Progress In Electromagnetics Research, PIER 54, 221–244, 2005

ELECTROMAGNETIC FIELD FROM A HORIZONTAL ELECTRIC DIPOLE IN THE SPHERICAL ELECTRICALLY EARTH COATED WITH N-LAYERED DIELECTRICS

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Abstract—In this paper, the electromagnetic field in air from a radiating horizontal electric dipole located in the homogeneous isotropic spherical electrically earth coated with N-layered dielectrics is investigated anew. The starting point is based on the formulas for the electromagnetic fields in air from vertical electric and magnetic dipoles situated in air above the surface of the earth coated with N-layered dielectrics. The complete explicit formulas are derived for the electromagnetic fields in the earth due to vertical electric and magnetic dipoles located in air. By using reciprocity theorem, the formulas are readily obtained for the six components of the electromagnetic field in air radiated by a horizontal electric dipole located in the earth.

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1. INTRODUCTION

The problem of the electromagnetic (EM) field from a dipole source located on or near the surface of the spherical earth has been a subject of interest for many years. The early classic harmonic-series solution for a dipole in the presence of a homogeneous sphere was begun by Watson [1]. The subsequent further developments were made by other pioneers [2–11], such as Van del Pol, Fock, Bremmer, Norton, and Wait. Despite the remarkable progress that had already been made on this problem, some aspects of it remained in the dark. The problem was recently reexamined by Houdzoumis [12, 13] and Margetis [14]. The exact formulas are obtained for the complete EM field on the surface of the spherical earth when vertical and horizontal electric dipoles are located on that surface.

When the spherical earth is covered with layered dielectrics, the derivations of the EM field radiated by a dipole source are in general much more complicated. In [15–17], the complete explicit formulas have been derived for the EM field of a dipole source over the spherical conducting or electrically earth coated with a dielectric layer. Furthermore, the general case of the EM field radiated by a dipole source near the surface of the spherical earth coated with N-layered dielectrics has been treated in [18].

In what follows, we will attempt to determine the EM field from a horizontal electric dipole located in the spherical electrically earth. The starting point is based on the formulas for the EM fields in air due to vertical electric and magnetic dipoles situated in air above the surface of the earth coated with N-layered dielectrics addressed in [18]. By using boundary conditions, the formulas of the EM fields in the earth due to vertical electric and magnetic dipoles located in air are derived readily. Based on the above results, using the reciprocity theorem, the formulas are obtained for the six components of the EM field from a horizontal electric dipole located in the earth. To illustrate the applications of the formulas obtained, computations are carried out when the layered dielectrics are composed of successive 2 spherically bounded layers, each with the thickness $l/2$.

2. THE EM FIELD IN THE EARTH DUE TO A VERTICAL ELECTRIC DIPOLE SITUATED IN AIR

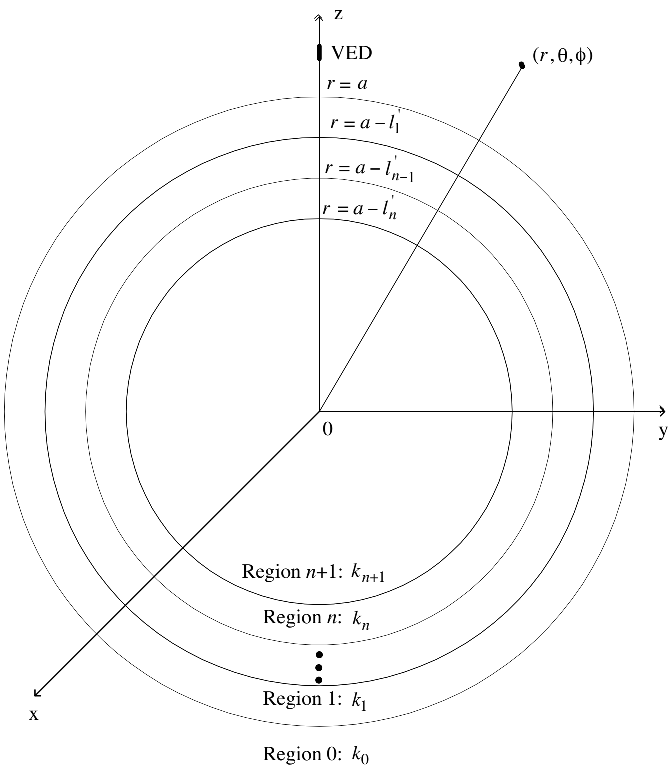

Figure 1. The vertical electric dipole at the height z_s in air above the surface of the spherical earth coated with N-layered dielectrics: $l_j' = l_1 + \cdots + l_j$, $j = 1, 2, ..., n$.

In this section we will start with the formulas for the EM field in air due to a vertical electric dipole over the surface of the spherical earth coated with N-layered dielectrics addressed in [18]. Assuming that dipole is represented by the current density $\widehat{z}Idl\delta(x)\delta(y)\delta(z-b)$, where $b = z_s + a$, and $z_s > 0$ denotes the height of the dipole above the surface of the spherical earth coated with N-layered dielectrics, the relevant geometry is illustrated in Fig. 1. The general case of spherical layering is that the layered dielectrics above the earth are composed of successive N spherically bounded layers, each with arbitrary thickness l_j, $j = 1, 2, ..., n$. The center of the sphere is the origin of a coordinate system (r, θ, ϕ). The air (Region 0, $r \geq a$) is characterized by the permeability μ_0, uniform permittivity ε_0, and conductivity $\sigma_0 = 0$. The N-layered dielectrics with the uniform thickness l consists of N spherical layers, e.g., $l = l_1 + l_2 + \cdots + l_n$. The dielectric in Region j $(a - l_j' \leq r \leq a - l_{j-1}')$ is characterized by the permeability μ_0, relative permittivity ε_{rj}, and conductivity σ_j. The earth (Region $n+1$, $r \leq a - l_n'$) is the dielectric medium characterized by the permeability μ_0, relative permittivity $\varepsilon_{r(n+1)}$, and conductivity σ_{n+1}. With use of a harmonic time factor $e^{-i\omega t}$, the wave numbers of those regions are

$$\begin{aligned} k_0 &= \omega\sqrt{\mu_0\varepsilon_0}, \\ k_j &= \omega\sqrt{\mu_0\left(\varepsilon_0\varepsilon_{rj} + i\sigma_j/\omega\right)}, \quad j = 1, 2, ..., n, \\ k_{n+1} &= \omega\sqrt{\mu_0\left(\varepsilon_0\varepsilon_{r(n+1)} + i\sigma_{n+1}/\omega\right)}. \end{aligned} \tag{1}$$

The complete formulas for the components $E_r^{(0)}$, $E_\theta^{(0)}$, and $H_\phi^{(0)}$ of the EM field in the air due to a vertical electric dipole at $(a + z_s, 0, 0)$, which is defined as the electric-type field, have been obtained in [18]. They are

$$\begin{aligned} \begin{bmatrix} E_r^{(0)} \\ E_\theta^{(0)} \\ H_\phi^{(0)} \end{bmatrix} &= \frac{iIdl\cdot\eta}{\lambda a}\cdot\frac{e^{i(ka\theta+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x} \\ &\times\begin{bmatrix} \sum_s \frac{F_s(z_s)\cdot F_s(z_r)}{t_s - q^2}\cdot e^{it_s x} \\ \sum_s \frac{iF_s(z_s)\cdot\frac{\partial F_s(z)}{k_0\partial z}|_{z=z_r}}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]}\cdot e^{it_s x} \\ \sum_s \frac{F_s(z_s)\cdot F_s(z_r)/\eta_0}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]}\cdot e^{it_s x} \end{bmatrix} \end{aligned} \tag{2}$$

Here, $\theta = d/a$, d is the great-circle distance between the dipole source and the observer, and a is the earth radius. $x = (k_0 a/2)^{1/3}\theta$ is a normalized range parameter, and $t_s (s = 1, 2, ...)$ are the roots of the mode equation.

$$W_2'(t) - qW_2(t) = 0, \tag{3}$$

where

$$q = i\frac{1}{\eta_0}\left(\frac{k_0 a}{2}\right)^{\frac{1}{3}} \cdot Z_0(a). \tag{4}$$

The surface impedance, $Z_0(a)$, at the boundary between Regions 0 and 1 has been obtained in [18]. It is

$$\begin{aligned} Z_0(r)\Big|_{r=a} = & -\frac{\omega\mu_0\gamma_1}{k_1^2} \cdot \tanh\Bigg\{ i\gamma_1 l_1 + \tanh^{-1}\left[\frac{\gamma_2 \cdot k_1^2}{\gamma_1 \cdot k_2^2} \cdot \tanh\Bigg[i\gamma_2 l_2 \right. \\ & + \tanh^{-1}\left[\frac{\gamma_3 \cdot k_2^2}{\gamma_2 \cdot k_3^2} \cdot \tanh\Bigg[i\gamma_3 l_3 + \cdots + \tanh^{-1}\left[\frac{\gamma_n \cdot k_{n-1}^2}{\gamma_{n-1} \cdot k_n^2} \right.\right. \\ & \left.\left.\left. \cdot \tanh\left[i\gamma_n l_n + \tanh^{-1}\left(\frac{k_n \cdot \triangle_g}{\gamma_n}\right)\right]\right] \cdots \right]\Bigg]\Bigg]\Bigg]\Bigg\}. \end{aligned} \tag{5}$$

The "height-gain" function $F_s(z)$ is defined by

$$F_s(z) = \frac{W_2(t_s - y)}{W_2(t_s)}, \tag{6}$$

where

$$y = \left[\frac{2\nu(\nu+1)}{a^3}\right]^{1/3} z \approx \left(\frac{2}{k_0 a}\right)^{1/3} k_0 z. \tag{7}$$

Here, W_2 is the Airy function of the second kind. z_r denotes the height of the observer above the surface of the earth coated with N-layered dielectrics. It is noted that z_s and z_r are somewhat less than the range, d, while the latter is less than the earth radius, a.

It is seen that, in Region j ($j = 1, 2, ..., n$), the non-zero components $E_r^{(j)}$, $E_\theta^{(j)}$, and $H_\phi^{(j)}$ of the EM field can be expressed in terms of the potential function, U_j, which is the solution of the scalar Helmholtz equation

$$(\nabla^2 + k_j^2)U_j = 0, \tag{8}$$

In spherical coordinates, because of symmetry, a solution for U_j is written in the form of

$$U_j = \frac{1}{r}R_j(r)\Phi(\theta). \tag{9}$$

It is well known that the azimuthal field variations can be expressed in terms of the Legendre functions of the first kind $P_\nu(\cos(\pi-\theta))$ and their derivatives [15–18]. The radial field variations are determined by the following differential equation.

$$\frac{d^2R_j(r)}{dr^2}+k_j^2\left[1-\frac{\nu(\nu+1)}{k_j^2r^2}\right]R_j(r)=0. \tag{10}$$

In Region j ($j=1,2,...,n$), $a-l_j^{'}\leq r\leq a-l_{j-1}^{'}$, $l_j^{'}\ll a$, we take $r\approx a$. Taking into account $\nu(\nu+1)\approx k_0^2a^2$, the solution of (10), which has been addressed specifically in [18], can be written as follows:

$$R_j(r)=B_je^{i\gamma_j\cdot[r-(a-l_j^{'})]}+C_je^{-i\gamma_j\cdot[r-(a-l_j^{'})]}. \tag{11}$$

where $\gamma_j=\sqrt{k_j^2-k_0^2}$, $j=1,2,...,n$. The surface impedance, $Z_j(r)$, at any point in Region j for the EM field of electric type can be expressed as follows:

$$\begin{aligned} Z_j(r) &= \frac{E_\theta^{(j)}}{H_\phi^{(j)}} = -\frac{\omega\mu_0\gamma_j}{k_j^2}\cdot\frac{B_je^{i\gamma_j\cdot[r-(a-l_j^{'})]}-C_je^{-i\gamma_j\cdot[r-(a-l_j^{'})]}}{B_ne^{i\gamma_n\cdot[r-(a-l_j^{'})]}+C_je^{-i\gamma_j\cdot[r-(a-l_j^{'})]}} \\ &= -\frac{\omega\mu_0\gamma_j}{k_j^2}\cdot\tanh\left[\ln\left(\frac{B_j}{C_j}\right)^{\frac{1}{2}}+i\gamma_j[r-(a-l_j^{'})]\right]. \end{aligned} \tag{12}$$

Taking into account the impedance boundary condition $Z_1(r)|_{r=a}=Z_0(r)|_{r=a}$ at $r=a$, which is between Regions 0 and 1, it is easily obtained

$$\begin{aligned} \triangle_{1E} = \frac{B_1}{C_1} = \exp&\left\{2\tanh^{-1}\left[\frac{\gamma_2\cdot k_1^2}{\gamma_1\cdot k_2^2}\cdot\tanh\left[i\gamma_2l_2+\tanh^{-1}\right.\right.\right. \\ &\left[\frac{\gamma_3\cdot k_2^2}{\gamma_2\cdot k_3^2}\cdot\tanh\left[i\gamma_3l_3+\cdots+\tanh^{-1}\left[\frac{\gamma_n\cdot k_{n-1}^2}{\gamma_{n-1}\cdot k_n^2}\right.\right.\right. \\ &\left.\left.\left.\left.\left.\left.\times\tanh\left[i\gamma_nl_n+\tanh^{-1}\left(\frac{k_n\cdot\triangle_g}{\gamma_n}\right)\right]\right]\cdots\right]\right]\right]\right]\right\}, \end{aligned} \tag{13}$$

With the boundary condition $Z_2(r)|_{r=a-l_1}=Z_1(r)|_{r=a-l_1}$ at $r=a-l_1$, we get

$$\triangle_{2E}=\frac{B_2}{C_2}=\exp\left\{2\tanh^{-1}\left[\frac{\gamma_3\cdot k_2^2}{\gamma_2\cdot k_3^2}\cdot\tanh\left[i\gamma_3l_3+\tanh^{-1}\left[\frac{\gamma_4\cdot k_3^2}{\gamma_3\cdot k_4^2}\right.\right.\right.\right.$$

$$\times \tanh\left[i\gamma_3 l_3 + \cdots + \tanh^{-1}\left[\frac{\gamma_n \cdot k_{n-1}^2}{\gamma_{n-1} \cdot k_n^2} \cdot \tanh\left[i\gamma_n l_n + \tanh^{-1}\left(\frac{k_n \cdot \triangle_g}{\gamma_n}\right)\right]\right] \cdots \right]\right]\right]\Bigg\}. \quad (14)$$

With the boundary condition at $r = a - l'_j$ ($l'_j = l_1 + l_2 + ... + l_j$), which is between Regions j and $j+1$, we obtain

$$\triangle_{jE} = \frac{B_j}{C_j} = \exp\left\{2\tanh^{-1}\left[\frac{\gamma_{j+1} \cdot k_j^2}{\gamma_j \cdot k_{j+1}^2} \cdot \tanh\left[i\gamma_j l_j + \tanh^{-1}\left[\frac{\gamma_{j+1} \cdot k_j^2}{\gamma_j \cdot k_{j+1}^2} \cdot \tanh\left[i\gamma_{j+1} l_{j+1} + \cdots + \tanh^{-1}\left[\frac{\gamma_n \cdot k_{n-1}^2}{\gamma_{n-1} \cdot k_n^2} \times \tanh\left[i\gamma_n l_n + \tanh^{-1}\left(\frac{k_n \cdot \triangle_g}{\gamma_n}\right)\right]\right] \cdots \right]\right]\right]\right]\right\}. \quad (15)$$

With the boundary condition at $r = a - l'_n$ ($l'_n = l_1 + l_2 + ... + l_n = l$), we find

$$\triangle_{nE} = \frac{B_n}{C_n} = \exp\left[2\tanh^{-1}\left(\frac{k_n \cdot \triangle_g}{\gamma_n}\right)\right]. \quad (16)$$

When the observer lies on the boundary between Regions 0 and 1, $z_r = 0$, $F_s(0) = 1$, $\partial F_s(z_r)/(k_0 \partial z)|_{z_r=0} = -iZ_0/\eta_0$. The complete formulas for the components E_r, E_θ, and H_ϕ of the EM field in Region 1 ($a - l_1 < r < a$) due to a vertical electric dipole at $(a + z_s, 0, 0)$ in Region 0 ($r > a$) can be derived readily.

$$\begin{bmatrix} E_r^{(1)} \\ E_\theta^{(1)} \\ H_\phi^{(1)} \end{bmatrix} = \frac{iIdl \cdot \eta}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \times \begin{bmatrix} \frac{k_0^2}{k_1^2} \cdot \triangle_{1E}^{(1)} \cdot \sum_s \frac{F_s(z_s)}{t_s - q^2} \cdot e^{it_s x} \\ \triangle_{1E}^{(2)} \cdot \sum_s \frac{F_s(z_s) \cdot Z_0(a)/\eta_0}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]} \cdot e^{it_s x} \\ \triangle_{1E}^{(3)} \cdot \sum_s \frac{F_s(z_s)/\eta_0}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]} \cdot e^{it_s x} \end{bmatrix}, \quad (17)$$

where

$$\begin{bmatrix} \triangle_{1E}^{(1)} \\ \triangle_{1E}^{(2)} \\ \triangle_{1E}^{(3)} \end{bmatrix} = \begin{bmatrix} \dfrac{\triangle_{1E} \cdot e^{i\gamma_1[r-(a-l_1)]} + e^{-i\gamma_1[r-(a-l_1)]}}{\triangle_{1E} \cdot e^{i\gamma_1 l_1} + e^{-i\gamma_1 l_1}} \\ \dfrac{\triangle_{1E} \cdot e^{i\gamma_1[r-(a-l_1)]} - e^{-i\gamma_1[r-(a-l_1)]}}{\triangle_{1E} \cdot e^{i\gamma_1 l_1} - e^{-i\gamma_1 l_1}} \\ \dfrac{\triangle_{1E} \cdot e^{i\gamma_1[r-(a-l_1)]} + e^{-i\gamma_1[r-(a-l_1)]}}{\triangle_{1E} \cdot e^{i\gamma_1 l_1} + e^{-i\gamma_1 l_1}} \end{bmatrix}. \tag{18}$$

On applying the boundary conditions at $r = a - l_1$, $r = a - l_2$,..., $r = a - l_n$, we can write the complete formulas for the components $E_r^{(n)}$, $E_\theta^{(n)}$, and $H_\phi^{(n)}$ of the EM field at the boundary $r = a - l$ $(l = l_1 + ... + l_n)$ radiated by a vertical electric dipole at $(a + z_s, 0, 0)$ in air (Region 0, $r > a$).

$$\begin{bmatrix} E_r^{(n)} \\ E_\theta^{(n)} \\ H_\phi^{(n)} \end{bmatrix}_{r=a-l} = \frac{iIdl \cdot \eta}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \times \begin{bmatrix} \dfrac{k_0^2}{k_n^2} \cdot \triangle_{nE}^{(1)} \cdot \displaystyle\sum_s \frac{F_s(z_s)}{t_s - q^2} \cdot e^{it_s x} \\ \triangle_{nE}^{(2)} \cdot \displaystyle\sum_s \frac{F_s(z_s) \cdot Z_0(a)/\eta_0}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]} \cdot e^{it_s x} \\ \triangle_{nE}^{(3)} \cdot \displaystyle\sum_s \frac{F_s(z_s)/\eta_0}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]} \cdot e^{it_s x} \end{bmatrix}, \tag{19}$$

where

$$\begin{bmatrix} \triangle_{nE}^{(1)} \\ \triangle_{nE}^{(2)} \\ \triangle_{nE}^{(3)} \end{bmatrix} = e^{i(\gamma_1 l_1 + \gamma_2 l_2 + ... + \gamma_n l_n)} \cdot \begin{bmatrix} \displaystyle\prod_{j=1}^{n} \frac{\triangle_{jE} + 1}{\triangle_{jE} \cdot e^{i2\gamma_j l_j} + 1} \\ \displaystyle\prod_{j=1}^{n} \frac{\triangle_{jE} - 1}{\triangle_{jE} \cdot e^{i2\gamma_j l_j} - 1} \\ \displaystyle\prod_{j=1}^{n} \frac{\triangle_{jE} + 1}{\triangle_{jE} \cdot e^{i2\gamma_j l_j} + 1} \end{bmatrix}. \tag{20}$$

Consequently, we obtain the following expressions for the EM field of electric type in Region $n+1$.

$$\begin{bmatrix} E_r^{(n+1)} \\ E_\theta^{(n+1)} \\ H_\phi^{(n+1)} \end{bmatrix}_{r=a-l-d_r} = \frac{iIdl\cdot\eta}{\lambda a}\cdot\frac{e^{i(k_0a\theta+k_{n+1}d_r+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x}$$
$$\times\begin{bmatrix} \frac{k_0^2}{k_{n+1}^2}\cdot\triangle_{nE}^{(1)}\cdot\sum_s\frac{F_s(z_s)}{t_s-q^2}\cdot e^{it_sx} \\ \triangle_{nE}^{(2)}\cdot\sum_s\frac{F_s(z_s)\cdot Z_0(a)/\eta_0}{(t_s-q^2)\left[1+\frac{t_s}{2}\left(\frac{2}{k_0a}\right)^{2/3}\right]}\cdot e^{it_sx} \\ \triangle_{nE}^{(3)}\cdot\sum_s\frac{F_s(z_s)/\eta_0}{(t_s-q^2)\left[1+\frac{t_s}{2}\left(\frac{2}{k_0a}\right)^{2/3}\right]}\cdot e^{it_sx} \end{bmatrix}. \tag{21}$$

3. THE EM FIELD IN THE EARTH DUE TO A VERTICAL MAGNETIC DIPOLE SITUATED IN AIR

In this section we will start with the formulas for the EM field in air due to a vertical magnetic dipole over the surface of the spherical earth coated with N-layered dielectrics. When the dipole source at $(a+z_s, 0, 0)$ is replaced by the vertical magnetic dipole with its moment $M = Ida$, and da is the area of the loop, the complete formulas for the components $E_\phi^{(0)}$, $H_r^{(0)}$, and $H_\theta^{(0)}$ of the EM field over the surface of the spherical earth coated with the N-layered dielectrics, which is defined as the magnetic-type field, have been derived in [18]. They are

$$\begin{bmatrix} E_\phi^{(0)} \\ H_r^{(0)} \\ H_\theta^{(0)} \end{bmatrix} = \frac{\omega\mu_0Ida}{\lambda a}\frac{e^{i(k_0a\theta+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x}\begin{bmatrix} \sum_s\frac{G_s(z_s)G_s(z_r)}{t_s-(q^h)^2}e^{it'_sx} \\ \frac{1}{\eta_0}\sum_s\frac{G_s(z_s)G_s(z_r)}{t_s-(q^h)^2}e^{it'_sx} \\ \frac{i}{\eta_0}\sum_s\frac{G_s(z_s)\frac{\partial G_s(z)}{k_0\partial z}|_{z=z_r}}{t_s-(q^h)^2}e^{it'_sx} \end{bmatrix}, \tag{22}$$

where

$$G_s(z) = \frac{W_2(t_s^{'} - y)}{W_2(t_s^{'})}, \tag{23}$$

is a "height-gain" function, and $W_2(t_s^{'})$ is the Airy function of the second kind. Here, $t_s^{'}(s = 1, 2, ...)$ are the roots of the following mode equation

$$W_2^{'}(t^{'}) - q^h W_2(t^{'}) = 0, \tag{24}$$

where

$$q^h = \frac{i\omega\mu_0}{k_0} \cdot \left(\frac{k_0 a}{2}\right)^{\frac{1}{3}} \cdot Y_0(a). \tag{25}$$

The surface admittance, $Y_0(a)$, at $r = a$ is expressed as follows:

$$\begin{aligned} Y_0(r)|_{r=a} \;=\; & -\frac{\gamma_1}{\omega\mu_0} \tanh\left\{ i\gamma_1 l_1 + \tanh^{-1}\left[\frac{\gamma_2}{\gamma_1} \cdot \tanh\left[i\gamma_2 l_2 \right.\right.\right. \\ & \left.\left.\left. + \tanh^{-1}\left[\frac{\gamma_3}{\gamma_2} \cdot \tanh\left[i\gamma_3 l_3 + \cdots + \tanh^{-1}\left[\frac{\gamma_n}{\gamma_{n-1}} \right.\right.\right.\right.\right.\right. \\ & \left.\left.\left.\left.\left.\left. \times \tanh\left[i\gamma_n l_n + \tanh^{-1}\left(-\frac{k_n}{\gamma_n \cdot \triangle_g}\right)\right]\right] \cdots \right]\right]\right]\right] \right\}. \end{aligned} \tag{26}$$

In Region j, the components $E_\phi^{(0)}$, $H_r^{(0)}$, and $H_\theta^{(0)}$ of the EM field can be expressed in terms of a potential function, V_j, which is the solution of the following scalar Helmholtz equation

$$(\nabla^2 + k_j^2)V_j = 0. \tag{27}$$

Similarly, the azimuthal field variations are expressed in terms of the Legendre functions of the first kind $P_\nu(\cos(\pi-\theta))$ and their derivatives, and the radial field variations are written as

$$R_j^{'}(r) = B_j^{'} e^{i\gamma_j \cdot [r-(a-l_j^{'})]} + C_j^{'} e^{-i\gamma_j \cdot [r-(a-l_j^{'})]}. \tag{28}$$

Then, the quantity $Y_j(r)$ of the surface admittance in Region j for the EM field of magnetic type can be arrived at the following expressions.

$$\begin{aligned} Y_j(r) \;=\; \frac{H_\theta^{(j)}}{E_\phi^{(j)}} \;&=\; -\frac{\gamma_j}{\omega\mu_0} \cdot \frac{B_j' e^{i\gamma_j [r-(a-l_j^{'})]} - C_n' e^{-i\gamma_j [r-(a-l_j^{'})]}}{B_j' e^{i\gamma_j [r-(a-l_j^{'})]} + C_j' e^{-i\gamma_j [r-(a-l_j^{'})]}} \\ &=\; \tanh\left\{ \ln\left(\frac{B_j'}{C_j'}\right)^{1/2} + i\gamma_j [r - (a - l_j')] \right\}. \end{aligned} \tag{29}$$

Taking into account the impedance boundary condition $Y_1(r)|_{r=a} = Y_0(r)|_{r=a}$ at $r = a$, it is easily obtained.

$$\begin{aligned}\triangle_{1M} = \frac{B_1'}{C_1'} &= \exp\left\{2\tanh^{-1}\left[\frac{\gamma_2}{\gamma_1}\cdot\tanh\left[i\gamma_2 l_2+\tanh^{-1}\left[\frac{\gamma_3}{\gamma_2}\right.\right.\right.\right.\\ &\quad\times\tanh\left[i\gamma_3 l_3+\cdots+\tanh^{-1}\left[\frac{\gamma_n}{\gamma_{n-1}}\cdot\tanh\left[i\gamma_n l_n\right.\right.\right.\\ &\quad\left.\left.\left.\left.\left.\left.\left.+\tanh^{-1}\left(-\frac{k_n}{\gamma_n\cdot\triangle_g}\right)\right]\right]\cdots\right]\right]\right]\right]\right\}, \end{aligned} \tag{30}$$

With the boundary condition $Y_2(r)|_{r=a-l'} = Y_1(r)|_{r=a-l'}$ at $r = a - l_1'$, we get

$$\begin{aligned}\triangle_{2M} = \frac{B_2'}{C_2'} &= \exp\left\{2\tanh^{-1}\left[\frac{\gamma_3}{\gamma_2}\cdot\tanh\left[i\gamma_3 l_3+\tanh^{-1}\left[\frac{\gamma_4}{\gamma_3}\right.\right.\right.\right.\\ &\quad\times\tanh\left[i\gamma_3 l_3+\cdots+\tanh^{-1}\left[\frac{\gamma_n}{\gamma_{n-1}}\cdot\tanh\left[i\gamma_n l_n\right.\right.\right.\\ &\quad\left.\left.\left.\left.\left.\left.\left.+\tanh^{-1}\left(-\frac{k_n}{\gamma_n\cdot\triangle_g}\right)\right]\right]\cdots\right]\right]\right]\right]\right\}, \end{aligned} \tag{31}$$

With the boundary condition at $r = a - l_j'$ ($l_j' = l_1 + l_2 + ... + l_j$), we obtain

$$\begin{aligned}\triangle_{jM} = \frac{B_j'}{C_j'} &= \exp\left\{2\tanh^{-1}\left[\frac{\gamma_{j+1}}{\gamma_j}\cdot\tanh\left[i\gamma_j l_j+\tanh^{-1}\left[\frac{\gamma_{j+1}}{\gamma_j}\right.\right.\right.\right.\\ &\quad\times\tanh\left[i\gamma_{j+1} l_{j+1}+\cdots+\tanh^{-1}\left[\frac{\gamma_n}{\gamma_{n-1}}\cdot\tanh\left[i\gamma_n l_n\right.\right.\right.\\ &\quad\left.\left.\left.\left.\left.\left.\left.+\tanh^{-1}\left(-\frac{k_n}{\gamma_n\cdot\triangle_g}\right)\right]\right]\cdots\right]\right]\right]\right]\right\}, \end{aligned} \tag{32}$$

With the boundary condition at $r = a - l$, ($l = l_1 + l_2 + ... + l_n$), we find

$$\triangle_{nM} = \frac{B_n'}{C_n'} = \exp\left[2\tanh^{-1}\left(-\frac{k_n}{\gamma_n\cdot\triangle_g}\right)\right]. \tag{33}$$

When the observer lies on the boundary between Regions 0 and 1, $z_r = 0$, $G_s(0) = 1$, $\partial G_s(z_r)/(k_0\partial z)|_{z_r=0} = -i\eta_0 Y_0(a)$, the complete formulas for the components $E_\phi^{(1)}$, $H_r^{(1)}$, and $H_\theta^{(1)}$ of the EM field in Region 1 ($a - l_1 < r < a$) due to a vertical magnetic dipole at $(a + z_s, 0, 0)$ in Region 0 ($r > a$) are obtained as follows:

$$\begin{bmatrix} E_\phi^{(1)} \\ H_r^{(1)} \\ H_\theta^{(1)} \end{bmatrix} = \frac{i\omega\mu_0 Ida}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \times \begin{bmatrix} \triangle_{1M}^{(1)} \cdot \sum_s \frac{G_s(z_s)}{t'_s - (q^h)^2} \cdot e^{it'_s x} \\ \triangle_{1M}^{(2)} \cdot \sum_s \frac{G_s(z_s)/\eta_0}{t'_s - (q^h)^2} \cdot e^{it'_s x} \\ \triangle_{1M}^{(3)} \cdot \sum_s \frac{G_s(z_s) \cdot Y_0(a)/\eta_0}{t'_s - (q^h)^2} \cdot e^{it'_s x} \end{bmatrix}, \quad (34)$$

where

$$\begin{bmatrix} \triangle_{1M}^{(1)} \\ \triangle_{1M}^{(2)} \\ \triangle_{1M}^{(3)} \end{bmatrix} = \begin{bmatrix} \frac{\triangle_{1M} \cdot e^{i\gamma_1[r-(a-l_1)]} + e^{-i\gamma_1[r-(a-l_1)]}}{\triangle_{1M} \cdot e^{i\gamma_1 l_1} + e^{-i\gamma_1 l_1}} \\ \frac{\triangle_{1M} \cdot e^{i\gamma_1[r-(a-l_1)]} + e^{-i\gamma_1[r-(a-l_1)]}}{\triangle_{1E} \cdot e^{i\gamma_1 l_1} + e^{-i\gamma_1 l_1}} \\ \frac{\triangle_{1M} \cdot e^{i\gamma_1[r-(a-l_1)]} - e^{-i\gamma_1[r-(a-l_1)]}}{\triangle_{1M} \cdot e^{i\gamma_1 l_1} - e^{-i\gamma_1 l_1}} \end{bmatrix}. \quad (35)$$

With the boundary conditions at $r = a - l'_1$, $r = a - l'_2$, ..., $r = a - l'_n$, we arrive at the following formulas for the components $E_r^{(n)}$, $E_\theta^{(n)}$, and $H_\phi^{(n)}$ of the EM field at the $r = a - l$ ($l = l_1 + ... + l_n$).

$$\begin{bmatrix} E_\phi^{(n)} \\ H_r^{(n)} \\ H_\theta^{(n)} \end{bmatrix}_{r=a-l} = \frac{\omega\mu_0 Ida}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \times \begin{bmatrix} \triangle_{nM}^{(1)} \cdot \sum_s \frac{G_s(z_s)}{t'_s - (q^h)^2} \cdot e^{it'_s x} \\ \triangle_{nM}^{(2)} \cdot \sum_s \frac{G_s(z_s)}{t'_s - (q^h)^2} \cdot e^{it'_s x} \\ \triangle_{nM}^{(3)} \cdot \sum_s \frac{G_s(z_s) Y_0(a)/\eta_0}{t'_s - (q^h)^2} \cdot e^{it'_s x} \end{bmatrix}, \quad (36)$$

where

$$\begin{bmatrix} \triangle_{nM}^{(1)} \\ \triangle_{nM}^{(2)} \\ \triangle_{nM}^{(3)} \end{bmatrix} = e^{i(\gamma_1 l_1+\gamma_2 l_2+\cdots+\gamma_n l_n)} \cdot \begin{bmatrix} \prod_{j=1}^{n} \frac{\triangle_{jM}+1}{\triangle_{jM}\cdot e^{i2\gamma_j l_j}+1} \\ \prod_{j=1}^{n} \frac{\triangle_{jM}+1}{\triangle_{jM}\cdot e^{i2\gamma_j l_j}+1} \\ \prod_{j=1}^{n} \frac{\triangle_{jM}-1}{\triangle_{jM}\cdot e^{i2\gamma_j l_j}-1} \end{bmatrix}. \quad (37)$$

Then, it follows the complete formulas for the EM field in Region $n+1$.

$$\begin{bmatrix} E_{\phi}^{(n+1)} \\ H_{r}^{(n+1)} \\ H_{\theta}^{(n+1)} \end{bmatrix}_{r=a-l-d_r} = \frac{\omega\mu_0 Ida}{\lambda a} \cdot \frac{e^{i(k_0 a\theta+k_{n+1}d_r+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}} \cdot \sqrt{\pi x} \times \begin{bmatrix} \triangle_{nM}^{(1)} \cdot \sum_s \frac{G_s(z_s)}{t_s-(q^h)^2} \cdot e^{it'_s x} \\ \triangle_{nM}^{(2)} \cdot \sum_s \frac{G_s(z_s)/\eta_0}{t'_s-(q^h)^2} \cdot e^{it_s x} \\ \triangle_{nM}^{(3)} \cdot \sum_s \frac{G_s(z_s)Y_0(a)/\eta_0}{t_s-(q^h)^2} \cdot e^{it'_s x} \end{bmatrix}. \quad (38)$$

4. THE EM FIELD IN AIR OF A HORIZONTAL ELECTRIC DIPOLE LOCATED IN THE EARTH

In this section, we attempt to formulate the formulas of the EM field in air due to a horizontal electric dipole, Ids he Ids^{he}, which is located at the depth d_s of the earth, as illustrated in Fig. 2. From the above expressions of the EM fields in the earth (Region $n+1$) from vertical electric and magnetic dipoles in air (Region 0), by using reciprocity theorem, it is fairly straightforward to arrive the expressions of the vertical electric field $E_r^{he(0)}$ and the vertical magnetic field $H_r^{he(0)}$ in air when a horizontal electric dipole is located in the earth.

$$E_r^{he(0)} = -\frac{i\eta Ids^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta+k_{n+1}d_s+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}} \cdot \sqrt{\pi x} \cdot \cos\phi$$

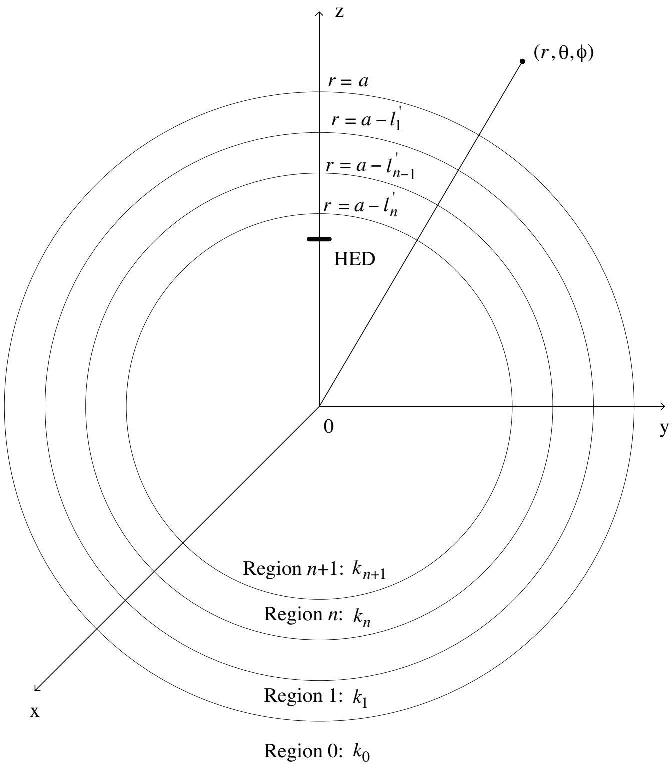

Figure 2. The horizontal electric dipole at the depth d_s in the spherical earth coated with N-layered dielectrics: $l_j' = l_1 + \cdots + l_j$, $j = 1, 2, ..., n$.

$$\times \sum_s \frac{\triangle_{nE}^{(2)} \cdot Z_0(a)/\eta_0 \cdot F_s(z_r)}{(t_s - q^2)\left[1 + \frac{t_s}{2}\left(\frac{2}{k_0 a}\right)^{2/3}\right]} \cdot e^{it_s x}, \tag{39}$$

$$H_r^{he(0)} = -\frac{iIds^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + k_{n+1} d_s + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \sin\phi$$

$$\times \sum_s \frac{\triangle_{nM}^{(1)} \cdot G_s(z_r)}{t_s' - (q^h)^2} \cdot e^{it_s' x}. \tag{40}$$

Here, ϕ is the orientation angle between the dipole source and the observer, and the horizontal dipole is located at $(a - l - d_s, 0, 0)$. In spherical coordinates, from Maxwell's equations, the rest four components $E_\theta^{he(0)}$, $E_\phi^{he(0)}$, $H_\theta^{he(0)}$, and $H_\phi^{he(0)}$ can be expressed in terms of $E_r^{he(0)}$ and $H_r^{he(0)}$.

$$\left(\frac{\partial^2}{\partial r^2}+k^2\right)(rH_\phi^{he(0)}) = i\omega\epsilon_0\frac{\partial E_r^{he(0)}}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial^2 H_r^{he(0)}}{\partial r\partial\phi}, \quad (41)$$

$$\left(\frac{\partial^2}{\partial r^2}+k^2\right)(rE_\theta^{he(0)}) = \frac{\partial^2 E_r^{he(0)}}{\partial r\partial\theta}+\frac{i\omega\mu_0}{\sin\theta}\frac{\partial H_r^{he(0)}}{\partial\phi}, \quad (42)$$

$$\left(\frac{\partial^2}{\partial r^2}+k^2\right)(rH_\theta^{he(0)}) = -\frac{i\omega\epsilon_0}{\sin\theta}\frac{\partial E_r^{he(0)}}{\partial\phi}+\frac{\partial^2 H_r^{he(0)}}{\partial r\partial\theta}, \quad (43)$$

$$\left(\frac{\partial^2}{\partial r^2}+k^2\right)(rE_\phi^{he(0)}) = \frac{1}{\sin\theta}\frac{\partial^2 E_r^{he(0)}}{\partial r\partial\phi}-i\omega\mu_0\frac{\partial H_r^{he(0)}}{\partial\theta}. \quad (44)$$

From (41), we get

$$\frac{\nu(\nu+1)}{r}(H_\phi^{he(0)}) = i\omega\epsilon_0\frac{\partial E_r^{he(0)}}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial^2 H_r^{he(0)}}{\partial r\partial\phi}. \quad (45)$$

Taking into account the relation $\nu(\nu+1)/r \approx k_0^2 a$, as a result of some straightforward operations we obtain the following expression for the component $H_\phi^{he(0)}$.

$$\begin{aligned} H_\phi^{he(0)} = & -\frac{iIds^{he}}{\lambda a}\cdot\frac{e^{i(k_0a\theta+k_{n+1}d_s+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x}\cdot\cos\phi \\ & \times\left[\sum_s\frac{\triangle_{nE}^{(2)}\cdot Z_0(a)/\eta_0\cdot F_s(z_r)}{t_s-q^2}\cdot e^{it_sx}+\frac{1}{k_0a\sin\theta}\right. \\ & \left.\times\sum_s\frac{\triangle_{nM}^{(1)}\cdot\frac{\partial G_s(z)}{k_0\partial z}\mid_{z=z_r}}{t_s'-(q^h)^2}\cdot e^{it_s'x}\right]. \end{aligned} \quad (46)$$

Similarly, we obtain

$$\begin{aligned} H_\theta^{he(0)} = & \frac{Ids^{he}}{\lambda a}\cdot\frac{e^{i(k_0a\theta+k_{n+1}d_s+\frac{\pi}{4})}}{\sqrt{\theta\sin\theta}}\cdot\sqrt{\pi x}\cdot\sin\phi \\ & \times\left[\frac{1}{k_0a\sin\theta}\sum_s\frac{\triangle_{nE}^{(2)}\cdot Z_0(a)/\eta_0\cdot F_s(z_r)}{(t_s-q^2)\left[1+\frac{t_s}{2}\left(\frac{2}{k_0a}\right)^{2/3}\right]}\cdot e^{it_sx}\right. \end{aligned}$$

$$- \sum_s \frac{\triangle_{nM}^{(1)} \cdot \frac{\partial G_s(z)}{k_0 \partial z} |_{z=z_r}}{t_s' - (q^h)^2} \cdot e^{it_s' x} \Bigg], \tag{47}$$

$$\begin{aligned} E_\theta^{he(0)} &= \frac{\eta I ds^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + k_{n+1} d_s + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \cos\phi \\ &\quad \times \Bigg[- \sum_s \frac{\triangle_{nE}^{(2)} \cdot Z_0(a)/\eta_0 \cdot \frac{\partial F_s(z)}{k_0 \partial z} |_{z=z_r}}{t_s - q^2} \cdot e^{it_s x} \\ &\quad + \frac{1}{k_0 a \sin\theta} \cdot \sum_s \frac{\triangle_{nM}^{(1)} \cdot G_s(z_r)}{t_s' - (q^h)^2} \cdot e^{it_s' x} \Bigg], \end{aligned} \tag{48}$$

$$\begin{aligned} E_\phi^{he(0)} &= \frac{i\eta I ds^{he}}{\lambda a} \cdot \frac{e^{i(k_0 a\theta + k_{n+1} d_s + \frac{\pi}{4})}}{\sqrt{\theta \sin\theta}} \cdot \sqrt{\pi x} \cdot \sin\phi \\ &\quad \times \Bigg[\frac{i}{k_0 a \sin\theta} \cdot \sum_s \frac{\triangle_{nE}^{(2)} \cdot Z_0(a)/\eta_0 \cdot \frac{\partial F_s(z)}{k_0 \partial z} |_{z=z_r}}{(t_s - q^2) \left[1 + \frac{t_s}{2} \left(\frac{2}{k_0 a} \right)^{2/3} \right]} \cdot e^{it_s x} \\ &\quad + \sum_s \frac{\triangle_{nM}^{(1)} \cdot G_s(z_r)}{t_s' - (q^h)^2} \cdot e^{it_s' x} \Bigg], \end{aligned} \tag{49}$$

where the superscript he designates that the EM field is radiated by a horizontal electric dipole. From the results obtained, it is seen that the EM field consists of the electric-type and magnetic-type terms when a horizontal electric dipole is located in the earth coated with N-layered dielectrics.

5. COMPUTATIONS AND ANALYSIS

To illustrate the complete formulas for the EM field in N-layered spherical regions from a dipole source, the computations are carried out when the layered dielectrics are composed of a succession of 2 spherically bounded layers, each with the thickness $l/2$. Assuming that $l = 100\,\text{m}$, the radius of the earth is $a = 6370\,\text{km}$, Region 1 $(a - l/2 < r < a)$ is characterized by the relative permittivity $\varepsilon_{r1} = 10$ and conductivity $\sigma_1 = 10^{-5}\,\text{S/m}$, Region 2 $(a - l < r < a - l/2)$ is characterized by the relative permittivity $\varepsilon_{r2} = 20$ and conductivity $\sigma_2 = 2 \times 10^{-5}$ S/m, and the earth (Region 3, $r < a - l$) is characterized by the relative permittivity $\varepsilon_{r3} = 100$ and conductivity $\sigma_3 = 4$ S/m, a vertical electric dipole is placed on the surface of the spherical earth coated with layered dielectrics, $z_s = 0$ m, and the observer is located at the depth $d_r = 1$ m below the layered dielectrics, the electric field $|E_r|$ in V/m and the magnetic field $|H_\phi|$ in A/m at $f = 100$ kHz, and $f = 200$ kHz are computed and plotted in Figs. 3 and 4, respectively. It is seen that the interference occurs for the EM field of electric type when the thickness of the layered dielectrics is larger than a certain value. Assuming that all parameters are same with those in Figs. 3 and 4, the electric field $|E_\phi|$ in V/m and the magnetic field $|H_r|$ in A/m due to unit vertical magnetic dipole at $f = 100$ kHz and $f = 200$ kHz are computed and plotted in Figs. 5 and 6, respectively. Obviously, the interference does not occur for the EM field of magnetic type.

Assuming that the parameters of all regions are same with those in Figs. 3 and 4, and the height of the observer and the depth of the dipole source are $z_r = d_s = 1$ m, the electric field $|E_r|$ in V/m and the

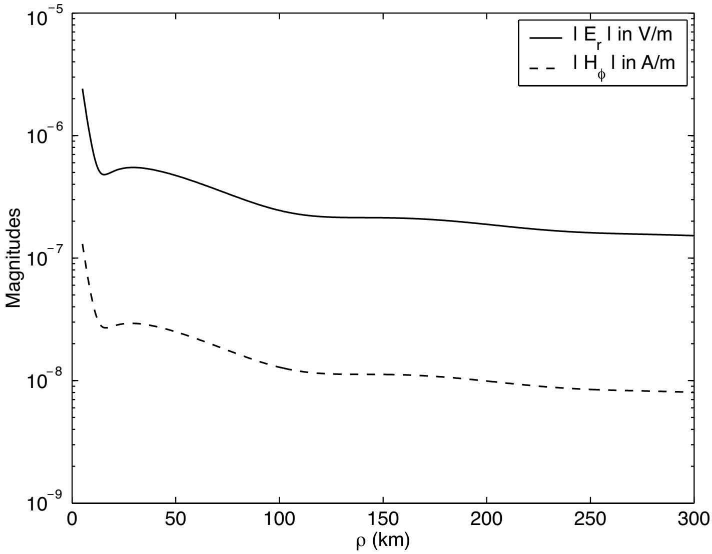

Figure 3. The electric field $|E_r|$ in V/m and the magnetic field $|H_\phi|$ in A/m at $f = 100$ kHz in the spherical earth due to unit vertical electric dipole versus the propagating distance ρ; $l = 100$ m, $a = 6370$ km, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}$ S/m, $\sigma_2 = 2 \times 10^{-5}$ S/m, $\sigma_3 = 4$ S/m, $d_r = 1$ m, and $z_s = 0$ m.

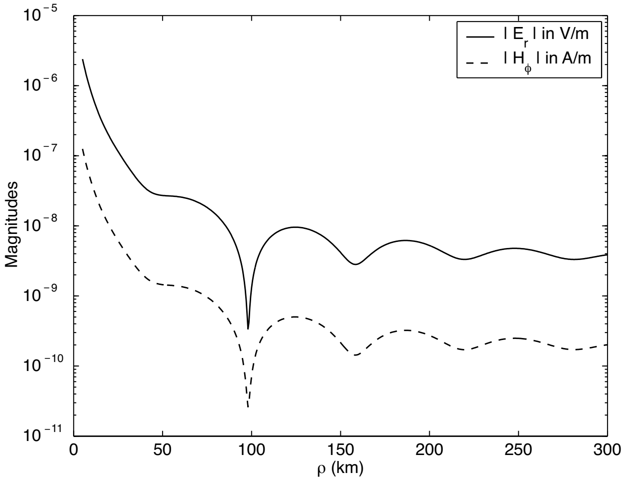

Figure 4. The electric field $|E_r|$ in V/m and the magnetic field $|H_\phi|$ in A/m at $f = 200\,\text{kHz}$ in the spherical earth due to unit vertical electric dipole versus the propagating distance ρ; $l = 100\,\text{m}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\text{S/m}$, $\sigma_3 = 4\,\text{S/m}$, $d_r = 1\,\text{m}$, and $z_s = 0\,\text{m}$.

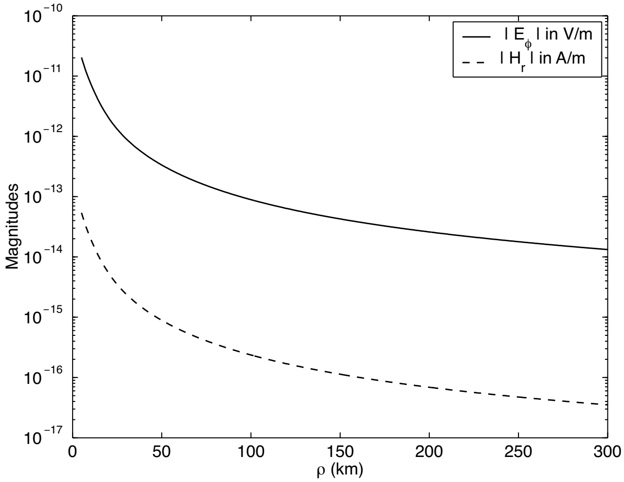

Figure 5. The electric field $|E_\phi|$ in V/m and the magnetic field $|H_r|$ in A/m in the spherical earth at $f = 100\,\text{kHz}$ due to unit vertical magnetic dipole versus the propagating distance ρ; $l = 100\,\text{m}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\text{S/m}$, $\sigma_3 = 4\,\text{S/m}$, $d_r = 1\,\text{m}$, and $z_s = 0\,\text{m}$.

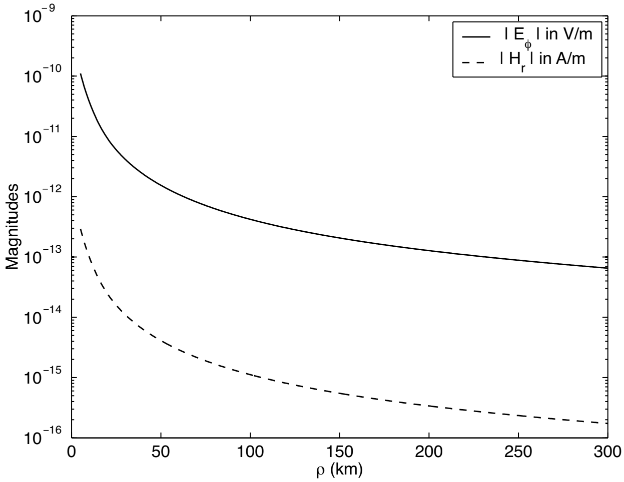

Figure 6. The electric field $|E_\phi|$ in V/m and the magnetic field $|H_r|$ in A/m in the earth at $f = 200\,\text{kHz}$ due to unit vertical magnetic dipole versus the propagating distance ρ; $l = 100\,\text{m}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\text{S/m}$, $\sigma_3 = 4\,\text{S/m}$, $d_r = 1\,\text{m}$, and $z_s = 0\,\text{m}$.

magnetic fields $|H_\phi|$ in A/m due to unit horizontal electric dipole at $f = 100\,\text{kHz}$ and $f = 200\,\text{kHz}$ are calculated and plotted in Figs. 7 and 8, respectively. In order to demonstrate the influence of the thickness of the layered dielectrics, the electric fields $|E_r|$ in V/m and the magnetic fields $|H_\phi|$ in A/m at $l = 60\,\text{m}$, $l = 100\,\text{m}$, and $l = 140\,\text{m}$ are shown in Figs. 9 and 10, respectively.

6. CONCLUSIONS

From the above derivations and computations, it is seen that the interference occurs for the EM field radiated by horizontal or vertical electric dipoles when the thickness of the layered dielectrics is larger than a certain value and the interference does not occur for the EM field radiated by vertical magnetic dipole. It has been demonstrated that the excitation efficiency of horizontal or vertical electric dipoles is much higher than that of a vertical magnetic dipole. The formulas and computations for the EM field radiated by electrically horizontal antenna located in the earth or vertical antenna in air have useful applications in the communication to the observer in the air or in the asphalt- and cement-coated spherical earth at low frequencies.

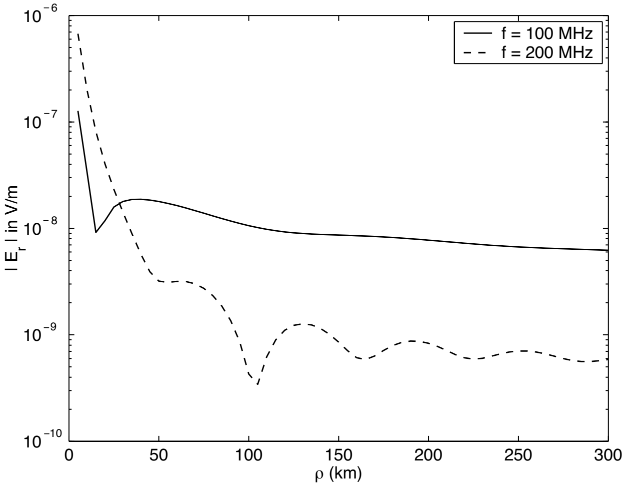

Figure 7. The electric field $|E_r|$ in V/m in the air at $f = 100\,\text{kHz}$ and $f = 200\,\text{kHz}$ due to unit horizontal electric dipole in the earth versus the propagating distance ρ; $l = 100\,\text{m}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\text{S/m}$, $\sigma_3 = 4\,\text{S/m}$, and $d_s = z_r = 1\,\text{m}$, and $\phi = 0$.

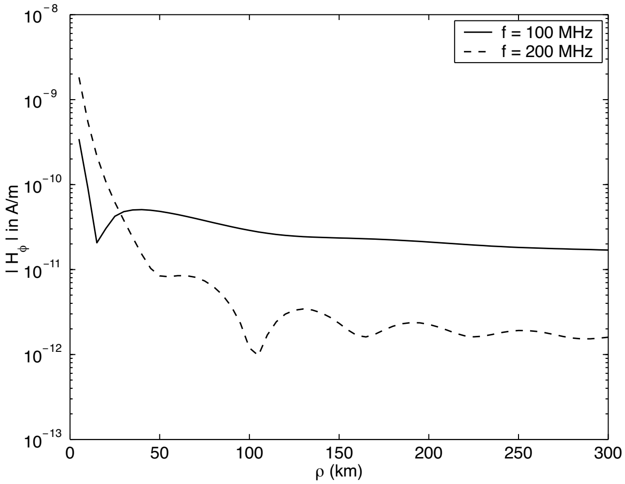

Figure 8. The magnetic field $|H_\phi|$ in A/m at $f = 100\,\text{kHz}$ and $f = 200\,\text{kHz}$ due to unit horizontal electric dipole versus the propagating distance ρ; $l = 100\,\text{m}$, $a = 6370\,\text{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\text{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\text{S/m}$, $\sigma_3 = 4\,\text{S/m}$, and $d_s = z_r = 1\,\text{m}$, and $\phi = 0$.

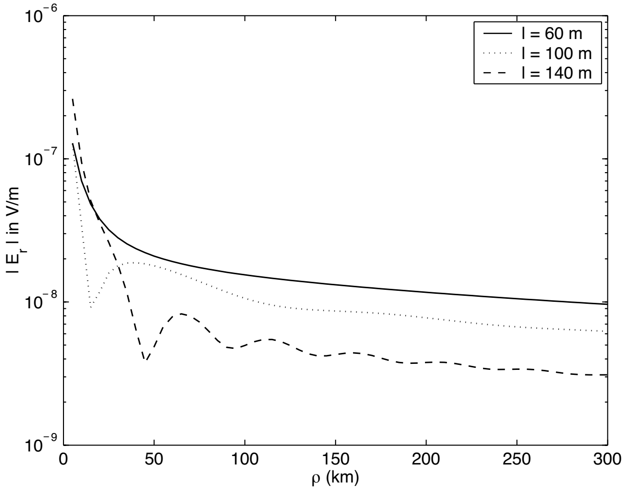

Figure 9. The electric field $|E_r|$ in V/m at $l = 60\,\mathrm{m}$, $l = 100\,\mathrm{m}$, and $l = 140$ m due to unit horizontal electric dipole versus the propagating distance ρ; $f = 100\,\mathrm{kHz}$, $a = 6370\,\mathrm{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\mathrm{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\mathrm{S/m}$, $\sigma_3 = 4\,\mathrm{S/m}$, and $d_s = z_r = 1\,\mathrm{m}$, and $\phi = 0$.

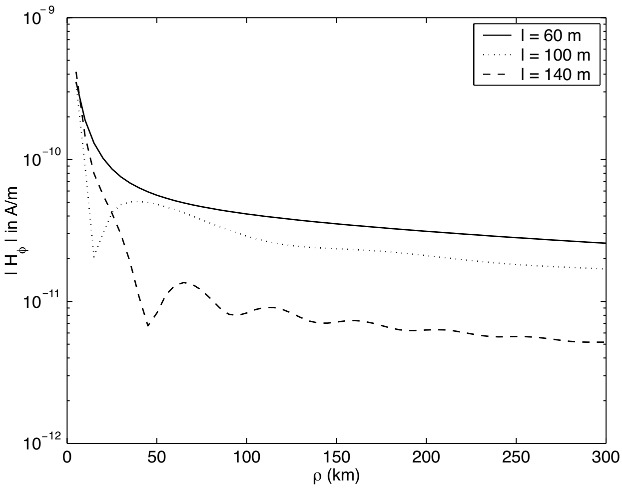

Figure 10. The magnetic field $|H_\phi|$ in A/m at $l = 60\,\mathrm{m}$, $l = 100\,\mathrm{m}$, and $l = 140\,\mathrm{m}$ due to unit horizontal electric dipole in the earth versus the propagating distance ρ; $f = 100\,\mathrm{kHz}$, $a = 6370\,\mathrm{km}$, $\varepsilon_{r1} = 10$, $\varepsilon_{r2} = 20$, $\varepsilon_{r3} = 100$, $\sigma_1 = 10^{-5}\,\mathrm{S/m}$, $\sigma_2 = 2 \times 10^{-5}\,\mathrm{S/m}$, $\sigma_3 = 4\,\mathrm{S/m}$, and $d_s = z_r = 1\,\mathrm{m}$, and $\phi = 0$.

REFERENCES

1. Watson, G. N., "The diffraction of radio waves by the earth," *Proc. the Royal Society*, Vol. A95, 83–99, 1918.
2. Van der Pol, B. and H. Bremmer, "The propagation of radio waves over a finitely conducting spherical earth," *Phil. Mag.*, Vol. 25, 817–834, 1938; Further note on above, Vol. 27, 261–275, 1939.
3. Fock, V. A., *Electromagnetic Diffraction and Propagation Problems*, 191–212, Pergamon Press, Ltd., Oxford, 1965.
4. Bremmer, H., *Terrestrial Radio Waves*, Elsevier, New York, 1949.
5. Bremmer, H., "Applications of operational calculus to ground-wave propagation, particularly for long waves," *IRE Trans. Antennas Porpag.*, Vol. AP-6, 267–274, 1958.
6. North, K. A., "The calculations of ground-wave field intensity over a finitely conducting spherical earth," *Proc. IRE*, Vol. 29, 623–639, 1941.
7. Wait, J. R., "Radiation and propagation from a vertical antenna over a spherical earth," *Journal of Research of National Bureau of Standards*, Vol. 56, 237–244, 1956.
8. Wait, J. R., "The electromagnetic field of a horizontal dipole in the presence of a conducting half-space," *Can. J. Phys.*, Vol. 39, 1017–1027, 1961.
9. Wait, J. R. and D. A. Hill, "Excitation of the HF surface wave by vertical and horizontal antennas," *Radio Sci.*, Vol. 14, 767–780, 1979.
10. Hill, D. A. and J. R. Wait, "Ground wave attenuation function for a spherical earth with arbitrary surface impedance," *Radio Sci.*, Vol. 15, No. 3, 637–643, 1980.
11. Wait, J. R., "The ancient and modern history of EM ground-wave propagation," *IEEE Antennas and Propagat. Mag.*, Vol. 40, No. 5, 7–24, 1998.
12. Houdzoumis, V. A., "Vertical electric dipole radiation over a sphere: character of the waves that propagate through the sphere," *J. Appl. Phys.*, Vol. 86, 3939–3942, 1999.
13. Houdzoumis, V. A., "Two modes of wave propagation manifested in vertical electric dipole radiation over a sphere," *Radio Sci.*, Vol. 35, No. 1, 19–29, 2000.
14. Margetis, D., "Radiation of horizontal electric dipole on large dielectric sphere," *J. Math. Phys.*, Vol. 43, 3162–3201, 2002.
15. Pan, W. Y. and H. Q. Zhang, "Electromagnetic field of a vertical electric dipole on the surface of a dielectric layer

overlay a global conductor," *Radio Sci.*, Vol. 38, No. 3, 1061, 10.1029/2002RS002689, 2003.

16. Li, K. and S.-O. Park, "Electromagnetic field in the air generated by a horizontal electric dipole located in the spherical electrically earth coated with a dielectric layer," *J. Electromagn. Waves Applicat.*, Vol. 17, No. 10, 1399–1417, 2003.
17. Li, K., S.-O. Park, and H.-Q. Zhang, "Electromagnetic field in the presence of a three-layered spherical region," *Progress In Electromagnetics Research*, J. A. Kong (ed.), Vol. 45, 103–121, EMW Publishing, Cambridge, Boston, 2004.
18. Li, K., S.-O. Park, and H.-Q. Zhang, "The electromagnetic field over the spherical earth coated with N-layered dielectric," *Radio Sci.*, Vol. 39, RS2008, doi:10.1029/2002RS002771, 2004.
19. Gradshteyn, I. S. and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, New York, 1980.

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