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COMPUTATIONS OF ELECTROMAGNETIC FIELDS RADIATED FROM COMPLEX LIGHTNING CHANNELS

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Abstract—In this paper, three methods for calculating electromagnetic fields radiated from complex lightning channels are discussed, which includes channel obliqueness, branches and tortuosity. By rotating and moving the coordinate system of the vertical channel, differential expressions of electromagnetic fields from the irregular channel can be derived from the conclusions of the straight and vertical channel. Through analyzing calculation results of two examples reveals that channel tortuosity and branch is to introduce the higher frequency content above 100 kHz into lightning electromagnetic fields.

1. INTRODUCTION

In most computations of lightning electromagnetic fields, the return-stroke channel is assumed to be straight and vertical [1], while it is known to be tortuous on scales ranging from less than 1 m to over 1 km [2]. In the case of natural lightning, only subsequent strokes were considered. The effects of channel tortuosity on return-stroke radiation fields may be studied theoretically using a piecewise linear representation of the lightning channel. In general, the effect of tortuosity is to introduce fine structure into the time-domain radiation field waveform and consequently to increase the higher frequency content above 100 kHz. At each kink, that is, point at which the linear segments joint, there is a change in the direction of the propagation of the current wave and such changes will introduce rapid variations in the radiation field. If consideration for branches, radiation fields should be overlaid with electromagnetic fields from main channel and branches. In Reference [3], we assume the lightning channel is a straight and vertical antenna, its electromagnetic fields are calculated with dipole method. Based on Reference [3], we present the methods to calculate electric fields and magnetic fields radiated from a tortuous channel with branches in this paper.

2. OBLIQUE LIGHTNING CHANNEL

In lightning engineering model, we first assume a lightning channel is oblique, and oblique angle is α (see Fig. 1). After rotating cylindrical coordinate system (see Fig. 2), channel's length is H^* and height of return-stroke current is h^*, the infinitesimal current element dz' can be looked as an electric dipole at a height z'^*, which moves upward along the channel at a speed v. The observation point P has a

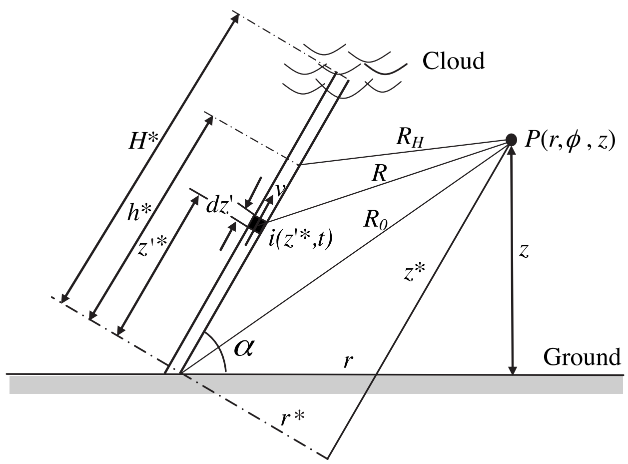

Figure 1. Model of oblique lightning channel.

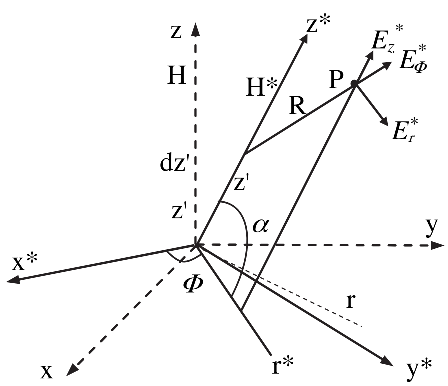

Figure 2. Rotating cylindrical coordinate system.

horizontal distance r from the lightning channel and a height z, and R is the distance from the electric dipole. The ground has a perfect conductivity and the air has a permittivity ε_0 and permeability u_0. The current $i(z'^*, t)$ in channel is a function varying with channel height and time [4]. If we find out corresponding relations between two coordinate systems, and substitute them into the formulations for the vertical channel in Reference [3], we will be able to obtain equations to calculate electromagnetic fields of oblique lightning channel.

According to geometric relations in Fig. 2, we can obtain

$$r* = \sqrt{r^2+z^2}\sin(\alpha - tg^{-1}\frac{z}{r}) \tag{1}$$

$$z* = \frac{r*}{tg(\alpha - tg^{-1}\frac{z}{r})} = \frac{\sqrt{r^2+z^2}\sin(\alpha - tg^{-1}\frac{z}{r})}{tg(\alpha - tg^{-1}\frac{z}{r})} \tag{2}$$

$$h* = \frac{h}{\sin\alpha} \tag{3}$$

$$H* = \frac{H}{\sin\alpha} \tag{4}$$

$$z'* = \frac{z'}{\sin\alpha} \tag{5}$$

where z, r, h, and z' denote the correlative distance in original coordinate system (x, y, z) respectively. So the distance from Point P to the bottom of return-stroke current is

$$R_0 = \sqrt{r^2+z^2} \tag{6}$$

The distance from Point P to the top of return-stroke current is

$$\begin{aligned} R_H &= \sqrt{r^{*2} + (h^* - z^*)^2} \\ &= \left\{\left(r^2+z^2\right)\sin^2\left(\alpha - tg^{-1}\frac{z}{r}\right) + \right. \\ &\quad \left.\left[\frac{h}{\sin\alpha} - \frac{\sqrt{r^2+z^2}\sin\left(\alpha - tg^{-1}\frac{z}{r}\right)}{tg\left(\alpha - tg^{-1}\frac{z}{r}\right)}\right]^2\right\}^{1/2} \end{aligned} \tag{7}$$

The distance from Point P to the dipole dz' is

$$\begin{aligned} R &= \sqrt{(z^* - z'^*)^2 + r^{*2}} \\ &= \left\{\left(r^2+z^2\right)\sin^2\left(\alpha - tg^{-1}\frac{z}{r}\right) + \right. \\ &\quad \left.\left[\frac{\sqrt{r^2+z^2}\sin\left(\alpha - tg^{-1}\frac{z}{r}\right)}{tg\left(\alpha - tg^{-1}\frac{z}{r}\right)} - \frac{z'}{\sin\alpha}\right]^2\right\}^{1/2} \end{aligned} \tag{8}$$

We still use dipole method to solve Maxwell's equations. Substituting r^*, h^* into the formulations for the vertical channel in Reference [3], analytical expressions of electric fields and magnetic fields at Point P $(r,\ \phi,\ 0)$ in rotating coordinate system (x^*, y^*, z^*) can be derived as following

$$H_\phi{}^*(r,0,t) = \frac{I_0}{2\pi}\left[\frac{h^*}{r^*(h^{*2}+r^{*2})^{1/2}} + \frac{r^*}{\frac{c}{v}(h^{*2}+r^{*2})+h^*(h^{*2}+r^{*2})^{1/2}}\right] \tag{9}$$

$$\begin{aligned} E_z{}^*(r,0,t) = \frac{I_0}{2\pi\varepsilon_0}\Bigg[&\frac{-th^*+\frac{2h^{*2}}{v}+\frac{r^{*2}}{v}}{(h^{*2}+r^{*2})^{3/2}} - \frac{1}{r^*v} \\ &-\frac{r^{*2}}{c^2\,(h^{*2}+r^{*2})^{3/2}\left(\frac{1}{v}+\frac{h^*}{c\sqrt{h^{*2}+r^{*2}}}\right)}\Bigg] \end{aligned} \tag{10}$$

where h^* denotes the height of lightning return-stroken current in rotating coordinate system, it can be calculated with Equation (3); r^* denotes horizontal distance between Point P and the point that lightning strik falls to the ground, it can be calculated with Equation (1).

Consequently, in order to evaluate electromagnetic fields of any point in lightning space, substituting z^*, z'^*, R into the formulations in Reference [3], we can get differential expressions of electric fields and magnetic fields in the rotating cylindrical coordinate system as following

$$\begin{aligned} dE_r &= \frac{dz'}{4\pi\varepsilon_0}\Bigg[\frac{3r^*\,(z^*-z'^*)}{R^5}\int_0^t i(t-R/c)dt + \frac{3r^*\,(z^*-z'^*)}{cR^4}i(t-R/c) \\ &\quad+\frac{r^*\,(z^*-z'^*)}{c^2R^3}\frac{\partial i(t-R/c)}{\partial t}\Bigg] \end{aligned} \tag{11}$$

$$\begin{aligned} dE_z &= \frac{dz'}{4\pi\varepsilon_0}\Bigg[\frac{2\,(z^*-z'^*)^2-r^{*2}}{R^5}\int_0^t i(t-R/c)dt \\ &\quad+\frac{2\,(z^*-z'^*)^2-r^{*2}}{cR^4}i(t-R/c) - \frac{r^{*2}}{c^2R^3}\frac{\partial i(t-R/c)}{\partial t}\Bigg] \end{aligned} \tag{12}$$

$$dB_\phi = \frac{\mu_0 dz'}{4\pi}\left[\frac{r^*}{R^3}i(t-R/c) + \frac{r^*}{cR^2}\frac{\partial i(t-R/c)}{\partial t}\right] \tag{13}$$

where R can be computed with Equation (8), z^* can be computed with Equation (2), z'^* can be computed with Equation (5), r^* can be computed with Equation (1).

3. LIGHTNING CHANNEL BRANCHES

3.1. Calculation Method

As we know, a lightning channel is irregular, so its general field intensity is the sum of electromagnetic fields from main channel and branches [5]. But the direction of either coordinate system is different, thus electric fields and magnetic fields should be added in vector. Assuming the main channel is straight and vertical, the branch is oblique (see Fig. 3), electromagnetic fields from the main channel can still be calculated with the formulations in Reference [3], while field intensity expressions of the branch is derived as following.

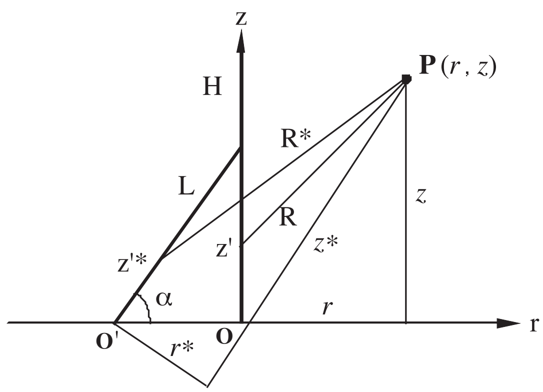

Figure 3. Cylindrical coordinate system for branches.

In Fig. 3, Point O is original point, its location is different from the oblique channel, we need to calculate the displacement at r direction once again. The horizontal distance between Point P and main channel is r, the distance between Point P and Point O' is $r+L\cos\alpha$, therefore

$$r^* = \sqrt{(r+L\cos\alpha)^2+z^2}\,\sin\left(\alpha - tg^{-1}\frac{z}{r+L\cos\alpha}\right) \tag{14}$$

$$z^* = \sqrt{(r+L\cos\alpha)^2+z^2}\,\cos\left(\alpha - tg^{-1}\frac{z}{r+L\cos\alpha}\right) \tag{15}$$

$$\begin{aligned} R^* &= \sqrt{\left(z^*-z'^*\right)^2+r^{*2}} \\ &= \left\{\left[(r+L\cos\alpha)^2+z^2\right]\sin^2\left(\alpha - tg^{-1}\frac{z}{r+L\cos\alpha}\right)\right. \\ &\quad \left.+\left[\sqrt{(r+L\cos\alpha)^2+z^2}\cos\left(\alpha - tg^{-1}\frac{z}{r+L\cos\alpha}\right)-\frac{z'}{\sin\alpha}\right]^2\right\}^{1/2} \end{aligned} \tag{16}$$

Replacing r^*, z^* and R in Equations (11), (12), (13) with above three expressions, field intensity from the branch at Point P can be written as

$$dE_r{}^* = \frac{dz'}{4\pi\varepsilon_0}\left[\frac{3r^*(z^*-z'^*)}{R^{*5}}\int_0^t i(t-R^*/c)dt + \frac{3r^*(z^*-z'^*)}{cR^{*4}}i(t-R^*/c) + \frac{r^*(z^*-z'^*)}{c^2R^{*3}}\frac{\partial i(t-R^*/c)}{\partial t}\right] \tag{17}$$

$$dE_z * = \frac{dz'}{4\pi\varepsilon_0}\left[\frac{2(z^*-z'^*)^2-r^{*2}}{R^{*5}}\int_0^t i(t-R^*/c)dt + \frac{2(z^*-z'^*)^2-r^{*2}}{cR^{*4}}i(t-R^*/c) - \frac{r^{*2}}{c^2R^{*3}}\frac{\partial i(t-R^*/c)}{\partial t}\right] \tag{18}$$

$$dB_\phi * = \frac{\mu_0 dz'}{4\pi}\left[\frac{r^*}{R^{*3}}i(t-R^*/c) + \frac{r^*}{cR^{*2}}\frac{\partial i(t-R^*c)}{\partial t}\right] \tag{19}$$

In above three expressions, R^* can be calculated with Equation (16), z^* can be calculated with Equation (15), r^* can be calculated with Equation (14).

Assuming E_z, E_r and B_ϕ represent electromagnetic fields of main channel, E_z^*, E_r^* and B_ϕ^* represent electromagnetic fields of branches (see Fig. 4). From this figure we may know that electronic fields of both channels have different directions, while the directions of magnetic fields are same. Therefore, general field intensity at Point P can be evaluated as following.

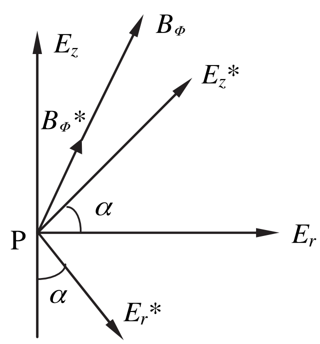

Figure 4. Overlay of electromagnetic fields.

Electronic field at z direction:

$$\sum E_z = E_z + E_z^* \sin\alpha - E_r^* \cos\alpha \tag{20}$$

magnetic field at r direction:

$$\sum E_r = E_r + E_z^* \cos\alpha - E_r^* \sin\alpha \tag{21}$$

general electronic field:

$$\sum E = \sqrt{\left(\sum E_z\right)^2 + \left(\sum E_r\right)^2} \tag{22}$$

general magnetic field:

$$\sum B_\phi = B_\phi + B_\phi^* \tag{23}$$

3.2. An Evaluation Example for Branches

We choose a vertical main channel with a branch as an example to evaluate field intensity and compare calculation results with a vertical channel without branches (see Fig. 5). The branch's oblique angle is 60° and its height is 100 m, whole main channel's height is 200 m. At first we solve electromagnetic fields of both channels respectively, then add their field intensity in vector, thus we will be able to get general field intensity. The branch's electromagnetic fields can be evaluated according to above oblique channel's formulations, while main channel's electromagnetic fields are still calculated with the method for vertical channel. The calculation results are shown in Fig. 6 and Fig. 7 from which we can know that values of main channel with a branch is more than those who have no branches, there are some high frequency contents at starting point of the waveform, and more are branches, more are high frequency contents.

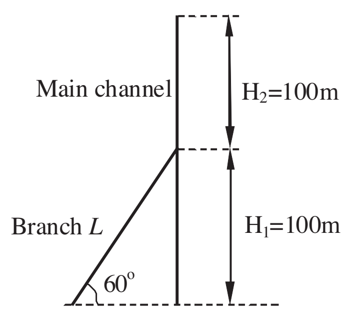

Figure 5. A vertical channel with a branch.

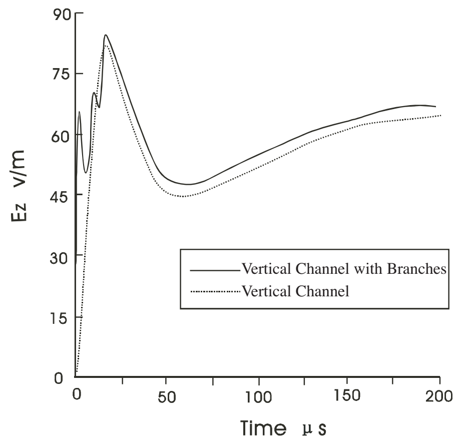

Figure 6. Vertical electronic field from branches ($r = 10\,\text{km}$, $z = 0$).

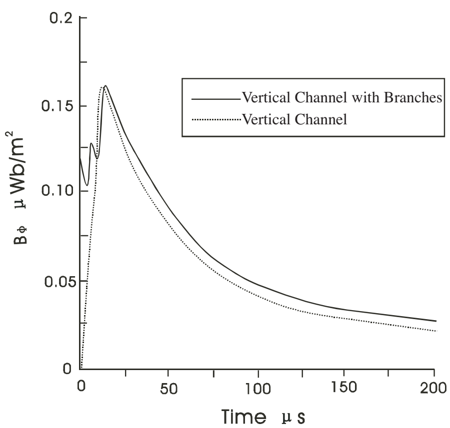

Figure 7. Azimuthal magnetic field from branches ($r = 10\,\text{km}$, $z = 0$).

4. LIGHTNING CHANNEL TORTUOSITY

4.1. Calculation Method

Actually nature lightning is tortuous and has many branches. In order to calculate its electromagnetic fields we may divide a tortuous channel into many linear segments which may be looked as oblique channels to proceed, so general field intensity can be obtained through adding electronic fields and magnetic fields of all linear segments. In Fig. 8, we assume that there is a tortuous lightning channel including two oblique linear segments which length are L and L_1, oblique angle are

α and α_1 respectively in cylindrical coordinate system. Thus Point P's electromagnetic fields from OO_1 can be evaluated directly with Equations (11), (12), (13), while field intensity from O_1O_2 can not be calculated directly with these three equations. Only after the coordinate system is moved to Point O_1, these three equations can be used.

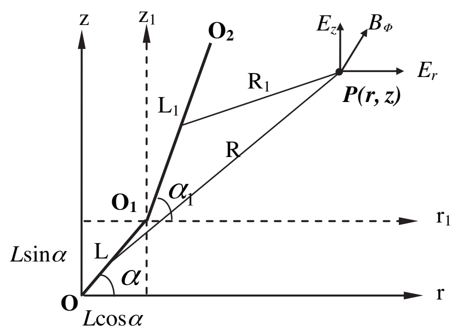

Figure 8. Coordinate system for tortuous channel.

In coordinate system z-o-r, Point O_1's coordinate is $(L\cos\alpha, L\sin\alpha)$, Point P's coordinate is (r, z). After this coordinate system is moved to Point O_1, Point O_1's coordinate is $(0, 0)$, Point P's coordinate is $(r\text{-}L\cos\alpha,\ z\text{-}L\sin\alpha)$. Therefore, replacing r in Equation (1) through (5) with r-$L\cos\alpha$, replacing z with z-$L\sin\alpha$, we can obtain the values of z^*, z'^*, R_1, r^* and h^* regarding channel O_1O_2 as following

$$r^* = \sqrt{(r-L\cos\alpha)^2+(z-L\sin\alpha)^2}\sin\left(\alpha_1 - tg^{-1}\frac{z-L\sin\alpha}{r-L\cos\alpha}\right) \quad (24)$$

$$z^* = \frac{\sqrt{(r-L\cos\alpha)^2+(z-L\sin\alpha)^2}\sin\left(a_1 - tg\frac{z-L\sin\alpha}{r-L\cos\alpha}\right)}{tg\left(\alpha_1 - tg^{-1}\frac{z-L\sin\alpha}{r-L\cos\alpha}\right)} \quad (25)$$

$$z'^* = \frac{z'-(z-L\sin\alpha)}{\sin\alpha_1} \quad (26)$$

$$\begin{aligned} R_1 &= \sqrt{r^{*2}+(z^*-z'^*)^2} \\ &= \left\{\left[(r-L\cos\alpha)^2+(z-L\sin\alpha)^2\right]\sin^2\left(a_1 - tg^{-1}\frac{z-L\sin\alpha}{r-L\sin\alpha}\right)\right. \\ &\left[\frac{(\sqrt{r-L\cos\alpha})^2+(z-L\sin\alpha)^2\sin\left(\alpha_1 - tg^{-1}\frac{z-L\sin\alpha}{r-L\cos\alpha}\right)}{tg\left(\alpha_1 - tg^{-1}\frac{z-L\sin\alpha}{r-L\cos\alpha}\right)}\right. \end{aligned}$$

$$-\frac{z'-(z-L\sin\alpha)}{\sin\alpha_1}\Bigg]^2\Bigg\}^{1/2} \tag{27}$$

$$\begin{aligned} h^* &= \frac{h}{\sin\alpha_1} \\ &= \frac{\xi}{(1-\xi^2)\sin\alpha_1}\left[ct-\xi(z-L\sin\alpha)-\right. \\ &\left.\sqrt{[\xi ct-(z-L\sin\alpha)]^2+(r-L\cos\alpha)^2(1-\xi)^2}\right] \end{aligned} \tag{28}$$

where $\xi = v/c$, c is the travel speed of electromagnetic wave which equals light speed $3\times10^8\,\mathrm{m/s}$ in air medium.

Substituting above expressions into Equations (11) (12) (13), Point P's field intensity from channel O_1O_2 can be written as

$$dB_\phi{}^* = \frac{\mu_0 dz'}{4\pi}\left[\frac{r^*}{R_1^3}i(t-R_1/c)+\frac{r^*}{cR_1^2}\frac{\partial i(t-R_1/c)}{\partial t}\right] \tag{29}$$

$$\begin{aligned} dE_r{}^* &= \frac{dz'}{4\pi\varepsilon_0}\left[\frac{3r^*(z^*-z'^*)}{R_1^5}\int_0^t i(t-R_1/c)dt\right. \\ &\left.+\frac{3r^*(z^*-z'^*)}{cR_1^4}i(t-R_1/c)+\frac{r^*(z^*-z'^*)}{c^2R_1^3}\frac{\partial i(t-R_1/c)}{\partial t}\right] \end{aligned} \tag{30}$$

$$\begin{aligned} dE_z{}^* &= \frac{dz'}{4\pi\varepsilon_0}\left[\frac{2(z^*-z'^*)^2-r^{*2}}{R_1^5}\int_0^t i(t-R_1/c)dt\right. \\ &\left.+\frac{2(z^*-z'^*)^2-r^{*2}}{cR_1^4}i(t-R_1/c)-\frac{r^{*2}}{c^2R_1^3}\frac{\partial i(t-R_1/c)}{\partial t}\right] \end{aligned} \tag{31}$$

where R_1 can be got from Equation (27), z^* can be got from Equation (25), z'^* can be got from Equation (26), r^* can be got from Equation (24).

Consequently, Point P' electronic fields and magnetic fields may be calculated respectively with different equations. Equations (11) (12) (13) should be used in the case of return-stroke current going through L, or time $t \le L/v$ (v is the speed of return-stroke current), Equations (29) (30) (31) should be used in the case of return-stroke current going through L_1, or time $t > L/v$.

4.2. An Evaluation Example for Tortuous Channel

In Fig. 9 we assume that a tortuous channel is composed of two linear oblique channels. From above discussion we know the first linear segment may be calculated with the equations of oblique channel directly, while the second segment can be calculated only after the coordinate system is moved.

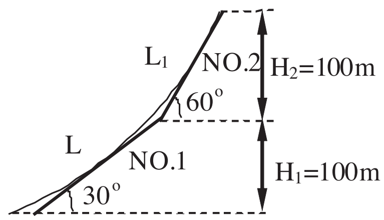

Figure 9. A tortuous channel.

In order to compare evaluation results with a vertical channel, we assume the oblique angle of NO. 1 segment α is 30°, the angle of NO. 2 segment α_1 is 60°, the height of each segment is 100 m and the height of the vertical channel is 200 m, their lightning current parameters are the same as Reference [6], return-stroke speed is 1.5×10^8 m/s [7]. Evaluation results are shown as Fig. 10 and Fig. 11, the real line denotes electromagnetic fields from the tortuous channel, the dashed line denotes electromagnetic fields from the vertical channel. From these two figures we can see that initial values of electronic fields and magnetic fields are different and the values at starting part of waveform of tortuous channel are much greater, which illuminate that there are

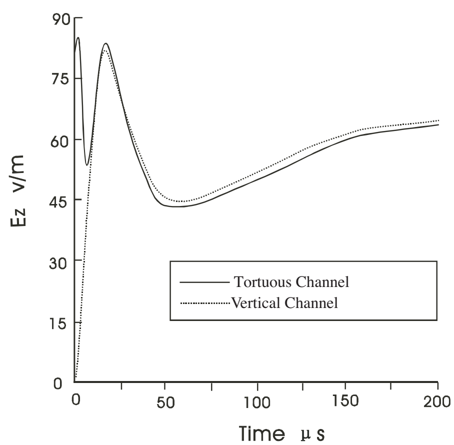

Figure 10. Vertical electronic field from tortuous channel ($r = 10$ km, $z = 0$).

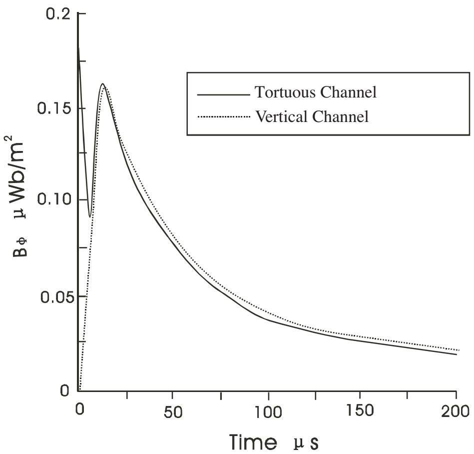

Figure 11. Azimuthal magnetic field from tortuous channel ($r = 10\,\mathrm{km}$, $z = 0$).

high frequency contents above 100 kHz emerged in electromagnetic fields. While initial values from the vertical channel are almost zero, which illuminate that there are no high frequency contents coming from a vertical channel. Otherwise, the values of electromagnetic fields from the vertical channel are greater than tortuous channel on the same conditions. Therefore channel tortuosity will introduce higher frequency contents into electromagnetic fields, and more are tortuosity, more are high frequency contents [8, 9].

5. CONCLUSION

In this paper, three methods for calculating electromagnetic fields radiated from complex lightning channels have been discussed, which includes channel obliqueness, branches and tortuosity. By rotating and moving the coordinate system of the vertical channel, differential expressions of electromagnetic fields from the oblique channel can be derived from the conclusions of the vertical channel. Electromagnetic fields from the channel branch may be evaluated through overlaying electronic fields and magnetic fields from a vertical main channel and oblique channels. The tortuous channel can be divided into many linear oblique segments from which field intensity can be added to obtain general electromagnetic fields from whole tortuous channel.

Through analyzing calculation results of two examples reveals that channel tortuosity and branches are to introduce the higher frequency contents above 100 kHz into electromagnetic fields, and more are tortuosity and branches, more are higher frequency contents. Meanwhile, field intensity from a channel with branches is greater than single vertical channel.

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