

ABOUT THE INTERFERENCE INDUCED BY ELECTRONS. WHY DOES THE ELECTRON BEHAVE AS A WAVE?

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Abstract—One of the most interesting and peculiar phenomena of Quantum Mechanics is the interference (I) induced by the electrons. Strangely enough, though the electrons are real particles, they often behave just like waves. From the point of view of the classical mechanics the I induced by the electrons is unexplainable, however it is solved mathematically using the formalism of quantum mechanics and applying Schrödinger's equation. The quantum solution of the problem is clear and elegant, especially from a mathematical point of view, however it still leaves some perplexities as to understand how exactly the phenomenon happens.

We will make a hypothesis trying to understand the undulation phenomenon of the electron: it is really a strange and mysterious phenomenon. Maybe if we consider that the electron, just as the baryons and the mesons, might be made of smaller particles (saving the integrity of the unity of the negative electrical charge and the other Laws of Conservation), we could understand more easily how a single electron can go through two close holes at the same time. Analogously we could better understand another very particular quantum phenomenon carried out mainly by electrons, that is the tunnel effect. In this case, though the particle does not have enough energy to go through the potential barrier, though it does not have any material possibility to pass through a layer which does not have any hole, after several "attempts" the particle will manage to pass through the barrier anyway, as it had dug a tunnel, or as it had managed to find a "breach" in the wall. In this phenomenon too, though we can explain it from a mathematical point of view, using the equations of the quantum mechanics, it is still not clear how actually the electron manages to have an undulation behaviour.

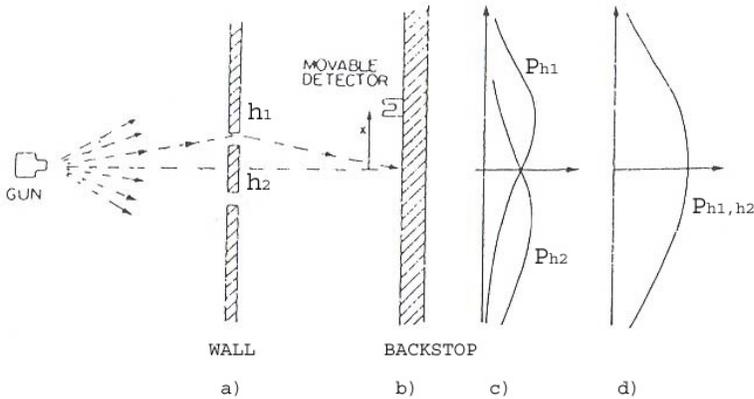


Figure 1.

1. INTRODUCTION

One of the most peculiar and interesting phenomena of the Quantum Mechanics is represented by the interference (I). We do not mean the phenomenon which takes place when the waves, going through two slits and without any measurement, make a characteristic I figure on a backstop. What we mean is the I induced by electrons. Strangely enough, though electrons are real particles, they often behave just as waves. This does not happen if they have to pass through just one slit, or if we want to be just spectators of the phenomenon. In this case the I will never take place, the electrons will behave exclusively like particles, just as it happens if we shoot normal bullets against a layer with two slits. Let's be more clear.

If we use a gun to shoot bullets against a board with two holes, after a certain number of shots, using a detector connected to the backstop where the bullets arrived, we can calculate the mean of the probability that the bullets pass through hole 1 (P_{h1}), or through hole 2 (P_{h2}) (see Fig. 1). It is interesting to notice that "the effect with both holes open (see Fig. 1d) is the sum of the effects with each hole open alone (see Fig. 1c). We shall call this result an observation of *no interference*" [1]. Indeed if we sum up P_{h1} and P_{h2} (Fig. 1c), they correspond exactly to the values of $P_{h1,h2}$ (Fig. 1d), that is: $P_{h1} + P_{h2} = P_{h1,h2}$.

Following the same experimental criteria, but using a source of water waves, as it has been demonstrated several times [2], the result will be a typical I figure (see Fig. 2) "if we cover one hole at a time and measure the intensity distribution at the absorber we find the rather

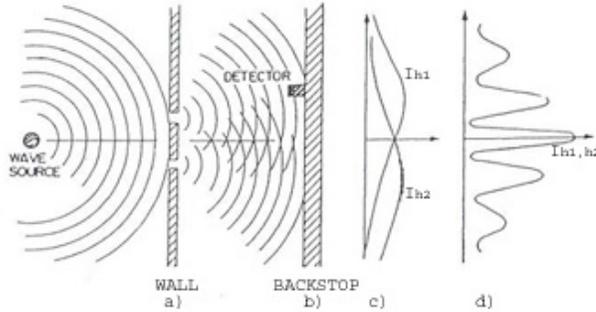


Figure 2.

simple intensity curves shown in part (c) of the figure (2). I_{h1} is the intensity of the wave from hole 1 (which we find by measuring when hole 2 is blocked off) and I_{h2} is the intensity of the wave from hole 2 (see when hole 1 is blocked). The intensity $I_{h1,h2}$ observed when both holes are open (See Fig. 2d) is certainly *not* the sum of I_{h1} and I_{h2} . We say that there is *interference* of the two waves. At some places (where the curve $I_{h1,h2}$ has its maxima) the waves are ‘in phase’ and the wave peaks add together to give a large amplitude and, therefore, a large intensity. We say that the two waves are “interfering constructively” at such places. There will be such constructive interference wherever the distance from the detector to one hole is a whole number of wavelengths larger (or shorter) than the distance from the detector to the other hole. At those places where the two waves arrive at the detector with a phase difference of π (where they are “out the phase”) the resulting wave motion at the detector will be the difference of the two amplitudes. The waves “interfere destructively”, and we get a low value for the wave intensity. We expect such low values wherever the distance between hole 2 and the detector by an odd number of half-wavelengths. The low value of $I_{h1,h2}$ in Fig. 2 corresponds to the places where the two waves interfere. At the detector the intensity of the wave passing through hole 1 is proportional to the mean squared height. In the same way the intensity of the wave coming from hole 2 is proportional to $|I_{h2}|^2$. Similarly, for hole 2 the intensity is proportional to $|I_{h2}|^2$. When both holes are open, the intensity is $|I_{h1} + I_{h2}|^3$ [1].

In 1927 Davisson and Germer shot a flux of electrons against a layer with two holes. They got a surprising result. Since electrons are particles they expected to have a result similar to what happens when we shoot bullets, as we can see in Fig. 1. Instead they realised that the impact position of the electron formed a *I* (interference) figure typical of the waves. For the first time it was demonstrated that the flux

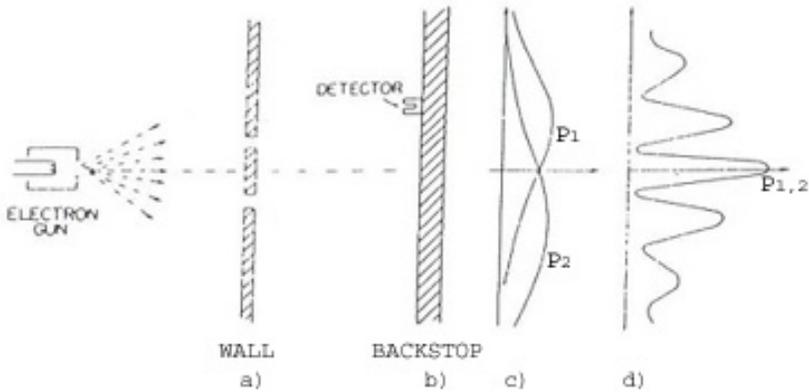


Figure 3.

of electrons behaved, against any prevision, as a wave [2]. Classical Physics was mined: the typical I figure of the waves took place also when electrons were shot one at a time, even one every ten seconds. Later the experiment has been produced several more times; it has always confirmed the same results: see Fig. 3 (though it would be the same if we referred to Fig. 2, since the two graphs are exactly the same). Feynman considers it as “the only mystery of Quantum Mechanics. A phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of Quantum Mechanics” [1].

2. DISCUSSION

We have then a particle, the electron, behaving just as a wave. Besides, if it is not observed it can pass simultaneously in both holes! How can just one particle, of course indivisible, to pass through both holes at the same time? It makes us think that the electron manages to split up, but how? It may seem madness, but why cannot we think that the electron is made of smaller particles? Up until some forty years ago the proton and the neutron were considered elemental particles. Later Gell-Mann’s intuition resulted winning, nowadays we normally teach that the hadrons are made of quarks (Qs). Namely baryons are made of three Qs and the mesons are made of a Q and an anti-quark. Why cannot it be the same for the lepton, such as the electron? Of course it is not easy to demonstrate! However the phenomenon of the I induced by the electron (where a single particle manages to pass through two

different holes) may represent an “indirect signal” that the electron is not a “real elemental” particle. How is it made then? It may be made of 2 (or 3) sub-particles. Awaiting for a possible more suitable word, at the moment we can call these sub-particles “leptoquarks” (LQs). Let’s proceed with order. To introduce this new concept and to make it possible and/or probable (at least theoretically acceptable) first of all we need to respect the main “Laws of Conservation” imposed by Physics. Among these Laws, about a dozen, we consider those more related to the electron, considered as an elemental particle, indivisible. Let’s analyze it.

- 1) The Law of Conservation of the Leptonic Number: the leptonic number of the electron is $+1$, it has to remain so. If we consider the electron as made of two LQs, we have a leptonic number of $+2$, so we have to assume the presence of an electronic antineutrino inside the electron, or associated to it. Since the antineutrino has a leptonic number of -1 we have as a final sum $+1$, which corresponds to the leptonic number of the electron.
- 2) Law of Conservation of the Electric Charge: as we all know the electric charge of the electron is -1 . Thus, since it has to remain unchanged, we will consider that one LQ has a fractional electric charge (similarly to the Qs!) of $-4/3$, whereas the other LQ should, as a consequence, carry an electrical charge of $+1/3$. In this way their sum will give us an electrical charge of -1 , that is the electron’s. Besides, since the electronic antineutrino has a null charge, it does not modify the final electrical charge.
- 3) Law of Conservation of the Angular Momentum: the electron is a fermion, indeed it has a half-integer spin of $\pm 1/2$. The electronic antineutrino has the same value (spin $\pm 1/2$), it should be the same for each of the two LQs, which should have spins rotating in contrary directions, so as to balance, without summing up. In this way it is the spin of the antineutrino to rotate in the same direction of the electron it belongs to. Thus we have a final half-integer spin of $\pm 1/2$ — in the same way of the electron — having its same rotation spin (but not a spin of $\pm 1\frac{1}{2}$). Besides with the two LQs having anti parallel spins it is respected Pauli’s Principle of Exclusion, which forbids two analogous fermions to have equal quantum state at the same time (though LQs have electrical charge differently fractionated and probably different masses)
- 4) Law of Conservation of the Baryonic Number: the electron is a lepton, thus it has a null baryonic number, that is zero. It is the same for the LQs and the electronic antineutrino (since it is a lepton too).

- 5)–6) Laws of Conservation of the Momentum (p) and of the mass (m): the momentum corresponds to the mass of the considered particle multiplied its velocity ($p = mv$) . In order not to break this rule, the probable LQs will have to keep the same momentum of the electron as a sum. Thus, if the two LQs have a mass approximately half of the electron, they should travel in the same direction and with a speed analogous to the electron; summing up the momentum of the LQs, it is obtained a value superimposing to the mass and the momentum of the electron they belong to.
- 7) Law of Conservation of the Energy: it is partially implied in point 6), since, as the mass of the electron should distribute more or less equally between the LQs, in the same way, for the Principle of Equivalence Mass-Energy (Einstein), the energy of the electron should divide in its probable components, however keeping its energetic value (0,511 MeV). Besides, the energy associated to the considered electronic antineutrino is so small that it does not take a significant quantity of energy from the system.

If we consider the existence of three LQs (just as the number of Qs inside the baryons), things become more complex. In this case we have a leptonic number of +3, thus in order to respect the law of conservation of the leptonic number, we need to associate another electronic antineutrino to the electron. Besides, to keep the unity of negative charge of the electron, it should not be wrong to think that two LQs carry $-2/3$ electric charge each, and one, on the contrary, carries a charge of $+1/3$. In this case, in order to respect the Law of Conservation of the Momentum, (as well as Pauli's Principle) the two LQs having equal electric charge should show anti parallel spins, the same should happen with the two antineutrinos. In this way the LQ with an electric charge of $+1/3$ should have the same spin direction of the electron it belongs to. In this way the law of conservation of the baryonic number would not be broken. As far as the law of conservation of the momentum is concerned, in the pattern with 3 LQs, their speed would not change (respect to the electron), neither would change the momentum. The latter would come from the sum of the three momentum of the three LQs, each about $1/3$ of the electron momentum in relation to their masses. Moreover it is not taken for granted that they have an equal mass. We all know that the Qs which form the most common baryons (2 Qs up and 1 Q down for the proton, or 2 Qs down and 1 Q up for the neutron) have different masses. This is true both in the pattern with two LQs and in the one with three LQs. Besides, the way the latter is presented it does not need the "colour" parameter, as it was necessary in the Qs pattern with baryons. Yet the pattern with 3 LQs appears much more complex than the one with

2 LQs, since it imposes more sub-particles inside the electron, two of which are electronic antineutrinos: this is a very unusual event, it is much less probable. It is implicit that, if what we assumed was true, the positron would be made of two anti-LQs plus an electronic neutrino (or, if ever, by three anti-LQs and two neutrinos).

If what we assumed about the existence of LQs is true, there is one more aspect to consider. As the Qs are kept together in the hadrons by the colour force, through the continuous exchange of gluons between the Qs, in the same way we can assume that with electrons electrically charged the LQs are kept together by a force which we can call "lepton force" or, in short, leptoforce (LF). According to Quantum Mechanics this force would act through a continuous exchange of messengers of the force, bosons, which would keep together the LQs in the electron. It is also possible that inside the electron there is a certain freedom of movement for the LQs: this "freedom" could explain those "strange" peculiar behaviours of the electron, as when it passes through two holes at the same time, or through a barrier. We cannot leave out the so called heavy leptons, such as the muon and the tauon. They could be made of heavier LQs, keeping unchanged the total electric charge. In this way a certain symmetry is kept with heavier Qs. The heavy leptons, however, belong to the second and third family of the Standard Model, they have a very short life, so we can consider them as extemporaneous particles. The electron, instead, is eternal.

It could be objected: why these probable LQs have never come out? Why a LQ has never been seen in particle accelerators, or in colliders? The answer could be: for the same reason it will never be possible to see a free Q, that is a particle carrying a colour charge, but just through an indistinct jet of hadrons (indeed each Q is eternally confined since a few moments after the Big Bang: that is that Q has not come out from its hadron for 13,7 billion of years). Thus, though the LF cannot be compared to the intensity, to the strength of the strong nuclear force (or colour force), also the LF apparently has not been prevaricated yet in a particle accelerator, so it has not been possible yet to point out the probable components of the electron. There is no need to try to describe the behaviour of the probable antineutrino since, as we all know, it does not leave any sign of its passage (but in very particular conditions).

Following this preamble, since it is certain that a single electron can go through two holes (close to each other) at the same time, it can be easier to imagine that it is its probable components that, "loosed" by the LF, manage for a short time to depart from each other (within the limits imposed by the LF) so that to go through the holes separately and at the same time: one of them through the first hole, the other (or

the other two) through the second one. It could be a lack of information of ours, but we do not know such an accurate and detailed experiment for the passage of the electron through three holes, placed directly on one first panel (not on a second layer placed behind one with two holes).

If what we assumed is true, we can examine with Feynman the I induced by the electrons. The latter are shot with an electron gun against a thin metal plate with two holes in it. In this experiment too, of course, there is another plate placed beyond the holes, which will serve as a backstop, next to it there is a detector, which might be a Geiger counter, or an electron multiplier connected to a loudspeaker. This allows us to hear all the clicks equal: “there are no half-clicks. The clicks come very erratically, as you have heard a Geiger counter operating” [1]. It could be seen as a typical quantum phenomenon of randomness, of irregularity, according to the different probabilities where and how the electrons will pass through the holes. “The size (loudness) of each click is always the same. If we lower the temperature of the wire in the gun, the rate of clicking slows down, but still each click sounds the same. We would notice also that if we put two separate detectors at the backstop, one or the other would click, but never both at once” [1]. Apparently this is in clear contradiction with what we said in the preamble, which is to disprove the fact that the electron may split up and go through the two holes simultaneously under shape of its LQs. Indeed, if we shoot the electrons separately, one at a time, we will never hear two simultaneous clicks in the two detectors. It is not true that the electron splits up then? No, it is, it could be possible: as soon as it goes beyond the barrier, through the two holes separately under shape of LQs, the electron “compacts” again just after, so that “whatever arrives at the backstop it arrives in *lumps*. All the *lumps* are the same size: only whole *lumps* arrive, and they arrive one at a time at the backstop. We shall say: electrons always arrive in identical lumps” [1]. It is just as once the particle has fulfilled its function, once it has reached its aim, (i.e., going through some holes in the best way possible, maybe the one with a less waste of energy) it compacts again, it “recombines” (so that to be also less vulnerable, who knows?), arriving whole to the backstop. The curve which we get is just the same of the graph of the waves, represented in Fig. 2. “Yes that is the way electrons go” [1]. Let’s just change the names, let’s substitute the I of the “intensity”, referred to the waves, with the P , indicating the “probability” an electronic lump will arrive at the backstop (see Fig. 3).

The result obtained with $P_{1,2}$ (Fig. 3d), that is the graph traced leaving both holes open, is not the sum of P_1 and P_2 (Fig. 3c), which

graph is obtained leaving open one hole at a time. Going along with the experiment with the waves, also shooting electrons we obtain a typical I pattern (Figs. 2 and 3, coincide perfectly), obtaining on the contrary a graph different from Fig. 1, when we shoot bullets. Indeed, differently from Figs. 1, “for electrons $P_{1,2} \neq P_1 + P_2$. How can such an I (interference) come about? Perhaps we should say: well, that means, presumably, that it is *not true* that the lumps go either through hole 1 or hole 2, because if they did, the probabilities should add. Perhaps they go in a more complicated way. *They split in half* and But no! They cannot, they always arrive in lumps Well, perhaps some of them go through 1, and then they go around through 2, and then around a few more times, or by some other complicated path” [1]. Well, it should be right so: as the electron passes, it can split in two or three and go through both slits. It will be argued that it is not possible: “only whole electrons always arrive at the backstop in identical lumps” [1]. Yes, it is true, the electrons arrive at the backstop in lumps, and all the same, but that is because they, probably, combine again in one structure just as soon as they pass the two slits. It is likely that the LF reunites the two (or three) LQs which probably make the electron: it puts them together, it “reins in”, so they will form a spatial whole, and every single electron will get on the backstop. “But notice! There are some points at which very few electrons arrive when *both* holes are open, but which receive many electrons if we close one hole, so closing one hole increased the number from the other. Notice, however, that at the centre of the pattern, $P_{1,2}$ is more than twice as large as $P_1 + P_2$. It is as though closing one hole decreased the number of electrons which come through the other hole. It seems hard to explain both effects by proposing that the electrons travel in complicated paths. It is all quite mysterious. And the more you look at it the more mysterious it seems. Many ideas have been concocted to try to explain the curve for $P_{1,2}$. None of them can get the right curve for $P_{1,2}$ in terms of P_1 and P_2 ” [1]. Yet, This could be easily explained introducing the concept that the electron is made of smaller particles, which can probably split away from each other (it should not be a real separation) as they go through the two holes separately. In this way it is easier to understand Fig. 3d. If we close a hole, the LQs which make each electron do not have any reason, or any opportunity, to split up, thus the electrons go through the open hole all together. The result is what we normally expect, adding P_1 and P_2 (Fig. 3c).

Let’s analyze it mathematically. “Yet, surprisingly enough, the mathematics for relating P_1 and P_2 to $P_{1,2}$ is extremely simple. For $P_{1,2}$ is just like the curve $I_{1,2}$ of Fig. 2d, and that was simple. What is going on at the backstop can be described by two complex numbers

that we can call ϕ_1 and ϕ_2 (they are functions of x , of course)" [1]. The x represents the distance from the centre of the backstop where the electron arrives. On their turn ϕ_1 and ϕ_2 are the "probability amplitude" that hole 1 or 2 is passed respectively. Whereas the probability that the event takes place is indicated with the module at the power of 2: $|\phi|^2$. "The absolute square of ϕ_1 gives the effects with only hole 1 open. That is, $P_1 = |\phi_1|^2$. The effects with only hole 2 open is given by ϕ_2 in the same way. That is, $P_2 = |\phi_2|^2$. The mathematics is the same as that we had for the water waves! (it is hard to see how one could get such a simple result from a complicated game of electrons going back and forth through the plate on same strange trajectory). We conclude the following: the electrons arrive in lumps, like particles, and the probability of arrival of these lumps is distributed like the distribution of intensity of a wave. It is in this sense that an electron behaves sometimes like a particle and sometimes like a wave" [1]. In conclusion, we learn from mathematics that the values of $P_{1,2}$ are certainly double respect to the result we get from the sum of $P_1 + P_2$. Let's have a look. Let's represent mathematically the graph in Fig. 3c:

$$P_1 + P_2 = P \quad \text{that is } |\phi_1|^2 + |\phi_2|^2 = P$$

Now, if we simplify further and give the value of 2 to ϕ , we get:

$|2|^2 + |2|^2 = 8$. That is P in Fig. 3c gives the result of 8. Whereas the graph in Fig. 3d, is mathematically expressed as follows:

$$P_{1,2} = |\phi_1 + \phi_2|^2$$

Since we have given the value of 2 to ϕ , we will have $|2 + 2|^2 = 16$. The result we get is just double. It is really surprising, but it is what actually happens. If we try to understand it, this result makes us think that with both holes open several electrons (or just a part of them, who knows?) may split up, divide, in this way going through both holes. That is, a single electron, acting through its probable components, manages to pass at the same time through both holes (then it compacts again just before arriving on the backstop). This result, exactly double than what we expect, if our interpretation is correct, may make us think that the electron is made of two sub-particles, rather than three (though we cannot be certain about it: we do not have a mathematical check with three holes).

Let us see now if an electron, splitting up, can actually pass through both holes at the same time. We use the same experiment, but just add a light source and place it just beyond the holes, at the same distance from them. Here it comes the so called "measurement paradox". What is it? If we light the two holes to watch which hole

the electron comes through, we will always see it pass for one hole at a time, in fact the pattern we get from the backstop is completely analogous to the one we have when we shoot bullets. The figure we get in fact is identical to Fig. 1. In short, if electrons are observed, they do not give the I any more. "Here is what we see: every time that we hear a click from our electron detector (at the backstop), we also see a flash of light either near hole 1 or near hole 2, but never both at once! And we observe the same result no matter where we put the detector. From this observation we conclude that when we look at the electrons we find that the electrons go either through one hole or the other. That is, although we succeeded in watching which hole our electron comes through, we no longer get the old interference curve $P_{1,2}$, but a new one, $P'_{1,2}$, showing no interference! If we turn out the light $P_{1,2}$ is restored. We must conclude that when we look at the electrons the distribution of them on the screen is different than when we do not look. Perhaps it is turning on our light source that disturbs things? It must be that the electrons are very delicate, and the light, when it scatters off the electrons, gives them a jolt that changes their motion. We know that the electric field of the light acting on a charge will exert a force on it. So perhaps we should expect the motion to be changed. Anyway, the light exerts a big influence on the electrons. By trying *to watch* the electrons we have changed their motion. That is, the jolt given to the electron, when the photon is scattered by it is such as to change the electron's motion enough so that if it might have gone to where $P_{1,2}$ was at a maximum it will instead land where $P_{1,2}$ was a minimum; that is why we no longer see the wavy interference effects. When we work up our data (computing the probabilities) we find these results: those seen by hole 1 have a distribution like P'_1 ; those seen by hole 2 have a distribution like P'_2 (so that those seen by either hole 1 or hole 2 have a distribution like $P'_{1,2}$); and those not seen at all have a wavy distribution just like $P_{1,2}$ of Fig. 3. *If the electrons are not seen, we have interference!*" [1]. But if the electrons are observed, they do not give the I any longer. "One might still like to ask: how does it work? What is the machinery behind the law? No one has found any machinery behind the law. No one can explain any more than we have just explained. No one will give you any deeper representation of the situation. We have no idea about a more basic mechanism from which these results can be deduced" [1]. What happened? Is this true only for the electrons or for all particles? Is it a general rule? Apparently it is: it is a consequence of the "paradox measurement". Thus, If we want to visualize, "to illuminate" the behaviour of a particle, especially if it is not heavy, we modify its system, since when we measure it we determine the "collapse of the wave function" of

the visualized particle. “A wave function not collapsed involves the idea, not very common, that a particle may be in more places (close to each other) at the same time, but it is not possible to become aware of, because the measurement locates a particle in a position or in another. The concept of the collapse of the wave function goes along with our experience because it assumes that the measurement forces the wave function to abandon the *quantum limbo* and choose one of the several opportunities offered by reality. Thus, if we measure the position of the electron, for instance, we make its wave function change shape suddenly. When we measure the position of the electron its peaks collapse, reducing to zero in all places where the particle are not, the probabilities increase to 100% in the only position where the particle is found with the measurement” [2]. That is why the figure of I disappears: the amplitude probabilities, related to the position where the electron could have been found (or its probable components), were compressed as the particle was observed, they reduced in one position. This is the “collapse of the wave function” or “reduction of the amplitudes”.

From what we have assumed, we can infer that the measurement, the visualization, that is lightening the electrons through the photons of the visible band, can bring the LQs near to each other, so that the distance between them is reduced (who knows? It may coincide with the “reduction of the amplitudes”). It is as though with an external input on the electron, the LF “alerted” and “assembled” the LQs, packing them in a narrow space. Why? Who knows? Maybe we can talk about something like a “surviving instinct” of the particle: as though, “touched” by a foreign body, as a sufficiently energetic photon, it may feel “threatened” and thus “withdraw into itself”. Who knows? Yet the collapse of the wave function is something real. We quote the interpretation given by Heisenberg: we do not know where the electron is before the measurement, this lack of knowledge reflects on the wave function, which describes it potentially present in different points. When we measure its position, our knowledge of it suddenly changes: in theory now we know where it is with accuracy. The sudden change of our knowledge determines as much sudden a change in the wave function which collapses and assumes the configuration of one crest” [2]. Therefore the sudden collapse of the wave function is not at all a surprising phenomenon, it is just a sharp and sudden change at a knowledge level [2].

On the other hand without measurement we have the I : it is as though the electron, not disturbed, kept the LQs with “slacken reins”, as though the string tight by the LF between the LQs was looser, (as it happens with the Qs in baryons, in the same way as the higher or lower

“tension” used by the colour force through the gluons). Thus, since the LQs are freer to move, those belonging to the same electron manage to go through the plate, passing simultaneously each for one hole, this explains the I we get. Somebody may ask: “why should it have such a behaviour? In case LQs do exist, why should they pass each for a hole? What is the goal? In case there is one. 1) The phenomenon of I is something real. 2) The I is a typical phenomenon of the wave, whereas it is a mysterious peculiarity which occurs with the electron, which is a particle, has a matter, a certain weight, though a light one. 3) If we visualize the electrons when they go through one hole or the other, just because of the “measurement paradox” the electrons pass through one hole at a time and we will not have the I . 4) if the electron is not disturbed, if the physical system is not modified, we will have the I . 5) a single undisturbed electron will be able to pass through the two holes simultaneously (contributing to the I). Maybe the most difficult point to explain is number 5. Maybe we can hope to understand this phenomenon introducing the concept of the electron being made of 2–3 minor particles. It is certain that the electrons arrive on the backstop as a whole. But as we said, once the electron, the particle, has gone beyond the obstacle, probably it does not have any reason to be spread in the space, thus it compacts again and allows us to detect it on the backstop. Why should it recombine? Who knows? May it depend on some natural rule innate in the microscopic world? Maybe we can consider it as a spontaneous behaviour rather than as a rule. Is it the spontaneous behaviour to follow the easiest and most linear path, the one with a “minor waste of energy”? May it also depend on quantum fluctuations inside the electron or on the physical composition and equilibrium of the electron, a peculiarity of the LF, which sometimes keeps the LQs more bound together and sometimes make them spread? It is as though when the electron is about to go through the layer with two close slits, the “energetic string” represented by the LF would stretch out, as the string of a bowstring. Thus the LQs — placed at the two ends of the string — manage to pass through the two holes at the same time. After that, when the necessity ends, the string loosens again, it is not stretched any more, so that the LQs gather and compact again. Who knows? It is really hard to say. We can just say that a similar behaviour is likely to happen with Qs, when the “energetic string” which keeps them together, represented by the colour force, is more stretched or more loose according to the necessities. Besides, the idea that the electron is not as simple as it appears is not something new. Protons and neutrons seem elemental particles too. Schrödinger already sensed some complexity about it. “Schrödinger advanced a hypothesis: maybe the *substance* the electrons are made

can be distributed in the space, forming the material support to the wave with which the electron can show itself. [2]. Why cannot we think that “such a substance of which electrons are made” is represented just by the LQs?

Besides, “in the last 80 years it has been proved the usefulness of the wave functions in the prevision and interpretation of several experimental data, nevertheless, there is not yet a way universally accepted to define them at a physical level: it is still a controversial point that the wave function of the electron is the electron itself, is associated to the electron and is a mathematical instrument to show the motion of the electron or the representation of what we can know about it. However what we know is that thanks to these waves the quantum mechanics has introduced, in an unexpected way, the probability in the physical laws” [2].

Anyway, the main point is to try to explain the I of the electron. “The interference effect occurs also when there is only one electron in each moment. A single electron can show the I . It can go through both slits and interfere with itself, if we may say so” [3]. Several experiments have shown that an elemental particle such as the electron, has also an undulation character. “The typical interference figure of the waves occurs also if we shoot the electrons separately, one by one, against the screen” [2]. If what we assumed is true, maybe they are the particles making the electron (which move inside it and are kept together by the LF) to propagate as waves: so some of them can pass through a slit some through the other. Thus, if two LQs separate to go each through a hole, it can be thought that the “energy tail” which keeps them together in the electron, represented by branches of the LF, may temporarily break and come together once it has gone through the holes. This tail should be “virtual”, that is not material, but energetic, since it is represented by a continuous flux of bosons which LQs exchange continuously. It should be just this continuous exchange of bosons, which we can call “leptonic bosons”, to represent the LF. That is there may be a more or less strict analogy with the gluons of the colour force: also in that case when the string tightens too much (as a consequence of an excessive removal of the Qs, within the limits of their freedom of movement) it breaks and comes together right after. After going through the holes separately, once the link between the LQs is fully restored, the electron compacts again and arrives as a whole on the backstop, equal to all the other electrons arriving there. That is the *probability amplitude* may tell us where to find the fundamental components of the particle, they move continuously inside the particle but we do not know in which part of the “field” of the particle they are in a specific moment. The *wave function* can help us foresee them

with “probability”. Besides, “it is not possible to see the wave functions directly” [2], maybe just because it is not possible to see directly the LQs, nor it is possible to isolate them (as it happens with the Qs, which can be detected only indirectly, statistically, through a jet of hadrons) [4–6]. We all know that the wave functions can be described mathematically, applying to Schrödinger’s Equation (1926):

$$H\psi(x, t) = i\hbar \frac{d\psi(x, t)}{dt},$$

where H is the Hamiltonian, \hbar is the Planck’s constant reduced or rationalized, corresponding to $h/2\pi$, i is the imaginary unity, x and t are respectively the spatial and temporal coordinates, d is the derivation index and ψ is the wave function. The latter comes from De Broglie’s intuition (1923), according to which, likewise electromagnetic radiations, material corpuscles, and particularly electrons, can in a way be associated to a wave. De Broglie’s relation is the following: $p = \frac{h}{\lambda}$ where p is the impulse or the momentum of the electron, h is the Planck’s constant and λ is the wave length of the particle. It is interesting to note that the De Broglie equation is exactly the same when we want to express mathematically the impulse of an electromagnetic wave (the photon). Besides, it can be useful to specify that: “it was Max Born who correctly interpreted the ψ of the Schrödinger equation in terms of a probability amplitude — that very difficult idea that the square of the amplitude is not the charge density but is only the probability per unit volume of finding an electron there, and that when you do find the electron some place the entire charge is there” [7]. Finding the electron means modify the system (as the measurement), thus the result is the collapse of the wave function. Hence the probable components of the electron condensate immediately, that is they gather in one place, concentrating in a very narrow space the entire electrical charge of the particle, whereas the other points of the field of the electron tend to zero, they disappear. Yes, we specified namely the *field of the electron*. Indeed, “It is possible to apply the idea of the field to the matter too. In a certain way the wave function can be interpreted as fields which action is to provide the “probability” that a certain material particle is in a certain region of the space. We can think of an electron as a particle (the one which leaves a mark on a phosphor screen or on backstop), but it can be thought or rather it has to be thought as a wave field too, which generates interference phenomena. There is a way to associate to the wave function of the electron its field (it is called *electron field*) similarly to the electromagnetic field where the role of the photon, however, is played by the electron itself” [2]. If the electron is made of smaller

particles (the LQs), kept together by the LF, it is possible to think, instead, that it is the electron itself to make in its whole the “electron field”. Besides, it seems quite incongruous that a particle interacts with an identical particle, using as a boson the same kind of particle. It could be much more logical, congruous and natural that the messenger of the electron field is the “leptonic boson” (corresponding to the gluon among the hadrons).

From what we said, it is also important “to emphasize a very important difference between classical and quantum mechanics. We have been thinking about the probability that an electron will arrive in a given circumstance. We have implied that in our experimental arrangement (or even in the best possible one) it would be impossible to predict exactly what would happen. We can only predict the odds! This would mean, if it is very true, that physics has given up on the problem of trying to predict exactly what will happen in a definite circumstance: Yes! Physics has given up. We do not know how to predict what would happen in a given circumstance, and we believe now that it is impossible that the only thing that can be predicted is the probability of different events. It must be recognized that this is a retrenchment in our earlier ideal of understanding nature. It may be a backward step, but no one has seen a way to avoid it. We make now a few remarks on a suggestion that has sometimes been made to try to avoid the description we have given: *perhaps the electron has some kind of internal works — some inner variables — that we do not yet know about*. Perhaps that is why we cannot predict what will happen. If we could look more closely at the electron, we could be able to tell where it would end up. So far as we know, that is impossible. We could still be in difficulty. Suppose we were to assume that *inside the electron there is some kind of machinery* that determines where it is going to end up” [1]. Thus we know that Feynman too started to “suspect” that the electron was not such a simple particle, elemental, he suspected that “inside the electron there could be some kind of machinery, some internal variable”, able to guide the path. Therefore, our hypothesis would not be conceptually too far. “That machinery must also determine which hole it is going to go through on its way” [1]. We may say that the electron follows the way which seems the easiest, the most linear and the most natural. If it finds only one hole open, it will go through this one. If it finds two open holes, it will probably, most of the times, make the *I*. How? This is the main question which inspired this article. It is very well proved by now that the electron induces the *I*, just as a wave, yet it is a particle. But this is explainable; it happens with other particles too, De Broglie guessed that as early as 1924. Besides, the energy of the electron

corresponds to an electromagnetic wave of a certain frequency such as the γ ray, in which it transforms when it collides with its antiparticle. In short, the fact that the electron behaves like a wave does not cause any astonishment! We know this. Yet one may wonder: how can a *single particle* — which is considered “elemental”, that is it cannot be divided in further smaller particles — *pass through two different holes*? As it had the gift of ubiquity. It seems more logical to infer that the electron splits temporarily in its probable components. The latter will go through the two holes contemporarily: a LQ through a hole, the other (or the other two) through the second one, then they will compact again in the electron and get on the backstop as a whole, as one particle. It could be an inborn characteristic of the electron to be able to go through two holes contemporarily, if it is true that when the electron travels the whole electron system, with its field and contents, moves. We could think that it is the electron field to go through the two holes contemporarily: it would precisely pass through them as a “field”. Besides we read: “But we must not forget that what is inside the electron should not be dependent on what we do, and in particular upon whether we open or close one of the holes. So if an electron, before it starts, has already made up its mind which hole it is going to use, and where it is going to land” [1]. Why should the electron have decided a priori which hole go through? It should be more likely that along the way the electron “feels” which way is easier; maybe it will respect the law of the minor waste of energy — “the Principle of Least Action” [8] — following the closest path, just as the photon follows the shortest and straightest path: “that out of all possible paths that it might take to get from one point to another, light takes the path which requires the *shortest time*” (Fermat’s Principle of Least Time, 1650) [9]. Or, in case the electron finds all “blocked up”, it will probably try, with “great effort”, that is with remarkable waste of energy, after several attempts, to exploit the “tunnel effect”. Let’s analyze shortly this phenomenon.

The “tunnel effect” (TE) is one of the oddest properties of the quantum world. It is a typically quantum effect: it cannot be calculated and justified applying the laws of the classical physics, the Newtonian one. The problem that this effect manages to solve is the calculus of probability that a particle, having a certain energy, manages to get over a barrier with an energetic potential higher than the energy of the particle itself. In this case, in fact, the laws of classical mechanics immediately give us a null result of probability. On the contrary, the formalism of the quantum mechanics allows us to calculate it confirming the experimental reality. Also in this case it is useful to apply Schrödinger equation, otherwise the phenomenon

would be unexplainable: from the calculus it appears the probability, small but well defined, to penetrate the barrier and, if this is not infinite in height and thickness, to pass through it and get beyond it. It is important to mention that the lower the height and/or the thickness of the barrier, as well as the lighter the particle, the higher the penetrability. This is why the electron is the most suitable particle to use the TE. It is because sub-atomic particles do not have to be considered as rigid objects, but a sort of *clouds* (the “electronic cloud”, the electron field), thus they are able to go through a thin barrier.

Thus, using the TE a particle, an atomic nucleon, has a probability not null to pass through a potential barrier even though the particle has a kinetic energy lower than the maximum height of the potential barrier. That is, a barrier which is absolutely impenetrable from a classical point of view (a particle which bumps against a barrier without having enough energy to pass it will not be able to go through it in any way), becomes penetrable for the quantum mechanics. What happens is that the penetration of a potential barrier, which is unexplainable from a corpuscular point of view, becomes possible as a undulation phenomenon, that is when the particle leaves its corpuscular aspect and gets a typically undulation behaviour. All things considered, it seems that the particle manages to go through the barrier only “as a wave”.

The TE has a wide range of applications, however it is mainly used in the electronic field. We owe to Esaki (1957) the construction of the first “tunnel diode”, which he made working on germanium semiconductor diodes, characterized by a very thin thickness of the junction. Esaki realized that electrons were able to penetrate very thin barriers (with a thickness around one hundred atoms) digging a tunnel in the layer they had to go through. It was a completely new phenomenon, impossible according to the laws of the classical physics. Esaki’s diode has been widely used with amplifiers, oscillators, trigger circuits, electronics switches. It was Josephson (1962) [10] who created a particular junction in semiconductor materials (Josephson Junction), which allows the transit of the electrons through the TE, with low consumptions and without releasing too much heat. “A very interesting situation was noticed by Josephson while analyzing what might happen at a junction between two superconductors. Suppose we have two superconductors which are connected by a thin layer of insulating material. Such an arrangement is now called a *Josephson junction*. If the insulating layer is thick, the electrons can’t get through; but if the layer is thin enough, there can be an appreciable quantum mechanics amplitude for electrons to jump across. This is just another example of the quantum-mechanical penetration of

a barrier. Josephson analyzed this situation and discovered that a number of strange phenomenon should occur" [7]. Josephson junction is very much used in semiconductors, i.e. in the memory cells of the computers. The last use of the TE is at the IBM of Zurich, a TE electronic microscope.

Though calculations confirm that such a peculiar phenomenon, as the TE, can occur, we do not know exactly how it actually happens; that is in which way it takes place. It is widely thought that, through a quantum fluctuation mechanism, the particle borrows the lacking energy, respect to the potential of energy to get through. As soon as the particle goes through the barrier, it will immediately release the mentioned energy. This mechanism is allowed by the quantum mechanics, the shortest the span of the energetic loan, the higher the probabilities that this phenomenon can occur. All clear! What is not clear at all is how the particle materially gets through the barrier. The borrowed energy can be worth both for the electrons and for the hadrons. The TE is at the bottom of some radioactive phenomena, such as α emission. That is even two protons and two neutrons, bound in one helium nucleon, thanks to the TE manage to get free from the nucleon with a high atomic weight which they belong to. This phenomenon, however, occurs very rarely, as we learn from the radioactive half time tables. Indeed, "since the particle α has two protons, it is pushed back by the whole charge of the protons thus the particle tries to escape but the wall around the nucleon prevents the particle from getting free. In this way the particle is trying to make a tunnel. It wants to penetrate the barrier because it wants to escape and of course, sooner or later it will manage to do that. How long will that take? Maybe some thousand of years, but we cannot be sure about that. It may *probably* take it thousands of years to escape, but it may also get free in any moment. There is no way to be sure about it: it is just a matter of probability" [3].

However, there must be a reason if the TE is more likely to happen with lighter particles such as the electron rather than with bigger ones. Someone may say that this happens because, compared to heavier particles, "the very small size of the electron implies that it undergoes a great influence of its associated wave. It is the *undulation nature of the electron* to allow it to go through the barrier" [11]. Apparently it is the very small mass of the electron to allow it to have a wider associated wave, this makes its undulation behaviour, as well as the TE, easier. On the contrary "those particles having a much bigger mass than the electron, have a much shorter *associated wave*" [12]. This limits a lot its potential undulation behaviour, which can be verified by a TE rarer than the electron and more and more infrequent as the mass of the

particle increases. Yes, the electron may have an undulation behaviour but it is still a “matter wave”; it has a mass of $0,511/c^2$ MeV, that is $9,11 \cdot 10^{-28}$ gr It will be said: the lacking energy is borrowed through a quantum fluctuation mechanism. That’s all right! Thus the electron will go through the barrier as a wave. It is clear but only till a certain extend. What happens to the mass while the particle goes through the layer? It could be said: the particle takes it with itself; besides the electron is the lightest particle having a mass. Right, but it still has a mass, it occupies a certain surface: how can it go through a barrier? But if we consider the electron as having 2 (or 3) smaller particles, it will be easier to understand this phenomenon. Maybe a particle with a mass half (or one third) of the electron may be able to pass through the barrier more easily. The mechanism should be analogous to what may happen with the I. That is, with the TE the electron may penetrate the barrier as a “field” (the electron field), separating its LQs (or rather getting “spread” with its LQs), in order to reduce the impact surface which will collide against the layer, in this way increasing the probabilities to find a passage through the “meshes” of the wall.

It is useful at this point to make a consideration. When we earlier tried to measure the mass of the probable LQs, we divided the mass of the electron and what may be the number of particles which could make the electron. However it may be not accurate and safe to follow this method, it might take us on the wrong way. If we measure the mass of what makes the proton (2 up Qs and 1 down Q), we get the value of 12 MeV. Yet, the mass of the proton is 938,9 MeV! That is the whole mass of the 3 Qs making the proton is only about one eightieth of the mass of the proton. In the same way (very indicatively), if the electron is made of LQs, we may have that the total mass of the LQs of every single electron may represent only about one eightieth of the mass of the electron. But we have to suppose that there may be 2 (or even three) LQs for each electron. Thus we should divide the mass of the electron for 160 (or 240). In the first case (maybe the most probable) we have: $511000 \text{ eV}/160 = 3193,75 \text{ eV}$. That is, if there are 2 LQs, for each of them we should have a mass of 3,193 KeV. In the second case we would have: $511000 \text{ eV}/240 = 2129,166 \text{ eV}$. That is, if there are three LQs for each electron, we have a mass of 2,129 KeV (it is implicit that every time we express the mass in energetic equivalent, at the denominator it is understood c^2 , that is the square of the speed of light).

Incidentally, we need to consider that where the electron weights 0,511 MeV, the Q up (the lightest) weights 3 MeV (a MeV is a million of electron volt), and the Q down is 6 MeV. This means that an electron

weights $1/6$ of a Q up and $1/12$ of a Q down. Hence, in case there are only two LQs which make the electron, we have that one LQ weights one thousand of the lightest Q (and about $1/320000$ of the mass of the proton).

Why all this? What is the use of these calculations and rates? Because a very small and light LQ, probably just $1/160$ of the mass of the electron, or even something like one thousand of the weight of a Q up, makes it “closer” to a wave, it increases the probabilities to “find a path in the layer” to pass through. This makes more acceptable and/or likely what we thought. If we think about it, the energy of a probable LQ, which may approximately correspond to 3200 eV , is just the energy of a “soft” x ray. That is a LQ would have the same “size”, at least energetic, of “less penetrating” x rays. Moreover, to that energy will have always to correspond a determined and specific frequency and wave length. As Giacconi and Tucker remind us, “x rays have the right wavelength to see the microscopic world” [13]. Thus, we may say, the electronic microscope allows us a vision as with x rays. It seems really an analogy, a real correspondence, between the behaviour of the electron in its all, and an x ray. That is a LQ is apparently superimposing with an EMW of a certain frequency. Thus, the energy of a LQ could explain its undulation behaviour, how the electron manages to appear and behave like a wave too.

Besides, the dimension of the particle must have a great meaning to the fulfilment of the TE. To confirm this we have the great difference in probability that this phenomenon may happen: it is more likely for the electron ($0,511\text{ MeV}$), and less likely for the proton or neutron (respectively 938 and $940/c^2\text{ MeV}$). It would take some thousand years to a particle α to pass spontaneously through a barrier: Its mass is $6,6 \cdot 10^{-24}\text{ gr}$. If we convert it in MeV, we have that the mass lost to get the fusion of a helium nucleon is about $0,63\%$ of the sum of the particle masses making it (two nuclei of deuterium) [12], that is the mass of a helium nucleon is about 3735 MeV . It could be objected: let's consider that the electron may go through the barrier as we assumed, but how can a baryon, or even a particle α , to penetrate the barrier in the same way of an electron? Well, the hadrons are made of Qs, this is not a hypothesis, it is verified! Thus it is possible that Qs use the same mechanism foreseen for the LQs. In this case the bigger difference in weight (Qs are certainly heavier than the hypothetical LQs), may explain why a particle made of Qs passes through a barrier with less probabilities than a LQ. Besides, we should also consider the difference in strength between the colour force and the LF. It should be much more likely that Qs (within their limits of movement) are kept “in check”, thus it is very difficult that a hadron has an undulation

behaviour, indeed, as we said earlier, it has a “much shorter *associated wave*” [12].

3. CONCLUSIONS

Summing up, there is a still mysterious phenomenon: the I induced by electron. It is not explainable from the point of view of the classical mechanics, but it is solved from a mathematical point of view, using the formalism of the quantum mechanics and applying to the phenomenon the solution of Schrödinger’s equation. The quantum solution to the problem is clear and elegant, especially from a mathematic point of view, but it leaves us some perplexity as for the understanding of the real fulfilment of the phenomenon.

It has been expressed a hypothesis to understand the I induced by the electron: it is really a peculiar phenomenon. According to Quantum Mechanics “when we talk about a situation as the electron going through the slits, we describe it with an *amplitude*. It is something similar to the waves; it is often called *wave function*. The *amplitude* can go through both slits and produce I , just as the water waves. But then where are the particles? Which slit do they actually go through? The *amplitude* does not tell you anything about it” [3]. Maybe if we consider that the electron might be made of smaller particles, just as the baryons and mesons, (safeguarding the integrity of the unitary negative charge, the conservation of the leptonic number, the conservation of the angular momentum etc.), this may help us understand how a single electron can go through two close holes at the same time. Besides, there is another very particular quantum phenomenon, which is explained with the quantum mechanics too, and which gives us some perplexities about how it really happens, that is the TE. The energy borrowed through spontaneous quantum fluctuations, though justifying the energetic compensation given in favour of the electron (or another particle) and allowing it to go through the barrier more easily, does not clarify how actually the electron pierces the barrier.

It can be easier to understand the TE phenomenon if we consider that it may have the same mechanism of the I . That is, if it is possible to divide the electron in smaller particles, as to be superimposing with a low energy x ray (soft x ray), as a consequence the reduction of the surface of the particle colliding with the barrier could make the penetration of the barrier itself easier.

On the other hand, it should not appear too unreal the idea that a LQ can be superimposing with an x ray, both as analogous energetic value, and in its real behaviour, that is as a wave. We all know

that the electron too is superimposing with an electromagnetic wave, though more energetic than an x ray: a γ ray in its case. Indeed, just as suggested by Dirac, in the collision between an electron and its antiparticle, there is the annihilation of the particles themselves, with a complete transformation of their mass in a couple of γ rays, having an energy exactly equal to the combined mass of the annihilated particles" [12]. Therefore, to an electron having a mass of $9,11 \cdot 10^{-28}$ gr which equivalent energetic is 0,511 MeV, will have to correspond an electromagnetic wave of $5,11 \cdot 10^5$ eV. Now, since in some circumstances we can consider the electron equivalent to a γ ray of a definite energy, we should be able to explain its undulation behaviour: both when the electron induces I , typical of the waves, and when the electron utilizes the TE (a phenomenon much easier to make for a wave than for a particle). Thus, there should not be any more any reason to introduce the concept of the LQs. The observation is right, though it does not tell us yet how a single particle manages to pass *simultaneously* through two holes: it seems more logic and convincing that in this case the electron splits, for a moment, in its probable components.

This is a willing attempt to understand some of the most unexplainable quantum phenomena. It is not possible to demonstrate this hypothesis; we should wait for more powerful colliders, such as the Large Hadron Collider, which is supposed to be prepared in Geneve in three years. Beyond any disprove or confirmation, the concept we showed should not be in contrast with what we know from Physics.

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