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STUDY ON THE OCCLUSIONS BETWEEN RAYS AND NURBS SURFACES IN OPTICAL METHODS

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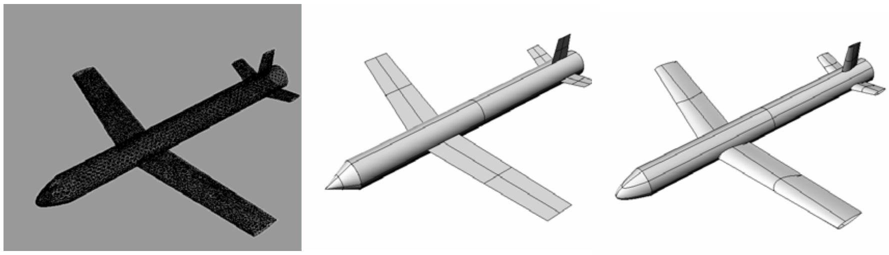
Abstract—An algorithm focusing on the occlusions between rays and NURBS surfaces is presented in this paper, where simple geometrical principle which makes the algorithm adequate in possible cases is used. When the ray starts from or ends on the surface, the self-shadowing is also taken into account. The algorithm given can be applied to optical methods like PO GTD or UTD in electromagnetic calculation where shadowing is of key problem and especially when the NURBS modeling is introduced.

1. INTRODUCTION

Within most optical methods of electromagnetic calculation, shadowing is necessary to consider to ensure the validity of results. In Physical Optics (PO), the shadowing areas must be determined to calculate the RCS and in Uniform geometrical Theory of Diffraction (UTD), if a traced ray is blocked by any part of the model but not considered, result of pattern is tend to be invalid. The complexity of shadowing is related to the models used. Complex models in optical methods are constructed by typical simple geometric objects, which obviously will cause incorrectness between the approached models and the real ones, or structured by triangular meshes, which will introduce more calculational complexity to get close enough to the real ones, see Fig. 1.

In the area of modeling, the Non-Uniform Rational B-Spline (NURBS) technique which can archive the real models easily and correctly is widely used as way of modeling in Computer Aided Geometrical Design (CAGD) and has been already included by more and more commercial CAD/CAM softwares like CATIA, UG, etc.

Due to its advantages in modeling, the NURBS technique has already been introduced into the area of calculation electromagnetism



model of triangluar mesh model of typical objects model of NURBS surfaces

Figure 1. A simplified aircraft modeled in different ways.

like PO and GTD method [1–3] where obviously shadowing is required. The introduction extends the application of these methods to arbitrary surfaces, but in those papers little has been commented on the aspect of shadowing. In the work of Sefi [4], an algorithm dealing with shadowing was presented by determining the intersection between rays and surfaces. But while studying the UTD method based on NURBS [5], this algorithm is found to be inadequate when the ray starts from or ends on the surface and also self-shadowing is not taken into account.

In this paper, based on the NURBS technique, the occlusions between rays and surfaces constructed by NURBS is studied and an algorithm which is adequate in possible cases is given where simple geometrical principle is introduced. The case when the ray starts from or ends on the surface where self-shadowing is included is taken into account and also is the case when the ray intersects with the surface on the extension. The algorithm given can be applied to those electromagnetic methods like PO GTD or UTD methods when the NURBS modeling is introduced and where shadowing is of key problem.

2. KNOWLEDGE OF NURBS

The occlusions occur because of the existence of surfaces, so it is necessary to know something about the surface studied and here the basic knowledge of NURBS is introduced. The mathematical expression of a NURBS surface is as follows [6]:

$$\boldsymbol{r}(\boldsymbol{u},\boldsymbol{v}) = \frac{\sum\limits_{i=0}^{n}\sum\limits_{j=0}^{m} \alpha_{ij}\boldsymbol{P}_{ij}N_p^i(u)N_q^j(v)}{\sum\limits_{i=0}^{n}\sum\limits_{j=0}^{m} \alpha_{ij}N_p^i(u)N_q^j(v)} \longrightarrow \begin{cases} u \in [0,1] \\ v \in [0,1] \end{cases} \tag{1}$$

where P_{ij} are the control points and α_{ij} are the corresponding weights. $N_p^i(t)$ are the normalized B-spline basis functions of degree p defined recursively as:

$$\begin{aligned} N_0^i(t) &= \begin{cases} 1 & t_i \le t \le t_{i+1} \\ 0 & \text{others} \end{cases} \\ N_p^i(t) &= \frac{t - t_i}{t_{i+p} - t_i} N_{p-1}^i(t) + \frac{t_{i+p+1} - t}{t_{i+p+1} - t_{i+1}} N_{p-1}^{i+1}(t) \\ \text{let } \frac{0}{0} &= 0 \end{aligned} \tag{2}$$

where t_i are the so-called knots forming a knot vector $\boldsymbol{T} = (t_0, t_1, \ldots, t_m)$.

When forming an algorithm, the NURBS format needs transforming to Bezier format. The underlying reason why a conversion is needed is the lack of simple numerically stable algorithms for determining derivatives for NURBS. Surfaces in NURBS format can be transformed to Bezier surfaces easily using the algorithm called Cox-De Boor algorithm [7]. A Bezier surface is defined as:

$$\boldsymbol{r}(\boldsymbol{u}, \boldsymbol{v}) = \frac{\sum\limits_{i=0}^{n} \sum\limits_{j=0}^{m} \alpha_{ij} \boldsymbol{P}_{ij} B_n^i(u) B_m^j(v)}{\sum\limits_{i=0}^{n} \sum\limits_{j=0}^{m} \alpha_{ij} B_n^i(u) B_m^j(v)} \longrightarrow \begin{cases} u \in [0, 1] \\ v \in [0, 1] \end{cases} \tag{3}$$

where $\boldsymbol{P}_i$ are the control points and α_i are the corresponding weights. $B_n^i(u)$ are the Bernstein polynomials of degree n defined as:

$$B_n^i(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i} \tag{4}$$

3. POSSIBLE CASES AND ALGORITHM

In optical methods, the targets are always electrically large and constructed by unions of surfaces, then occlusion is not exist only when none of the surfaces blocks the ray, namely, all the surfaces should be considered. For a surface, both shadowing and self-shadowing are to be considered.

Self-shadowing appears when the ray starts from or ends on the surface as in Fig. 2 and this always happens in PO and UTD method.

According to (1)–(4), the normal of 'middle' is easy to achieve, then if the inequation below is satisfied, the ray is blocked by the surface itself as in (b):

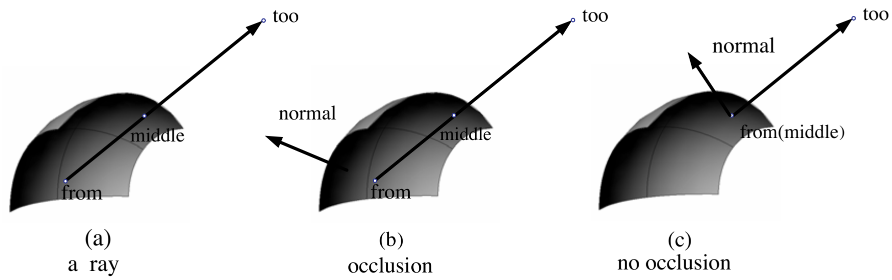


Figure 2. Self-shadowing and way to solve it.

$$(\boldsymbol{too} - \boldsymbol{from}) \cdot \boldsymbol{normal} < 0 \tag{5}$$

The surface is part of the target, which makes it a potential occluder to the ray. Several possible cases are illustrated below.

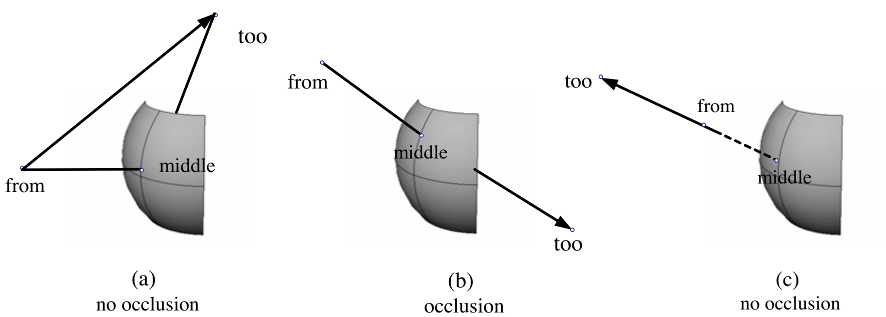


Figure 3. Occlusion of a surface to a ray.

Considering all the cases shown in Fig. 3, the distances between 'from', 'too' and 'middle' are managed, which yields the function for further use:

$$fun(u, v) = |\boldsymbol{middle} - \boldsymbol{from}| + |\boldsymbol{too} - \boldsymbol{middle}| - |\boldsymbol{too} - \boldsymbol{from}| \tag{6}$$

where 'middle' is on the surface and (u, v) are the parametric coordinates of 'middle' on the surface, which yields that fun varies with (u, v). As it is known to all, the addition of length of any two sides of a triangular is larger than the other one. Then what should be done next is to find the minimum of fun. Once a minimum is found, an occlusion exists if the minimum is below certain tolerance close enough to zero (like 0.0001 in this paper) or the ray passes the surface without interaction. To minimize fun, CGM is used and applied to the two parametric coordinates of the surface, CGM requires the knowledge of the partial derivatives of fun, given by:

$$\begin{aligned}\frac{\partial fun(u,v)}{\partial u} &= \left(\frac{(\boldsymbol{middle}-\boldsymbol{from})}{|\boldsymbol{middle}-\boldsymbol{from}|}+\frac{(\boldsymbol{too}-\boldsymbol{middle})}{|\boldsymbol{too}-\boldsymbol{middle}|}\right)\cdot\frac{\partial\boldsymbol{middle}}{\partial u}\\ \frac{\partial fun(u,v)}{\partial v} &= \left(\frac{(\boldsymbol{middle}-\boldsymbol{from})}{|\boldsymbol{middle}-\boldsymbol{from}|}+\frac{(\boldsymbol{too}-\boldsymbol{middle})}{|\boldsymbol{too}-\boldsymbol{middle}|}\right)\cdot\frac{\partial\boldsymbol{middle}}{\partial v}\end{aligned} \tag{7}$$

Also, CGM requires the knowledge of a initial point which can be achieved by using a regular grid of sample points. The sample point s_{ij} are uniformly located in the parametric space of the surface $\boldsymbol{r}(\boldsymbol{u},\boldsymbol{v})$, and computed as follows:

$$s_{ij} = \boldsymbol{r}(i\Delta u, j\Delta v), \quad i = 0, 1, \ldots, N_{samp} \tag{8}$$

where $\Delta u = u_{\max}/N_{samp}$ and $\Delta v = v_{\max}/N_{samp}$ and usually a N_{samp} of about 25 is enough. Usually, the third case in Fig. 2 is tend to be ignored which will cause incorrectness, but it is obviously involved within the functions above. The flow-chart of the algorithm is given below:

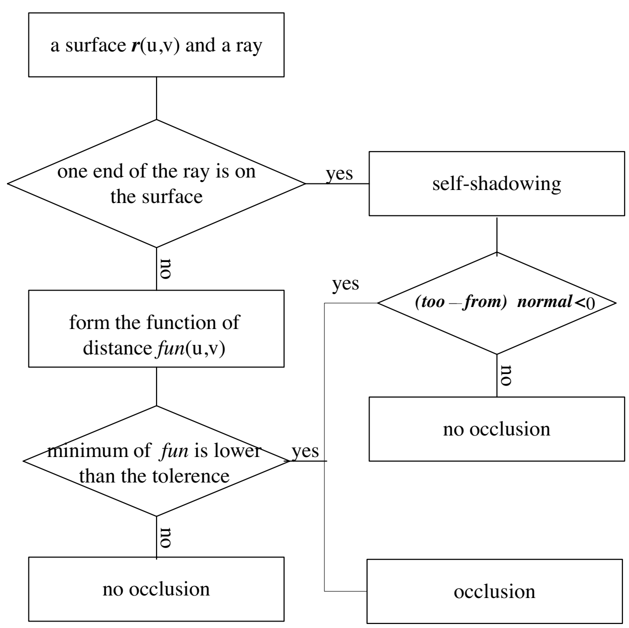


Figure 4. Flow-chart of the algorithm.

Figure 5 shows an example of the algorithm where a bigger plane is placed as background, and the other two surfaces are placed in front. The rays start from (5,5,0) and end at the union of points on the bigger plane. Applying the algorithm to the system and the cast shadow of the front surfaces can be seen.

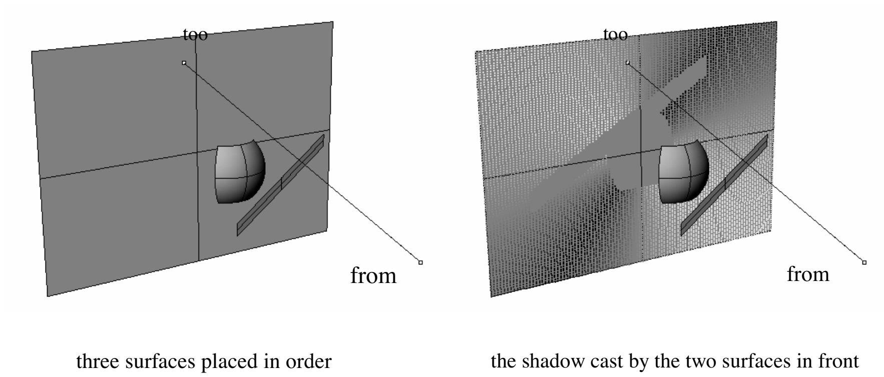


Figure 5. Example of occlusion.

4. DISCUSSION AND CONCLUSION

In the use of the algorithm, a special case appears when the ray is in the tangent plane of the surface. Case shown in Fig. 6 is special because it is hard to decide if the ray is blocked or not, so it is supposed to depend on the need, that is, different options are taken according to different methods. If it is considered not to be an occlusion, then a test is done, that is if $(\boldsymbol{too} - \boldsymbol{from}) \cdot \boldsymbol{normal} = 0$, the ray is not taken as blocked.

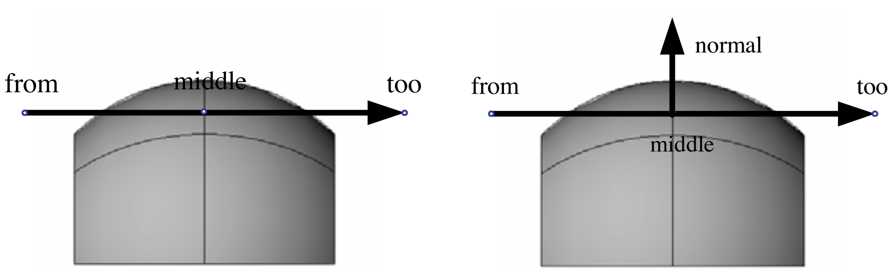


Figure 6. A special case.

Shadowing between given rays and surfaces needs considering in most optical methods of electromagnetic calculation. In this paper, the occlusions between rays and surfaces constructed by NURBS are studied and an algorithm introducing simple geometrical principle is given. Both the case when the ray intersects with the surface on the extension and self-shadowing are taken into account. The algorithm can be applied to electromagnetic methods like PO or UTD methods when the NURBS modeling is introduced and where shadowing is of key problem.

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