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ANALYSIS OF GUIDED MODES IN ASYMMETRIC LEFT-HANDED SLAB WAVEGUIDES

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Abstract—The guided modes in a left-handed material (LHM) asymmetric slab waveguide are studied in this paper. Dispersion properties of electromagnetic guided waves are discussed by introducing three normalized parameters. The guidance conditions of guided modes in waveguide are determined by using a graphical method. Then we put emphasis on the surface wave modes with respect to different waveguide parameters and structures. The power flux in LHM asymmetric waveguide according to the *Poynting* vector is investigated in the end.

1. INTRODUCTION

Left-handed material (LHM), whose permittivity ε and permeability μ are simultaneously negative, has been firstly introduced by Veselago [1] in 1968. It posses a number of peculiar properties that different from normal dielectrics, such as *Negative Refraction, Reversed Cerenkov Radiation* and *Reversed Doppler Effect*. Pendry [2, 3] proposed the possibility that the LHM can be fabricated by some novel artificial microstructured metallic materials. After the fabrication of LHM over the microwave frequency by Smith [4, 5] in 2001, it opens up a heat discussion around the world because of its diverse potential application. One of attractive studies about LHM is the propagation of electromagnetic waves in LHM waveguides.

Zhang et al. [6] theoretically investigated the electromagnetic properties of multi-layer LHM waveguide, and Kong et al. [7] demonstrated a unique negative lateral shift for a Gaussian beam reflected from a grounded LHM slab, which is distinctly different from a shift caused by a regular grounded slab. In LHM waveguides, there exists a sort of unique electromagnetic waves termed as "*surface waves*" [8] which cannot exist in normal waveguides. The surface waves appear when the wave number becomes purely imaginary. Cory et al. [9] investigated the electromagnetic guided waves of LHM symmetric slab waveguide. Shadrivov et al. [10] studied the linear guided waves propagating in a slab waveguide made of LHM, and they revealed many peculiar properties, including the absence of fundamental modes and the sign-varying energy flux. In particular, they predicted the existence of novel types of guided waves with a *dipole-vortex structure* of the *Poynting* vector. Kim et al. [11] theoretically analyzed guidance characteristics of circular LHM rod waveguide including the dispersion and power confinement characteristics. Mohammed et al. [12] derived and analyzed the anomalous dispersion relation of grounded LHM slab waveguide and compared with that of conventional grounded dielectric slab waveguide. They also got the cut-off frequencies of guided modes in such a waveguide. These investigations of LHM waveguides can pave the way for the design of new filters and resonances. S. F. Mahmoud [13] also analyzed the same structure and showed that, unlike a slab with positive parameters, the dominant mode can have evanescent fields on both sides of the interface between the slab and the surrounding air. J. He and S. He [14] studied the dispersion properties of the guided modes propagating along a dielectric slab waveguide with a left-handed material substrate and they also showed that both the oscillating and surface modes can propagate very slowly along such a waveguide. If the thickness of the core layer is chosen appropriately, the propagation speed of the guided waves can even approach zero. K. L. Tsakmakidis et al. [15] identify and classify all surface plasmon polarition (so-called SPP) eigenmodes supported by generalized asymmetric slab heterostructures. They pursued a rigorous analytical study and proved that a total of 30 solutions to the involved characteristic equation giving the SPP eigenmodes can exist for all choices of the refractive index distribution, constitutive parameters ε and μ, and the thickness of the core.

In this paper, we introduce 3 normalized parameters — the normalized frequency V, the normalized propagation constant $p2$ and the rate of asymmetric Δ — to discuss the guided modes of an asymmetric slab waveguide with a core of LHM sandwiched between two normal dielectric slabs. Dispersion properties of electromagnetic waves including guided modes and surface wave modes with different waveguide parameters are discussed. We determine the guidance conditions of guided modes in waveguide by using a graphical method and find that the guided mode in LHM waveguide is very different from that in normal dielectric waveguide such as the absence of fundamental mode. Then we put emphasis on the discussion of surface wave modes with respect to different waveguide parameters and structures by analyzing the dispersion equation using normalized parameters hereinbefore and we found that the existence of the guided modes depends much on the parameters and structures of the waveguide. We lastly analyze the power flux in waveguide according to the *Poynting* vector by introduce *the normalized power*. We find that when the frequency of guided mode reaches to a certain value, the net power flow of the guided mode can be 0.

2. ANALYSIS OF GUIDED MODES

Consider an asymmetric waveguide structure with a core of LHM sandwiched between two normal dielectric slabs as shown in Figure 1. LHM layer in region 2 has the thickness of d, and the two claddings marked with region 1 and region 3 are normal dielectric slabs that have positive $\varepsilon_1, \mu_1, \varepsilon_3$ and μ_3, respectively. The coordinate is shown in Figure 1. We also assume that $\varepsilon_3\mu_3 > \varepsilon_1\mu_1$, which means the refractive index of region 3 is bigger than that of region 1. Taking the transverse electric (TE) mode into consideration (results for TM waves can be obtained from duality), the electric field is polarized along the y-axis. We can see that the electric field E_y can be expressed as $E_y(x, z, t) = E(x)\exp[-i(\omega t - \beta z)]$, where β is the propagation constant along the z-axis. The electric fields in three regions can be written as

$$E(x) = \begin{cases} A_1 \exp[-\alpha_1(x-d)], & x > d \\ A_2 \cos(k_2 x + \varphi), & 0 \le x \le d \\ A_3 \exp(\alpha_3 x), & x < 0 \end{cases} \tag{1}$$

where $\alpha_1 = \sqrt{\beta^2 - k_0^2 n_1^2}$ and $\alpha_3 = \sqrt{\beta^2 - k_0^2 n_3^2}$ are the evanescent rates in region 1 and region 3. $k_2 = \sqrt{k_0^2 n_2^2 - \beta^2}$ is the transverse wave number in region 2, which can either be real or imaginary. n_1, n_2, and n_3 are the refractive index in three regions separately. k_0 is the wave number in vacuum, $\varphi = \tan^{-1}(-\mu_2\alpha_3/(\mu_3 k_2))$ is the phase shift of guided mode in region 2.

To find the guidance conditions, the boundary conditions at $x = 0$ and $x = d$ are applied, and the dispersion equation of guided modes can be obtained as follow

$$k_2 d = m\pi + \tan^{-1}\left(\frac{\mu_2\alpha_1}{\mu_1 k_2}\right) + \tan^{-1}\left(\frac{\mu_2\alpha_3}{\mu_3 k_2}\right) \tag{2}$$

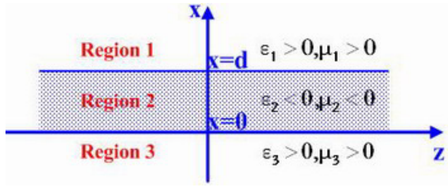

Figure 1. Geometry for an asymmetric LHM slab waveguide. Region 2 is LHM, regions 1 and 3 are normal dielectrics.

Here we introduce three normalized parameters

$$\begin{cases} V = k_0 d\sqrt{n_2^2 - n_3^2} \\ p^2 = \dfrac{N^2 - n_3^2}{n_2^2 - n_3^2} \\ \Delta = \dfrac{n_3^2 - n_1^2}{n_2^2 - n_3^2} \end{cases} \tag{3}$$

where V is the normalized frequency which increases with the frequency increases. p^2 is the normalized propagation constant whose value range is $[0, 1]$, when $p^2 = 0$, the guided mode vanish. N is the effective refractive index $N = \beta/k_0$. Δ is a parameter depicts the rate of asymmetric in a three layers waveguide. When $\Delta = 0$, it means a symmetric waveguide. From these transformations we can get a new normalized dispersion equation:

$$V(1-p^2)^{\frac{1}{2}} = m\pi + \tan^{-1}\left[\frac{\mu_2}{\mu_3}\left(\frac{p^2}{1-p^2}\right)^{\frac{1}{2}}\right] + \tan^{-1}\left[\frac{\mu_2}{\mu_1}\left(\frac{p^2+\Delta}{1-p^2}\right)^{\frac{1}{2}}\right] \tag{4}$$

For a given parameters, the dispersion curve is shown in Figure 2. We can see from Figure 2 that for a given frequency V, we can get p^2, and then the propagation constant of guided mode is determined and that with the increase of V, the LHM waveguide can accommodate more and more high-order guided modes. When V is small enough, all the guided modes in waveguide are forbidden. That is to say each guided mode in LHM waveguide has a cutoff frequency. For a certain range of V ($V(p^2 = 1) \sim V(p^2 = 0)$), the mode TE_1 appears. The TE_1 guided mode can not coexist with any other higher-order modes. This is a novel property indicate a possible bandpass filtering effect of the LHM waveguide without apparent uses of resonant structures. We can also see that the guided mode TE_0 cannot be found for any V in the

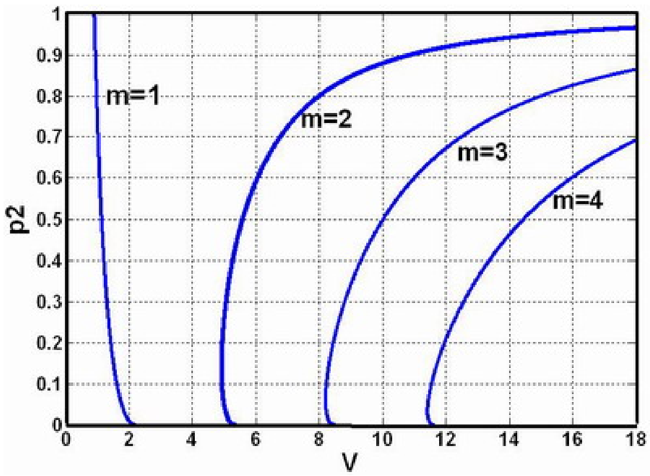

Figure 2. The normalized dispersion curve of TE guided mode with $\varepsilon_1 = 1,\ \mu_1 = 1,\ \varepsilon_2 = -2,\ \mu_2 = -2,\ \varepsilon_3 = 2,\ \mu_3 = 1$.

LHM waveguide. It's very different from the conventional dielectric waveguides whose TE_0 modes can exists if beyond a cutoff frequency.

The distributions of electric field of guided mode TE_1, TE_2 and TE_3 are shown in Figure 3. The properties of them are very similar to those of conventional guided modes in normal dielectric waveguides. As the figure show, the electric field is oscillatory in region 2 while it becomes evanescent outside the region 2. m represent the number of intersections between the fields and the x-axis in region 2. We can also see from the figure that the peaks of transverse distribution are asymmetric because of its asymmetric structure.

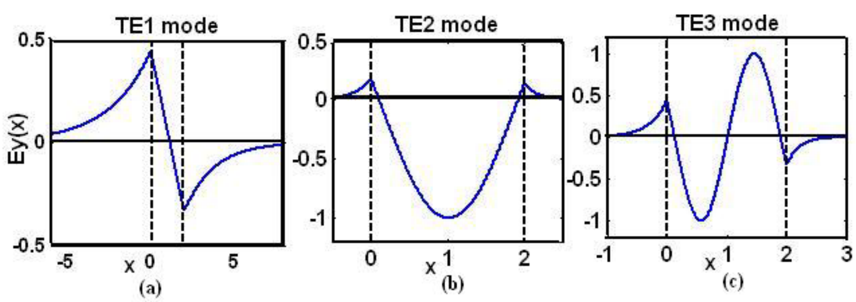

Figure 3. Distribution of amplitudes of electric field components of guided modes, where (a) $V = 1.1363$ (b) $V = 10$ (c) $V = 10$. m represent the number of intersections between the fields and the x-axis in region 2.

3. ANALYSIS OF SURFACE WAVE MODES

In LHM waveguide, there exists a sort of unique electromagnetic waves termed as the "surface waves" that decay exponentially away from, but propagate along, the two dielectric interfaces. As we know, the TE surface wave modes exist when the magnetic permeability of two dielectrics have different signs, while for TM surface wave modes the electric permittivity of two dielectrics should be of different signs.

When the surface wave modes exist in the LHM waveguide, the propagation constant of region 2 is purely imaginary ($k_2 = i\alpha_2$). Substitute it into the dispersion equation, we get

$$\alpha_2 d = \tanh^{-1}\left[-\frac{\left(\frac{\mu_2\alpha_1}{\mu_1\alpha_2}\right)+\left(\frac{\mu_2\alpha_3}{\mu_3\alpha_2}\right)}{1+\left(\frac{\mu_2\alpha_1}{\mu_1\alpha_2}\right)\left(\frac{\mu_2\alpha_3}{\mu_3\alpha_2}\right)}\right] \tag{5}$$

Here, the $\tan^{-1}$ in equation (2) is replaced by $\tanh^{-1}$. We discuss the existence of surface wave modes in LHM waveguide due to different waveguide parameters as follow.

3.1. $\varepsilon_2\mu_2 > \varepsilon_3\mu_3$

We introduce three normalized parameters V, p^2, Δ analogous to the analysis of guided modes above, then Equation (5) becomes

$$V(p^2-1)^{\frac{1}{2}} = \tanh^{-1}\left[\frac{\left(-\frac{\mu_2}{\mu_1}\right)\left(\frac{p^2+\Delta}{p^2-1}\right)^{\frac{1}{2}}+\left(-\frac{\mu_2}{\mu_3}\right)\left(\frac{p^2}{p^2-1}\right)^{\frac{1}{2}}}{1+\left(-\frac{\mu_2}{\mu_1}\right)\left(\frac{p^2+\Delta}{p^2-1}\right)^{\frac{1}{2}}\left(-\frac{\mu_2}{\mu_3}\right)\left(\frac{p^2}{p^2-1}\right)^{\frac{1}{2}}}\right] \tag{6}$$

Here the value range of p^2 is $[1, +\infty)$. For a given waveguide parameter, we can discuss the existence of surface wave modes with two different cases as follow:

(1) $-\mu_2/\mu_3 \geq 1$. For a given waveguide parameter, we can plot the dispersion curve of surface wave modes as shown in Figure 4. Under this circumstance there exists only one surface wave mode below a certain frequency. This frequency can be known by setting $p^2 = 1$. The subgraph represents the distribution of amplitude of electric field in waveguide. We can see that the amplitude of electric field in all regions decay exponentially away from the dielectric interface. Since there isn't any intersection between the distribution curve and the x-axis in region 2, we can name this mode as the "*even mode*".

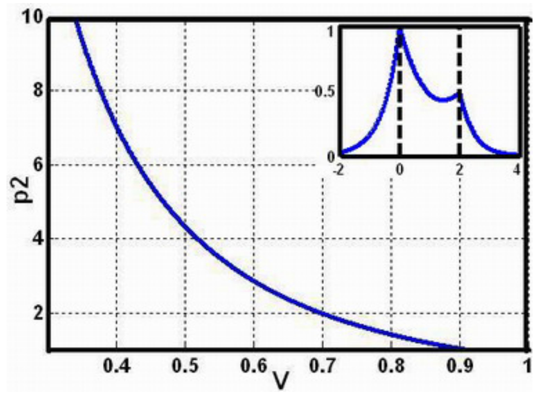

Figure 4. Normalized dispersion curve of the surface wave modes with $\varepsilon_1 = 1,\ \mu_1 = 1,\ \varepsilon_2 = -2,\ \mu_2 = -2,\ \varepsilon_1 = 2,\ \mu_1 = 1,\ d = 2\,\text{cm}$. The subgraph represents the distribution of amplitudes of electric field.

(2) $-\mu_2/\mu_3 < 1$. For a given waveguide parameter, we can plot the dispersion curves in Figure 5 with different value of $-\mu_2/\mu_3$. We can see from the figure that even mode exists at all value of V, however another mode which called the "*odd mode*" appear only when a threshold V is exceeded. As shown in figure, when $-\mu_2/\mu_3 = 0.5$, $V = 4$, there exist two surface waves solutions marked with A and B. A represent the even mode whose amplitude of electric field is shown in the left subgraph in Figure 5 and B represent the odd mode whose amplitude of electric field is shown in the right subgraph in Figure 5. We can also see from the figure that the cutoff frequency of odd mode increases when the value of $-\mu_2/\mu_3$ decreases.

3.2. $\varepsilon_2\mu_2 < \varepsilon_3\mu_3$

Here, we introduce $U = -iV$, p^2, Δ, where the value of V is purely imaginary, then the dispersion equation becomes

$$U(1-p^2)^{\frac{1}{2}} = \tanh^{-1}\left[\frac{\left(-\frac{\mu_2}{\mu_1}\right)\left(\frac{p^2+\Delta}{p^2-1}\right)^{\frac{1}{2}} + \left(-\frac{\mu_2}{\mu_3}\right)\left(\frac{p^2}{p^2-1}\right)^{\frac{1}{2}}}{1+\left(-\frac{\mu_2}{\mu_1}\right)\left(\frac{p^2+\Delta}{p^2-1}\right)^{\frac{1}{2}}\left(-\frac{\mu_2}{\mu_3}\right)\left(\frac{p^2}{p^2-1}\right)^{\frac{1}{2}}}\right] \tag{7}$$

Here equation (7) is different from equation (6), and the value range of p^2 is $(-\infty, 0]$. For a given waveguide parameter, we can also discuss the existence of surface wave modes in two different cases as follow:

(1) $-\mu_2/\mu_3 \geq 1$. For a given waveguide parameter, the dispersion curve is shown in Figure 6(a). We can see from the figure that the even mode exists at all value of U, while there is no odd mode in waveguide.

(2) $-\mu_2/\mu_3 < 1$. For a given waveguide parameter, the dispersion curve is shown in Figure 6(b). We can see from the figure that there

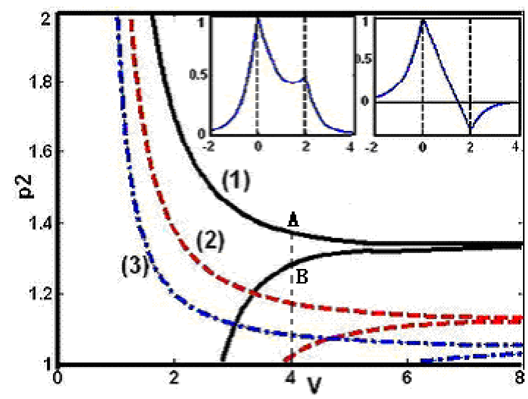

Figure 5. Normalized dispersion curves of the surface wave modes with $n_1 = 1$, $n_2 = -2$, $n_3 = \sqrt{2}$ where (1) $-\mu_2/\mu_3 = 0.5$ (2) $-\mu_2/\mu_3 = 1/3$ (3) $-\mu_2/\mu_3 = 0.2$. The subgraph represents the distribution of amplitudes of electric field.

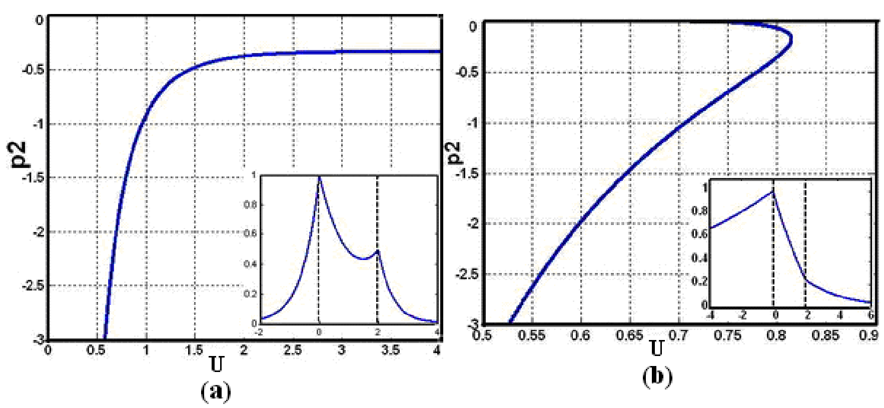

Figure 6. Normalized dispersion relationship of the surface wave modes with $n_1 = 1$, $n_2 = -\sqrt{2}$, $n_3 = 2$, (a) $-\mu_2/\mu_3 = 2$ (b) $-\mu_2/\mu_3 = 0.5$. The subgraph represents the distribution of amplitudes of electric field.

only exists even mode below a certain U and when the value of U falls into a certain range, there exist two even modes that have different sign of power flux. There is no odd mode in waveguide at any value of U.

From this section we can come to the conclusion that in LHM asymmetric slab waveguide there exists surface wave mode, and that it is decay exponentially away from dielectric interfaces of core and claddings. The existence of the surface wave mode depends on the selection of parameters and structures of waveguides. The surface wave mode is a novel wave mode that can not established in normal dielectric materials.

4. POWER FLUX IN THE WAVEGUIDE

Energy flux in waveguide is an important parameter. We know that the energy flux is characterized by the Poynting vector averaged over the period T and it can be defined as $S = \frac{1}{2}\mathrm{Re}\,[E \times H^*]$. For a monochromatic plane wave, the energy flux is directed along the z-axis. We can analyze the transverse profile of S_z of guided modes for given waveguide parameters. We know that $S_z = \beta E^2(x)/2\omega\mu(x)$, so the normalized distribution of energy flux is shown in Figure 7. Here we plot the distributions of TE_1, TE_2 and TE_3 guided modes. We can see from the figure that the energy distribution of LHM asymmetric slab waveguide is very different from that of normal dielectric waveguide. The direction of energy flux in region 2 (with $\mu_2 < 0$) is opposite to that in regions 1 and 3 (with $\mu > 0$). That is to say the *Poynting* vector in LHM layer is anti-parallel to the phase-velocity vector.

The total power flux in LHM asymmetric waveguide can be obtained as an integral of the *Poynting* vector

$$P = P_1 + P_2 + P_3 = \int_d^{+\infty} S_z dx + \int_0^d S_z dx + \int_{-\infty}^0 S_z dx \qquad (8)$$

where P_1, P_2 and P_3 are the power flux in region 1, 2 and 3. Substitute equation (1) into equation (8), we can get the total power flux in region 1, 2 and 3 separately

$$\begin{cases} P_1 = \dfrac{A_2^2 \beta \cos^2(\alpha_2 d + \varphi)}{4\omega\mu_1 k_1} \\ P_2 = \dfrac{A_2^2 \beta}{8\omega\mu_2\alpha_2}[\sin(2\alpha_2 d + 2\varphi) - \sin(2\varphi) + 2\alpha_2 d] \\ P_3 = \dfrac{A_2^2 \beta \cos^2 \varphi}{4\omega\mu_3 k_3} \end{cases} \qquad (9)$$

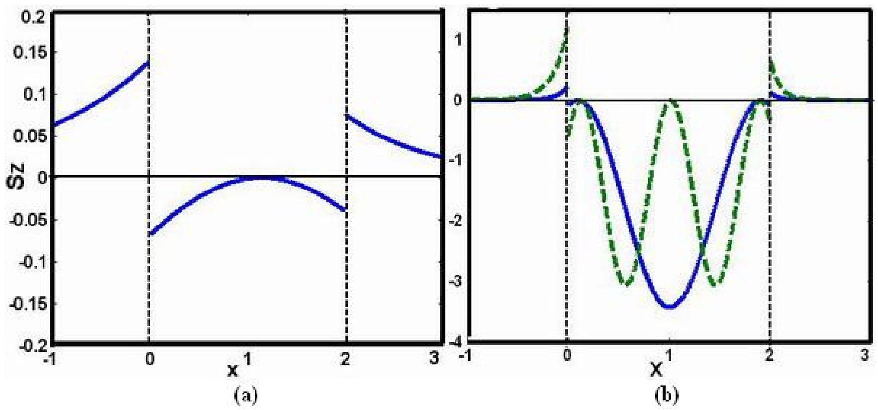

Figure 7. Distribution of energy flux along the transverse profile of waveguide for a given waveguide parameters, where (a) TE_1: $V = 1.1363$ (b) the dashed line is TE_2, the real line is TE_3: $V = 10$.

To investigating the net power flux in waveguide we use the *normalized power* which was introduced in Ref. [10]. The *normalized power* flux can be expressed as

$$\overline{P} = \frac{P_1 + P_2 + P_3}{|P_1| + |P_2| + |P_3|} \tag{10}$$

When $\overline{P} < 0$, the net total power flow of the guided mode is anti-parallel with the direction of phase flow and this wave is called the backward wave. We can get the opposite result when $\overline{P} > 0$ and we call this wave the "*forward wave*". We take the TE_2 TE2 guided mode as shown in Figure 8, which is zoomed in partly from Figure 2. We can see from Figure 8 that when $m > 1$ there exist a negative slope portion in dispersion curve which is very different from conventional waveguide. That is to say when V falls into a certain region, there are two TE_m modes. We call this region "the special region" shaded in Figure 8. Just as shown in figure, when $V = 5$ there exists two solutions of guided mode marked with B and C, and the point $B$is forward wave and the point C is backward wave.

We use the normalized power $\overline{P}$ to analyze the net power flux in waveguide with different guidance solutions in Figure 8. From calculation we know that when the guidance solution falls between A and D, the normalized power $\overline{P} > 0$. In this case, the portion of the guided mode inside the LHM slab (region 2) shows the *Poynting* vector to be anti-parallel to the direction of the modal phase flow and the portion of the guided mode outside the LHM slab (region 1 and

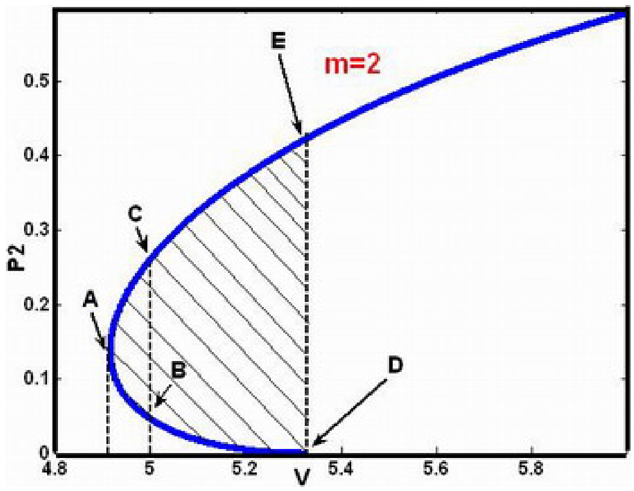

Figure 8. Normalized dispersion curve when $m = 2$. $\varepsilon_1 = 1,\ \mu_1 = 1,\ \varepsilon_2 = -2,\ \mu_2 = -2,\ \varepsilon_3 = 2,\ \mu_3 = 1$.

region 3) show the Poynting vector to be parallel with the phase flow. But the total power flow of the part of mode in region 2 is smaller than that of the total power flow outside the region 2. In this case, the net power flux of the guided mode is parallel with the direction of the phase flow. That is why we call it "*forward wave*". When the curve reaches to A (the slope of this point is $+\infty$), $\overline{P} = 0$. In this case, the net power flow of the guided mode is 0. After that, when the curve shift to the position with positive slope, the portion of the guided mode inside the LHM slab (region 2) shows the *Poynting* vector to be anti-parallel to the direction of the phase flow, and the portion of the guided mode outside the LHM slab (region 1 and region 3) show the Poynting vector to be parallel with the phase flow. But in this case the net power flux of guided mode is anti-parallel with the direction of the phase flow which is different from "forward wave". We can call this wave "backward wave".

5. CONCLUSION

We have analyzed above the electromagnetic properties of LHM asymmetric slab waveguide. We determine the guidance conditions of guided modes in waveguide by using a graphical method. There are some novel properties in LHM waveguide: (i) There is no fundamental node-less mode in LHM waveguide at all. (ii) The first-order guided modes exist only in a particular range of V, and it cannot coexist with any other higher-order modes. (iii) When $m > 1$, the dispersion curves of guided modes have a portion with negative slope, it appears a little bit below the cutoff frequency.

Apart form the guided mode, there exist surface wave modes in LHM waveguide. The properties of surface wave modes are analyzed in detail for given waveguide parameters and structures. We found that the surface waves decay exponentially away from, but propagate along, the dielectric interfaces of core and claddings. The existence of the guided modes depends on the parameters and structures of the waveguide.

We analyzed lastly the power flux in LHM waveguide and found that the power flux in region of LHM is negative while it is positive outside the LHM slab and that the net total power flux in waveguide determined by the parameters of waveguide as well as the frequency.

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