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ON A STUDY OF DIFFRACTION AND DISPERSION MANAGED SOLITON IN A CYLINDRICAL MEDIA

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Abstract—Analytical and numerical techniques are developed for the analysis of the solitary pulse propagation in a cylindrical media, where both the dispersion and diffraction management are known to exist. In the first part we treat the situation when there is no diffraction management by a simple analytical approach and show that it is possible to control both spatial and temporal width over one period of the dispersion map. An important out put of our analytical treatment is that we can predict the value of length of the second link and the amount of group velocity dispersion there if the initial conditions are given. Unfortunately since the spatial chirp can not be controlled, the treatment can not be repeated a second time. So for a long distance propagation a different treatment in needed. We then show that for long period of transmission, it is necessary to introduce diffraction management term, the best form turns out to be periodic function of the distanced travelled. The detailed variation of spatial and temporal width, chirp, and amplitude are explicitly given.

1. INTRODUCTION

One of the major goals in the study of nonlinear wave interactions in optics is the generation of pulses that are localized in all transverse dimensions of space as well as in time− spatiotemporal solitons [1]. In the present day literature they are dubbed as light bullets [2]. The most important problem associated with such structures in more than one dimension is their instability due to the existence of collapse [3]. Also it has not been possible to observe such spatiotemporal solitons in 3D (in bulk) [4] or in 2D (in planar wave guide) [5] experimentally. Though some experimental evidence has been obtained in 3D crystals with $\chi^{(2)}$ nonlinearity [6]. It was thought that a 2D spatial cylindrical soliton can be stabilized in a bulk layered medium with opposite sign of Kerr nonlinearity in adjacent layers [7], corresponding to self focussing and self defocussing. A similar phenomena is also envisioned in case of Bose-Einstien condensation. Actually the prerequisite for the generation of stable completely localized spatiotemporal soliton is the existence of a self focussing nonlinearity, anomalous GVD and one or more process which can prevent collapse. Some study in this respect was done by Abdullaev et al. [8], who considered periodically varying dispersion. Some what similar considerations were done by Matuszewski et al. [9] also. In spite of some difficulties in the experimental realizations of such spatiotemporal solitons, several authors have suggested means for the stabilization and propagation of them in fiber.

The quest for spatiotemporal solitons or light bullets faces two main challenges: first, physically relevant models of nonlinear optical systems, based on evolution equations that allow stable three dimensional propagation, ought to be identified, second suitable materials should be found where such models can be implemented. Thus advances in both theoretical and experimental directions are necessary. In the first part of the present communication we have analyzed the case of spatiotemporal soliton in two dimension with cylindrical symmetry when usual dispersion management is taken into account. We have developed an analytical method for the analysis of the different stages of management [10]. Our result indicates that it is possible to keep both the spatial and temporal width of the pulse in control for at least one period of dispersion management even in absence of diffraction management. But for a long distance transmission it is shown in the second part that one should have a diffraction management term. Actually there can be three form of diffraction management term, of which it is observed that a purely periodic form given the best possible result.

2. FORMULATION

The local amplitude u of the electromagnetic wave propagating along z (in a suitably dimensionless units obeys)

$$iu_z + \frac{1}{2}\left[\bigtriangledown_\perp^2 u + \beta_2(z)u_{tt}\right] + \gamma|u|^2 u = 0 \tag{1}$$

where we have considered a three dimensional situation and $\bigtriangledown_\perp^2$ operates on the transverse co-ordinates (x, y), t is the temporal variable, $\beta_2(z)$ is the usual GVD (Group velocity dispersion) coefficient one finds for dispersion management

$$\beta_2(z) = \begin{cases} D_+ > 0 & 0 \leq z \leq L_+ \\ D_- < 0 & L_+ \leq z \leq L_+ + L_- \equiv L \end{cases}$$

which repeats periodically with L. With cylindrical symmetry equation(1) can be re written as

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)u + \beta_2\frac{\partial^2 u}{\partial t^2}\right] + \gamma|u|^2 u = 0 \tag{2}$$

The Lagrangian corresponding to equation (2) is given as;

$$L = \int_{-\infty}^{\infty}\int_{0}^{\infty} dtdr\left\{\frac{i}{2}\left(u\frac{\partial u^*}{\partial z} - u^*\frac{\partial u}{\partial z}\right)r + \frac{1}{2}\left|\frac{\partial u}{\partial r}\right|^2 r + \frac{1}{2}\beta_2\left|\frac{\partial u}{\partial t}\right|^2 r - \frac{\gamma}{2}|u|^4 r\right\} \tag{3}$$

We further more assume that the propagating pulse has a gaussian form and so can be written as;

$$u = A(z)exp\left\{i\phi(z) - \frac{1}{2}\left[\frac{r^2}{R^2(z)} + \frac{t^2}{W^2(z)}\right] + \frac{i}{2}\left[b(z)r^2 + c(z)t^2\right]\right\} \tag{4}$$

Due to the condition of energy conservation $\frac{dP}{dz} = 0$ and corresponding normalization we get;

$$A^2 = \frac{2}{\sqrt{\pi}}\frac{P}{WR^2}$$

which can be used to rewrite u as

$$u = \left(\frac{2}{\sqrt{\pi}}\right)^{1/2}\left(\frac{P}{WR^2}\right)^{1/2} exp\left\{i\phi - \frac{1}{2}\left[\frac{r^2}{R^2} + \frac{t^2}{W^2}\right] + \frac{i}{2}\left[br^2 + ct^2\right]\right\}$$

Substituting u in the Lagrangian and integrating; we get

$$\begin{aligned} L \;=\; & P\left(\frac{d\phi}{dz} + \frac{1}{2}R^2\frac{db}{dz} + \frac{1}{4}W^2\frac{dc}{dz}\right) + \frac{P}{2}\left(\frac{1}{R^2} + b^2R^2\right) \\ & + \frac{P}{4}\beta_2\left(\frac{1}{W^2} + c^2W^2\right) - \frac{\gamma}{2\sqrt{2\pi}}\frac{P^2}{WR^2} \end{aligned} \tag{5}$$

Considering Euler-Lagrange equations with respect to the parameters (ϕ, b, c, W, R) we arrive at

$$\frac{dR}{dz} = b\ R \tag{6}$$

$$\frac{db}{dz} = \left(\frac{1}{R^4} - b^2\right) - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W R^4} \tag{7}$$

$$\frac{dW}{dz} = \beta_2\, c\, W \tag{8}$$

$$\frac{dc}{dz} = \beta_2\left(\frac{1}{W^4} - c^2\right) - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R^2 W^3} \tag{9}$$

It may be noted that R is the spatial width, b the spatial chirp, W the corresponding temporal width, and c the temporal chirp.

In the usual approach one usually takes recourse to numerical simulation of equations (6)–(9) and studies the variation of pulse parameters. But in this formulation we want to adopt a different methodology and try to solve this system in an analytical manner. We start with some analytical ansatz for all the variables; in polynomial form

$$R^2(z) = c_0^{(1)} + c_1^{(1)}\, z + c_2^{(1)}\, z^2 + \text{...........}, \qquad 0 \le z \le L_1 \tag{10}$$

We start from $z = 0$;

$$c_0^{(1)} = R^2(z=0) = R_0^2 \tag{11}$$

Differentiating equation (10) and using equation (6), at $z = 0$,

$$c_1^{(1)} = 2R_0^2\, b_0 \tag{12}$$

and differentiating again equation (10) and using equation(6) and equation (7), we get

$$c_2^{(1)} = \frac{1}{R_0^2} + b_0^2\, R_0^2 - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_0 R_0^2} \tag{13}$$

With a similar expression for $W^2(z)$;

$$W^2(z) = a_0^{(1)} + a_1^{(1)}\, z + a_2^{(1)}\, z^2 + \text{...........}, \qquad 0 \le z \le L_1 \tag{14}$$

adopting a similar procedure we get at $z = 0$

$$a_0^{(1)} = W^2(z=0) = W_0^2 \tag{15}$$

along with

$$a_1^{(1)} = 2\beta_2^{(1)}\, c_0 W_0^2 \tag{16}$$

$$a_2^{(1)} = \frac{\left(\beta_2^{(1)}\right)^2}{W_0^2} + \left(\beta_2^{(1)}\right)^2 c_0^2\, W_0^2 - \frac{\beta_2^{(1)}}{\sqrt{2\pi}}\frac{\gamma P}{R_0^2 W_0} \tag{17}$$

So with these normalizations we get for the first section of the fibre, i.e., for $0 \leq z \leq L_1$

$$R^2(z) = R_0^2 + \left(2\, R_0^2\, b_0\right) z + \left\{ \frac{1}{R_0^2} + b_0^2 R_0^2 - \frac{1}{\sqrt{2\pi}} \frac{\gamma P}{W_0 R_0^2} \right\} z^2 \tag{17}$$

$$\begin{aligned} W^2(z) \;=\; & W_0^2 + \left(2\beta_2^{(1)} W_0^2\, c_0\right) z \\ & + \left\{ \frac{\left(\beta_2^{(1)}\right)^2}{W_0^2} + \left(\beta_2^{(1)}\right)^2 c_0^2\, W_0^2 - \frac{\beta_2^{(1)}}{\sqrt{2\pi}} \frac{\gamma P}{R_0^2 W_0} \right\} z^2 \end{aligned} \tag{18}$$

Now for the second section that is $z \geq L_1$ we set;

$$R^2(z) \;=\; c_0^{(2)} + c_1^{(2)}(z - L_1) + c_2^{(2)}(z - L_1)^2 \tag{19}$$

$$W^2(z) \;=\; a_0^{(2)} + a_1^{(2)}(z - L_1) + a_2^{(2)}(z - L_1)^2 \tag{20}$$

$$\text{for } \; L_1 \leq z \leq L_1 + L_2$$

The continuity of the temporal chirp $c(z)$ implies that a jump $W_z(z)$ when passing from one section to the other; that is

$$\left. \begin{aligned} W_z(L_1^-) = \beta_2^{(1)}\, c(L_1) W(L_1) \\ W_z(L_1^+) = \beta_2^{(2)}\, c(L_1) W(L_1) \end{aligned} \right\} \tag{21}$$

where L_1^+ and L_1^- are the limits of $z = L_1$ from the left and right. So that

$$\frac{W_z(L_1^+)}{W_z(L_1^-)} = \frac{\beta_2^{(2)}}{\beta_2^{(1)}} \tag{22}$$

For spatial width similarly we have

$$\left. \begin{aligned} R_z(L_1^-) = b(L_1) R(L_1) \\ R_z(L_1^+) = b(L_1) R(L_1) \end{aligned} \right\} \tag{23}$$

So that there is a continuity of $R_z(L_1^+) = R_z(L_1^-)$. Treating equations (19) in the same way as before, but this time putting $z = L_1$, we get

$$c_0^{(2)} = R^2(L_1) = R_1^2$$

Differentiating equations(19) we obtain

$$2R(z)R_z(z) = c_1^{(2)} + 2c_2^{(2)}(z - L_1)$$

so

$$c_1^{(2)} = 2R(L_1)R_z(L_1^-) \tag{24}$$
$$= 2R(L_1)R_z(L_1^+) \tag{25}$$

which can be found from equation(17). In fact

$$c_1^{(2)} = c_{12} = 2R_0^2\, b_0 + 2\left(\frac{1}{R_0^2} + R_0^2 b_0^2 - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_0 R_0^2}\right) L_1 \tag{25}$$

On the other hand

$$2R(z)R_{zz}(z) + 2R_z^2(z) = 2c_2^{(2)}$$

which leads to

$$c_2^{(2)} = \frac{1}{R_1^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_1 R_1^2} + \frac{c_{12}^2}{4R_1^2} \tag{26}$$

Similarly for the temporal width we get using equation (20)

$$a_0^{(2)} = W^2(L_1) = W_1^2 \tag{27}$$

And differentiating equation (20) at $z = L_1$ we have

$$a_1^{(2)} = \frac{\beta_2^{(2)}}{\beta_2^{(1)}} 2W(L_1)W_2(L_1^-) \tag{28}$$

which immediately leads to

$$\begin{aligned} a_1^{(2)} &= a_{12} \\ &= 2\beta_2^{(2)} W_0^2 c_0^2 + 2\left\{\frac{\beta_2^{(1)}\beta_2^{(2)}}{W_0^2} + \beta_2^{(1)}\beta_2^{(2)} W_0^2 c_0^2 - v\frac{1}{\sqrt{2\pi}}\beta_2^{(2)}\frac{\gamma P}{W_0 R_0^2}\right\} L_1 \end{aligned} \tag{29}$$

And in a similar manner,

$$a_2^{(2)} = W_1\left\{\frac{\left(\beta_2^{(2)}\right)^2}{W_1^3} - \frac{\beta_2^{(2)}}{\sqrt{2\pi}}\frac{\gamma P}{R_1^2 W_1^2}\right\} + \frac{a_{12}^2}{4W_1^2} \tag{30}$$

Therefore in the second link i.e., in the region $L_1 \le z \le L_1 + L_2$, the form of $R^2(z)$ and $W^2(z)$ are given by

$$R^2(z) = R_1^2 + c_{12}(z - L_1) + \left\{\frac{1}{R_1^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R_1^2 W_1} + \frac{c_{12}^2}{4R_1^2}\right\}(z - L_1)^2 \tag{31}$$

and

$$W^2(z) = W_1^2 + a_{12}(z-L_1) + \left\{ \frac{\left(\beta_2^{(2)}\right)^2}{W_1^2} - \frac{\beta_2^2}{\sqrt{2\pi}} \frac{\gamma P}{R_1^2 W_1} + \frac{a_{12}^2}{4W_1^2} \right\} (z-L_1)^2 \tag{32}$$

where c_{12} and a_{12} are given by equation(25) and equation(29) respectively. The objective here to suggest the dispersion map for the system consisting of concatenated links of fibers in which the pulse at the end is identical to the input pulse. Here we consider usual dispersion map as given below

$$\beta_2(z) = \begin{cases} \beta_2^{(1)} > 0 & 0 \le z \le L_1 \\ \beta_2^{(2)} < 0 & L_1 \le z \le L_1 + L_2 \\ \beta_2^{(1)} > 0 & L_1 + L_2 < z < 2L_1 + L_2 \equiv L \end{cases}$$

which repeats periodically with the period L. We now use the symmetry of the temporal width about the points $W_z = 0$. So that between first and second link we get

$$W(z = L_1 + L_2) = W_2 = W(z = L_1) = W_1 \tag{33}$$

$$R(z = L_1 + L_2) = R_2 = R(z = L_1) = R_1 \tag{34}$$

which when coupled with equations (32) yields;

$$a_{12}L_2 = -\left\{ \frac{\left(\beta_2^{(2)}\right)^2}{W_1^2} - \frac{\beta_2^{(2)}}{\sqrt{2\pi}} \frac{\gamma P}{R_1^2 W_1^2} + \frac{a_{12}^2}{4W_1^2} \right\} L_2^2 \tag{35}$$

It is reasonable to suppose that the nonlinear effects are small in the second link because the peak power decreases in the first link and the second link is usually short in comparison. Therefore we are justified in treating the second one as linear. We define;

$$\left.\begin{array}{c} \Gamma = a_{12}L_2, \qquad \triangle_1 = \beta_2^{(1)} L_1, \qquad \triangle_2 = \beta_2^{(2)} L_2 \\ F_1 = \dfrac{\gamma P}{W_0} L_1, \qquad F_2 = \dfrac{\gamma P}{W_0} L_2 \end{array}\right\} \tag{36}$$

So that we can rewrite equations (35) as;

$$\Gamma W_1^2 = \left(\triangle_2^2 + \frac{\Gamma^2}{4} \right) \tag{37}$$

where

$$\Gamma = a_{12}\,L_2$$
$$= 2\,\triangle_2\left(W_0^2c_0 - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right) + 2\,\triangle_1\,\triangle_2\left(\frac{1}{W_0^2} + c_0^2W_0^2\right)$$

and

$$W_1^2 = W_0^2 + \triangle_1\left(2W_0^2c_0 - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right) + \triangle_1^2\left(\frac{1}{W_0^2} + c_0^2W_0^2\right)$$

along with;

$$\triangle_2 = \frac{N}{D} \tag{38}$$

where

$$\begin{aligned} N \;=\; & -\left[2\left(W_0^2c_0 - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right) + 2\,\triangle_1\left(\frac{1}{W_0^2} + c_0^2W_0^2\right)\right] \\ & \times\left[W_0^2 + \triangle_1\left(2W_0^2c_0 - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right) + \triangle_1^2\left(\frac{1}{W_0^2} + c_0^2W_0^2\right)\right] \end{aligned} \tag{39}$$

$$D \;=\; 1 + \left\{\left(W_0^2c_0^2 - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right) + \triangle_1\left(\frac{1}{W_0^2} + c_0^2W_0^2\right)\right\}^2 \tag{40}$$

From these equations we can determine the dispersion constant $\beta_2^{(2)}$ for the second link. Again from the equations of the spatial width we get

$$R_2^2 = R_1^2 + \tau + \left\{\frac{L_2^2}{R_1^2} + \frac{\tau^2}{4R_1^2}\right\} \tag{41}$$

with $\tau = c_{12}L_2$ and

$$R_1^2 = R_0^2 + (2R_0^2b_0)L_1 + \left\{L_1^2\left(\frac{1}{R_0^2} + R_0^2b_0^2\right) - \frac{1}{\sqrt{2\pi}}\frac{F_1L_1}{R_0^2}\right\} \tag{42}$$

From which we can solve for L_2

$$L_2 = \frac{N_1}{D_1} \tag{43}$$

where N_1 and D_1 are given as;

$$\begin{aligned} N_1 = & -\left[2R_0^2b_0 + 2\left\{L_1\left(\frac{1}{R_0^2} + R_0^2b_0\right) - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right\}\right] \\ & \times\left[R_0^2 + \left(2R_0^2b_0\right)L_1 + \left\{L_1^2\left(\frac{1}{R_0^2} + R_0^2b_0\right) - \frac{1}{\sqrt{2\pi}}\frac{F_1L_1}{R_0^2}\right\}\right] \end{aligned} \tag{44}$$

$$D_1 = 1+\left\{R_0^2 b_0 + L_1\left(\frac{1}{R_0^2} + r_0^2 B_0^2\right) - \frac{1}{\sqrt{2\pi}}\frac{F_1}{R_0^2}\right\}^2 \tag{45}$$

Equation (43) gives us the length of the second link in the dispersion management.

We now consider the third section of the fiber, for which we write;

$$W^2(z) = a_0^{(3)} + a_1^{(3)}(z - L_1 - L_2) + a_2^{(3)}(z - L_1 - L_2)^2 \tag{46}$$

Proceeding as before one gets

$$a_0^{(3)} = W_2^2 \tag{47}$$

$$\begin{aligned} a_1^{(3)} &= a_{13} \\ &= -\left[\left(2\beta_2^{(1)} W_0^2 c_0\right) + 2\left\{\frac{\left(\beta_2^{(1)}\right)^2}{W_0^2} + \left(\beta_2^{(1)}\right)^2 c_0^2 W_0^2 - \frac{1}{\sqrt{2\pi}}\beta_2^{(1)}\frac{\gamma P}{R_0^2 W_0}\right\}\right] \end{aligned} \tag{48}$$

and

$$a_2^{(3)} = \frac{\left(\beta_2^{(1)}\right)^2}{W_2^2} - \frac{\beta_2^{(1)}}{\sqrt{2\pi}}\frac{\gamma P}{R_2^2 W_2} + \frac{a_{13}^2}{4W_2^2} \tag{49}$$

A similar expansion for the square of the spatial width leads to

$$R^2(z) = c_0^{(3)} + c_1^{(3)}(z - L_1 - L_2) + c_2^{(3)}(z - L_1 - L_2)^2 \tag{50}$$

where

$$c_0^{(3)} = R_2^2, \quad c_1^{(3)} = c_{13} = -\left[\left(2R_0^2 b_0\right) + 2\left(\frac{1}{R_0^2} + R_0^2 b_0 - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_0 R_0^2}\right)\right] \tag{51}$$

along with;

$$c_2^{(3)} = \frac{1}{R_2^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_2 R_2^2} + \frac{c_{13}^2}{4R_2^2} \tag{52}$$

So collecting our results we can summarize the form of $b(z)$ and $c(z)$ for the various sections of the fibers as follows;

For $\quad 0 \le z \le L_1$

$$b(z) = \frac{1}{R^2(z)}\left[R_0^2 b_0 + \left(\frac{1}{R_0^2} + R_0^2 b_0^2 - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_0 R_0^2}\right) z\right] \tag{53}$$

$L_1 \le z \le L_1 + L_2$

$$\begin{aligned} b(z) &= \frac{1}{R^2(z)}\left[\frac{c_{12}}{2} + (z - L_1)\left\{\frac{1}{R_1^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_1 R_1^2} + \frac{c_{12}^2}{4R_1^2}\right\}\right] \qquad (54) \\ & L_1 + L_2 \leq z \leq L_1 + L_2 + L_1 \\ b(z) &= \frac{1}{R^2(z)}\left[\frac{c_{13}}{2} + (z - L_1 - L_2)\left\{\frac{1}{R_2^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W_2 R_2^2} + \frac{c_{13}^2}{4R_2^2}\right\}\right] \end{aligned}$$

(55)

Similarly;

For; $0 \leq z \leq L_1$

$$c(z) = \frac{1}{W^2(z)}\left[W_0^2\, c_0 + z\left\{\frac{\beta_2^{(1)}}{W_0^2} + \beta_2^{(1)} c_0^2 W_0^2 - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R_0^2 W_0}\right\}\right] \qquad (56)$$

$L_1 \leq z \leq L_1 + L_2$

$$c(z) = \frac{1}{W^2(z)}\left[\frac{a_{12}}{2\beta_2^{(2)}} + (z - L_1)\left\{\frac{\beta_2^{(2)}}{W_1^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R_1^2 W_1} + \frac{a_{12}^2}{4\beta_2^{(2)} W_1^2}\right\}\right] \qquad (57)$$

and lastly for $L_1 + L_2 \leq z \leq L_1 + L_2 + L_1$

$$c(z) = \frac{1}{W^2(z)}\left[\frac{a_{13}}{2\beta_2^{(1)}} + (z - L_1 - L_2)\left\{\frac{\beta_2^{(1)}}{W_2^2} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R_2^2 W_2} + \frac{a_{13}^2}{4\beta_2^{(1)} W_2^2}\right\}\right] \qquad (58)$$

Using these analytical results and the possible set of numerical values given below Figure 1, we compute the physical characteristics of the pulse over the one period of the dispersion management. It is observed that though spatial and temporal width of the pulse remains bounded yet the chirp can not be totally controlled. It is this fact which prevents us to repeat the procedure a second time and as such for a long transmission length we require a effective means of diffraction management. In the following section we introduce such an effect and try to visualize the propagation numerically.

2.1. Diffraction Management

For total control of the pulse we introduce a diffraction management function into the equation of motion which now reads

$$iu_z + \frac{1}{2}\left[\kappa(z)\, \bigtriangledown_\perp^2\, u + \beta_2(z) u_{tt}\right] + \gamma |u|^2 u = 0 \qquad (59)$$

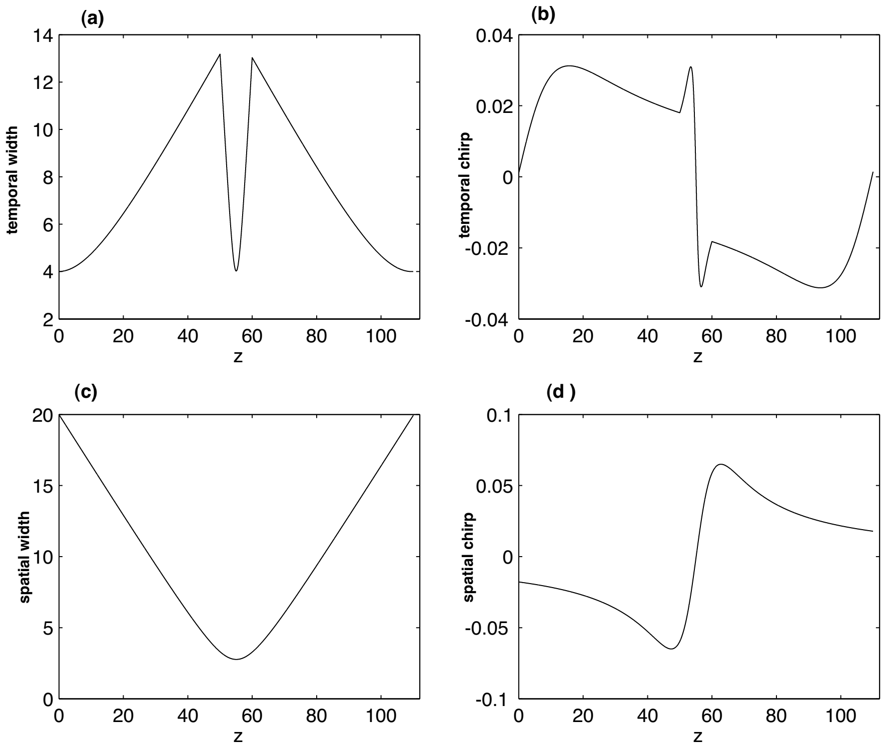

Figure 1. Evolution of (a)- temporal width , (b)- temporal chirp, (c)- spatial width (d)- spatial chirp in one analytically designed dispersion map without loses, for $P = 0.1$, $\gamma = 2.314$, $L_2 = 10.0118$, $\beta_2^{(2)} = -10.0386$.

with the same form of the pulse as given in equation(4) we get the following equations for the parameters

$$\frac{dR}{dz} = \kappa b\, R \tag{60}$$

$$\frac{db}{dz} = \kappa\left(\frac{1}{R^4} - b^2\right) - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W R^4} \tag{61}$$

$$\frac{dW}{dz} = \beta_2\, c\, W \tag{62}$$

$$\frac{dc}{dz} = \beta_2\left(\frac{1}{W^4} - c^2\right) - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R^2 W^3} \tag{63}$$

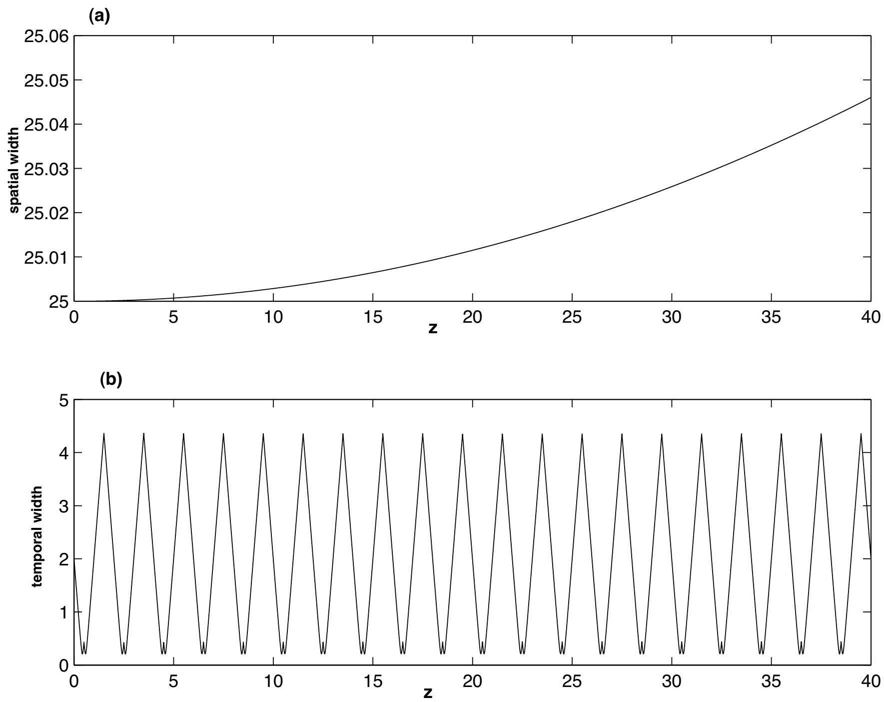

Figure 2. Numerical simulations of the propagation of (a)- spatial width, (b)- temporal width of the pulse through the 20th cycle for $P = 0.1,\ \gamma = 2.314$.

For convenience we define new variables $v(z) = b(z)R(z),\ T(z) = c(z)W(z)$, for which the equations out to be

$$\frac{dR}{dz} = \kappa(z)v(z) \tag{64}$$

$$\frac{dv}{dz} = \frac{\kappa(z)}{R^3(z)} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{W(z)R^3(z)} \tag{65}$$

$$\frac{dW}{dz} = \beta_2(z)T(z) \tag{66}$$

$$\frac{dT}{dz} = \frac{\beta_2(z)}{W^3(z)} - \frac{1}{\sqrt{2\pi}}\frac{\gamma P}{R^2(z)W^2(z)} \tag{67}$$

Since the form of $\kappa(z)$ turns out to be of nonplynomial type our above analytical approach can not be applied in this case and we take

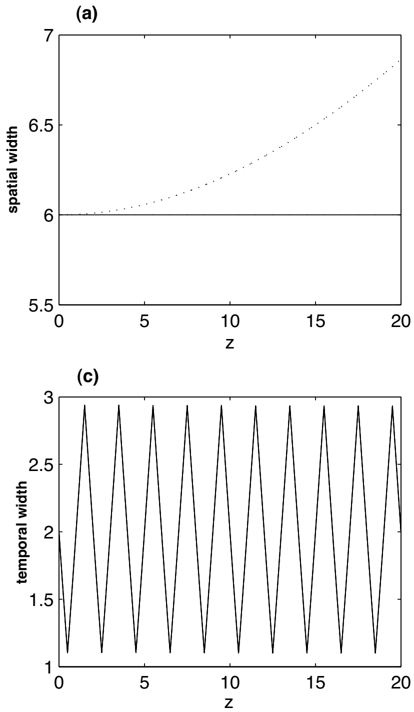

Figure 3. Numerical simulation of evolution of (a)- spatial width, (b)- $v(z)$, (c)- temporal width, (d)- $T(z)$ of the spatiotemporal soliton through 20th cycle. Dashed line axis correspond to $\kappa = constant = 1.0$, solid line axis corresponds a step function same as $\beta_2(z)$ of dispersion map.

recourse to a numerical simulation. In this situation it is observed that $\kappa(z)$ can be considered to be either a step function same as $\beta_2(z)$ of dispersion management or a purely periodic profile or a function which has got partly periodic character. Since until now the experimental observations regarding the spatio-temporal soliton in more than two dimension are rare. We have considered all the above possibilities in detail. For example when $\kappa(z)$ is of the same type as $\beta_2(z)$, we have exhibited the corresponding variation of spatial and temporal parameters v and T in Figures 3(a) to 3(d). In Figures 3(a) and 3(b) we have also shown the situation when κ is constant. The important point to note is that both spatial parameter v remains steady when κ is of the type of $\beta_2(z)$, but they go on increasing with constant κ for long distance propagation. The corresponding pulse intensity is

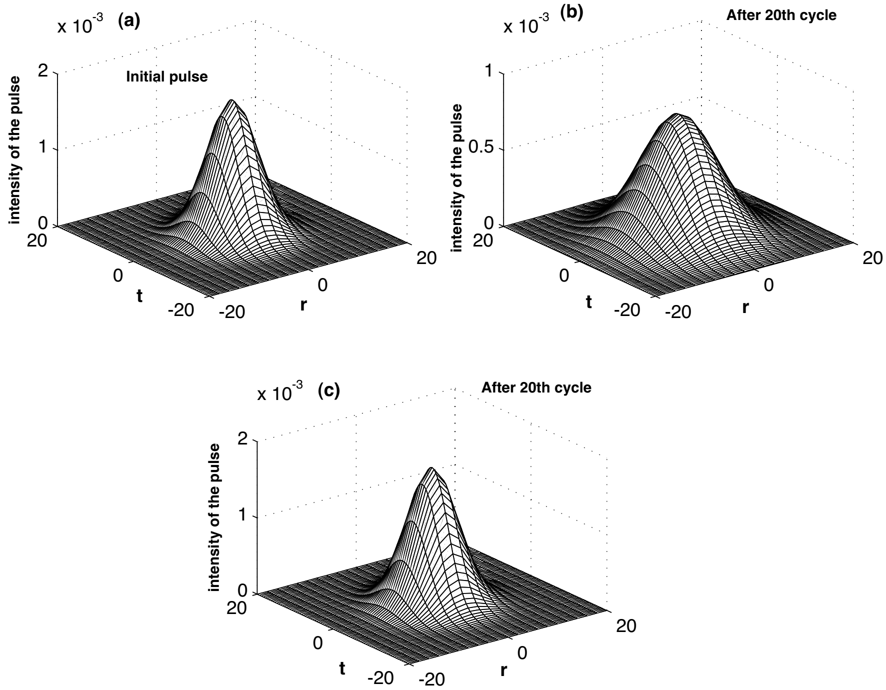

Figure 4. Intensity distribution in the 3D soliton (b)- $\kappa = constant = 1.0$ (c)- κ is a step function same as $\beta_2(z)$ of dispersion map.

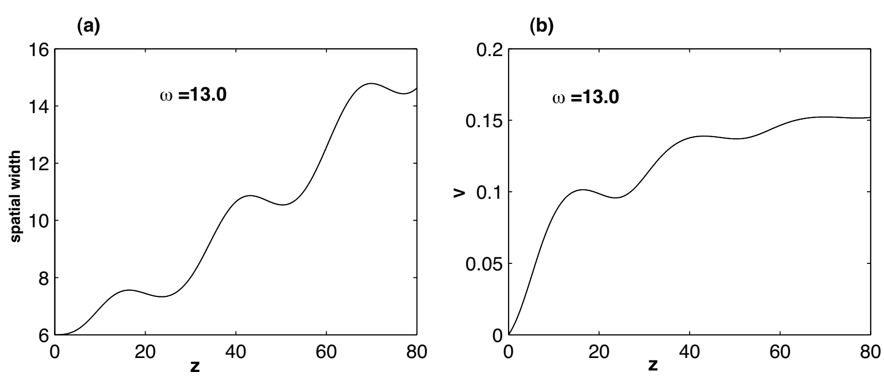

Figure 5. Variation of (a)- spatial width, (b)- $v(z)$ of the 3D soliton through 40th cycle with $\kappa(z) = 1 + g_1 \sin(\omega\, z)$.

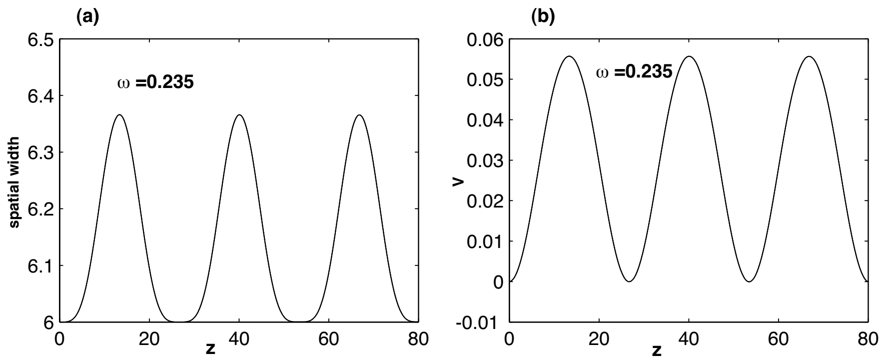

Figure 6. Variation of (a)- spatial width, (b)- $v(z)$ of the 3D soliton through 40th cycle with $\kappa(z) = g_1 \sin(\omega z)$.

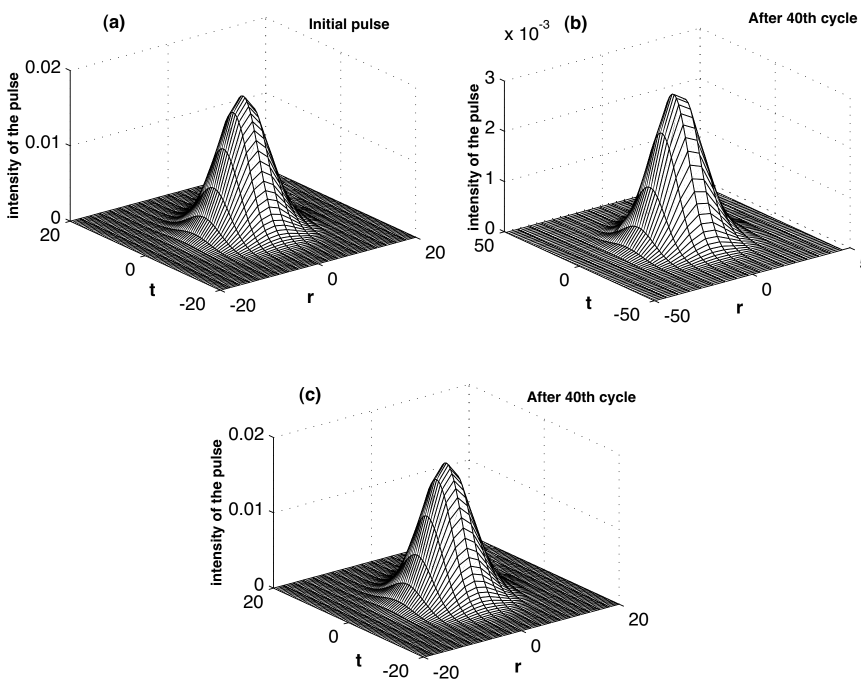

Figure 7. Intensity distribution in the 3D soliton with (b)- $\kappa(z) = 1 + g_1 \sin(\omega z)$. (c)- $\kappa(z) = 1 + g_1 \sin(\omega z)$.

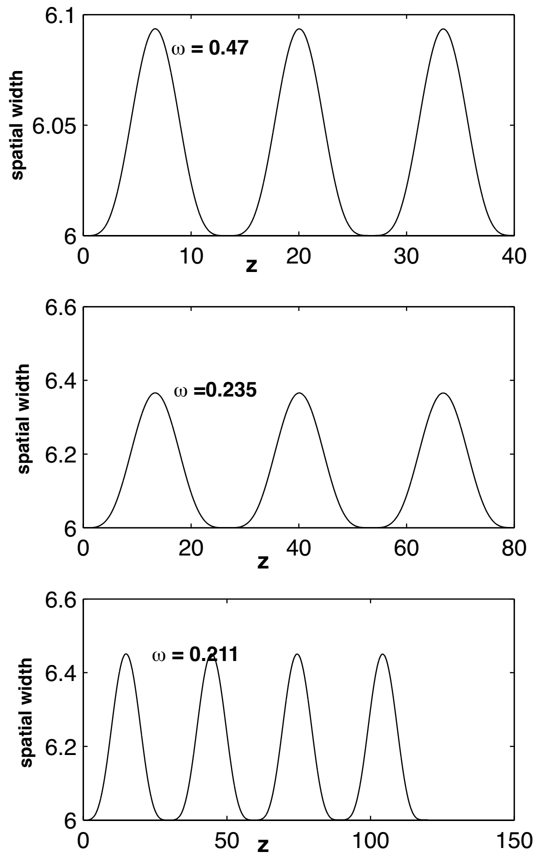

Figure 8. The spatial width of the 3D soliton vs propagation distance z from numerical solution of the coupled set of equation (64) to (67) with $\kappa(z) = 1 + g_1 \sin(\omega\, z)$ for different ω.

in Figures 4(a) to 4(b), where the broadening is quite evident. On the other hand for the case when $\kappa(z)$ is purely or partly periodic the corresponding variations of pulse parameters are displayed in Figures 5(a), 5(b) and 6(a), 6(b). From which it is quite transparent that the purely periodic profile for κ, that is $\kappa = g \sin(\omega z)$ is the most suitable one with a special value of ω, which keeps control over the pulse parameters. For each of these situation we show the corresponding intensity profiles in Figures 7(a) to 7(c), where the pulse shape shows to be controlled.

In fact one can try to compare the general problem of spatiotemporal soliton with the problem of optical lattice. Because of the concept of diffraction management, which is actually periodic in the propagation direction while in case of optical lattice one has such variation of refractive index in the transverse direction. Incidentally one can mention that an important approach for the stabilization of the multi dimensional solitons has also been developed on the basis of Gross-Petavesvei equation including a periodic potential representing a sort of optical lattice. Such ideas needs further explorations and only then one can be sure about the actual stabilization effects of spatiotemporal soliton.

3. DISCUSSIONS AND CONCLUSIONS

In our above analysis we have studied the basic problem of a spatio-temporal soliton in a cylindrical media, when there is both GVD and diffraction. It is observe that a very simple analytic approach can be developed so that one can have idea of a priori values of GVD and length of the dispersion map. It is demonstrated that even in absence of any diffraction management, the spatial and temporal width, both, can be contained within bounds, but only for one complete cycle of the map. So that for a long haul transmission of such a soliton, a suitable from of diffraction management was assumed, and from the simulated result one can visualize that a purely periodic nature for $\kappa(z)$ is the most suitable one. On the other hand due to the lack of enough experimental evidence it is difficult to have a first hand idea of the behaviour of such solitons. Still our analysis may suggest some possible avenues for future study.

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