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THE ASYMPTOTIC SOLUTION OF AN INTEGRAL EQUATION FOR MAGNETIC CURRENT IN A PROBLEM OF WAVEGUIDES COUPLING THROUGH NARROW SLOTS

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Abstract—Based on the asymptotic method of averaging, an approximate analytical solution of the integral equation concerning a magnetic current in slot-hole coupling apertures of electrodynamic volumes, which differ profitably from the known ones in literature, has been obtained. The formulas for the currents and characteristics scattering of transverse and longitudinal slots in common broad and narrow walls of rectangular waveguides are given. The comparison to results obtained by other methods and experimental data has been done.

1. INTRODUCTION

The problem of electromagnetic coupling of two waveguides through apertures in their common walls is a classical problem which attracted the attention of many investigators starting from the paper written by Bethe in 1944 [1]. The narrow slots with the length of $2L$ commensurable with the operating wavelength of λ have especially been studied [2–10]. The investigations of the slots located both in broad and narrow walls of rectangular waveguides have been conducted by different methods, namely: analytical [2, 3], variational [4, 5], numerical [6–8], among which the most effective methods are the moments method and its particular case known as Galerkin's method, the equivalent circuits method [9, 10], and also the finite elements method and the moments method [11].

At present, commercial finite elements software (e.g., "Ansoft's HFSS and Designer", "CST Microwave Studio", "Zeland" and other) is available to solve such problems. However, these programs need intensive memory and sometimes they are very slow, for example, in electrically long slots and multi-slot systems analysis. That is why there exists a need to develop approximate methods which provide fast, sufficiently accurate calculations of simple waveguide-slot structures.

On the other hand, the approximate methods, mentioned above, have some drawbacks. The known analytical solutions have a limited range of applicability ($kL \cong \pi/2$, where $k = 2\pi/\lambda$) and the variational and equivalent circuits methods suppose the presence of the information a priori about the distribution function of the equivalent slot magnetic current. Even an approximation of this information is unknown (for example, for electrically longitudinal slots) in some cases.

In this paper the asymptotic method of averaging has been used to obtain the general approximate analytical expression for the magnetic current in the slot applied both for the adjusted slots ($kL = n\pi/2$, $n = 1, 2, 3\ldots$) and for the unadjusted ones ($kL \neq n\pi/2$) coupling two waveguides of different cross-section sizes in a common case which are exited by the arbitrary field of impressed sources. We also suggest perfect functions for the induced magnetomotive forces method, which provide a rather satisfactory current approximation in electrically long longitudinal slots and in the system of transverse slots.

2. PROBLEM FORMULATION AND INITIAL INTEGRAL EQUATIONS

Let two volumes (limited by ideally conducting flat surfaces) be coupled between each other by the slot in the common unlimited thin wall. Using the boundary condition of the tangential magnetic field continuity on the S_{sl} surface of the coupling aperture, we obtain the following integral equation concerning the equivalent magnetic current (time dependence $e^{i\omega t}$ throughout the paper is used) [12]:

$$(graddiv + k^2) \int\limits_{S_{sl}} \hat{G}_m^{\Sigma}(\vec{r}, \vec{r}\,') \vec{J}^m(\vec{r}\,') d\vec{r}\,' = -i\omega \vec{H}_0^{\Sigma}(\vec{r}). \tag{1}$$

Here: $\vec{r}$ is the observation point radius-vector; $\vec{r}\,'$ is the source radius-vector; $\vec{J}^m(\vec{r})$ is the magnetic current surface density on the aperture; $\hat{G}_m^{\Sigma}(\vec{r}, \vec{r}\,') = \hat{G}_m^{e}(\vec{r}, \vec{r}\,') + \hat{G}_m^{i}(\vec{r}, \vec{r}\,')$, $\hat{G}_m^{e,i}(\vec{r}, \vec{r}\,')$ are the magnetic dyadic Green's functions; $\hat{H}_0^{\Sigma}(\vec{r}) = \vec{H}_0^{i}(\vec{r}) - \vec{H}_0^{e}(\vec{r})$ are the impressed sources fields in the internal (index "i" for the region 1 and 2) and the external (index "e" for regions 3 and 4) volumes.

The $\hat{G}_m^{e,i}(\vec{r}, \vec{r}\,')$ functions are the following [13]:

$$\hat{G}_m^{e,i}(\vec{r}, \vec{r}\,') = \hat{I} G(\vec{r}, \vec{r}\,') + \hat{G}_{0m}^{e,i}(\vec{r}, \vec{r}\,'), \tag{2}$$

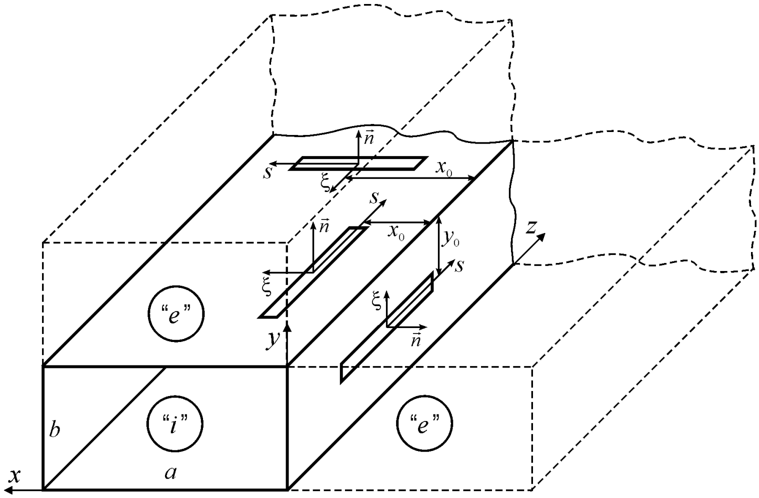


Figure 1. The problem formulation and the symbols used.

where $\hat{I}$ is the unit dyadic, $G(\vec{r}, \vec{r}\,') = e^{-ik|\vec{r}-\vec{r}\,'|}/|\vec{r}-\vec{r}\,'|$ is the Green's free space function and $\hat{G}^{e,i}_m(\vec{r}, \vec{r}\,')$ are the regular everywhere dyadic functions providing satisfaction of boundary conditions for $\hat{G}^{e,i}_m(\vec{r}, \vec{r}\,')$ functions on the internal surface of the volumes coupled.

The equation (1) is rather difficult to analyze in a general case, however, for the narrow slots ($d/2L \ll 1$, $d/\lambda \ll 1$), where d is the slot width, the equation is sufficiently simplified. In this case the slot current can be written in the following way (index "m" is omitted):

$$\vec{J}(\vec{r}) = \vec{e}_s J(s)\chi(\xi), \quad J(\pm L) = 0, \quad \int\limits_{\xi} \chi(\xi) d\xi = 1, \tag{3}$$

where s and ξ are the longitudinal and transverse local slot coordinates (Figure 1); $\vec{e}_s$ is the unit vector; $\chi(\xi)$ is the set function accounting the peculiarities of the electrostatic field on the slot edge [14]:

$$\chi(\xi) = \frac{1/\pi}{\sqrt{(d/2)^2 - \xi^2}}. \tag{4}$$

Thus, the $\vec{J}(\vec{r})$ current problem in the $\vec{H}^{\Sigma}_0(\vec{r})$ field given reduces to the determination of the $J(s)$ current distribution function.

Let us consider the slot to be rectilinear and the impressed field in the external volume to be absent, that is $\vec{H}^{e}_0(\vec{r}) = 0$. Then substituting

(3) and (4) into (1) we get:

$$\left(\frac{d^2}{ds^2}+k^2\right)\int\limits_{-L}^{L} J(s')\left[G_s^e(s,s')+G_s^i(s,s')\right]ds' = -i\omega H_{0s}^i(s), \quad (5)$$

$$G_s^{e,i}(s,s') = 2\frac{e^{-ikR(s,s')}}{R(s,s')} + G_{0s}^{e,i}(s,s'), \quad R(s,s') = \sqrt{(s-s')^2+(d/4)^2}. \quad (6)$$

Here we take into account the fact that for the sources on the flat surface $\hat{G}^{e,i}(s,\xi;s',\xi') = 2\hat{I}G(s,\xi;s',\xi') + \hat{G}_0^{e,i}(s,\xi;s',\xi')$ we have $G(s,s') = \int_\xi G(s,\xi;s',\xi')\chi(\xi')d\xi'$, $G_0^{e,i}(s,s') = \int_\xi G_0^{e,i}(s,\xi;s',\xi')\chi(\xi')d\xi'$.

It must be noted that in the kernel (6) of the integral equation (5) the approximate expression for $|\vec{r}-\vec{r}'|$ is the transverse coordinate dependence is chosen in the form of $(\xi-\xi')^2 \cong (d/4)^2$, as it is usually used in the vibrator antenna theory [15] and it is precisely the form for the slots on metallic surfaces [14, 16].

Isolating the logarithmic peculiarity in the equation (5) analogically with [2, 15] we obtain:

$$\int\limits_{-L}^{L} J(s')\frac{e^{-ikR(s,s')}}{R(s,s')}ds' = J(s)\Omega(s) + \int\limits_{-L}^{L}\frac{J(s')e^{-ikR(s,s')}-J(s)}{R(s,s')}ds', \quad (7)$$

where $\Omega(s) = \int_{-L}^{L}\frac{ds'}{R(s,s')}$. Suppose due to [2] $\Omega(s) \approx \Omega(0) = 2\ln\frac{8L}{d}$, we obtain the integral-differential equation with a small parameter:

$$\frac{d^2J(s)}{ds^2} + k^2J(s) = \alpha\{i\omega H_{0s}(s) + F[s,J(s)] + F_0[s,J(s)]\}, \quad (8)$$

where $\alpha = \frac{1}{8\ln(d/(8L))}$ is the natural small ($|\alpha| \ll 1$) parameter of the problem; $H_{0s}(s)$ is the component of the field of the impressed sources on the slot axis;

$$F[s,J(s)] = 4\left[-\frac{dJ(s')}{ds'}\frac{e^{-ikR(s,s')}}{R(s,s')}\bigg|_{-L}^{L} + \left(\frac{2dJ(s)}{ds}+J(s)\frac{d}{ds}\right)\frac{1}{R(s,s')}\right] + 4\int\limits_{-L}^{L}\frac{\left[\frac{d^2J(s')}{ds'^2}+k^2J(s')\right]e^{-ikR(s,s')} - \left[\frac{d^2J(s)}{ds^2}+k^2J(s)\right]}{R(s,s')}ds' \quad (9)$$

is the slot own field in the infinite screen;

$$F_0[s, J(s)] = -\frac{dJ(s')}{ds'}\left[G^e_{0s}(s, s') + G^i_{0s}(s, s')\right]\Big|_{-L}^{L} + \int_{-L}^{L}\left[\frac{d^2J(s')}{ds'^2} + k^2 J(s')\right]\left[G^e_{0s}(s, s') + G^i_{0s}(s, s')\right] ds' \quad (10)$$

is the slot own field repeatedly reflected from the coupled volumes walls.

We must note that the inequality $|\alpha| \ll 1$ is valid in sufficiently wide limits of the ratio $(d/2L)$ variation: for example, for $(d/2L) = 0.1$ we have $|\alpha| = 0.034$ and for $(d/2L) = 0.3$ we have $|\alpha| = 0.048$.

3. ASYMPTOTIC SOLUTION OF AN INTEGRAL EQUATION FOR CURRENT

Due to the constants variation method [17] let us change the variables:

$$\begin{aligned} J(s) &= A(s)\cos ks + B(s)\sin ks, \\ \frac{dJ(s)}{ds} &= -A(s)k\sin ks + B(s)k\cos ks, \\ &\quad \left(\frac{dA(s)}{ds}\cos ks + \frac{dB(s)}{ds}\sin ks = 0\right), \quad (11) \\ \frac{d^2J(s)}{ds^2} + k^2 J(s) &= -\frac{dA(s)}{ds}\sin ks + \frac{dB(s)}{ds}\cos ks \\ &= \frac{\alpha}{k}\{i\omega H_{0s}(s) + F_N[s, J(s)]\}. \end{aligned}$$

The equation (8) goes to the next system of the integral-differential equations for the unknown functions $A(s)$ and $B(s)$:

$$\begin{aligned} \frac{dA(s)}{ds} &= -\frac{\alpha}{k}\left\{i\omega H_{0s}(s) + F_N\left[s, A(s), \frac{dA(s)}{ds}, B(s), \frac{dB(s)}{ds}\right]\right\}\sin ks, \\ \frac{dB(s)}{ds} &= +\frac{\alpha}{k}\left\{i\omega H_{0s}(s) + F_N\left[s, A(s), \frac{dA(s)}{ds}, B(s), \frac{dB(s)}{ds}\right]\right\}\cos ks, \end{aligned} \quad (12)$$

where $F_N = F + F_0$ and it is the slot full own field.

The equations obtained are fully equivalent to the equation (8) and they are the system of integral-differential equations of standard type, unsolved for a derivative. The right-hand parts of these equations are proportional to the α small parameter. Therefore, the $A(s)$ and $B(s)$

functions in the right-hand parts of the equations (12) can be regarded as slowly changing functions. To solve the system of the equations in (12), it is possible to use the asymptotic method of averaging the application ground of which is given in [18, 19]. Then when we put the system of the equations (12) in accordance with the simplified system [18] where in the right-hand parts of the equations $\frac{dA(s)}{ds}=0$, $\frac{dB(s)}{ds}=0$. When we make partial averaging in [19] along the s explicitly entered variable, we obtain two equations of the first approximation:

$$\begin{aligned}\frac{d\overline{A}(s)}{ds} &= -\alpha\left\{\frac{i\omega}{k}H_{0s}(s)+\overline{F}_N[s,\overline{A},\overline{B}]\right\}\sin ks,\\ \frac{d\overline{B}(s)}{ds} &= +\alpha\left\{\frac{i\omega}{k}H_{0s}(s)+\overline{F}_N[s,\overline{A},\overline{B}]\right\}\cos ks,\end{aligned}\tag{13}$$

where

$$\begin{aligned}\overline{F}_N[s,\overline{A},\overline{B}] &= \left[\overline{A}(s')\sin ks'-\overline{B}(s')\cos ks'\right]G_s^{\Sigma}(s,s')\Big|_{-L}^{L},\\ G_s^{\Sigma}(s,s') &= 4\frac{e^{-ikR(s,s')}}{R(s,s')}+G_{0s}^{e}(s,s')+G_{0s}^{i}(s,s')\\ &= G_s^{e}(s,s')+G_s^{i}(s,s').\end{aligned}\tag{14}$$

Integrating the system (13) and substituting the obtained values $\overline{A}(s)$ and $\overline{B}(s)$ as the approximating functions for $A(s)$ and $B(s)$ in (11), we get the most general asymptotic expression for the narrow slot current in the arbitrary position relative to coupled volumes walls:

$$\begin{aligned}J(s) &= \overline{A}(-L)\cos ks+\overline{B}(-L)\sin ks\\ &\quad+\alpha\int\limits_{-L}^{s}\left\{\frac{i\omega}{k}H_{0s}(s')+\overline{F}_N[s',\overline{A},\overline{B}]\right\}\sin k(s-s')ds'.\end{aligned}\tag{15}$$

To define the constants $\overline{A}(\pm L)$ and $\overline{B}(\pm L)$, it is necessary to use the boundary conditions (3) and the symmetrical conditions [15] which are uniquely connected with the slot excitation technique. Then taking into consideration the symmetrical (index "s") and the antisymmetrical (index "a") current components at arbitrary excitation $H_{0s}(s)=H_{0s}^{s}(s)+H_{0s}^{a}(s)$ of the slot with accuracy not more than the terms of α^2 order, we finally have:

$$J(s) = J^s(s)+J^a(s)=\alpha\frac{i\omega}{k}\left\{\int\limits_{-L}^{s}H_{0s}(s')\sin k(s-s')ds'\right.$$

$$
-\frac{\sin k(L+s)\int\limits_{-L}^{L} H_{0s}^{s}(s')\sin k(L-s')ds'}{\sin 2kL+\alpha N^{s}(kd,2kL)}
-\frac{\sin k(L+s)\int\limits_{-L}^{L} H_{0s}^{a}(s')\sin k(L-s')ds'}{\sin 2kL+\alpha N^{a}(kd,2kL)}\Bigg\}, \tag{16}
$$

where $N^s(kd, 2kL)$ and $N^a(kd, 2kL)$ are the functions of the slot own field which are equal, respectively:

$$
\begin{aligned}
N^{s}(kd,2kL) &= \int\limits_{-L}^{L}\Big[G_{s}^{\Sigma}(s,-L)+G_{s}^{\Sigma}(s,L)\Big]\sin k(L-s)ds,\\
N^{a}(kd,2kL) &= \int\limits_{-L}^{L}\Big[G_{s}^{\Sigma}(s,-L)-G_{s}^{\Sigma}(s,L)\Big]\sin k(L-s)ds,
\end{aligned} \tag{17}
$$

and which are completely defined by Green's functions of the coupled volumes representing infinite and half-infinite waveguides, resonators and etc.

It is necessary to note that near the resonance ($\sin 2kL \approx 0$) of the main contribution to the current amplitude is made by the functions of the slot own field $N^s(kd, 2kL)$ and $N^a(kd, 2kL)$ which take into account both the basic oscillation mode and high wave modes in the surroundings of the slot. Stevenson [2] obtained the expression for the magnetic current in a case of the arbitrary oriented narrow slot situated on the common wall of rectangular waveguides by the iterations method, which King used for thin vibrators [15]. However, in [2] (unlike the formula (16)) the $kL = \pi/2$ assumption was made for the current distribution function and the slot own field function.

As an example, let us consider the coupling of two identical rectangular waveguides by the $\{a \times b\}$ cross sections through the symmetrical transverse slot in the common broad wall and via the longitudinal slot in the common narrow wall. We also consider the coupling of two mutually perpendicular waveguides in the H-plane through the longitudinal/transverse slot in the common broad wall.

3.1. Symmetrical Transverse Slot in a Common Broad Wall of Waveguides

In this case $H_{0s}(s) = H_{0s}^s(s) = H_0 \cos(\pi s/a)$. Taking into account that for coupling of two waveguides of equal sizes $N^s = 2W^s$, $N^a = 2W^a$, we get:

$$J(s) = -\alpha H_0 \frac{i\omega \left\{ \cos ks \cos\left(\frac{\pi L}{a}\right) - \cos kL \cos\left(\frac{\pi s}{a}\right) \right\}}{\gamma^2 \left[\cos kL + \alpha 2W_t^s(kd, kL)\right]}, \tag{18}$$

where $\gamma^2 = k^2 - (\pi/a)^2$, H_0 is the amplitude of the incident H_{10}-wave, falling from $z = -\infty$ (a region 1).

The $|S_{11}|$ reflection coefficient, the $|S_{12}|$ transmission ones in the first waveguide and the transmission coefficients $|S_{13}|$ and $|S_{14}|$ in the second waveguide equal, respectively:

$$\begin{aligned} |S_{11}| &= |S_{13}| = |S_{14}| = \left| \frac{4\pi\alpha f(kL, \pi L/a)}{abk\gamma[\cos kL + \alpha 2W_t^s(kd, kL)]} \right|, \\ |S_{12}| &= \left| 1 - \frac{4\pi\alpha f(kL, \pi L/a)}{iabk\gamma[\cos kL + \alpha 2W_t^s(kd, kL)]} \right|. \end{aligned} \tag{19}$$

3.2. Longitudinal Slot in a Common Narrow Wall of Waveguides

For a longitudinal slot, the field projection of impressed sources on the slot axis equals $H_{0s}(s) = H_0 \exp(-i\gamma s)$ and we have

$$\begin{aligned} J(s) &= J^s(s) + J^a(s) \\ &= \alpha H_0 \frac{i\omega}{(\pi/a)^2} \left\{ e^{-i\gamma s} - \frac{\cos ks \cos \gamma L}{\cos kL + \alpha 2W_{ln}^s(kd, kL)} \right. \\ &\quad \left. + i \frac{\sin ks \sin \gamma L}{\sin kL + \alpha 2W_{ln}^a(kd, kL)} \right\}. \end{aligned} \tag{20}$$

For the reflection, transmission, and coupling coefficients we obtain the following expressions:

$$\begin{aligned} |S_{11}| &= |S_{13}| = \left| \frac{4\pi\alpha}{abk\gamma} \left\{ \frac{f^s(kL, \gamma L)}{\cos kL + \alpha 2W_{ln}^s(kd, kL)} \right. \right. \\ &\quad \left. \left. + \frac{f^a(kL, \gamma L)}{\sin kL + \alpha 2W_{ln}^a(kd, kL)} - 2kL \frac{\sin 2\gamma L}{2\gamma L} \right\} \right|, \end{aligned} \tag{21}$$

$$|S_{12}| = \left|1 + \frac{i4\pi\alpha}{abk\gamma}\left\{\frac{f^s(kL,\gamma L)}{\cos kL + \alpha 2W^s_{ln}(kd,kL)} + \frac{f^a(kL,\gamma L)}{\sin kL + \alpha 2W^a_{ln}(kd,kL)} - 2kL\right\}\right|, \; |S_{14}| = |S_{12} - 1|.$$

3.3. Longitudinal/Transverse Slot in a Common Broad Wall of Waveguides

In this case, the current distribution depends upon where the waveguide excitation sources are situated. If the H_{10} incident wave is propagating in the waveguide for which the coupling slot is transverse, then we get:

$$J(s) = -\alpha H_0 \frac{i\omega\left\{\cos ks \cos\left(\frac{\pi L}{a}\right) - \cos kL \cos\left(\frac{\pi s}{a}\right)\right\}}{\gamma^2\left\{\cos kL + \alpha\left[W^s_t(kd,kL) + W^s_{lb}(kd,kL)\right]\right\}}. \tag{22}$$

If the slot for the exciting field is longitudinal, then we have:

$$\begin{aligned} J(s) &= J^s(s) + J^a(s) \\ &= -\alpha H_0 \frac{i\omega \cos\frac{\pi x_0}{a}}{(\pi/a)^2}\left\{\frac{(\cos ks \cos\gamma L - \cos kL \cos\gamma s)}{\cos kL + \alpha\left[W^s_{lb}(kd,kL) + W^s_t(kd,kL)\right]}\right. \\ &\quad \left. - i\frac{(\sin ks \sin\gamma L - \sin kL \sin\gamma s)}{\sin kL + \alpha\left[W^s_{lb}(kd,kL) + W^a_t(kd,kL)\right]}\right\}. \end{aligned} \tag{23}$$

For the current in (22) the coupling coefficients are defined by the expressions (19), where it is necessary to make the following change $2W^s_t \to W^s_t + W^s_{lb}$. For the current in (23) they equal, respectively, (the H_{10} incidence wave spreads from the region 3 into the region 4):

$$\begin{aligned} |S_{33}| &= \left|\frac{4\pi\alpha\cos^2\frac{\pi x_0}{a}}{abk\gamma}\left\{\frac{f^s_1(kL,\gamma L)}{\cos kL + \alpha\left[W^s_{lb}(kd,kL) + W^s_t(kd,kL)\right]}\right.\right. \\ &\quad \left.\left. + \frac{f^a_1(kL,\gamma L)}{\sin kL + \alpha\left[W^a_{lb}(kd,kL) + W^a_t(kd,kL)\right]}\right\}\right|, \end{aligned} \tag{24}$$

$$|S_{34}| = \left|1 - \frac{4\pi\alpha\cos^2\frac{\pi x_0}{a}}{iabk\gamma}\left\{\frac{f^s(kL,\gamma L)}{\cos kL + \alpha\left[W^s_{lb}(kd,kL) + W^s_t(kd,kL)\right]}\right.\right.$$

$$- \left. \frac{f^a(kL,\gamma L)}{\sin kL + \alpha\left[W_{lb}^a(kd,kL)+W_t^a(kd,kL)\right]}\right\}\Bigg|,$$

$$|S_{31}|^2 = |S_{32}|^2 = \frac{1}{2}\left(1-|S_{33}|^2-|S_{34}|^2\right).$$

The expressions for W_t^s, W_t^a, W_{ln}^s, W_{ln}^a, W_{lb}^s, W_{lb}^a, f, f^s, f^a, f_1^s, f_1^a functions are represented in Appendix A.

3.4. Finite Thickness of Coupling Region Account

The h finite thickness of the wall between coupled volumes can be taken into account due to [20, 21] which introduces the d_e slot effective width concept. Then at the $(h/\lambda) \ll 1$, precisely up to the terms of the $\{(hd)/\lambda^2\}$ order, we have [20]:

$$\frac{h}{d} \ll 1: \quad d_e = d\left(1-\frac{1}{\pi}\frac{h}{d}\ln\frac{d}{h}\right); \quad \frac{h}{d} \gtrsim 1: \quad d_e = d\left(\frac{8}{\pi e}e^{-\frac{\pi}{2}\frac{h}{d}}\right). \tag{25}$$

The expression in [21] is a good approximation for both cases:

$$d_e \cong d\,e^{-\frac{\pi h}{2d}}. \tag{26}$$

The $|S_{13}|$ coefficient calculations using the approximate ratios of (25) and (26) within the limits of $0 \le h/2L \le 0.2$ coincide with the results, obtained in [22], where the account of the rectangular waveguide wall thickness has been made by solving two coupling integral equations with method of moments.

4. NUMERICAL RESULTS AND DISCUSSION

The plots of the dependences of the $|S_\Sigma|^2 = |S_{13}|^2 + |S_{14}|^2$ coupling coefficient from the length of the symmetrical transverse slot in the common infinitely thin broad wall of two identical rectangular waveguides are in Figure 2. The dependences have been calculated by different methods. It is seen that the calculations made by means of the averaging, variotional [4] and moments methods [6] give the values of the slot resonance length (slot "shortening") of $2L \cong 0.47\lambda$. The results obtained by using the quasi-static [3] and equivalent circuits (the "reaction" method [9]) methods lead to the resonance value of $2L = 0.5\lambda$ that does not correspond the reality. We note that Figure 2 gives the results of solving the problem with the help of the moment method as a numerical experiment.

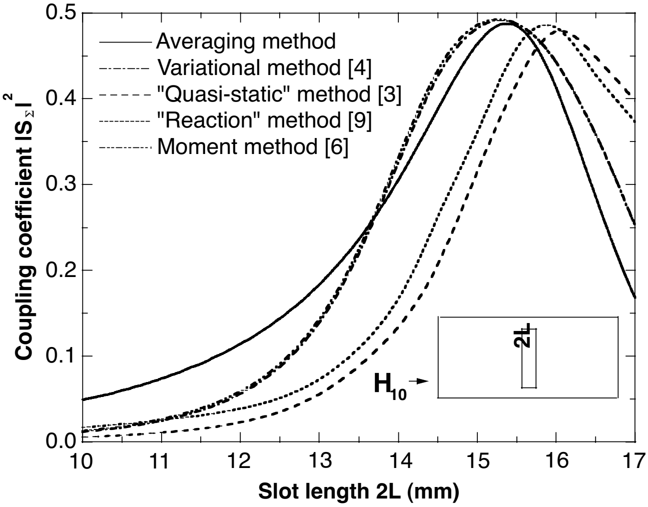
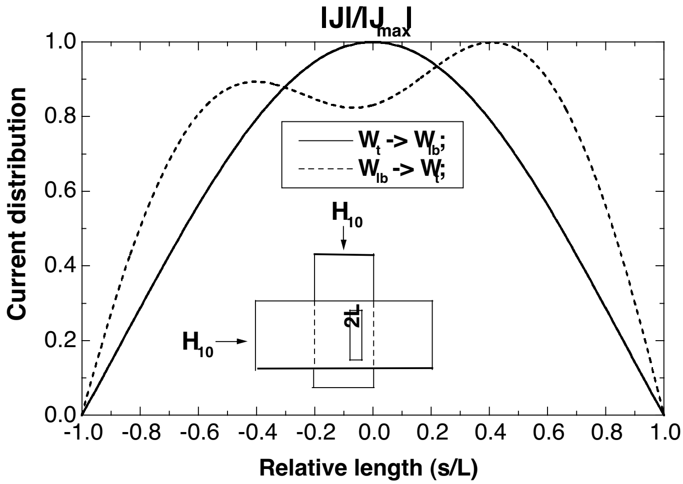


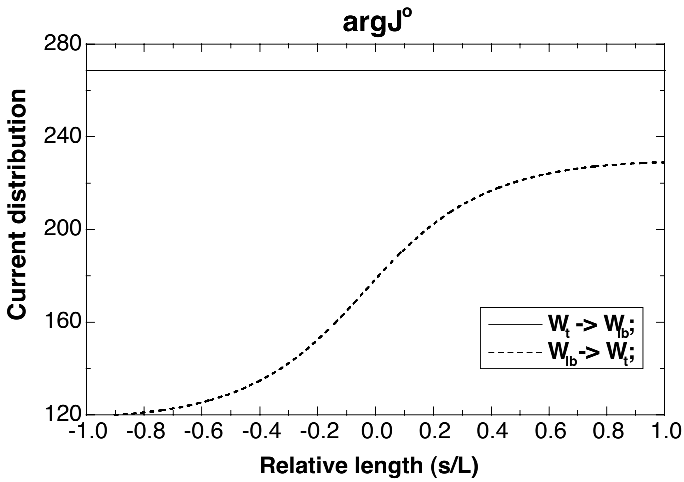
Figure 2. The coupling coefficient dependence from the symmetrical transverse slot length in the common broad wall of two rectangular waveguides at: $a = 22.86$ mm, $b = 10.16$ mm, $d = 1.5875$ mm, $\lambda = 32.0$ mm, $h = 0.0$ mm.

In Figure 3 there are the plots of the amplitude-and-phase distribution of the $J(s) = |J(s)|e^{i \arg J(s)}$ current along the transverse-longitudinal slot in the common infinitely thin broad wall of two mutually perpendicular waveguides. It is seen that if the slot for the H_{10} excitation wave is transverse ($W_t \to W_{lb}$), then the amplitude distribution of current is purely symmetrical and the current phase is constant along the slot length. For another case of the excitation ($W_{lb} \to W_t$), the amplitude distribution of the current is sufficiently asymmetric and the current phase changes along the slot. The current distribution curves are of the same kind in the case of two waveguides coupling through the longitudinal slot in the common broad wall of the finite thickness of the rectangular waveguides in Figure 4. The calculations have been made according to the formula (23) (where there is the $W_t^{s,a} \to W_{lb}^{s,a}$ change) and by Galerkin's method: $J(s) = \sum\limits_{n=1}^{N} J_n \sin \frac{n\pi(L+s)}{2L}$, taking into account 2, 6 and 12 basis functions. We can see that the change of the external volume sufficiently increases the contribution of the $J^a(s)$ current antisymmetrical component into the general distribution function.

Thus, the obtained asymptotic solution of the integral equation

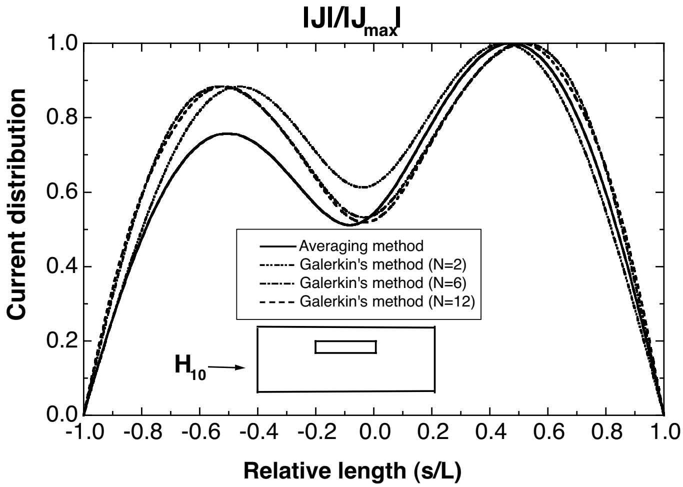


(a)

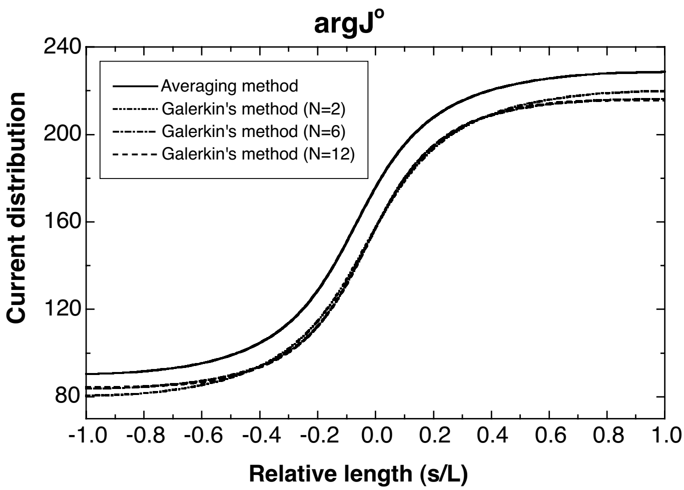


(b)

Figure 3. The current distribution along the transverse/longitudinal (W_t and W_{lb} respectively) slots in the common broad wall of two mutually perpendicular waveguides at: $a = 22.86\,\text{mm}$, $b = 10.16\,\text{mm}$, $d = 1.5875\,\text{mm}$, $\lambda = 25.8\,\text{mm}$, $2L = 20\,\text{mm}$, $x_0 = 1.43\,\text{mm}$ is the slot axis position, $h = 0.0\,\text{mm}$.



(a)



(b)

Figure 4. The current distribution along the longitudinal slot in the common broad wall of two rectangular waveguides at: $a = 22.86\,\text{mm}$, $b = 10.16\,\text{mm}$, $d = 1.5875\,\text{mm}$, $\lambda = 25.8\,\text{mm}$, $2L = 20\,\text{mm}$, $x_0 = 1.43\,\text{mm}$ is the slot axis position, $h = 2.0\,\text{mm}$.

concerning the magnetic current in slot-hole coupling apertures allows us to obtain analytical expressions for the current to the first approximation which is valid for various ratios between the wavelength and a longitudinal size of the slot. The given numerical results demonstrate the efficiency and the effectiveness of such a solution. However, there are some quantitative differences between the calculated values of electrodynamics characteristics of the slot coupling apertures, which have been obtained by using the above-mentioned asymptotic formulas and numerical methods. These differences may be removed by using the magnetic current expression obtained from asymptotic solution of the integral equation with the help of the averaging method in combination with other analytical methods, for example, with the induced magnetomotive forces method as it was proposed by the authors in [23, 24].

5. INDUCED MAGNETOMOTIVE FORCES METHOD WITH BASIS FUNCTIONS OF THE AVERAGING METHOD FOR ANALYSIS OF COUPLING SLOTS IN WAVEGUIDES

While applying the moments and Galerkin's methods to analyse single and multi-slot elements of the coupling of electrodynamic volumes, different basis and weight functions can be used: the piecewise [6], the piecewise linear and piecewise sinusoidal [25], the trigonometric [7, 8, 28, 29] and Gegenbauer polynomials [26, 27]. In these cases it is necessary to solve the system of linear algebraic equations (SLAE) of the N-order, where N is a number of linearly independent basis functions. The system matrix elements (N^2-in all) cannot always be obtained analytically, and calculation time increases proportionally to N^3 [30].

For slots system, the order of SLAE increases proportionally to the number of slots. Therefore it is necessary, to our minds, to approximate the distribution of the slot equivalent magnetic current by one or two functions (it depends on an excitation character) as it was done, for example, in [5, 9, 10]. When only one approximating function exists for every slot in a multi-slot system, Galerkin's method gets the name "induced magnetomotive forces method" (IMMFM). In this case the solution is more accurate when the approximating functions describing the slot magnetic current distribution are more accurate. In [5, 9, 10] the half-wave and wave sinusoidal functions were used for the slot with the $2L \leq \lambda$ length. For longer slots it is necessary to increase the number of functions.

We suggest better functions of the current distribution for

IMMFM that gives sufficiently satisfactory approximation of the current in longer longitudinal slots and in the transverse slots system. These functions have been obtained in Chapter 3 (formula (18) for transverse slots and formula (23) for longitudinal slots) when we solved the integral equation for the slot magnetic current by the asymptotic method of averaging.

5.1. Longitudinal Slot in a Common Broad Wall of Waveguides

Generally, the projection of the $H_{0s}(s)$ impressed field on the slot axis and the $J(s)$ magnetic current in it can be represented with two components — symmetrical and antisymmetrical ones along the slot with the respect to its center — $H_{0s}(s) = H^s_{0s}(s) + H^a_{0s}(s)$, $J(s) = J^s(s) + J^a(s)$. Owing to this, we can have the following integral-differential equation for the current (5) in a narrow linear slot:

$$\left(\frac{d^2}{ds^2} + k^2\right) \int\limits_{-L}^{L} [J^s(s') + J^a(s')][G^e_s(s, s') + G^i_s(s, s')]ds' = -i\omega \left[H^s_{0s}(s) + H^a_{0s}(s)\right]. \tag{27}$$

Let us represent the current as unknown amplitudes and distribution functions fixed:

$$J(s) = J^s_0 f^s(s) + J^a_0 f^a(s), \tag{28}$$

where the $f^s(s)$ and $f^a(s)$ functions must satisfy the following boundary conditions: $f^s(\pm L) = 0$, $f^a(\pm L) = 0$. From (28) we have only two unknown amplitudes J^s_0 and J^a_0. They can be obtained from two independent equations with the respect of J^s_0 and J^a_0 that we have using IMMFM:

$$\begin{aligned} J^s_0 \left[Y^e_s(kd, kL) + Y^i_s(kd, kL)\right] &= M_s(kL), \\ J^a_0 \left[Y^e_a(kd, kL) + Y^i_a(kd, kL)\right] &= M_a(kL), \end{aligned} \tag{29}$$

where:

$$Y^{e,i}_{s,a}(kd, kL) = \frac{1}{\omega} \int\limits_{-L}^{L} f^{s,a}(s) \left[\left(\frac{d^2}{ds^2} + k^2\right) \int\limits_{-L}^{L} f^{s,a}(s') G^{e,i}_{s,a}(s, s') ds'\right] ds \tag{30}$$

are the external and inner partial slot admittances and

$$M_{s,a}(kd, kL) = -i \int\limits_{-L}^{L} f^{s,a}(s) H^{s,a}_{0s}(s) ds \tag{31}$$

are the partial magnetomotive forces.

For the longitudinal slot in the broad wall of the rectangular waveguide due to (23), the $f^s(s)$ and $f^a(s)$ basis functions have the following forms:

$$\begin{aligned} f^s(s) &= \cos ks \cos\gamma L - \cos kL \cos\gamma s, \\ f^a(s) &= \sin ks \sin\gamma L - \sin kL \sin\gamma s. \end{aligned} \tag{32}$$

5.2. Two Symmetrical Transverse Slots in a Common Broad Wall of Waveguides

For the two slots in a waveguide wall (Figure 5), one can obtain the system of two coupled integral-differential equations for the $J_1(s_1)$ and $J_2(s_2)$ magnetic currents in the first and the second slots:

$$\begin{cases} \left(\dfrac{d^2}{ds_1^2}+k^2\right)\left[\displaystyle\int_{-L_1}^{L_1} J_1(s_1')G^{\Sigma}_{s_1}(s_1,s_1')ds_1' + \int_{-L_2}^{L_2} J_2(s_2')G^{\Sigma}_{s_1}(s_1,s_2')ds_2'\right] \\ \qquad = -i\omega H_{0s_1}(s_1), \\ \left(\dfrac{d^2}{ds_2^2}+k^2\right)\left[\displaystyle\int_{-L_2}^{L_2} J_2(s_2')G^{\Sigma}_{s_2}(s_2,s_2')ds_2' + \int_{-L_1}^{L_1} J_1(s_1')G^{\Sigma}_{s_2}(s_2,s_1')ds_1'\right] \\ \qquad = -i\omega H_{0s_2}(s_2), \end{cases} \tag{33}$$

where: $G^{\Sigma}_{s_{1,2}}(s_{1,2},s_{1,2}') = G^{e}_{s_{1,2}}(s_{1,2},s_{1,2}') + G^{i}_{s_{1,2}}(s_{1,2},s_{1,2}')$, $H_{0s_1}(s_1)$ and $H_{0s_2}(s_2)$ are the projections of the field of the impressed sources to the slot axes.

As in a previous case the currents in every slot can be written in the following way:

$$J_1(s_1) = J_{01}f_1(s_1),\ J_2(s_2) = J_{02}f_2(s_2);\ f_1(\pm L_1) = 0,\ f_2(\pm L_2) = 0. \tag{34}$$

Due to the induced magnetomotive forces method used for two slots system, we transform (33) into the following algebraic equations system relative to J_{01} and J_{02} unknown amplitudes:

$$\begin{cases} J_{01}Y^{\Sigma}_{11}(kd_1,kL_1) + J_{02}Y^{\Sigma}_{12}(kL_1,kL_2) = M_1(kL_1), \\ J_{01}Y^{\Sigma}_{21}(kL_2,kL_1) + J_{02}Y^{\Sigma}_{22}(kL_2,kL_2) = M_2(kL_2). \end{cases} \tag{35}$$

Here

$$Y^{\Sigma}_{mn}(kL_m,kL_n) = \frac{1}{\omega}\int_{-L_m}^{L_m} f_m(s_m)\left[\left(\frac{d^2}{ds_m^2}+k^2\right)\int_{-L_n}^{L_n} f_n(s_n')G^{\Sigma}_{s_m}(s_m,s_n')ds_n'\right]ds_m,$$

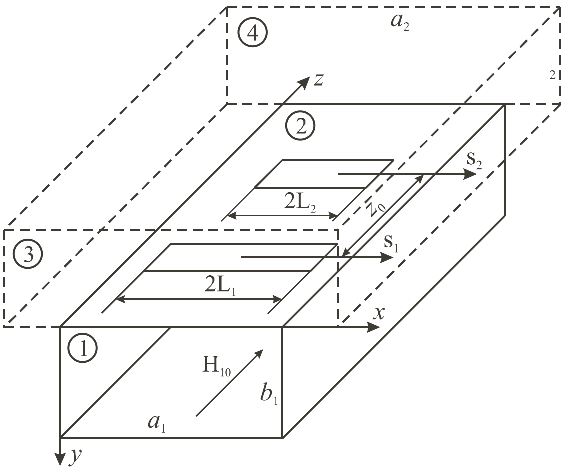


Figure 5. The slots system in the waveguides common wall.

$$(m, n = 1, 2) \qquad (36)$$

are the eigen $(m = n)$ and mutual $(m \neq n)$ slots admittances, respectively;

$$M_m(kL_m) = -i \int_{-L_m}^{L_m} f_m(s_m) H_{0sm}(s_m) ds_m, \quad (m = 1, 2) \qquad (37)$$

are the magnetomotive forces.

In the case of two symmetrical transverse slots, the currents in them are also symmetrical. They can be represented as ($k_c = 2\pi/\lambda_c$, λ_c is the cut-off H_{10} wavelength) due to (18):

$$f_m(s_m) = \cos k s_m \cos k_c L_m - \cos k L_m \cos k_c s_m, \quad (m = 1, 2). \qquad (38)$$

Generally, slots can be located at the φ angle to the longitudinal waveguide axis. Then according to the general solution of the integral equation (16) for the current, the basis functions of IMMFM have the forms:

$$\begin{aligned} f^s(s) &= \frac{\cos ks \cos k_2 L - \cos kL \cos k_2 s}{(\sin\varphi + (k_c/\gamma)\cos\varphi)^2} e^{ik_c x_0} \\ &\quad - \frac{\cos ks \cos k_1 L - \cos kL \cos k_1 s}{(\sin\varphi - (k_c/\gamma)\cos\varphi)^2} e^{-ik_c x_0}, \\ f^a(s) &= \frac{\sin ks \sin k_2 L - \sin kL \sin k_2 s}{(\sin\varphi + (k_c/\gamma)\cos\varphi)^2} e^{ik_c x_0} \\ &\quad + \frac{\sin ks \sin k_1 L - \sin kL \sin k_1 s}{(\sin\varphi - (k_c/\gamma)\cos\varphi)^2} e^{-ik_c x_0}, \end{aligned} \tag{39}$$

where: $k_1 = k_c \sin\varphi + \gamma\cos\varphi$, $k_2 = k_c \sin\varphi - \gamma\cos\varphi$, x_0 is the distance between the narrow waveguide wall and the slot center. Let us note that at $\varphi = 0$, the formulas (39) are transformed into (32) and at $\varphi = \pi/2$ into (38).

If the coupling between the waveguides is made by the multi-slot structure consisting of the M symmetrical transverse slots, then the current distribution function in each of them is chosen due to (38). Hence, the order of SLAE (35) increases relatively, the slots conductivities and the magnetomotive forces can be calculated owing to the same formulas (36) and (37) in this case.

5.3. Coupling Coefficients

Using (32) and (38) we can obtain unknown amplitudes J_0^s, J_0^a, J_{01}, J_{02} from (29) and (35), respectively. It gives us the opportunity to obtain energy characteristics of the coupling slot elements.

For the longitudinal slot in the common broad wall of rectangular waveguides, we have:

$$\begin{aligned} |S_{11}| &= |S_{13}| = \left| \frac{4\pi^3 \cos^2 \dfrac{\pi x_0}{a}}{\omega a^3 b\gamma} \left[\tilde{J}_0^s F^s(kL) + \tilde{J}_0^a F^a(kL) \right] \right|, \\ |S_{12}| &= \left| 1 - \frac{4\pi^3 \cos^2 \dfrac{\pi x_0}{a}}{i\omega a^3 b\gamma} \left[\tilde{J}_0^s F^s(kL) - \tilde{J}_0^a F^a(kL) \right] \right|, \quad |S_{14}| = |S_{12} - 1|, \end{aligned} \tag{40}$$

where

$$\tilde{J}_0^s = \frac{F^s(kL)}{Y_s^e(kd,kL) + Y_s^i(kd,kL)}, \quad \tilde{J}_0^a = \frac{F^a(kL)}{Y_a^e(kd,kL) + Y_a^i(kd,kL)}. \tag{41}$$

For two symmetrical transverse slots in the common broad wall of rectangular waveguides, the following expressions have been obtained:

$$\begin{aligned} |S_{11}| &= |S_{13}| = \left| \frac{4\pi\gamma}{abk^2} \left[\tilde{J}_1 F(kL_1) + e^{-i\gamma z_0} \tilde{J}_2 F(kL_2) \right] \right| \\ |S_{12}| &= \left| 1 - \frac{4\pi\gamma}{iabk^2} \left[\tilde{J}_1 F(kL_1) + e^{i\gamma z_0} \tilde{J}_2 F(kL_2) \right] \right|, \ |S_{14}| = |S_{12} - 1|, \end{aligned} \tag{42}$$

where $\tilde{J}_1 = \frac{F(kL_1)Y_{22}^{\Sigma} - e^{-i\gamma z_0} F(kL_2) Y_{12}^{\Sigma}}{Y_{11}^{\Sigma} Y_{22}^{\Sigma} - (Y_{12}^{\Sigma})^2}$, $\tilde{J}_2 = \frac{e^{-i\gamma z_0} F(kL_2) Y_{11}^{\Sigma} - F(kL_1) Y_{12}^{\Sigma}}{Y_{11}^{\Sigma} Y_{22}^{\Sigma} - (Y_{12}^{\Sigma})^2}$.

The expressions for the Y_s, Y_a, Y_{mn} admittances and the F^s, F^a, F functions are given in Appendix B.

5.4. Numerical Results

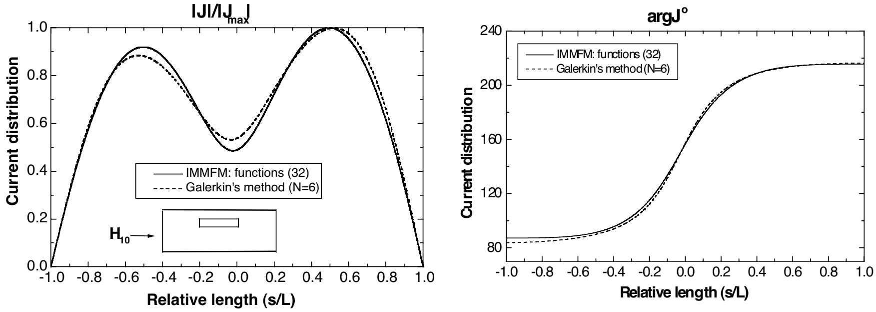
The current distribution curves in the case of two rectangular waveguides coupled through the longitudinal slot in the common broad wall of the finite thickness are given in Figures 6a, b. The calculations have been made according to the formulas (28), (32) and by the Galerkin's method taking into account 6 basis functions. In the Figure 6c, the plots of the $|S_{\Sigma}|^2 = |S_{13}|^2 + |S_{14}|^2$ coupling coefficients dependences of the longitudinal slot in the broad waveguide wall due to its electrical length are presented. In the Figures 7, 8, the coupling coefficients dependences are given for the system of two identical, rectangular symmetrical transverse slots where the distance between them equals z_0. The calculations have been made with the following representations of the magnetic current: a) in the form of $J(s) = \sum_{n=1}^{N} J_n \sin \frac{n\pi(L+s)}{2L}$ (Galerkin's method), b) with the use of the functions (32) and (38), c) using the following approximations:

$$J(s) = J_0^s \cos(\pi s/2L) + J_0^a \sin(\pi s/L) \text{ (for the longitudinal slot [5]);} \tag{43}$$

$$J_m(s_m) = J_{0m} \cos(\pi s_m/2L_m), \ m = 1, 2 \text{ (for two transverse slots system).} \tag{44}$$

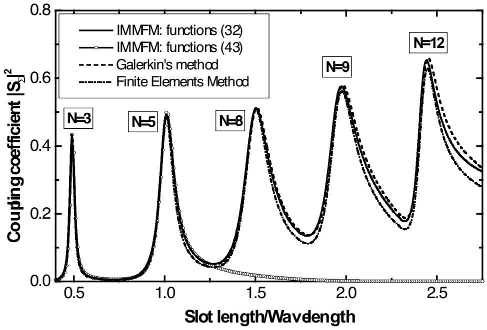
The Figure 6c also gives the calculated values obtained by using the finite elements method (FEM) due to the program "CST Microwave Studio".

Hence, the approximation (32) when only two basis functions are used, gives good match with the results obtained by using Galerkin's and the finite elements methods for the longitudinal slots with the electrical length up to $2L/\lambda \leq 2.75$ (when it is necessary to use 12 basis functions for the Galerkin's method); meanwhile the (43)



a) **b)**

Figure 6a, b. The current distribution along the longitudinal slot in the common broad wall of two rectangular waveguides at: $a = 22.86\,\text{mm}$, $b = 10.16\,\text{mm}$, $d = 1.5875\,\text{mm}$, $\lambda = 25.8\,\text{mm}$, $2L = 20\,\text{mm}$, $x_0 = 1.43\,\text{mm}$ is the slot axis position, $h = 2.0\,\text{mm}$.



c)

Figure 6c. The coupling coefficient dependence from the relative length of the longitudinal slot in the common broad wall of two rectangular waveguides at: $a = 23\,\text{mm}$, $b = 10\,\text{mm}$, $d = a/15$, $x_0 = a/6$, $\lambda/\lambda_c = 0.625$, $h = 2.0\,\text{mm}$.

approximation is satisfactory for only up to $2L/\lambda \leq 1.25$. In the case of two transverse slots, the (38) approximation is good, too. It gives satisfactory coincidence with the results of Galerkin's method and the experimental data in different parts of the band of the operating length of the H_{10}-wave, especially at the resonance points. The (44) basis functions change the resonance frequency values. We think that (43) and (44) functions describe the slot current distribution less accurately than the (32) and (38) functions because they do not have the λ, λ_c and λ_g (wavelength in the waveguide) values in the distribution, the ratios of which between each other, and the slot length supposes the

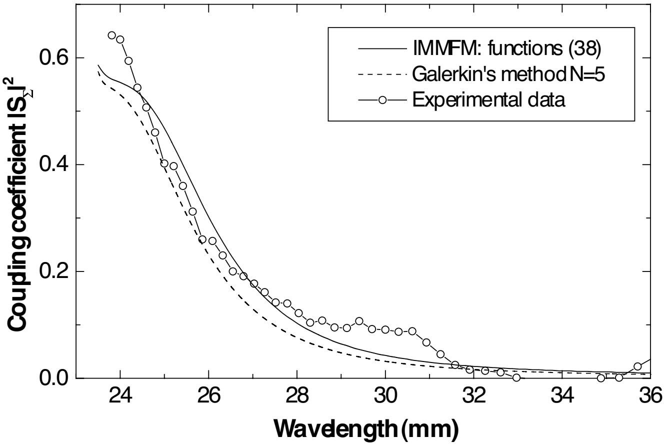


Figure 7. The coupling coefficient dependence from the wavelength for two symmetrical transverse slots in the common broad wall of two rectangular waveguides at: $a = 23\,\text{mm}$, $b = 10\,\text{mm}$, $d_1 = d_2 = 2.0\,\text{mm}$, $2L_1 = 2L_2 = 10.6\,\text{mm}$, $z_0 = 10.0\,\text{mm}$, $h = 1.0\,\text{mm}$.

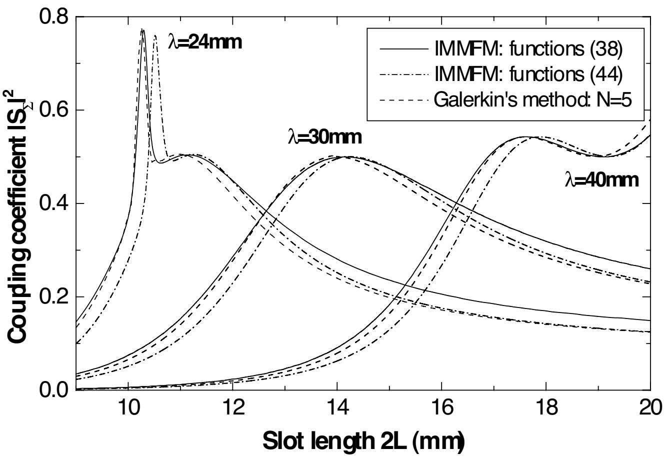


Figure 8. The coupling coefficient dependence from the wavelength for two symmetrical transverse slots in the common broad wall of two rectangular waveguides at: $a = 23\,\text{mm}$, $b = 10\,\text{mm}$, $d_1 = d_2 = 1.6\,\text{mm}$, $2L_1 = 2L_2 = 2L$, $z_0 = 2\lambda/3$, $h = 0.0\,\text{mm}$.

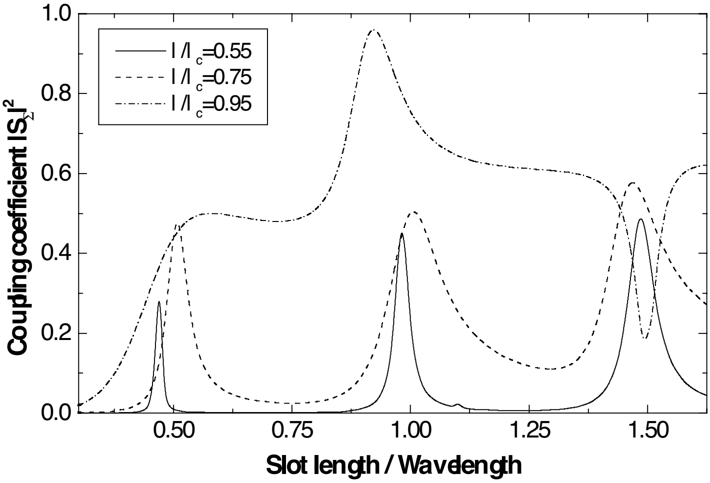


Figure 9. The coupling coefficient dependence from the relative length of the longitudinal slot in the common broad wall of two rectangular waveguides at: $a = 23\,\text{mm}$, $b = 10\,\text{mm}$, $d = a/15$, $x_0 = a/4$, $h = 2.0\,\text{mm}$.

formation of the magnetic current amplitude-phase distribution and energy characteristics of the coupling slot holes.

The latter is proved by the plots given in the Figure 9 where the curves of the coupling coefficient dependences of two identical waveguides through the longitudinal slot in their common broad wall from $2L/\lambda$ are represented at different values of λ/λ_c. It is clear that if the λ/λ_c ratio increases then the Q-factor of the $|S_\Sigma|^2 = f(2L/\lambda)$ resonance curves decreases. At definite ratio between $2L/\lambda$ and λ/λ_c, practically the full power of the initial wave from one waveguide to the other one can be transmitted.

The important results have been obtained in the case of the multi-slot coupling through transverse symmetrical slots. The Figure 10 gives the dependences of the $|S_{11}|$, $|S_{12}|$, $|S_{13}|$, $|S_{14}|$ and $|S_\Sigma|^2$ coupling coefficients from the wavelength for the system consisting of 16 slots of equal length, the distance between which equals $z_{0m} = \lambda_g^{res}/4$, where λ_g^{res} is the waveguide wavelength, which corresponds to the λ_{res} resonance wavelength of the single slot ($\lambda_{res} = 33.7\,\text{mm}$ for $2L = 16\,\text{mm}$). As it is seen from the plots, the power in this case, entering the first shoulder of the main waveguide (a region 1) is divided into 4 equal parts in sufficiently wide band of wavelengths ($\Delta\lambda/\lambda_{res} = 0.15$). Note, that here we take into account full interaction

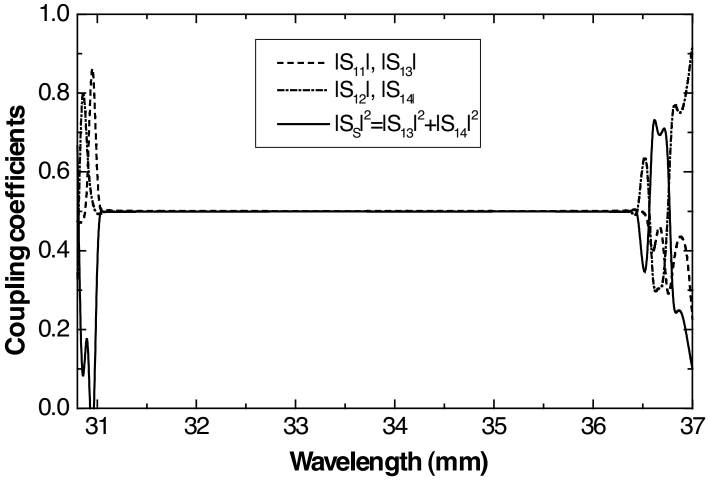


Figure 10. The coupling coefficients dependence from the wavelength for the system of 16 transverse slots in the common broad wall of two rectangular waveguides at: $a = 23\,\text{mm}$, $b = 10\,\text{mm}$, $d_m = 1.6\,\text{mm}$, $2L_m = 16\,\text{mm}$, $z_{0m} = 12.4\,\text{mm}$, $h = 0.2\,\text{mm}$.

between all slots for a finite thickness of waveguides walls and the calculation time is far less then when Galerkin's or the finite elements methods are used.

6. CONCLUSION

This paper presents a new asymptotic solution of the integral equation for the magnetic current in the problem of the electromagnetic coupling between waveguides via the narrow slots in their common walls. As a result, we have obtained the approximate analytical expressions for the slot magnetic current for the adjusted and unadjusted slots coupling two waveguides, generally, without concrete definition of the exiting field of the impressed sources. The analytical formulas for the current distribution function obtained by the averaging method can be used with other calculation methods (for example, the induced magnetomotive forces method) which suppose the availability of the information a priory about the current distribution function. They are groundless when this distribution is unknown even approximately.

APPENDIX A.

The functions of the slot own field for the case of coupling of two infinite rectangular waveguides are given below:

$$W_t^s(kd,kL) = \frac{8\pi}{ab}\sum_{m,n}\frac{\varepsilon_n e^{-k_z d/4}}{k_z(k^2-k_x^2)}\sin^2\frac{m\pi}{2} \times \cos k_xL(k\sin kL\cos k_xL - k_x\cos kL\sin k_xL), \quad \text{(A1)}$$

$$W_t^a(kd,kL) = -\frac{8\pi}{ab}\sum_{m,n}\frac{\varepsilon_n e^{-k_z d/4}}{k_z(k^2-k_x^2)}\cos^2\frac{m\pi}{2} \times \sin k_xL(k\cos kL\sin k_xL - k_x\sin kL\cos k_xL), \quad \text{(A2)}$$

$$W_{lb}^s(kd,kL) = \frac{4\pi}{ab}\sum_{m,n}\frac{\varepsilon_m\varepsilon_n\cos k_xx_0\cos k_x(x_0+d/4)}{k_z(k_x^2+k_y^2)} \times e^{-k_zL}[k_z\cos kL sh k_zL + k\sin kL ch k_zL], \quad \text{(A3)}$$

$$W_{lb}^a(kd,kL) = \frac{4\pi}{ab}\sum_{m,n}\frac{\varepsilon_m\varepsilon_n\cos k_xx_0\cos k_x(x_0+d/4)}{k_z(k_x^2+k_y^2)} \times e^{-k_zL}[k_z\sin kL ch k_zL - k\cos kL sh k_zL], \quad \text{(A4)}$$

$$W_{ln}^s(kd,kL) = \frac{4\pi}{ab}\sum_{m,n}\frac{\varepsilon_m\varepsilon_n\cos k_yy_0\cos k_y(y_0+d/4)}{k_z(k_x^2+k_y^2)} \times e^{-k_zL}[k_z\cos kL sh k_zL + k\sin kL ch k_zL], \quad \text{(A5)}$$

$$W_{ln}^a(kd,kL) = \frac{4\pi}{ab}\sum_{m,n}\frac{\varepsilon_m\varepsilon_n\cos k_yy_0\cos k_y(y_0+d/4)}{k_z(k_x^2+k_y^2)} \times e^{-k_zL}[k_z\sin kL ch k_zL - k\cos kL sh k_zL]. \quad \text{(A6)}$$

$$f\left(kL,\frac{\pi}{a}L\right) = 2\cos\frac{\pi}{a}L\frac{\sin kL\cos\frac{\pi}{a}L - \frac{\pi}{ka}\cos kL\sin\frac{\pi}{a}L}{1-(\pi/ka)^2} - \frac{\cos kL}{(2\pi/ka)}\left(\sin\frac{2\pi L}{a}+\frac{2\pi L}{a}\right). \quad \text{(A7)}$$

$$f^s(kL,\gamma L) = 2\cos\gamma L\frac{\sin kL\cos\gamma L-(\gamma/k)\cos kL\sin\gamma L}{1-(\gamma/k)^2},$$
$$f^a(kL,\gamma L) = 2\sin\gamma L\frac{\cos kL\sin\gamma L-(\gamma/k)\sin kL\cos\gamma L}{1-(\gamma/k)^2}. \quad \text{(A8)}$$

$$\begin{aligned} f_1^s(kL,\gamma L) &= f^s(kL,\gamma L) - \frac{\cos kL}{2(\gamma/k)}(\sin 2\gamma L + 2\gamma L), \\ f_1^a(kL,\gamma L) &= f^a(kL,\gamma L) - \frac{\sin kL}{2(\gamma/k)}(\sin 2\gamma L - 2\gamma L). \end{aligned} \tag{A9}$$

In the (A1)–(A9) expressions there are the following symbols: $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$, $k_z = \sqrt{k_x^2 + k_y^2 - k^2}$, $(m, n = 0, 1, 2 \dots)$; $\gamma = \sqrt{k^2 - (\pi/a)^2}$; $\varepsilon_{m,n} = 1$ at $m, n = 0$, $\varepsilon_{m,n} = 2$ at $m, n \neq 0$; x_0 and y_0 are the slots axes coordinates.

APPENDIX B.

The longitudinal slot admittances:

$$\begin{aligned} Y_s^i(kd,kL) = \frac{4\pi}{\omega ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} & \varepsilon_m \varepsilon_n \cos k_x x_0 \cos k_x \left(x_0 + \frac{d}{4}\right) \\ & \times \left\{ \left[\cos\gamma L \left(\frac{k}{k_z}\sin kL - \cos kL\right)\right] F^s(k_z L) \right. \\ & - \frac{\cos kL}{k_z^2 + \gamma^2} \left[\left(k_z^2 + k^2\right) \left(\frac{\gamma}{k_z}\sin\gamma L - \cos\gamma L\right) F^s(k_z L) \right. \\ & \left.\left. + \left(\frac{\pi}{a}\right)^2 F^s(kL) \right] \right\}, \end{aligned} \tag{B1}$$

$$F^s(kL) = 2\cos\gamma L \frac{k\sin kL\cos\gamma L - \gamma\cos kL\sin\gamma L}{(\pi/a)^2} - \cos kL \frac{\sin 2\gamma L + 2\gamma L}{2\gamma}, \tag{B2}$$

$$\begin{aligned} F^s(k_z L) = & \frac{\cos\gamma L}{k_z^2 + k^2} \left[k_z \cos kL \left(1 - e^{-2k_z L}\right) + k\sin kL(1 + e^{-2k_z L})\right] \\ & - \frac{\cos\gamma L}{k_z^2 + \gamma^2} \left[k_z \cos\gamma L \left(1 - e^{-2k_z L}\right) + \gamma\sin\gamma L(1 + e^{-2k_z L})\right] \end{aligned} \tag{B3}$$

$$\begin{aligned} Y_a^i(kd,kL) = \frac{4\pi}{\omega ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} & \varepsilon_m \varepsilon_n \cos k_x x_0 \cos k_x \left(x_0 + \frac{d}{4}\right) \\ & \times \left\{ \left[-\sin\gamma L \left(\frac{k}{k_z}\cos kL + \sin kL\right)\right] F^a(k_z L) \right. \\ & + \frac{\sin kL}{k_z^2 + \gamma^2} \left[\left(k_z^2 + k^2\right) \left(\frac{\gamma}{k_z}\cos\gamma L + \sin\gamma L\right) F^a(k_z L) \right. \end{aligned}$$

$$+\left(\frac{\pi}{a}\right)^2 F^a(kL)\Big]\Big\}, \tag{B4}$$

$$F^a(kL)=2\sin\gamma L\frac{k\cos kL\sin\gamma L-\gamma\sin kL\cos\gamma L}{(\pi/a)^2}-\sin kL\frac{\sin 2\gamma L-2\gamma L}{2\gamma}, \tag{B5}$$

$$F^a(k_zL)=\frac{\sin k_gL}{k_z^2+k^2}\left[k_z\sin kL\left(1+e^{-2k_zL}\right)-k\cos kL(1-e^{-2k_zL})\right]$$
$$-\ \frac{\sin kL}{k_z^2+\gamma^2}\left[k_z\sin\gamma L\left(1+e^{-2k_zL}\right)-\gamma\cos kL(1-e^{-2k_zL})\right]. \tag{B6}$$

The eigen ($m = n$) and mutual ($m \neq n$) slot admittances of the transverse slots system:

$$Y_{mm}(kd_m, kL_m) = \frac{4\pi}{\omega ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{k_z}e^{-k_z\frac{d_m}{4}}\sin^2\frac{m\pi}{2}$$
$$\times\left[\boldsymbol{I}(kL_m)\cos k_cL_m-\boldsymbol{I}(k_cL_m)\cos kL_m\right]^2, \tag{B7}$$

$$Y_{mn}(kL_{m,n}, kL_{n,m}) = Y_{nm}(kL_{n,m}, kL_{m,n})$$
$$= \frac{4\pi}{\omega ab}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\varepsilon_n(k^2-k_x^2)}{k_z}e^{-k_zz_0}\sin^2\frac{m\pi}{2}$$
$$\times\left[\boldsymbol{I}(kL_m)\cos k_cL_m-\boldsymbol{I}(k_cL_m)\cos kL_m\right]$$
$$\times\left[\boldsymbol{I}(kL_n)\cos k_cL_n-\boldsymbol{I}(k_cL_n)\cos kL_n\right], \tag{B8}$$

$$\boldsymbol{I}(kL) = 2\frac{k\sin kL\cos k_xL-k_x\sin k_xL\cos kL}{k^2-k_x^2};$$

$$F(kL) = 2\cos k_cL\frac{\sin kL\cos k_cL-(k_c/k)\cos kL\sin k_cL}{1-(k_c/k)^2}$$
$$-\cos kL\frac{\sin k_cL+2k_cL}{(2k_c/k)}. \tag{B9}$$

The rest symbols are the same as in the (A1)–(A9) formulas.

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