*Progress In Electromagnetics Research, PIER 64, 135–148, 2006*

# ABSORBING PROPERTIES OF A NEGATIVE PERMITTIVITY LAYER PLACED ON A REFLECTING GRATING

**O. P. Kusaykin, P. N. Melezhik, A. Ye. Poyedynchuk and O. S. Troschylo**

Usikov Institute for Radiophysics and Electronics
National Academy of Sciences of Ukraine
12 Proskury, Kharkiv, 61085 Ukraine

**Abstract**—Theoretical results on the electromagnetic properties of such a structure as a negative-permittivity layer placed on a reflecting grating are discussed. Effect of this structure resonant absorption of the excitation electromagnetic energy is studied in a wide range of problem parameters. The sufficient conditions of the effect appearance are determined by solving the dielectric-layer eigenoscillation problem. A comparative analysis of the absorption by the negative- and the positive-permittivity structures is made to reveal the characteristic features of the effect. Phenomenon of the absorption resonance splitting is shown in the case of plane wave oblique incidence on the structure. The $E$-polarized plane wave diffraction is considered to find out essential difference of the resonant absorption in the $E$ and $H$ cases.

## 1. INTRODUCTION

A considerable amount of research in the recent decades has been devoted to electromagnetic properties of such complex media as chiral materials, omega-medium, bi-anisotropic medium [1–18]. Serious efforts have been directed towards building microwave composites whose effective permittivity and permeability can take negative values [9, 10, 14]. In 1967, V. G. Veselago showed that the Poynting vector in the medium of this kind is opposite to the phase velocity direction [14]. Recently D. R. Smith and others [9, 10] have produced such a composite material for the microwave region and experimentally demonstrated the anomalous refraction effect.

Artificial materials with one constitutive parameter (permittivity or permeability) being negative have attracted much interest as well [15, 16]. A study of their electromagnetic properties has shown (see [17, 18]) that in combination with various periodic structures they can absorb the incident electromagnetic wave energy in resonant way. This property can be used for creating novel devices, non-reflecting coatings and microwave antenna components.

First we will consider the eigenoscillations of a dielectric layer with a negative-valued real part of the permittivity. On this basis, the resonance behavior of the layer absorption will be studied in the situation when it is placed on a reflecting diffraction grating.

In the most general case, the frequency dependence of the permittivity of the composite material should be taken into account. Therefore, our study of the simplified problem presented here can be considered as a first step in the analysis of the general problem.

## 2. DIFFRACTION PROBLEM FORMULATION

The diffraction problem is formulated for the oblique incidence of a plane $H$-polarized unit-amplitude electro-magnetic wave on an isotropic layer of the thickness $h_2$ infinite along the $OX$ and $OY$ axes (see Fig. 1). The incident angle is $\varphi$ and the time dependence is $\exp(-i\omega t)$. The layer permittivity is $\mathrm{Re}\varepsilon + i\mathrm{Im}\varepsilon$, with $\mathrm{Re}\varepsilon < -1$ and $\mathrm{Im}\varepsilon > 0$. We will assume that the dispersion is absent, i.e., $\varepsilon$ does not depend on frequency. The layer is placed on an infinite perfect electrically conducting (PEC) reflecting grating consisting of rectangular grooves of the depth $h_1$ and width $d$ cut with period $l$.

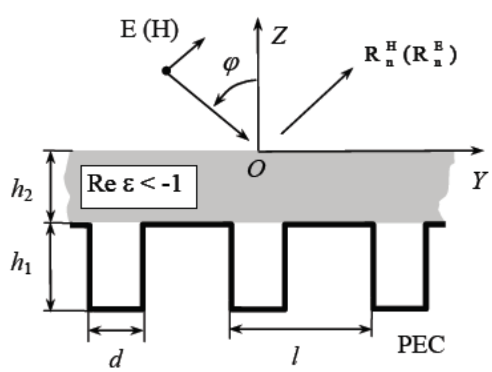



**Figure 1.** Problem geometry.

Assume that the incident field is $H$-polarized, hence the only non-vanishing component of the magnetic field vector is $H_x$. One can easily prove that the diffraction field is $H$-polarized as well. Satisfying the quasi-periodicity condition and the radiation condition at infinity, one can expand the total field $H_x$ component outside the structure as

$$H_x = e^{i\frac{2\pi}{l}\left(k_0^y y - k_0^z z\right)} + \sum_{n=-\infty}^{\infty} R_n^H e^{i\frac{2\pi}{l}\left(k_n^y y + k_n^z z\right)} \qquad (z > 0). \qquad (1)$$

In the $E$-polarization case, the total field outside the structure can be expanded into similar series as

$$E_x = e^{i\frac{2\pi}{l}\left(k_0^y y - k_0^z z\right)} + \sum_{n=-\infty}^{\infty} R_n^E e^{i\frac{2\pi}{l}\left(k_n^y y + k_n^z z\right)} \qquad (z > 0), \qquad (2)$$

where $k_n^y = (n + æ \sin\varphi$ and $k_n^z = \sqrt{æ^2 - k_n^{y2}}$ are the propagation constants of the $n$-th space harmonics along the $OY$ and $OZ$ axes, respectively, $æ = l/\lambda$, with $\lambda$ being the incident field wavelength in free space. The coefficients $R_n^H$ and $R_n^E$ are the sought complex amplitudes of the harmonics of the diffraction field.

By using the mode matching method and the generalized scattering matrix technique [19], the problem can be reduced to a set of linear algebraic equations of the second kind, which enables one to build an effective numerical algorithm for finding the reflection coefficients $R_n^{H(E)}$.

## 3. EIGENVALUE PROBLEM

For a better insight into the features of the formulated above diffraction problem, we will first analyze the eigenvalue characteristics of a lossless negative-permittivity layer. This problem is important because the eigenvalues coincide with the pole-type singularities of the analytic continuation of the diffraction problem solution in the complex frequency domain [20]. In this case, the dielectric layer can be considered as an open resonant structure with complex eigenfrequencies and relevant eigenoscillations. The discussion will concern two-dimensional (2-D) eigenoscillations depending on time and $y$ coordinate as $\exp[i(\Phi y - \omega t)]$, where $\Phi$ is the independent real-valued parameter of the problem and $\omega$ is the circular frequency.

Outside the layer, the eigenoscillations must satisfy the radiation condition (see [21]), which implies that the complex eigenfrequencies belong to the two-sheet Riemann surface of the function $\sqrt{k^2 - \Phi}$, where $k = \omega/c$ and $c$ is the velocity of light in vacuum. The eigenvalue problem solution satisfies a homogeneous set of linear algebraic equations that has a nontrivial solution if and only if the determinant equals zero, the sought complex eigenfrequencies being the roots of the characteristic equation [12]. In the $H$-polarization case ($E_x = 0$), this equation takes the form

$$F_1 F_2 = 0, \tag{3}$$

where $F_1 = (\varepsilon R - 1)\exp(ik_s h_2) - (\varepsilon R + 1)$, $F_2 = (\varepsilon R - 1)\exp(ik_s h_2) + (\varepsilon R + 1)$, $k_s = \sqrt{k^2\varepsilon - \Phi^2}$ is the layer wave number along the $OZ$ axis, and $R = \sqrt{k^2 - \Phi^2}/k_s$.

Evidently, the characteristic Equation (3) has two sets of independent solutions, which are the roots of the functions $F_1$ and $F_2$. These roots can be determined numerically by the standard Newton method.

The analysis of the layer eigenvalue problem in the $H$-polarization case shows that characteristic Equation (3) has two solutions in the form of two surface oscillations in the real $k$ domain. In Fig. 2a, these solutions are presented by solid lines ($F_1 = 0$ and $F_2 = 0$) as the surface oscillation eigenfrequencies versus the problem parameter $\Phi$. The dotted lines correspond to $k = \Phi$ and $k = \Phi\sqrt{(\varepsilon + 1)/\varepsilon}$. The curve $k = \Phi$ cuts off the sector $k > \Phi$, where the surface eigenoscillations cannot exist. The curve $k = \Phi\sqrt{(\varepsilon + 1)/\varepsilon}$ corresponds to the surface eigen-oscillation (surface polariton [22]) of the corresponding dielectric half-space.

Notice that $F_1 = 0$ gives the odd oscillations and $F_2 = 0$ gives the even ones, with respect to the layer median line. For $F_1 = 0$, there exists some value $\Phi_c = \ln[(\varepsilon - 1)/(\varepsilon + 1)]/h_2$, at which eigenfrequency is zero. The frequency is purely imaginary if $\Phi < \Phi_c$, and it is purely real if $\Phi > \Phi_c$. As $\Phi \to \infty$, the frequencies of both surface eigenoscillations asymptotically tend to the surface polariton frequency, $k = \Phi\sqrt{(\varepsilon + 1)/\varepsilon}$.

Besides the mentioned above surface oscillations, there is an infinite number of volumetric oscillations exponentially increasing with the distance from the layer. They correspond to purely imaginary eigenfrequencies on the "nonphysical" Riemann sheet of the function $\sqrt{k^2 - \Phi^2}$. In Fig. 2(a), they are seen in the half-space $\mathrm{Im}k < -1$ (dashed curves). From the characteristic Equation (3) it follows that, if $\Phi = 0$, the eigenfrequencies of these oscillations are given by the formula $k_p = -i\left[2\mathrm{arctg}\left(\sqrt{|\varepsilon|}\right) + \pi p\right]/h_2\sqrt{|\varepsilon|}\,(p = 0, 1, 2, \ldots)$.

In the case of the $E$-polarized oscillations, the characteristic equation takes the form

$$F_1^{\sim} F_2^{\sim} = 0, \tag{4}$$

where $F_1^{\sim} = (R-1)\exp(ik_s h_2)+(R+1)$ and $F_2^{\sim} = (R-1)\exp(ik_s h_2)-(R+1)$. In the real-$k$ domain, this equation has a unique solution for $F_2^{\sim} = 0$. It corresponds to the so-called leaky wave (the solid line in Fig. 2(b)). The peculiarity of this type of wave is the existence of both normal, if $\Phi h_2 < 2.2$, and anomalous, if $\Phi h_2 > 2.2$, dispersion domains.

## 4. DIFFRACTION PROBLEM

$H$-polarization

In this section, we will present the results of the numerical analysis of the plane $H$-wave diffraction problem for the structure shown in Fig. 1, in the single-mode regime.

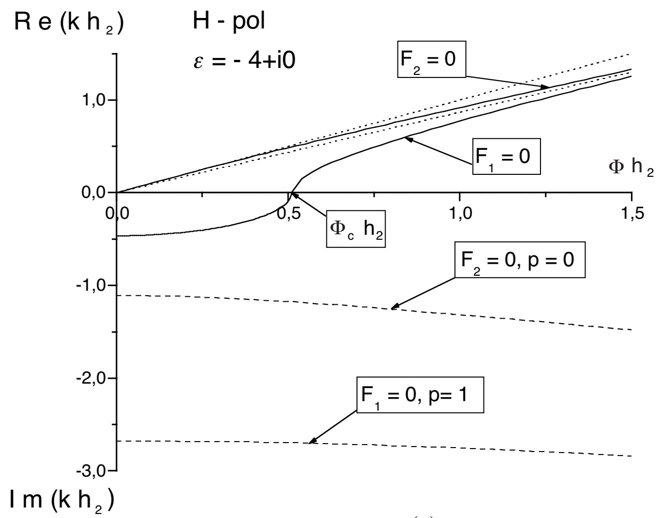
As follows from (1) and (2), the reflected field in the frequency domain $0 < æ < 1/(1+\sin\varphi)$ is a superposition of the zeroth propagating wave harmonic and infinite number of surface harmonics. The latter exponentially decay away from the structure along the $OZ$ axis but travel along the structure (axis $OY$) with the phase velocities $V_n^{\phi} = \pm\frac{æ}{n\pm æ\sin\varphi}c, (n = 1, 2\ldots)$. Hence, far from the structure the reflected field amplitude is given by the zeroth harmonic amplitude, $R_0^{H(E)}$.

The equal-value lines of $|R_0^H|$ (reflection coefficient) on the plane of the normalized layer thickness and normalized frequency, are presented in Fig. 3 for two different permittivity values: (a) $\varepsilon = -4 + i10^{-3}$ and (b) $\varepsilon = -1.1 + i10^{-3}$ ($\theta = d/l$ is the relative groove width of the grating). The dotted ($F_1$) and dashed ($F_2$) curves come from the eigenvalue problem solution under the condition that the layer surface waves are synchronous to the grating surface harmonics. The synchronism condition takes the form of identifies

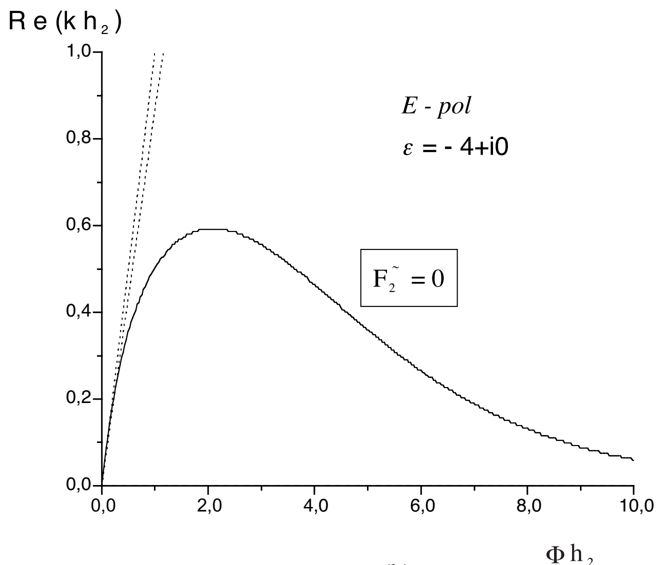
$$\left\{\begin{array}{lcl} kh_2 & = & 2\pi æ h_2/l \\ \Phi h_2 & = & 2\pi(n\pm æ\sin\varphi)h_2/l \end{array}\right. \quad (n = 1, 2, \ldots), \tag{5}$$

where $kh_2$ is the layer eigenfrequency and $\Phi h_2$ is the free parameter of the considered problem (see Fig. 2(a)).

When $|\mathrm{Re}\varepsilon|$ is large enough (Fig. 3(a)), the curves obtained from condition (5) and the maximum absorption curves of the incident wave ($|R_0^H| \ll 1$) demonstrate a good agreement, both qualitative and quantitative. This suggests that the absorption effect is caused by the resonances on the layer eigenfrequencies and by the synchronism the layer surface waves and the grating harmonics, which have the same phase velocity. However, if $\mathrm{Re}\varepsilon$ is close to $-1$ (Fig. 3(b)), the

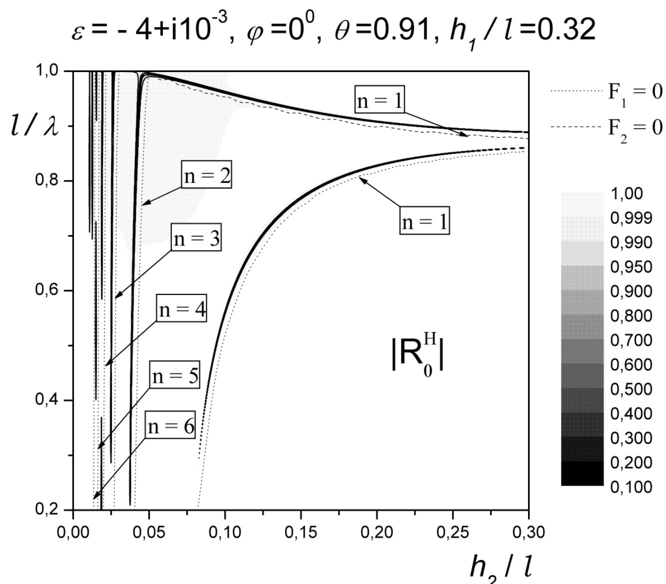



(a)

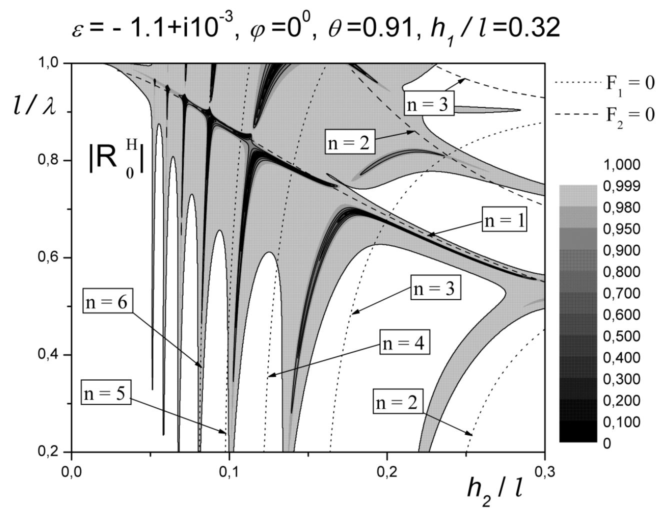



(b)

**Figure 2.** The layer eigenmode characteristics in the lossless case: (a) $H$-polarization, (b) $E$-polarization.




(a)




(b)

**Figure 3.** Reflection coefficient behavior in the $H$-polarization case ($\mathrm{Re}\varepsilon < -1$).

agreement is only qualitative. In our opinion, this is caused by the singularity at Re$\varepsilon = -1$.

Using the expression for $\Phi_c$ (see Fig. 2(a)) and condition (5), one finds out that in the single-mode domain, if the relative thickness $h_2/l > \ln\left[(\varepsilon - 1)/(\varepsilon + 1)\right]/2\pi$, there are two resonance absorption lines corresponding to $n = 1$. This corresponds to the $h_2/l > 0.08$ domain in Fig. 3(a). As the layer grows thicker, the lines asymptotically approach the polariton eigenfrequency, æ $= \sqrt{(\varepsilon + 1)/\varepsilon}$, so that æ $= 0.866$ for $\varepsilon = -4$. In the $h_2/l < 0.08$ domain, one observes a set of resonance absorption lines, which are nearly parallel to the frequency axis. In this domain, the quality-factors of the absorption resonances are lower than in the $h_2/l > 0.08$ domain.

As the permittivity real part approaches $-1$, the resonance absorption lines shift to the area of the larger layer thickness value (Fig. 3(b)). The general run of the resonance curves on the plane $(l/\lambda,\ h_2/l)$ diversifies suggesting more possibilities for the effect application. In particular, the resonant absorption breaks down near the intersection of the curves coming from condition (5). We attribute this situation to the parametric mode coupling between eigenoscillations of the layer-grating structure.

For comparison, the wave diffraction by a positive-permittivity dielectric layer laying on the same diffraction grating also demonstrates absorption resonances. Fig. 4 presents the equal-value lines of the reflection coefficient absolute value for the same parameters as in Fig. 3(a) but with the positive real part of the permittivity. The dashed curves plot the half-wavelength resonance condition, æ $= m/2\sqrt{\varepsilon}/(h_2/l),\ m = 1, 2, \ldots$ for the layer thickness. The solid curves marked with arrows display the dependences coming from (5) for the eigenvalue problem analysis performed in [12] for a similar lossless dielectric layer.

As seen from Fig. 4, the resonance absorption curves practically coincide with the eigenvalue lines of the dielectric layer provided that the layer surface waves are synchronous with the grating surface harmonics. The resonance absorption effect does not differ from that of the negative-permittivity layer in either amplitude or quality. The only but quite significant, in our opinion, dissimilarity is that any resonance absorption in the long-wavelength domain of frequencies, æ $\leq 1/\sqrt{\varepsilon}$ (this is æ $\leq 0.5$ in Fig. 4), is not possible for the layer with a positive Re$\varepsilon$. This property is explained by the fact that the relationship $k = \Phi/\sqrt{\varepsilon}$ defines the surface wave asymptote of a conventional dielectric waveguide. As can be seen from Fig. 3, a negative-permittivity layer is free from this limitation in the long-wavelength domain.

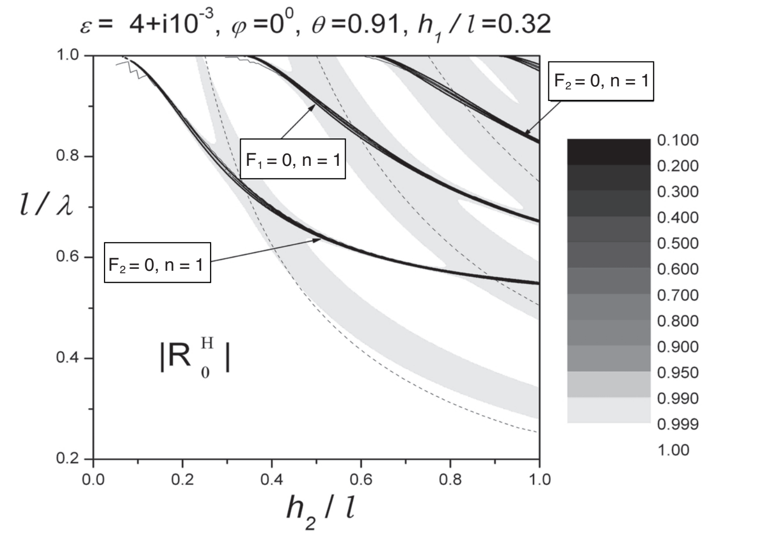



**Figure 4.** Reflection coefficient behavior in the case of ordinary dielectric, $H$-polarization.

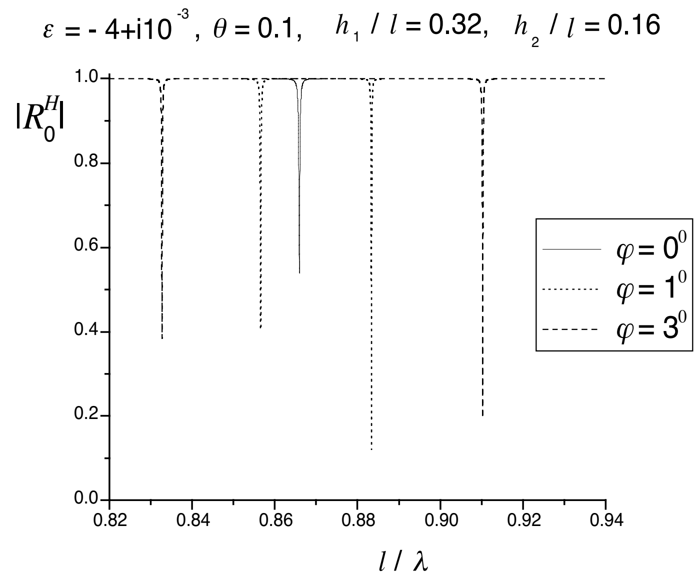
Note also the presence of nonresonance absorption domains (light-grey color in Fig. 4), where the reflection coefficient level does not go below $|R_0^H| < 0.995$, i.e., the absorption is of the order of the imaginary part of the permittivity. The fact that the half-wavelength resonance lines are located within the mentioned domains supports this conclusion.

So far, the normal incidence ($\varphi = 0$) of the plane wave has been considered. In Fig. 5, the amplitudes of the zeroth and the $\pm 1$ harmonics are plotted depending on the normalized frequency for different values of the incidence angle.

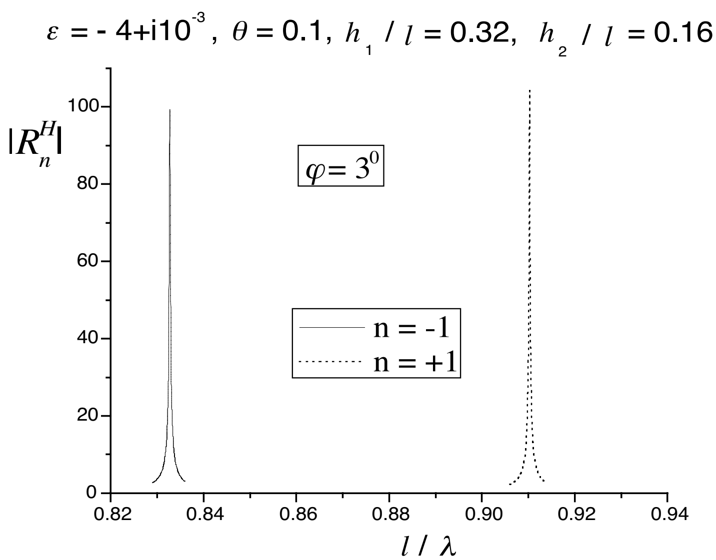
Fig. 5(a) shows that the absorption resonance splits in two ones at $\varphi \neq 0$. The explanation is that the grating surface harmonics of different-sign $n$ split in the phase velocity (see (1)). Note that the resonant absorption effect is accompanied by a sharp amplitude increase of the corresponding surface harmonic of the diffraction field. In support, see Fig. 5(b), where the normalized amplitudes of the $\pm 1$ harmonics reach hundreds of the units at the maximum-absorption frequencies.

$E$-polarization

Consider now the results of the diffraction problem solution in the case of the $E$-polarized plane wave incidence ($H_x = 0$) on a




(a)




(b)

**Figure 5.** Oblique wave incidence on the structure.

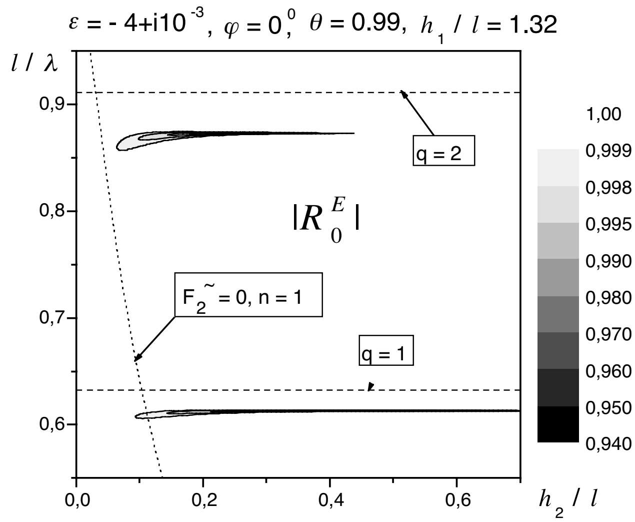



**Figure 6.** Behavior of the reflection coefficient in the $E$-polarization case (Re$\varepsilon < -1$).

dielectric layer supported by a diffraction grating (Fig. 1) and having Re$\varepsilon$ negative. In Fig. 6, the equal-value lines of the module of the reflection coefficient are plotted on the plane with coordinates of the normalized layer thickness and the normalized frequency. The dotted line comes from the eigenvalue problem solution in the $E$-polarization case (Fig. 2(b)) with the synchronism condition (5) held.

If the parameters are as indicated in Fig. 6 and the layer thickness is fixed, one can see two absorption resonances in the single-mode domain. The level of the reflection coefficient does not go below $|R_0^E| = 0.94$. The nature of the absorption in the $E$- and $H$-polarization cases is quite different that is clear from disagreement of the dotted line with any of the resonance absorption curves in Fig. 6. This is explained by the different character of the negative-permittivity layer eigenwave in the $E$-polarization case. This is a leaky wave, which cannot be synchronized with the surface harmonics of the diffraction grating.

On the other hand, the resonance absorption curves qualitatively agree with the dashed lines in Fig. 6. They correspond to the values $æ_{r,q} = 0.5\sqrt{(r/\theta)^2 + (ql/h_1)^2}$ $(r, q = 1, 2\ldots)$ obtained from the half-wavelength resonance condition of the zeroth harmonic in terms of the groove depth. For the grating parameters as those in Fig. 6, the mentioned condition gives $æ_{1,1} = 0.632$ and $æ_{1,2} = 0.912$.

## 5. CONCLUSION

In this paper, using the mode-matching method and the generalized scattering matrix technique has solved the problem of plane electromagnetic wave diffraction by the structure consisting of a negative-permittivity layer and a reflection diffraction grating. The effect of the resonance absorption has been studied numerically in a wide range of parameter values. Proceeding from the eigenvalue problem solution for a dielectric layer, sufficient conditions of this effect appearance has been determined, and the nature of the effect has been established, in the $H$-polarization case, as a resonant conversion of the excitation wave into the surface oscillations of the structure.

A comparative analysis of the diffraction by the structures with negative and positive values of permittivity has revealed the characteristic features of the absorption effect. The effect of absorption resonance splitting in the case of the plane wave oblique incidence on the structure has been demonstrated.

The $E$-polarized plane wave diffraction has been also considered to find out that the character of the resonance absorption essentially differs from that in the $H$-polarization case.

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