

## PERFECT ELECTROMAGNETIC CONDUCTOR (PEMC) AND FRACTIONAL WAVEGUIDE

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**Abstract**—Fractional curl operator has been utilized to study the fractional perfect electromagnetic conducting waveguide. The fractional perfect electromagnetic conducting waveguide may be regarded as intermediate step between the two given waveguides. One of the waveguide is composed of perfect electromagnetic conducting (PEMC) walls while other is dual of it as (DPEMC). Corresponding fields and surface impedance have been determined and boundary conditions for DPEMC surface have been derived.

### 1. INTRODUCTION

Recently concept of perfect electromagnetic conductor (PEMC) as generalization of the perfect electric conductor (PEC) and perfect magnetic conductor (PMC) [1–6] has been introduced and has attracted the attention of many researchers [7–11]. It is well known that PEC boundary may be defined by the conditions

$$\mathbf{n} \times \mathbf{E} = 0, \quad \mathbf{n} \cdot \mathbf{B} = 0$$

While PMC boundary may be defined by the boundary conditions

$$\mathbf{n} \times \mathbf{H} = 0, \quad \mathbf{n} \cdot \mathbf{D} = 0$$

The PEMC boundary conditions are of the more general form

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = 0, \quad \mathbf{n} \cdot (\mathbf{D} - M\mathbf{B}) = 0$$

where  $M$  denotes the admittance of the PEMC boundary. It is obvious that PMC corresponds to  $M = 0$ , while PEC corresponds to  $M = \pm\infty$ .

Another generalization of PEC and PMC reveals from the concept of fractional curl operator, i.e.,  $(\nabla \times)^\alpha$  [12]. The boundary is known as fractional dual interface with PEC and PMC as the two special situations of the fractional dual interface [13–18]. For a PEC interface placed in a medium having intrinsic impedance  $\eta_0$  as original situation, the surface impedance of the interface which may be regarded as intermediate step between the PEC and PMC may be written as [12]

$$Z_{fd} = j\eta_0 \tan\left(\frac{\alpha\pi}{2}\right)$$

where  $fd$  stands for fractional dual. These results may be obtained using the following relations [12]

$$\begin{aligned}\mathbf{E}_{fd} &= \frac{1}{(jk_0)^\alpha} (\nabla \times)^\alpha \mathbf{E} \\ \eta_0 \mathbf{H}_{fd} &= \frac{1}{(jk_0)^\alpha} (\nabla \times)^\alpha \eta_0 \mathbf{H}\end{aligned}$$

where  $k_0 = \omega\sqrt{\mu_0\epsilon_0}$  and  $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ . It may be noted that above two equations are Faraday-Ampere's Maxwell equations with fractionalized curl operator. Above relations yield solutions which may be regarded as intermediate step between the solution  $(\mathbf{E}, \eta_0 \mathbf{H})$  and  $(\eta_0 \mathbf{H}, -\mathbf{E})$ , when the value of fractional parameter changes between zero and one.

In present discussion, our interest is to determine the fractional dual solutions for a PEMC parallel plate waveguide using the fractional curl operator. Corresponding impedance of the walls of parallel plate waveguide has been determined and boundary conditions for DPEMC, which is a limiting case of fractional dual surface for  $\alpha = 1$ , have been derived.

## 2. FRACTIONAL DUALITY FOR PEMC PARALLEL PLATE WAVEGUIDE

Consider a parallel plate waveguide with PEMC walls at  $y = 0$  and  $y = b$ . The space between the plates is filled with air and the propagation is along  $z$ -axis while the dimensions of the plates are considered infinite along  $x$ -axis. Field solutions inside the guide can be proposed using the field equations for PEC walls guide and then applying the transformation given in [3], that is

$$\begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} = \frac{1}{(M\eta_0)^2 + 1} \begin{bmatrix} M\eta_0 & -\eta_0 \\ \frac{1}{\eta_0} & M\eta_0 \end{bmatrix} \begin{bmatrix} \mathbf{E}_d \\ \mathbf{H}_d \end{bmatrix} \quad (1)$$

where  $\mathbf{E}_d, \mathbf{H}_d$  are the solutions for a PEC plates waveguide and  $\mathbf{E}, \mathbf{H}$  are the solutions for PEMC plates waveguide.

Fields inside the waveguide may be considered as combination of two TEM plane waves bouncing back and forth obliquely between the two conducting plates. That is for TM mode propagating along  $z$ -axis in a PEC plates waveguide we can write

$$\mathbf{E}_d = \mathbf{E}_{1d} + \mathbf{E}_{2d} \quad (2a)$$

$$\eta_0 \mathbf{H}_d = \eta_0 \mathbf{H}_{1d} + \eta_0 \mathbf{H}_{2d} \quad (2b)$$

where  $(\mathbf{E}_{1d}, \mathbf{H}_{1d})$  are the electric and magnetic fields associated with one plane wave and are given below

$$\mathbf{E}_{1d} = \frac{A_n}{2} \left( -j\hat{z} - \frac{j\beta}{h}\hat{y} \right) \exp(jhy - j\beta z) \quad (3a)$$

$$\eta_0 \mathbf{H}_{1d} = \hat{x} \frac{jk}{h} \frac{A_n}{2} \exp(jhy - j\beta z) \quad (3b)$$

while electric and magnetic fields  $(\mathbf{E}_{2d}, \mathbf{H}_{2d})$  associated with second plane wave and are given below

$$\mathbf{E}_{2d} = \frac{A_n}{2} \left( j\hat{z} - \frac{j\beta}{h}\hat{y} \right) \exp(-jhy - j\beta z) \quad (3c)$$

$$\eta_0 \mathbf{H}_{2d} = \hat{x} \frac{jk}{h} \frac{A_n}{2} \exp(-jhy - j\beta z) \quad (3d)$$

After applying transformation given in equation (1), equation (3) can be written as

$$\begin{aligned} \mathbf{E}_1 &= B \left[ M\eta_0 \left( -\hat{z} - \frac{\beta}{h}\hat{y} \right) - \frac{k}{h}\hat{x} \right] \exp(jhy - j\beta z) \\ \mathbf{H}_1 &= \frac{B}{\eta_0} \left[ \left( -\hat{z} - \frac{\beta}{h}\hat{y} \right) + M\eta_0 \frac{k}{h}\hat{x} \right] \exp(jhy - j\beta z) \\ \mathbf{E}_2 &= B \left[ M\eta_0 \left( \hat{z} - \frac{\beta}{h}\hat{y} \right) - \frac{k}{h}\hat{x} \right] \exp(-jhy - j\beta z) \\ \mathbf{H}_2 &= \frac{B}{\eta_0} \left[ \left( \hat{z} - \frac{\beta}{h}\hat{y} \right) + M\eta_0 \frac{k}{h}\hat{x} \right] \exp(-jhy - j\beta z) \end{aligned} \quad (4)$$

where

$$B = \frac{jA_n}{2((M\eta_0)^2 + 1)}$$

Total fields inside the guide can be written as

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\
&= 2B \left[ -\frac{k}{h} \cos(hy) \hat{x} - M\eta_0 \frac{\beta}{h} \cos(hy) \hat{y} - jM\eta_0 \sin(hy) \hat{z} \right] \exp(-j\beta z)
\end{aligned} \tag{5a}$$

$$\begin{aligned}
\eta_0 \mathbf{H} &= \eta_0 \mathbf{H}_1 + \eta_0 \mathbf{H}_2 \\
&= 2 \frac{B}{\eta_0} \left[ M\eta_0 \frac{k}{h} \cos(hy) \hat{x} - \frac{\beta}{h} \cos(hy) \hat{y} - j \sin(hy) \hat{z} \right] \exp(-j\beta z)
\end{aligned} \tag{5b}$$

Fields  $\mathbf{E}_1$  and  $\mathbf{H}_1$  given by equations (4) are related through the Maxwell equations as

$$\mathbf{k}_1 \times \mathbf{E}_1 = \eta_0 \mathbf{H}_1 \tag{6a}$$

$$\mathbf{k}_1 \times \eta_0 \mathbf{H}_1 = -\mathbf{E}_1 \tag{6b}$$

Similarly

$$\mathbf{k}_2 \times \mathbf{E}_2 = \eta_0 \mathbf{H}_2 \tag{6c}$$

$$\mathbf{k}_2 \times \eta_0 \mathbf{H}_2 = -\mathbf{E}_2 \tag{6d}$$

where

$$\mathbf{k}_1 = \frac{1}{(jk)} (-jh\hat{y} + j\beta\hat{z})$$

$$\mathbf{k}_2 = \frac{1}{(jk)} (jh\hat{y} + j\beta\hat{z})$$

In order to fractionalize these fields we may expand the fields in terms of eigen vectors of the cross product operators ( $\mathbf{k}_1 \times$ ) and ( $\mathbf{k}_2 \times$ ) and then fractional fields can be found by fractionalizing the eigen values of the cross product operators as

$$\mathbf{E}_{1fd} = [(a_1)^\alpha P_1 \mathbf{A}_1 + (a_2)^\alpha Q_1 \mathbf{A}_2 + (a_3)^\alpha R_1 \mathbf{A}_3] \exp(jhy - j\beta z) \tag{7a}$$

$$\mathbf{H}_{1fd} = [(a_1)^\alpha P_2 \mathbf{A}_1 + (a_2)^\alpha Q_2 \mathbf{A}_2 + (a_3)^\alpha R_2 \mathbf{A}_3] \exp(jhy - j\beta z) \tag{7b}$$

where

$$\begin{aligned}
\mathbf{A}_1 &= \frac{1}{\sqrt{2}} \left[ \hat{x} - j\frac{\beta}{k} \hat{y} - j\frac{h}{k} \hat{z} \right], & a_1 &= j \\
\mathbf{A}_2 &= \frac{1}{\sqrt{2}} \left[ \hat{x} + j\frac{\beta}{k} \hat{y} + j\frac{h}{k} \hat{z} \right], & a_2 &= -j \\
\mathbf{A}_3 &= -j\frac{h}{k} \hat{y} + j\frac{\beta}{k} \hat{z}, & a_3 &= 0
\end{aligned}$$

and

$$\begin{aligned}
 P_1 &= \frac{B}{\sqrt{2}} \left[ -jM\eta \frac{k}{h} - \frac{k}{h} \right] \\
 Q_1 &= \frac{B}{\sqrt{2}} \left[ jM\eta \frac{k}{h} - \frac{k}{h} \right] \\
 R_1 &= 0 \\
 P_2 &= \frac{B}{\eta\sqrt{2}} \left[ -j\frac{k}{h} + M\eta \frac{k}{h} \right] \\
 Q_2 &= \frac{B}{\eta\sqrt{2}} \left[ j\frac{k}{h} + M\eta \frac{k}{h} \right] \\
 R_2 &= 0
 \end{aligned}$$

Using these values in equation (7), we get

$$\begin{aligned}
 \mathbf{E}_{1fd}(y, z) &= B \frac{k}{h} \left[ \left\{ M\eta_0 \sin\left(\frac{\alpha\pi}{2}\right) - \cos\left(\frac{\alpha\pi}{2}\right) \right\} \hat{x} \right. \\
 &\quad \left. - \frac{j\beta}{k} \left\{ -jM\eta_0 \cos\left(\frac{\alpha\pi}{2}\right) - j \sin\left(\frac{\alpha\pi}{2}\right) \right\} \hat{y} \right. \\
 &\quad \left. - j\frac{h}{k} \left\{ -jM\eta_0 \cos\left(\frac{\alpha\pi}{2}\right) - j \sin\left(\frac{\alpha\pi}{2}\right) \right\} \hat{z} \right] \exp(jhy - j\beta z)
 \end{aligned} \tag{8a}$$

$$\begin{aligned}
 \mathbf{H}_{1fd}(y, z) &= \frac{B}{\eta_0} \frac{k}{h} \left[ \left\{ M\eta_0 \cos\left(\frac{\alpha\pi}{2}\right) + \sin\left(\frac{\alpha\pi}{2}\right) \right\} \hat{x} \right. \\
 &\quad \left. - j\frac{\beta}{k} \left\{ jM\eta_0 \sin\left(\frac{\alpha\pi}{2}\right) - j \cos\left(\frac{\alpha\pi}{2}\right) \right\} \hat{y} \right. \\
 &\quad \left. - j\frac{h}{k} \left\{ jM\eta_0 \sin\left(\frac{\alpha\pi}{2}\right) - j \cos\left(\frac{\alpha\pi}{2}\right) \right\} \hat{z} \right] \exp(jhy - j\beta z)
 \end{aligned} \tag{8b}$$

Similarly we can write

$$\begin{aligned}
 \mathbf{E}_{2fd}(y, z) &= B \frac{k}{h} \exp(-j\alpha\pi) \\
 &\quad \times \left[ \left\{ -M\eta_0 \sin\left(\frac{\alpha\pi}{2}\right) - \cos\left(\frac{\alpha\pi}{2}\right) \right\} \hat{x} \right. \\
 &\quad \left. - \frac{j\beta}{k} \left\{ -jM\eta_0 \cos\left(\frac{\alpha\pi}{2}\right) + j \sin\left(\frac{\alpha\pi}{2}\right) \right\} \hat{y} \right. \\
 &\quad \left. + j\frac{h}{k} \left\{ -jM\eta_0 \cos\left(\frac{\alpha\pi}{2}\right) + j \sin\left(\frac{\alpha\pi}{2}\right) \right\} \hat{z} \right] \exp(-jhy - j\beta z) \tag{9a}
 \end{aligned}$$

$$\begin{aligned}
\mathbf{H}_{2fd}(y, z) &= \frac{B}{\eta_0} \frac{k}{h} \exp(-j\alpha\pi) \\
&\times \left[ \left\{ M\eta_0 \cos\left(\frac{\alpha\pi}{2}\right) - \sin\left(\frac{\alpha\pi}{2}\right) \right\} \hat{x} \right. \\
&- j \frac{\beta}{k} \left\{ -jM\eta_0 \sin\left(\frac{\alpha\pi}{2}\right) - j \cos\left(\frac{\alpha\pi}{2}\right) \right\} \hat{y} \\
&\left. + j \frac{h}{k} \left\{ -jM\eta_0 \sin\left(\frac{\alpha\pi}{2}\right) - j \cos\left(\frac{\alpha\pi}{2}\right) \right\} \hat{z} \right] \exp(-jhy - j\beta z) \quad (9b)
\end{aligned}$$

Adding these components we get the fractionalized fields as given below

$$\begin{aligned}
\mathbf{E}_{fd}(y, z) &= \frac{A_n}{((M\eta_0)^2 + 1)} \frac{k}{h} \exp(-j\frac{\alpha\pi}{2}) \\
&\times \left[ \{-M\eta_0 S_1 S_2 - jC_1 C_2\} \hat{x} \right. \\
&- \frac{j\beta}{k} \{M\eta_0 C_1 C_2 + jS_1 S_2\} \hat{y} \\
&\left. + \frac{h}{k} \{M\eta_0 C_1 S_2 - jS_1 C_2\} \hat{z} \right] \exp(-j\beta z) \quad (10a)
\end{aligned}$$

$$\begin{aligned}
\eta_0 \mathbf{H}_{fd}(y, z) &= \frac{A_n}{((M\eta_0)^2 + 1)} \frac{k}{h} \exp(-j\frac{\alpha\pi}{2}) \\
&\times \left[ \{jM\eta_0 C_1 C_2 - S_1 S_2\} \hat{x} \right. \\
&+ \frac{\beta}{k} \{-M\eta_0 S_1 S_2 - jC_1 C_2\} \hat{y} \\
&\left. + j \frac{h}{k} \{M\eta_0 S_1 C_2 - jC_1 S_2\} \hat{z} \right] \exp(-j\beta z) \quad (10b)
\end{aligned}$$

where

$$\begin{aligned}
C_1 &= \cos\left(\frac{\alpha\pi}{2}\right), & C_2 &= \cos\left(hy + \frac{\alpha\pi}{2}\right) \\
S_1 &= \sin\left(\frac{\alpha\pi}{2}\right), & S_2 &= \sin\left(hy + \frac{\alpha\pi}{2}\right)
\end{aligned}$$

We can see from equations (5) and (10) that when  $\alpha = 0$

$$\mathbf{E}_{fd} = \mathbf{E} \quad \eta_0 \mathbf{H}_{fd} = \eta_0 \mathbf{H}$$

and for  $\alpha = 1$

$$\mathbf{E}_{fd} = \eta_0 \mathbf{H} \quad \eta_0 \mathbf{H}_{fd} = -\mathbf{E}$$

For  $M = \infty$

$$\mathbf{E}_{fd} = \mathbf{E}_{fd} |_{\text{PEC}} \quad \eta_0 \mathbf{H}_{fd} = \eta_0 \mathbf{H}_{fd} |_{\text{PEC}}$$

For  $M = 0$

$$\mathbf{E}_{fd} = \mathbf{E}_{fd} |_{\text{PMC}} \quad \eta_0 \mathbf{H}_{fd} = \eta_0 \mathbf{H}_{fd} |_{\text{PMC}}$$

where  $\mathbf{E}_{fd} |_{\text{PEC}}$  and  $\mathbf{E}_{fd} |_{\text{PMC}}$  means fractional dual fields corresponding to waveguide with PEC and PMC walls respectively.

Surface impedance  $\underline{\underline{Z}}_{fd}$  may be written in a matrix form as

$$\underline{\underline{Z}}_{fd} = \begin{bmatrix} Z_{fdxx} & Z_{fdxz} \\ Z_{fdzx} & Z_{fdzz} \end{bmatrix}$$

where

$$\begin{aligned} Z_{fdxx} &= \frac{E_{fdx}}{H_{fdx}} = j\eta_0 \left\{ \frac{M\eta_0 S_1 S_2 + jC_1 C_2}{M\eta_0 C_1 C_2 + jS_1 S_2} \right\} \\ Z_{fdxz} &= -\frac{E_{fdx}}{H_{fdz}} = j\eta_0 \frac{k}{h} \left\{ \frac{M\eta_0 S_1 S_2 + jC_1 C_2}{M\eta_0 S_1 C_2 - jC_1 S_2} \right\} \\ Z_{fdzx} &= \frac{E_{fdz}}{H_{fdx}} = -j\eta_0 \frac{h}{k} \left\{ \frac{M\eta_0 C_1 S_2 - jS_1 C_2}{M\eta_0 C_1 C_2 - S_1 S_2} \right\} \\ Z_{fdzz} &= \frac{E_{fdz}}{H_{fdz}} = -j\eta_0 \left\{ \frac{M\eta_0 C_1 S_2 - jS_1 C_2}{M\eta_0 S_1 C_2 - jC_1 S_2} \right\} \end{aligned}$$

For  $\alpha = 0$  and  $0 < M < \infty$

$$\underline{\underline{Z}} = \eta_0 \begin{bmatrix} -\frac{1}{M\eta_0} & j\frac{k}{h} \cot(hy) \\ -j\frac{h}{k} \tan(hy) & M\eta_0 \end{bmatrix}$$

For  $\alpha = 1$  and  $0 < M < \infty$

$$\underline{\underline{Z}} = \eta_0 \begin{bmatrix} M\eta_0 & j\frac{k}{h} \cot(hy) \\ -j\frac{h}{k} \tan(hy) & -\frac{1}{M\eta_0} \end{bmatrix}$$

For  $M = \infty$  and  $0 < \alpha < 1$

$$\underline{\underline{Z}} = \eta_0 \begin{bmatrix} j \tan(\frac{\alpha\pi}{2}) \tan(hy + \frac{\alpha\pi}{2}) & j\frac{k}{h} \tan(hy + \frac{\alpha\pi}{2}) \\ -j\frac{h}{k} \tan(hy + \frac{\alpha\pi}{2}) & -j \cot(\frac{\alpha\pi}{2}) \tan(hy + \frac{\alpha\pi}{2}) \end{bmatrix}$$

For  $M = 0$  and  $0 < \alpha < 1$

$$\underline{\underline{Z}} = \eta_0 \begin{bmatrix} j \cot(\frac{\alpha\pi}{2}) \cot(hy + \frac{\alpha\pi}{2}) & -j\frac{k}{h} \cot(hy + \frac{\alpha\pi}{2}) \\ +j\frac{h}{k} \cot(hy + \frac{\alpha\pi}{2}) & -j \tan(\frac{\alpha\pi}{2}) \cot(hy + \frac{\alpha\pi}{2}) \end{bmatrix}$$

Also it can be seen from equation (10) that boundary conditions for the DPEMC surface are

$$\mathbf{n} \times (M\eta_0^2 \mathbf{H} - \mathbf{E}) = 0 \quad , \quad \mathbf{n} \cdot (\mathbf{B} + M\eta_0^2 \mathbf{D}) = 0$$

### 3. DISCUSSION AND CONCLUSIONS

It is noted that for  $\alpha = 0$ , we are dealing with a perfect electromagnetic conducting parallel plate waveguide carrying TM mode in z-direction. This may be interpreted as the superposition of two plane waves bounding back and forth obliquely between the two plates. As fractional parameter  $\alpha$  takes values from zero towards unity, there are two activities happening. One is the electric and magnetic field vectors are being rotated by an angle  $\alpha\pi/2$  in the counterclockwise direction. Other is that perfect electromagnetic conductor (PEMC) is changing to dual to perfect electroamgnetic conductor (DPEMC). That is parallel plates are intermediate step of PEMC and DPEMC . As  $\alpha$  becomes equal to unity rotation angle becomes equal to  $\pi/2$  and PEMC parallel plates changes to DPEMC parallel plates satisfying the boundary condition

$$\mathbf{n} \times (M\eta_0^2\mathbf{H} - \mathbf{E}) = 0 \quad , \quad \mathbf{n} \cdot (\mathbf{B} + M\eta_0^2\mathbf{D}) = 0$$

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