

SURFACE INTEGRAL EQUATION METHOD FOR GENERAL COMPOSITE METALLIC AND DIELECTRIC STRUCTURES WITH JUNCTIONS

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Abstract—The surface integral equation method is applied for the electromagnetic analysis of general metallic and dielectric structures of arbitrary shape. The method is based on the EFIE-CFIE-PMCHWT integral equation formulation with Galerkins type discretization. The numerical implementation is divided into three independent steps: First, the electric and magnetic field integral equations are presented and discretized individually in each non-metallic subdomain with the RWG basis and testing functions. Next the linearly dependent and zero unknowns are removed from the discretized system by enforcing the electromagnetic boundary conditions on interfaces and at junctions. Finally, the extra equations are removed by applying the wanted integral equation formulation, and the reduced system is solved. The division into these three steps has two advantages. Firstly, it greatly simplifies the treatment of composite objects with multiple metallic and dielectric regions and junctions since the boundary conditions are separated from the discretization and integral equation formulation. In particular, no special junction basis functions or special testing procedures at junctions are needed. Secondly, the separation of the integral equation formulation from the two previous steps makes it easy to modify the procedure for other formulations. The method is validated by numerical examples.

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1. INTRODUCTION

Composite structures of metallic and homogeneous dielectric materials have many important applications e.g., in the radar technology, antenna design and microwave engineering. Surface integral equation with the method of moments (MoM) [1] is a popular and powerful numerical method for the analysis of such objects. The method is well documented for single and isolated metallic and dielectric objects. However, the treatment of the field problem of a general composite structure with multiple material junctions remains challenging. Many earlier papers on this topic only consider rotationally or axially symmetric objects, e.g., [2–7], or if general 3D objects are considered, [8–14], either the metallic surfaces and interfaces of dielectric objects are not allowed to touch each other, or the question of the modeling the surface currents and testing procedure at junctions is more or less omitted. There are, however, several papers which discuss the general junction problem in detail, e.g., [5, 15, 16], [17] pp. 443–447, and [18].

In treating the field problem of a composite structure with the surface integral equation method, the main steps are the choice of the integral equation formulation, discretizing the surface integral equations by the basis and testing functions and enforcing electromagnetic boundary conditions at interfaces and junctions. In this procedure, usually, the junction problem, i.e., treatment of the

basis functions and testing procedure at the junctions, is the main difficulty, and it has been treated in several ways.

Some papers like [5, 10] and [18] use triangular half-basis functions at the junctions to expand the surface currents and to test the equations. The continuity of the surface currents is enforced by equating appropriate unknowns associated with the half-basis functions. However, in this approach the testing at junctions, without producing unnecessary line integrals, easily becomes complicated and often the topic is not discussed in detail or it is omitted. In addition, using different basis and testing functions on different parts of the surfaces complicates the practical implementation. In [15] surface currents at the junctions are expanded with so called multiplets, i.e., combinations of the original basis functions associated to a junction. But the testing at the junctions is again problematic because it cannot be done directly with the multiplets.

A straightforward way to avoid complications with the surface current expansion and testing is the use of the (full) Rao-Wilton-Glisson (RWG) [19] basis functions in both operations. In [16] the surface currents are expanded by the RWG functions, but the testing, enforcing the boundary conditions and the integral equation formulation are implemented by certain generalized testing currents expanded in the RWG functions. Combining the testing, the boundary conditions and the formulation this way makes the treatment of the junctions somewhat complicated and, in particular, the testing becomes fully dependent on the used integral equation formulation. In [17], pp. 443–447, the RWG basis functions are used both in the expansion and in the testing while the boundary conditions and the enforcing of the integral equation formulation are handled with the Number of Unknowns Reduction (NOUR) scheme. In NOUR first the homogeneous parts of the composite body are considered as isolated bodies and then they are merged together and the extra unknowns are removed. This approach, however, becomes complicated with multiple metallic and dielectric junctions and in [17] they are not considered.

EFIE-PMCHWT is the usual formulation for general composite structures [9, 11–17] and [18]. In this formulation Electric Field Integral Equation (EFIE) is applied on metallic surfaces and the Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) formulation [20] is applied on the dielectric interfaces, but it is not sufficient for removing the interior resonances if the structure includes closed metallic surfaces. In that case, in addition, on those surfaces the Combined Field Integral Equation (CFIE) [21] must be applied.

In this paper we treat general composite structures consisting of open and closed metallic surfaces and homogeneous dielectric

subdomains. All types of interface and surface junctions are allowed. The method is based on EFIE-CFIE-PMCHWT formulation to remove internal resonances (EFIE on open metallic surfaces, CFIE on closed metallic surfaces and PMCHWT formulation on the dielectric interfaces), and the numerical implementation is organized into three independent steps. Though the composite structures with junction problems are largely discussed in the literature, we believe that our approach in the presented generality is novel. The task division into three steps is the following:

- Step 1:** Present and discretize the electric and magnetic field integral equations in each homogeneous dielectric subdomain separately. Perform the discretization by the Galerkin method with the oriented RWG basis and testing functions.
- Step 2:** Enforce the boundary conditions on the metallic surfaces, the dielectric interfaces and at the junctions.
- Step 3:** Enforce the wanted integral equation formulation.

The first step leads to a system matrix of a diagonal block structure. The second step amounts to removing and combining unknowns leaving more equations than unknowns. In the third step we reduce the number of equations to that of the unknowns by combining, or removing, the extra equations according to the applied integral equation formulation.

In practice, we condense the last two steps into a few simple bookkeeping rules by which the final system matrix is directly assembled in an efficient way and without ever forming the matrices of the intermediate stages. In particular, the removed columns and rows of the system matrix are never computed in practice. However, we find it instructive to describe the treatment of the above three steps in terms of the discretized system matrix, because from that description the simple rules for the efficient system matrix assembly can easily be derived.

A similar division of the task into three steps was presented in [10] for penetrable structures without junctions; apparently this task division was not suggested for the general case, because the paper treats the junctions with half-RWG basis functions; anyway, a discussion on the testing and other details at general metal-dielectric junctions is omitted in the paper.

The procedure of this paper, i.e., why the discretization can be performed in subdomains before applying the boundary conditions and the integral equation formulation, can be shortly justified as follows: The discretization of the EFIEs and MFIEs separately in each subdomain by the Galerkin method only means replacing the original equations in subdomains by the approximative equations

obtained by restricting the surface currents to the function subspace spanned by the basis functions and by orthogonally projecting the original equations into the same subspace, see e.g., [22], Chapter 13. Therefore, the boundary conditions and the formulation, without changing their physical meaning, can be applied to the approximative equations equally well as to the original ones. On the other hand, the approximated equations are presented equivalently by the discretized system equations, and accordingly, the boundary conditions and the formulation can be directly applied to the system equations.

2. STEP ONE: PRESENTATION AND DISCRETIZATION OF SURFACE INTEGRAL EQUATIONS IN DIELECTRIC SUBDOMAINS

We consider the time harmonic electromagnetic waves in a piecewise homogeneous medium with the time factor $e^{-i\omega t}$. Let the computational domain be divided into homogeneous dielectric, or penetrable, and perfectly conducting (PEC) domains D_1, \dots, D_K as in Figure 1. The constant electromagnetic parameters of the domains are denoted by ϵ_j, μ_j and σ_j , where $\epsilon_j = \epsilon_{rj}\epsilon_0 + i\sigma_j/\omega$ denotes complex permittivity. A PEC object is defined by setting $\sigma_j = \infty$. In addition, let D_0 denote the (unbounded) background, let S_j denote the surface of $D_j, j = 0, \dots, K$, and let S_{jl} denote the interface of domains D_j and D_l . The PEC domains are further classified as closed and open domains. The latter ones represent thin metallic plates with zero volumes.

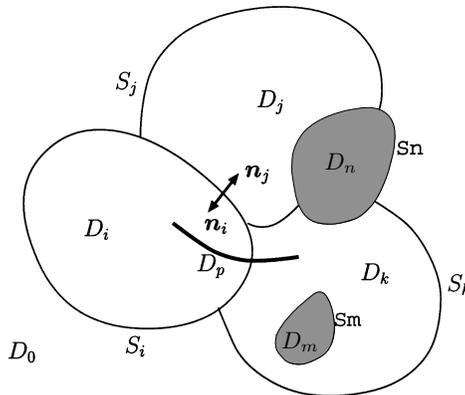


Figure 1. Piecewise homogeneous medium with open (D_p) and closed metallic objects (D_m, D_n). Metallic objects are denoted by shading.

2.1. Presentation of EFIEs and MFIEs

Let $\mathbf{E}_j^p, \mathbf{H}_j^p$ denote the primary or incident field of the sources in a domain D_j . The total field in D_j can be expressed as a sum of the incident field and the induced secondary field $\mathbf{E}_j^s, \mathbf{H}_j^s$ as

$$\mathbf{E}_j = \mathbf{E}_j^p + \mathbf{E}_j^s, \quad \mathbf{H}_j = \mathbf{H}_j^p + \mathbf{H}_j^s. \quad (1)$$

We define the equivalent electric and magnetic surface current densities on the surface S_j as

$$\mathbf{J}_j = \mathbf{n}_j \times \mathbf{H}_j \quad \text{and} \quad \mathbf{M}_j = -\mathbf{n}_j \times \mathbf{E}_j, \quad (2)$$

where \mathbf{n}_j is the inner unit normal of S_j pointing into the interior of D_j . If $\sigma_j < \infty$, the induced electromagnetic fields, $\mathbf{E}_j^s, \mathbf{H}_j^s$, generated by the currents \mathbf{J}_j and \mathbf{M}_j , can be expressed at point $\mathbf{r} \in D_j$ as [10, 15]

$$\mathbf{E}_j^s(\mathbf{J}_j, \mathbf{M}_j)(\mathbf{r}) = -\frac{1}{i\omega\epsilon_j}(\mathbf{D}_j\mathbf{J}_j)(\mathbf{r}) - (\mathbf{K}_j\mathbf{M}_j)(\mathbf{r}) \quad (3)$$

$$\mathbf{H}_j^s(\mathbf{J}_j, \mathbf{M}_j)(\mathbf{r}) = -\frac{1}{i\omega\mu_j}(\mathbf{D}_j\mathbf{M}_j)(\mathbf{r}) + (\mathbf{K}_j\mathbf{J}_j)(\mathbf{r}). \quad (4)$$

The integral operators \mathbf{D}_j and \mathbf{K}_j are defined as [10, 15]

$$(\mathbf{D}_j\mathbf{F})(\mathbf{r}) = \nabla \int_{S_j} G_j(\mathbf{r}, \mathbf{r}') \nabla'_s \cdot \mathbf{F}(\mathbf{r}') dS' + k_j^2 \int_{S_j} G_j(\mathbf{r}, \mathbf{r}') \mathbf{F}(\mathbf{r}') dS', \quad (5)$$

$$(\mathbf{K}_j\mathbf{F})(\mathbf{r}) = \int_{S_j} \nabla G_j(\mathbf{r}, \mathbf{r}') \times \mathbf{F}(\mathbf{r}') dS', \quad (6)$$

where $\nabla'_s \cdot$ denotes surface divergence of a tangential vector field in the primed coordinates, and

$$G_j(\mathbf{r}, \mathbf{r}') = \frac{e^{ik_j|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \quad (7)$$

is the homogeneous space Green's function and k_j is the wave number of domain D_j .

Next we write the needed integral equations on the boundary surfaces S_j of the domains D_j , $j = 0, \dots, K$. Let \mathbf{r} be a point in D_j . Letting $\mathbf{r} \rightarrow S_j$ and using the definition of the surface currents, we get for the tangential components,

$$\left(\mathbf{E}_j^p + \mathbf{E}_j^s \right)_{\text{tan}} = \mathbf{n}_j \times \mathbf{M}_j, \quad (8)$$

$$\left(\mathbf{H}_j^p + \mathbf{H}_j^s \right)_{\text{tan}} = -\mathbf{n}_j \times \mathbf{J}_j, \quad (9)$$

for all j , $j = 0, \dots, K$. By expressing the secondary fields in terms of the surface currents with the integral representations (3) and (4) in (8) and (9), and by using the tangential boundary values of the operators \mathbf{K} and \mathbf{D} , [23], we get the following equations on the surface S_j

$$\left(-\frac{1}{i\omega\varepsilon_j}(\mathbf{D}_j\mathbf{J}_j) - (\mathbf{K}_j\mathbf{M}_j) - \frac{1}{2}\mathbf{n}_j \times \mathbf{M}_j \right)_{\text{tan}} = -(\mathbf{E}_j^p)_{\text{tan}} \quad (10)$$

$$\left(-\frac{1}{i\omega\mu_j}(\mathbf{D}_j\mathbf{M}_j) + (\mathbf{K}_j\mathbf{J}_j) + \frac{1}{2}\mathbf{n}_j \times \mathbf{J}_j \right)_{\text{tan}} = -(\mathbf{H}_j^p)_{\text{tan}}, \quad (11)$$

for all j , $j = 0, \dots, K$. Equation (10) is called the Electric Field Integral Equation (EFIE) and (11) is called the Magnetic Field Integral Equation (MFIE). The PMCHWT formulation can be handled with EFIEs and MFIEs. For the CFIE formulation we, in addition, need to form the cross product of \mathbf{n}_j and MFIE and get the equation

$$-\frac{1}{i\omega\mu_j}\mathbf{n}_j \times (\mathbf{D}_j\mathbf{M}_j) + \mathbf{n}_j \times (\mathbf{K}_j\mathbf{J}_j) - \frac{1}{2}\mathbf{J}_j = -\mathbf{n}_j \times \mathbf{H}_j^p, \quad (12)$$

which is called as nMFIE in this paper.

2.2. Basis Functions

For the discretization of the equations (10)–(12) we use the Galerkin method and the Rao-Wilton-Glisson (RWG) basis functions [19]. In order to define the basis functions suppose that the surfaces are divided into planar triangular elements so that the elements on S_{jl} and S_{lj} on the opposite sides of the interface are equal and the meshes of the surfaces match.

An RWG basis function \mathbf{f}_n assigned to an edge e_n of the triangular mesh is defined as

$$\mathbf{f}_n(\mathbf{r}') = \begin{cases} \frac{L_n}{2A_n^+}(\mathbf{r}' - \mathbf{p}_n^+), & \mathbf{r}' \in T_n^+, \\ -\frac{L_n}{2A_n^-}(\mathbf{r}' - \mathbf{p}_n^-), & \mathbf{r}' \in T_n^-, \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

Here A_n^+ is the area of the triangle T_n^+ , L_n is the length of the common edge e_n and \mathbf{p}_n^\pm is the “free” vertex of T_n^\pm . An RWG function is displayed in Figure 2.

Though the RWG function \mathbf{f}_n is not continuous on the surface S_j , it still has the following partial continuity property: the component of

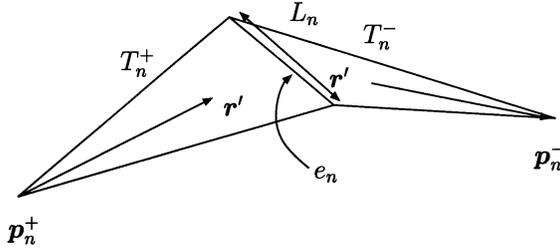


Figure 2. An RWG function \mathbf{f}_n assigned to the edge e_n .

\mathbf{f}_n normal to the edge e_n is continuous across the edge, and on the outer boundary of the triangle-pair $T_n^+ \cup T_n^-$ the component of \mathbf{f}_n normal to that boundary vanishes and therefore is continuous across that boundary. This partial continuity property of RWG functions makes it possible to expand surface currents in the RWG functions and preserve the surface current continuity across edges and junctions.

We finish this section by orienting the RWG functions which will be used in the discretization. Let e_n be an edge on the interface S_{jl} where only two domains D_j and D_l meet. Then two RWG functions \mathbf{f}_{n1} and \mathbf{f}_{n2} are assigned to that edge and they are on the opposite sides of S_{jl} . We now orientate them so that they flow into opposite directions, i.e., $\mathbf{f}_{n1} = -\mathbf{f}_{n2}$ see Figure 3. By this orientation the boundary conditions of the surface currents can be easily satisfied, as we will see later. If e_n is a junction edge, where more than two subdomains D_j meet, then all the RWG functions assigned to e_n are oriented so that on the opposite sides of the interfaces meeting e_n they again flow into opposite directions, i.e, they all flow past the edge either clockwise or counterclockwise, as illustrated in Figure 3.

2.3. Discretization by the Galerkin Method and RWG Basis Functions

In our approach the integral equations (10)–(12) of the previous section are discretized and converted into matrix equations by the Galerkin method with the oriented RWG basis functions separately in each domain. Let \mathbf{f}_n , $n = 1, \dots, N^{(j)}$ be the RWG basis functions defined on the triangular mesh on the boundary surface S_j of the subdomain D_j , where $N^{(j)}$ is the number of basis functions on the surface S_j . The unknown electric and magnetic surface current densities on the surface

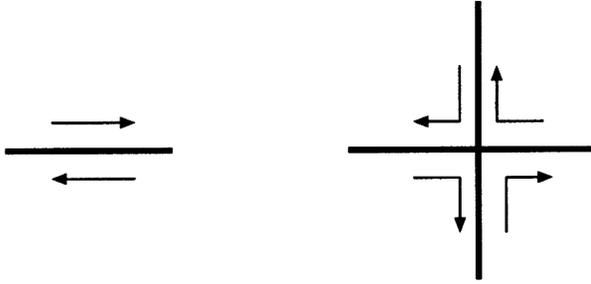


Figure 3. Orientation of the basis functions at an edge on the interface of two domains (a junction of two surfaces) and on the junction of four surfaces. Arrows indicate positive direction of the basis functions.

S_j are expanded in terms of the oriented basis functions \mathbf{f}_n

$$\mathbf{J}_j = \sum_{n=1}^{N^{(j)}} \alpha_n^{(j)} \mathbf{f}_n, \quad \mathbf{M}_j = \sum_{n=1}^{N^{(j)}} \beta_n^{(j)} \mathbf{f}_n. \quad (14)$$

The basis functions \mathbf{f}_n in the above expansions of \mathbf{J}_j and \mathbf{M}_j are called electric and magnetic, respectively. Next the equations (10)–(12) are converted into a matrix equation via a testing procedure. The EFIEs and nMFIEs are tested with the electric testing functions, and the MFIEs are tested with the magnetic testing functions. Since we apply the Galerkin method, these two testing functions are the same, i.e., the oriented RWG functions. By substituting the current approximations (14) into (10)–(12) and by testing with the oriented RWG functions \mathbf{f}_m , $m = 1, 2, \dots, N^{(j)}$, we get the following matrix equation in domain D_j

$$\begin{bmatrix} Z^{EJ(j)} & Z^{EM(j)} \\ Z^{HJ(j)} & Z^{HM(j)} \\ Z^{nHJ(j)} & Z^{nHM(j)} \end{bmatrix} \begin{bmatrix} I^{J(j)} \\ I^{M(j)} \end{bmatrix} = \begin{bmatrix} V^{E(j)} \\ V^{H(j)} \\ V^{nH(j)} \end{bmatrix}. \quad (15)$$

To decrease the singularity of the operator \mathbf{D} , the gradient is transformed into the testing function. By using the properties of the RWG basis functions, integrating by parts and the Gauss divergence theorem in the operator \mathbf{D} , the elements of the impedance matrix are given by [10]

$$Z_{mn}^{EJ(j)} = \frac{1}{i\omega\epsilon_j} \int_{T_m} \nabla_s \cdot \mathbf{f}_m(\mathbf{r}) \int_{T_n} G_j(\mathbf{r}, \mathbf{r}') \nabla'_s \cdot \mathbf{f}_n(\mathbf{r}') dS' dS$$

$$+i\omega\mu_j \int_{T_m} \mathbf{f}_m(\mathbf{r}) \cdot \int_{T_n} G_j(\mathbf{r}, \mathbf{r}') \mathbf{f}_n(\mathbf{r}') dS' dS, \quad (16)$$

$$\begin{aligned} Z_{mn}^{HJ(j)} &= \int_{T_m} \mathbf{f}_m(\mathbf{r}) \cdot \int_{T_n} \nabla G_j(\mathbf{r}, \mathbf{r}') \times \mathbf{f}_n(\mathbf{r}') dS' dS \\ &\quad + \frac{1}{2} \int_{T_m} \mathbf{f}_m(\mathbf{r}) \cdot (\mathbf{n}_j \times \mathbf{f}_n(\mathbf{r})) dS, \end{aligned} \quad (17)$$

$$Z_{mn}^{EM(j)} = -Z_{mn}^{HJ(j)} \quad \text{and} \quad Z_{mn}^{HM(j)} = \frac{\varepsilon_j}{\mu_j} Z_{mn}^{EJ(j)}, \quad (18)$$

$$\begin{aligned} Z_{mn}^{nHJ(j)} &= \int_{T_m} (\mathbf{n}_j \times \mathbf{f}_m(\mathbf{r})) \cdot \int_{T_n} \nabla G_j(\mathbf{r}, \mathbf{r}') \times \mathbf{f}_n(\mathbf{r}') dS' dS \\ &\quad - \frac{1}{2} \int_{T_m} \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}) dS, \end{aligned} \quad (19)$$

$$Z_{mn}^{nHM(j)} = \frac{1}{i\omega\mu_j} \left(\int_{\partial T_m^+} + \int_{\partial T_m^-} \right) \mathbf{m}_m \cdot (\mathbf{n}_j \times \mathbf{f}_m(\mathbf{r})) \int_{T_n} G_j(\mathbf{r}, \mathbf{r}') \nabla'_s \cdot \mathbf{f}_n(\mathbf{r}') dS' dl \quad (20)$$

where in the line integrals in (20) over the boundary curves ∂T_m^+ and ∂T_m^- of the triangles T_m^+ and T_m^- the vector \mathbf{m}_m is the unit outer normal of these boundary curves. In addition, $T_n = T_n^+ \cup T_n^-$ is the support of \mathbf{f}_n and $T_m = T_m^+ \cup T_m^-$ is the support of \mathbf{f}_m . For the numerical evaluation of these matrix elements with the singularity extraction technique we refer to [24].

The vectors $I^{J(j)}$ and $I^{M(j)}$ in (15) are the coefficient vectors of \mathbf{J}_j and \mathbf{M}_j , including the coefficients $\alpha_1^{(j)}, \dots, \alpha_{N(j)}^{(j)}$ and $\beta_1^{(j)}, \dots, \beta_{N(j)}^{(j)}$, and $V^{E(j)}$, $V^{H(j)}$ and $V^{nH(j)}$ are the excitation vectors

$$V_m^{E(j)} = - \int_{T_m} \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{E}_j^p(\mathbf{r}) dS, \quad (21)$$

$$V_m^{H(j)} = - \int_{T_m} \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{H}_j^p(\mathbf{r}) dS, \quad (22)$$

$$V_m^{nH(j)} = \int_{T_m} (\mathbf{n}_j \times \mathbf{f}_m(\mathbf{r})) \cdot \mathbf{H}_j^p(\mathbf{r}) dS. \quad (23)$$

Above $m, n = 1, \dots, N^{(j)}$, runs through all basis and testing functions on the surface S_j and all matrix blocks in (15) are square.

By repeating above testing procedure for all boundary surfaces S_j of the domains D_j , $j = 0, \dots, K$, we obtain the following block diagonal matrix equation

$$\begin{bmatrix} Z^{(0)} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & Z^{(K)} \end{bmatrix} \begin{bmatrix} I^{(0)} \\ \vdots \\ I^{(K)} \end{bmatrix} = \begin{bmatrix} V^{(0)} \\ \vdots \\ V^{(K)} \end{bmatrix}, \quad (24)$$

where

$$Z^{(j)} = \begin{bmatrix} Z^{EJ(j)} & Z^{EM(j)} \\ Z^{HJ(j)} & Z^{HM(j)} \\ Z^{nHJ(j)} & Z^{nHM(j)} \end{bmatrix}, \quad (25)$$

$$I^{(j)} = \begin{bmatrix} I^{J(j)} \\ I^{M(j)} \end{bmatrix} \quad (26)$$

$$V^{(j)} = \begin{bmatrix} V^{E(j)} \\ V^{H(j)} \\ V^{nH(j)} \end{bmatrix}. \quad (27)$$

This is the initial discretized system matrix equation, which presents only the (approximative) integral equations (10)–(12) in each subdomain and not the entire scattering and transmission field problem, because the interactions between the dielectric domains have not yet been taken into account via the boundary conditions. Also extra equations must be removed by applying the the integral equation formulation. We start now carrying out those two tasks.

3. STEP 2: ENFORCING THE BOUNDARY CONDITIONS

The solutions of the integral equations (10)–(12) of Section 2.1 must satisfy electromagnetic boundary conditions on the domain interfaces and at the junctions. In the previous sections these solutions, namely the equivalent surface current densities, were defined domain-wise without taking into account the boundary conditions. Due to the these conditions, the discretization of the equations leads to system equations which include linearly dependent unknowns, related to \mathbf{J} and \mathbf{M} on the interfaces of dielectric domains, and zero unknowns related to \mathbf{M} on the surfaces of metallic objects. In this section we find definite rules how the extra unknowns and basis functions are removed by enforcing the boundary conditions.

3.1. Boundary Conditions

The formulation of Section 2.1 includes two kind of surfaces, metallic and dielectric ones. A surface of a PEC object is called a metallic surface and an interface of two homogeneous penetrable domains with $\sigma < \infty$ is called a dielectric surface, or interface. Note that the interfaces of homogeneous non-metallic domains are called dielectric surfaces although the domains may be magnetic, too, i.e., $\mu_r > 1$. The metallic surfaces are further classified as closed and open surfaces, depending whether the surface circumvents a 3D body or not.

The boundary conditions of the electromagnetic fields directly determine how the surface currents behave on the metallic and dielectric surfaces. The boundary conditions are

1. $\mathbf{n} \times \mathbf{E}$ and $\mathbf{n} \cdot \mathbf{H}$ vanish on metallic surfaces,
2. $\mathbf{n} \times \mathbf{E}$ and $\mathbf{n} \times \mathbf{H}$ are continuous across dielectric surfaces.

The first boundary condition immediately yields the important result: the magnetic current $\mathbf{M} = -\mathbf{n} \times \mathbf{E}$ vanishes on metallic surfaces. In the following subsections we study what other properties the boundary conditions imply for the surface currents, in particular, at junctions of various types.

3.2. Classification of Edges and Junctions

For treating the boundary conditions and the integral equation formulation, we classify the edges and junctions into the following three cases. Let e_n be an edge on the triangular mesh. It is called a junction, if more than two domains or surfaces meet at e_n . Otherwise, we call e_n a single edge, or just an edge. We classify edges and junctions e_n as follows:

1. *Dielectric edge or junction:* e_n lies on an intersection of two or more dielectric surfaces but does not meet any metallic surfaces.
2. *Metallic edge or junction:* e_n lies on intersection of open or closed metallic surfaces but does not meet any dielectric surfaces.
3. *Composite metal-dielectric junction:* e_n lies on an intersection of at least one open or closed metallic surface and at least one dielectric surface.

Next we consider the boundary conditions in these three cases of edges and junctions.

3.3. Dielectric Edge or Junction

Let us next consider the surface currents and their expansions in the basis functions on a dielectric interface S_{jl} between two dielectric domains D_j and D_l . The boundary conditions directly imply that

$$\mathbf{J}_j = -\mathbf{J}_l, \quad \text{and} \quad \mathbf{M}_j = -\mathbf{M}_l. \quad (28)$$

This follows from the obvious fact that the surface currents are related to the fields by $\mathbf{J} = \mathbf{n} \times \mathbf{H}$ and $\mathbf{M} = -\mathbf{n} \times \mathbf{E}$ and the tangential components of the fields are continuous across a dielectric interface and the normal vectors \mathbf{n} point into opposite directions. Because the two basis functions assigned to an edge on S_{jl} and on the opposite sides of S_{jl} , flow into opposite directions (orientation of the basis functions), in the surface current representations they must have equal coefficients. This is illustrated in Figure 4.

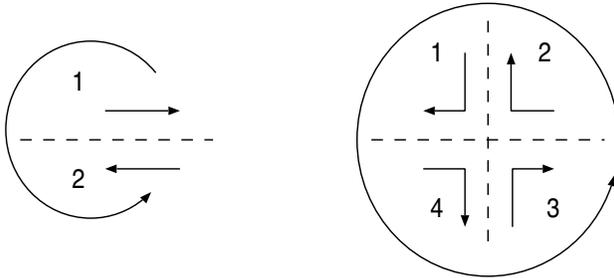


Figure 4. Combination of the unknowns associated to the electric and magnetic basis functions assigned at an edge on the interface of two dielectric domains and on the junction of four dielectric domains. Dashed lines denote dielectric surfaces and curved arrows indicate the basis functions with the same unknown coefficient. Numbers indicate dielectric domains.

This analysis generalizes to the dielectric junctions as follows. Let e_n be a dielectric edge or junction where two or more dielectric domains meet. Because the components of \mathbf{E} and \mathbf{H} , parallel to e_n , are also tangential components on the surfaces which meet at e_n , those components are continuous across the edge e_n into all directions. This implies that the components of the surface currents $\mathbf{J}_j = \mathbf{n}_j \times \mathbf{H}$ and $\mathbf{M}_j = -\mathbf{n}_j \times \mathbf{E}$, normal to the edge e_n , are continuous across the edge on an interface S_j which meets e_n . This is the (partial) continuity property of the surface currents on the dielectric edges and junctions, i.e., Kirchoff's laws for the surface currents. Furthermore, because of this continuity and (28), the normal components of all currents \mathbf{J}_j and

M_j , on surfaces S_j which meet e_n , must have the same absolute value at e_n , respectively. In order to make the current expansions in the oriented RWG functions to behave similarly at e_n , all basic functions of the same type, electric or magnetic, which are assigned to e_n , must have the same coefficient. Note that only those basis functions which are assigned to e_n contribute to the normal component on e_n . We get now our first rule for reducing the number of unknowns:

Rule 1: The unknown coefficients of the oriented basis functions of the same type assigned to the same dielectric edge or junction must have the same value and, hence, these unknowns must be combined into a single unknown.

Thus, at an edge or a junction of M dielectric surfaces, there are only two independent unknowns, one for \mathbf{J} and another for \mathbf{M} . For the system equations this rule implies that the columns of the system matrix associated with the basic functions of the same type and assigned to the same dielectric edge or junction are combined. This is illustrated in Figure 4.

3.4. Metallic Edge or Junction

Next we consider the surface currents and their expansions in the oriented RWG functions at a junction of M metallic surfaces. The surfaces may be open or closed. Since the boundary conditions imply that \mathbf{M} vanishes on metallic surfaces, the magnetic basis functions associated to e_n will be removed. In addition, since the magnetic field is not necessarily continuous across metallic surfaces, \mathbf{J} must have independent values on the opposite sides of the metallic surfaces. This, in turn, implies that the unknown coefficients associated to the electric basis functions assigned on a metallic junction or edge can not be combined. Naturally the electric basis functions inside closed metallic objects are removed and such basis functions should not ever be created.

There is one exception to the above rule, namely if all metallic surfaces associated to e_n are open and e_n is completely in the interior of one homogeneous dielectric domain. Consider first an open metallic surface S inside a homogeneous dielectric subdomain D_j . As already is mentioned, \mathbf{J} must have independent values on the opposite sides of S . Let us denote these currents by \mathbf{J}_1 and \mathbf{J}_2 . Let e_n be an edge on S so that e_n is completely in the interior of D_j , and let \mathbf{f}_1 and \mathbf{f}_2 be the basis functions assigned to e_n with unknown coefficients α_1 and α_2 . In a homogeneous medium, however, these two basis functions produce the same fields with opposite signs due to the orientation of the basis functions. Therefore, in order to avoid linear dependence of

the columns of the system matrix, the fields of the basis functions must be presented by only one of them, say that of \mathbf{f}_1 , and $\alpha_1 - \alpha_2$ can be considered as a new single unknown.

The above analysis generalizes for junctions of an arbitrary number of open metallic surfaces inside a homogeneous medium. Note also that we do not need the basis functions which are assigned to a boundary edge of an open metallic surface and lying completely inside a homogeneous dielectric domain, because \mathbf{J} has no component normal to such a boundary edge. Therefore, those electric basic functions and unknowns associated with them can be removed. In summary, we get our second rule:

Rule 2: At a metallic edge or junction meeting no dielectric surfaces, all the magnetic basis functions in the expansion of \mathbf{M} , with their unknowns, must be removed. In addition, if all surfaces assigned to e_n are open metallic surfaces and e_n lies completely in the interior of a homogeneous dielectric domain, one of the electric basis functions in the expansion of \mathbf{J} must be removed.

For the system equations this rule means that those columns of the system matrix, which are associated with the removed unknowns, will be removed. *Rule 2* is illustrated in Figure 5.

3.5. General Metal-Dielectric Junction

Next let e_n a general metal-dielectric junction. Because \mathbf{M} vanishes on metallic surfaces, then at e_n the component of \mathbf{M} , normal to e_n , also vanishes, and so the magnetic basis functions assigned at e_n and the unknowns associated with them are removed. Again due to the fact that the magnetic field is not continuous across open metallic surfaces, \mathbf{J} must have two independent values on the opposite sides of an open metallic surface. This implies that in the expansion of \mathbf{J} the unknown coefficients of the basis functions assigned to e_n on the opposite sides of metallic surfaces should be considered as independent unknowns, too. By combining this with the previous *Rule 2*, we obtain our third rule:

Rule 3: In the expansions of \mathbf{J} the unknown coefficients of the electric basis functions assigned to a general metal-dielectric junction between two metallic surfaces must have the same value and the corresponding unknowns must be combined to a single unknown. In the expansions of \mathbf{M} the magnetic basis functions, with their unknowns, assigned to a general metal-dielectric junction must be removed.

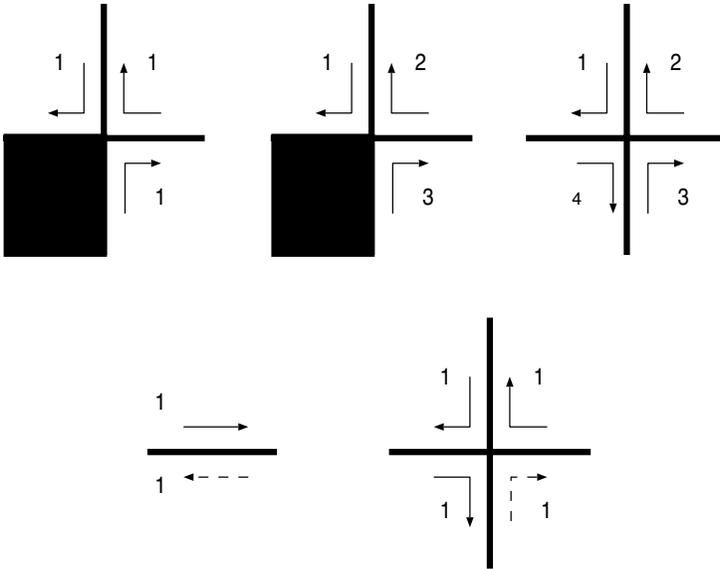


Figure 5. Basis functions and their unknowns at metallic junctions. Solid lines denote open metallic surfaces, closed metallic domains are denoted by black and the unknowns associated to the electric basis functions denoted with dashed lines are removed. Numbers indicate domains. Note that the unknowns can not be combined in the three cases at the top.

For the system equations this rule means that those columns, which are associated with the unknowns to be combined, will added together, and those columns are removed, which are associated with the magnetic basis functions to be removed. Figure 6 illustrates treatment of the electric unknowns at composite metal-dielectric junctions.

4. STEP 3: EFIE-CFIE-PMCHWT FORMULATION

After having removed the extra unknowns from the system equations by enforcing the boundary conditions, the remaining system has more equations than unknowns. In order to get a well-defined system, the number of equations has to be reduced to that of the remaining unknowns. The way how this reduction is done depends on the integral equation formulation. In this paper we apply the EFIE-CFIE-PMCHWT formulation, where EFIE and CFIE are applied on the metallic surfaces depending whether the surface is open or closed, and

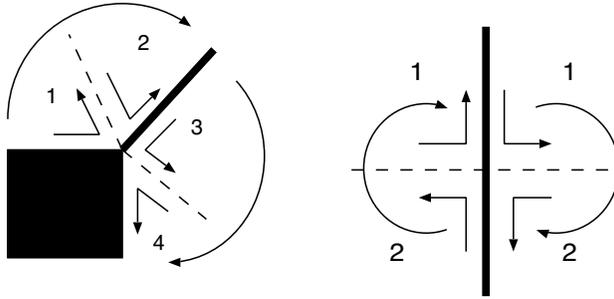


Figure 6. Basis functions and unknowns at composite metal-dielectric junctions. Thick lines denote open metallic surfaces, dashed lines denote dielectric surfaces, closed metallic objects are denoted by black and curved arrows indicate the basis functions with the same unknown coefficient. Numbers indicate domains.

PMCHWT is applied on the dielectric surfaces.

In this section we describe how the EFIE-CFIE-PMCHWT formulation is enforced on the reduced system equations by removing and combining certain equations. This gives a natural generalization of the PMCHWT formulation for both multiple dielectric and composite metal-dielectric junctions.

4.1. PMCHWT at Dielectric Edge or Junction

For a single edge on an interface of two dielectric domains the PMCHWT formulation is a summation of the EFIEs and MFIEs, respectively, defined on the opposite sides of the interface [20]. The PMCHWT formulation is generalized for dielectric junctions as follows. Let next e_n be a dielectric edge or a junction. We sum the adjacent EFIEs and MFIEs, respectively, as follows,

$$\sum_{m=1}^M \text{EFIE}_n^{(m)}, \quad \text{and} \quad \sum_{m=1}^M \text{MFIE}_n^{(m)}, \quad (29)$$

where M is the number of domains meeting at e_n , and $\text{EFIE}_n^{(m)}$ and $\text{MFIE}_n^{(m)}$ are the discretized EFIEs and MFIEs in those domains tested with the oriented RWG functions assigned to e_n . Equation (29) is a direct generalization of the conventional PMCHVVT equations with $M = 2$. It is also the same formulation as e.g., in [15] and [16], though the treatment is different in those papers. For the system equations, (29) means that the system equations $\text{EFIE}_n^{(m)}$ and $\text{MFIE}_n^{(m)}$ are added

together, respectively. This, in turn, corresponds to combining the rows of the matrix equation.

4.2. EFIE and CFIE at Metallic Edges and Junctions

First let e_n be an open metallic edge or a junction of open metallic surfaces. By *Rule 2* we have already removed all unknowns associated with the magnetic basis functions assigned to e_n . Since we apply EFIE on open metallic surfaces, now we also remove all MFIEs and nMFIEs, which are tested with the basis functions assigned to e_n , from the system equations.

If e_n is completely inside one dielectric domain, by *Rule 2* we have also removed one of the unknowns associated with the electric basis functions assigned to e_n . Thus, now we, in addition, remove that EFIE from the system equations which was tested with the corresponding electric basis function. Thereafter, if e_n is an edge or a junction where M open metallic surfaces meet inside one dielectric domain, there remain $M - 1$ electric equations for $M - 1$ (combined) electric unknowns associated with e_n .

Next let e_n be a single edge on a closed metallic surface or at a junction of a closed metallic surface and at least one open metallic surface. Again by *Rule 2* we remove all the unknowns associated with the magnetic basis function assigned to e_n . The CFIE formulation [21] at e_n means that the EFIE and nMFIE tested by the remaining electric basis function and denoted by $\text{EFIE}_n^{(l)}$ and $\text{nMFIE}_n^{(l)}$, respectively, are combined as

$$\alpha \text{EFIE}_n^{(l)} + (1 - \alpha) \eta_l \text{nMFIE}_n^{(l)}, \quad (30)$$

where $\eta_l = \sqrt{\mu_l/\varepsilon_l}$ and α is a coupling coefficient, which we set $\alpha = 1/2$. Accordingly, there remain one (combined) system equation and one (electric) unknown at e_n and we have also removed the MFIE, which is tested with the removed magnetic basis function, from the system equations.

4.3. EFIE and CFIE on General Metal-Dielectric Junction

Finally we consider the formulation at a general metal-dielectric junction e_n . Also here by *Rule 2* we have already removed the magnetic unknowns associated with the magnetic basis functions assigned to e_n . Accordingly, we remove also all MFIEs tested by those magnetic basis functions.

If e_n does not meet any closed metallic surfaces, then the EFIEs assigned to e_n are combined between any two metallic surfaces by

summing the corresponding EFIEs together

$$\sum_{m=1}^M \text{EFIE}_n^{(m)}, \quad (31)$$

where M is the number of dielectric domains between the two metallic surfaces. In that case we remove also all nMFIEs tested by those electric basis functions associated to e_n .

If e_n lies on a closed metallic surface, then the EFIEs and nMFIEs assigned to e_n are combined between any two metallic surfaces by summing the corresponding CFIEs as

$$\sum_{m=1}^M \text{CFIE}_n^{(m)}, \quad (32)$$

where M is again the number of dielectric domains between the two metallic surfaces. Due to *Rule 3* we in both above cases arrive at the same reduced number of equations and unknowns at e_n .

5. NUMERICAL VALIDATION

In this section the developed method is verified by considering numerical examples. We consider scattering by inhomogeneous dielectric and composite objects and analysis of microstrip and dielectric resonator antennas. The example cases are illustrated in Figure 7.

First consider an inhomogeneous dielectric sphere made of two hemispheres illuminated with an axially incident plane wave. The relative electric permittivity of the hemisphere at the top, ε_{r1} , is fixed as 4 and the relative electric permittivity of the second hemisphere at the bottom, ε_{r2} , is varied from 4 to 8. The electrical size of the sphere is $k_0 r = 1$, where k_0 is the free space wave number (wave number of the exterior) and r is the radius of the sphere. Figure 8 shows the electric and magnetic surface current densities on the surface of the sphere along a circumferential arc. Along the arc, two current components, J_θ , J_ϕ and M_θ , M_ϕ , normalized with the incident magnetic field and electric field, respectively, are displayed similarly as in [26] and [25]. In the case $\varepsilon_{r1} = \varepsilon_{r2}$, the solution of our method is compared to the Mie series solution and to the numerical solution of a homogeneous sphere and the results show a good agreement.

Then we consider the same inhomogeneous dielectric sphere, but in this case the second hemisphere at the bottom is PEC. Figure 9 shows the equivalent electric and magnetic surface currents

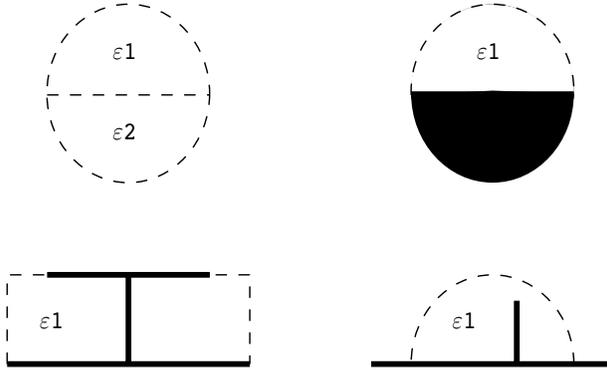


Figure 7. Example geometries from left to right and from top to bottom: An inhomogeneous dielectric sphere made of two homogeneous hemispheres, a composite sphere made of a dielectric hemisphere and metallic hemisphere, a rectangular microstrip antenna and a hemispherical dielectric resonator antenna. The spheres are illuminated with a plane wave from the top (from the side of ε_{r1}) and the antennas are excited with coaxial lines.

on the surface of the sphere similarly as in Figure 8. The currents are calculated with the EFIE-PMCHWT and CFIE-PMCHWT formulations.

Consider next a rectangular microstrip antenna with a finite ground plane and substrate [27]. The size of the patch is 30×20 mm and the size of the ground plane is 50×40 mm, both centered to the origin. The antenna is fed by a coaxial line with inner radius of 0.22 mm and outer radius of 1.4 mm at point $(0, -2.5, 0)$ mm. The relative epsilon of the substrate is 10 and the height of the substrate is 6.35 mm. Figure 10 shows the calculated input impedance of the antenna as a function of the frequency. The same antenna is considered also in [28] with an infinite ground plane and with a lossy (infinite) substrate ($\varepsilon_r = 10.2$, $\tan\delta = 0.001$). Figure 10 shows the results of the present method with a lossy substrate, too. For comparison the results obtained with the multilayered medium approach of [29] and [30] are also presented. In that method, the ground plane and the substrate are modeled as planar infinite layers with an appropriate layered medium Green's function. We repeated the calculations of Figure 10 with a larger 100×80 mm ground plane and substrate, too, using the method of this paper. Figure 10 shows that the results calculated with the large substrate and ground plane agree better with the results obtained with the layered model of [29] and [30] than the results calculated with the

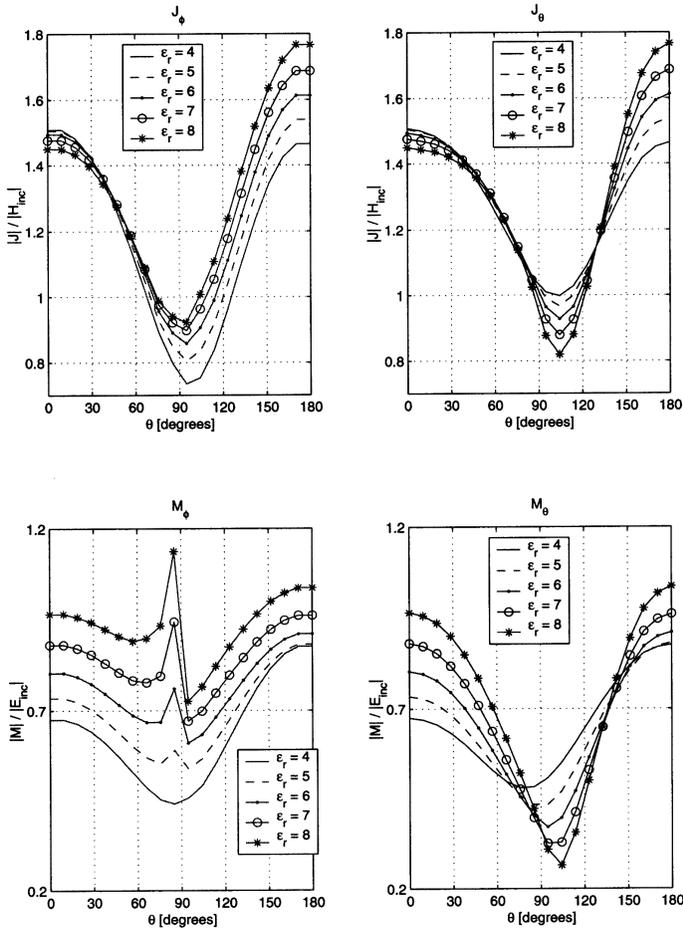


Figure 8. The electric and magnetic current densities on the surface of an inhomogeneous sphere made of two hemisphere with $\epsilon_{r1} = 4$ and $\epsilon_{r2} = 4, 5, 6, 7, 8$, illuminated with an axially incident plane wave.

original dimensions. Thus, we may conclude that the finite size of the substrate and ground plane has a clear effect on the computed input impedances.

Then we consider a hemispherical dielectric resonator antenna (HSDRA) [31,15]. The hemisphere of radius $r = 0.0254\text{m}$ is placed over a finite PEC plane of size $l_1 \times l_2$, where $l_1 = l_2 = c * r$ with a constant multiplier c , both centered to the origin. The relative epsilon of the sphere is 8.9 and $\tan\delta = 0.0038$. The antenna is fed by a 50 Ohm

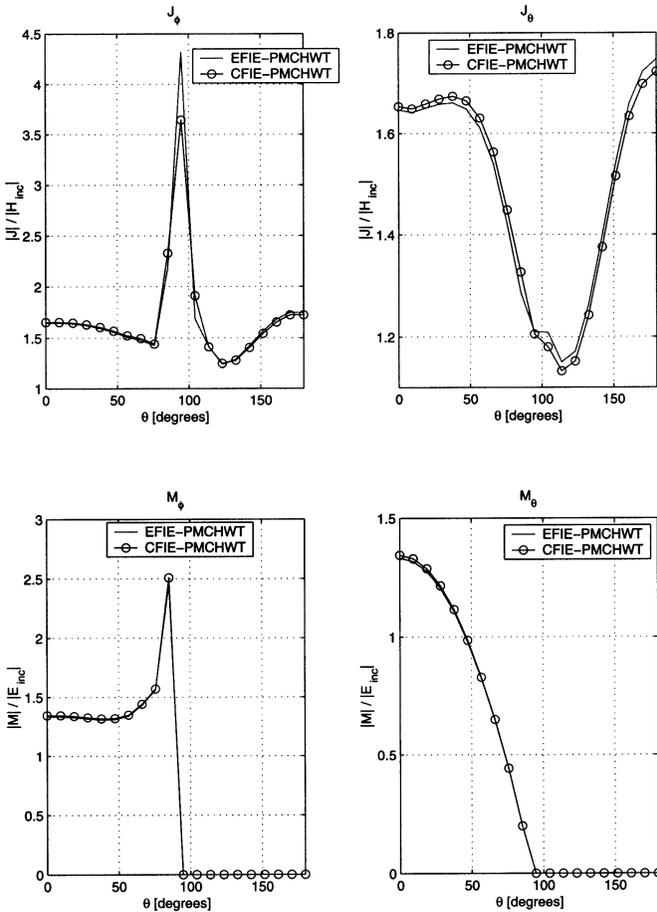


Figure 9. The electric and magnetic current densities on the surface of an inhomogeneous sphere made of two hemisphere with $\varepsilon_{r1} = 4$ and $\sigma_2 = \infty$, illuminated with an axially incident plane wave.

coaxial line of inner radius 0.75 mm placed at point $(0, 0.0174, 0)$ m. Height of the probe is 0.0152 m. Figure 11 shows the input impedance of the antenna as a function of the frequency with $c = 2$ and $c = 3$. The results compare reasonable well with the ones presented in [31] and [15]. The differences may be explained by the fact we apply finite ground plane whereas in [31] and [15] the ground plane is considered by mirroring as an infinite one. Figure 11 shows that the results of the present method approaches the ones of [15] when the size of the ground plane is enlarged.

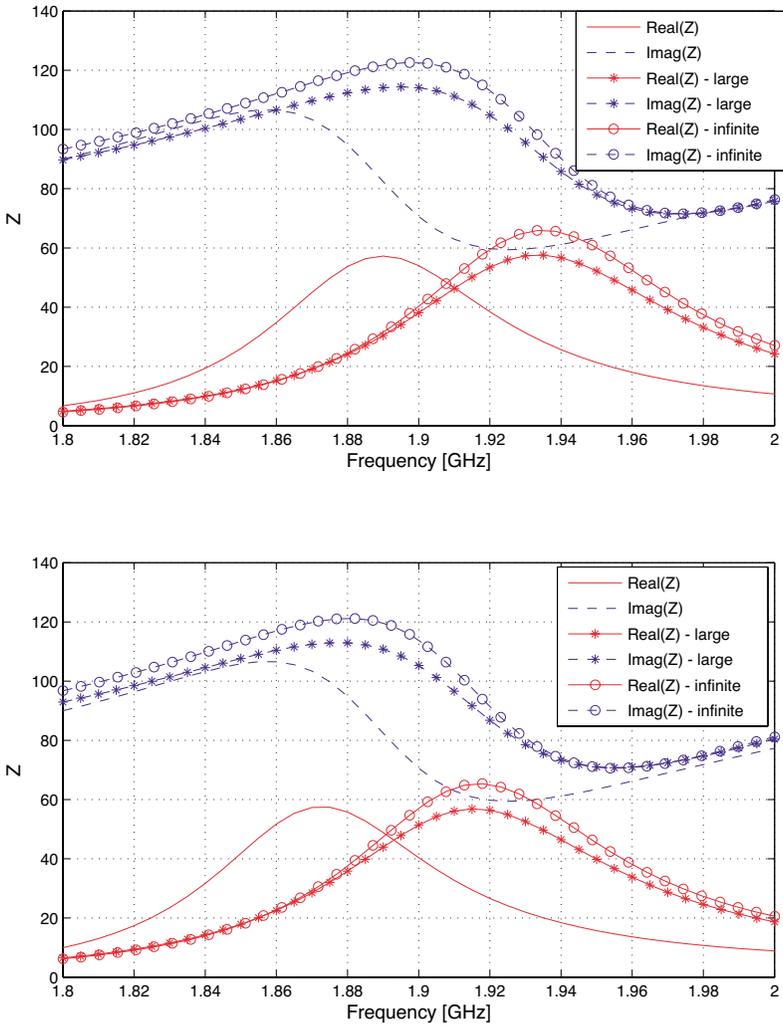


Figure 10. Input impedance of a rectangular microstrip antenna fed by a coaxial line as a function of the frequency. Real part is denoted by a solid line and imaginary part is denoted by a dashed line. At the upper figure the substrate has $\epsilon_r = 10$, $\tan\delta = 0$ and at the lower figure $\epsilon_r = 10.2$, $\tan\delta = 0.001$. The results obtained by the layered model [29, 30], are denoted by circles and the results calculated with a large 100×80 mm ground plane and substrate are denoted by stars.

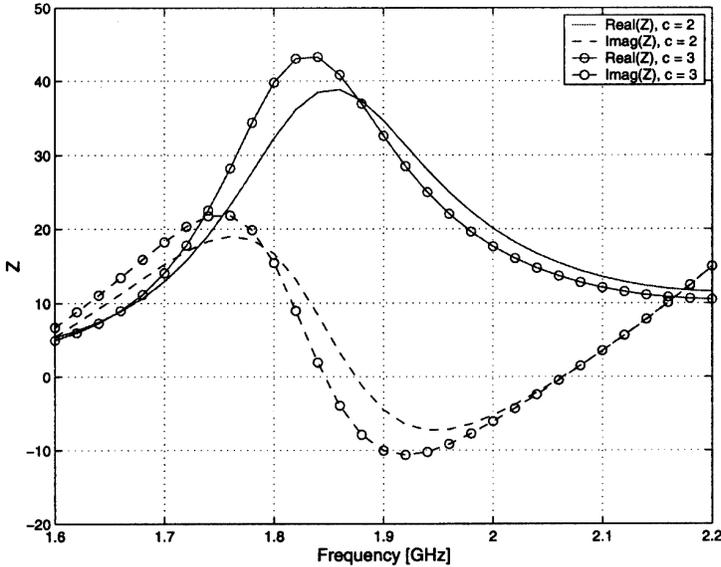


Figure 11. Input impedance of a HSDRA fed by a coaxial line as a function of frequency. Real part of the impedance is denoted by a solid line and imaginary part is denoted by a dashed line. The results obtained with the larger ground plane of size $3r \times 3r$ are denoted by circles.

6. CONCLUSIONS

In this paper electromagnetic analysis of complex three dimensional structures made of composite metallic and homogeneous dielectric regions is considered with the surface integral equation method. The method is based on the EFIE-CFIE-PCMHWI formulation and the Galerkin method with the oriented RWG basis and testing functions. An efficient three steps numerical procedure is presented. As the first step, the discretization and testing of the integral equations are performed individually in subdomains. In the second step, the electromagnetic boundary conditions are enforced by removing zero unknowns and combining linearly dependent unknowns. As the last step, the number of equations is reduced by enforcing the wanted integral equation formulation.

The great benefit of our approach is that it separates the expansion of the surface currents in the basis functions, the testing and the integral equation formulation from the enforcement of the

boundary conditions, thus making the treatment of junctions much more straightforward and easier. In particular, no special junction basis functions or special testing procedures at junctions are needed. Also separating the formulation from the first two steps, makes it much easier to modify the procedure for other integral equation formulations. In practice, the procedure can be condensed into simple bookkeeping rules by which the final system matrix can be directly assembled in an efficient way. The presented approach is validated with numerical examples.

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