{\tan^2(x) + \tanh^2(y)} \tag{7b}$$

Equation (3) becomes

$$W^2 + Z^2 = a^2 \tag{8}$$

or in component form

$$u^2 - v^2 + x^2 - y^2 = a^2 \tag{9a}$$

$$uv + xy = 0 \tag{9b}$$

Eliminating the variable W in (6) and (8), we obtain

$$Z^2 \left[\mu_r^2 + \cot^2(Z)\right] = \mu_r^2 a^2 \tag{10}$$

Substitute (5) into (10), after a rigorous derivation, the term $x \tan(x)$ can be rewritten in the form

$$x \tan(x) = \frac{\mu_r^2 a^2 x^2 y^2 P}{(x^2 + y^2)^2 N} > 0 \tag{11}$$

where

$$N = [1 + \tan^2(x)] y \tanh(y)[1 - \tanh^2(y)] > 0 \tag{12}$$

$$P = [\tan^2(x) + \tanh^2(y)]^2 > 0 \tag{13}$$

From (7a) and (11), conclusion can be readily drawn that u is positive for all values of $x \neq 0$ and $y \neq 0$. Consequently, the real part of α_1

is always positive for the complex solutions. Therefore, the complex solutions of TE modes are located on the proper Riemann sheet and they are all surface wave solutions.

On the other hand, from (7) and (9), it's seen that, if $x_1 + iy_1$ is a solution, then $x_1 - iy_1$, $-x_1 + iy_1$, and $-x_1 - iy_1$ are also solutions. This means the complex solutions of TE modes are symmetrically distributed in x-y plane. Hence we need consider only positive values of x and y. According to (11), solutions with positive values of x are located within the region $n\pi < x < n\pi + \frac{1}{2}\pi$, where $n = 0, 1, 2, \ldots$.

Based on the above analysis, the first three solutions for $a = 3$, $\mu_r = -1.6$, $x, y > 0$ are found by numerical method and listed in Table 1:

Table 1. Three complex solutions of TE modes for $a = 3$, $\mu_r = -1.6$.

n	1	2	3
$Z_n = x_n + iy_n$	$1.17\pi + 0.904i$	$2.02\pi + 0.870i$	$3.01\pi + 0.790i$
$W_n = u_n + iv_n$	$1.39 - 2.38i$	$0.984 - 5.60i$	$0.832 - 8.95i$

4. COMPLEX GUIDED WAVE SOLUTIONS OF TM MODES

In a similar procedure, we find the complex guided wave solutions for TM modes:

Let $\alpha_1 h$ be the complex variable

$$\alpha_1 h = W = u + iv \tag{14}$$

and let $k_2 h$ be the complex variable

$$Z = x + iy. \tag{15}$$

We can rewrite equation (1b) in component form

$$u = -\frac{1}{\varepsilon_r}\frac{y\tanh(y)[1+\cot^2(x)] - x\cot(x)[1-\tanh^2(y)]}{\cot^2(x)+\tanh^2(y)} \tag{16a}$$

$$v = \frac{1}{\varepsilon_r}\frac{x\tanh(y)[1+\cot^2(x)] + y\cot(x)[1-\tanh^2(y)]}{\cot^2(x)+\tanh^2(y)} \tag{16b}$$

and rewrite equation (3) in component form

$$u^2 - v^2 + x^2 - y^2 = a^2 \tag{17a}$$

$$uv + xy = 0 \tag{17b}$$

Following the similar procedure in Section 3, the term $x\cot(x)$ can be rewritten in the form

$$x\cot(x) = -\frac{(x^2+y^2)^2 N}{\varepsilon_r^2 a^2 y^2 Q} < 0 \tag{18}$$

where

$$N = [1+\tan^2(x)]y\tanh(y)[1-\tanh^2(y)] > 0 \tag{19}$$

$$Q = [1+\tan^2(x)\tanh^2(y)]^2 > 0 \tag{20}$$

As can be seen, u is positive for all values of $x \neq 0$ and $y \neq 0$. Hence, it can be concluded that the real part of α_1 is always positive for the complex solutions. Therefore, the complex solutions of TM modes are located on the proper Riemann sheet and they are all surface wave solutions.

Similarly, only the positive values of x and y need to be considered. From (18), solutions with positive values of x are located within the region $n\pi + \frac{1}{2}\pi < x < n\pi + \pi$, where $n = 0, 1, 2, \ldots$. The first three solutions for $a = 3$, $\varepsilon_r = -2.5$, $x, y > 0$ are found by numerical method and listed in Table 2:

Table 2. Three complex solutions of TM modes for $a = 3$, $\varepsilon_r = -2.5$.

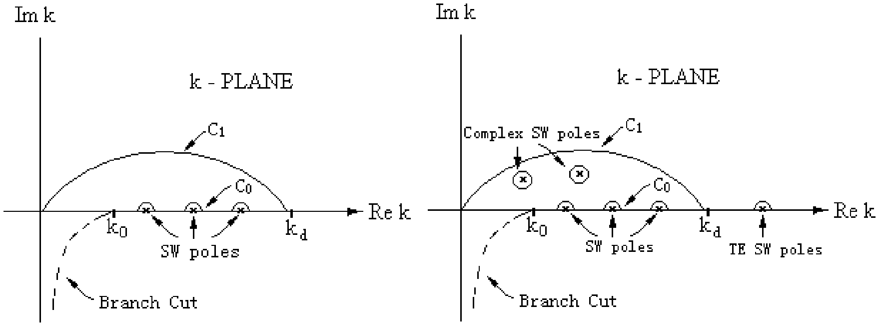
n	1	2	3
$Z_n = x_n + iy_n$	$1.52\pi + 0.554i$	$2.51\pi + 0.463i$	$3.501\pi + 0.442i$
$W_n = u_n + iv_n$	$1.11 - 5.82i$	$0.782 - 11.4i$	$0.719 - 16.5i$

5. DISCUSSIONS AND CONCLUSIONS

At this point, we have analyzed the complex guided wave solutions for a grounded slab made of MTMs. In the following, the importance of these peculiar solutions in solving surface wave excitation and microstrip problems will be presented. It is well known that the spatial-domain Green's functions in these problems can be given in the integral form:

$$G(\vec{r}) = \int_0^\infty \frac{F(k,\vec{r})}{T_e(k)T_m(k)} dk \tag{21}$$

The expressions of T_e and T_m have been given in Eq. (1). The zeros of T_e and T_m correspond to the TE and TM poles of the integrand,



(a) Conventional grounded slab (b) Grounded slab made of MTMs

Figure 2. Surface wave poles in k-plane.

respectively. For a conventional grounded slab, it is known that TE and TM surface wave poles only locate within the region $k_0 < k < k_0\sqrt{\mu_r\varepsilon_r}$ on the $\mathrm{Re}(k)$ axis. A nice way to avoid the integration near the poles is to deform the origin contour C_0 to the contour C_1 (as shown in Figure 2a), which is above the real k axis on the proper Riemann sheet. However, for a grounded slab made of MTMs, from Sections 3 and 4, we know that, in addition to the real surface wave poles (this is not the subject of this paper), there also exists an infinite number of complex TE and TM surface wave poles on the proper Riemann sheet. These complex poles will not directly contribute to the integral along the real axis. However, if the integration contour is deformed above the real axis in the proper Riemann sheet (as C_1 in Figure 2b), there is a possibility of capturing a finite number of complex surface wave poles, as shown in Figure 2b, and the calculation of residues at the captured poles is required.

In conclusion, we have considered a grounded slab made of MTMs and explored the complex guided wave solutions of both TE and TM modes. The important difference between the complex solutions of ordinary and MTMs grounded slab has been found analytically. For MTMs grounded slab, the complex solutions are distributed on the proper Riemann sheet and they are all surface wave solutions. Hence, in the problems of microstrip and surface wave excitation, there is a possibility of capturing a finite number of complex poles as the integration contour of Greens' functions is deformed in the proper Riemann sheet. In addition, it has been found that the complex solutions are symmetrically distributed in x-y plane and they can only exist in specific regions. Some numerical examples of complex solutions have been given to verify analytical results of their properties.

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