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FDTD MODELING OF ARBITRARY LINEAR LUMPED NETWORKS USING PIECEWISE LINEAR RECURSIVE CONVOLUTION TECHNIQUE

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Abstract—Based on the piecewise linear recursive convolution (PLRC) technique, FDTD modeling of Arbitrary linear lumped networks is studied in this paper, including one-port networks and two-port networks. Their general FDTD iterative formulations are obtained. Firstly, the admittance parameters in Laplace domain of lumped network are written as a summation form of several rational fractions; then the time domain admittance parameters can be obtained by means of inverse Fourier transform technique. Finally the time domain results are directly incorporated into the Maxwell-Ampere's difference equation using the PLRC technique. It is worth pointing out that this approach preserves the second-order accuracy and the explicit nature of the conventional FDTD method. The proposed technique can be extended to model arbitrary linear multi-port lumped networks. To show the validity of the proposed algorithm, we analyze two microstrip circuits including lumped networks. The results are compared with those obtained from the Z-transform technique and the good agreement is achieved.

1. INTRODUCTION

Analysis of microwave hybrid circuits including passive and active elements is not an easy work, because the electromagnetic interaction among passive and active elements of the circuits must be accurately modeled to predict the overall performance. The global electromagnetic simulators are indispensable for such circuits, and among the currently available techniques for global-domain electromagnetic analysis, the finite-difference time-domain (FDTD) method seems to be the best candidates [1], because lumped circuit elements can be incorporated and wide frequency-band performance can be obtained with only one FDTD run.

For the FDTD method, the focus is placed on how to introduce a lumped element into a FDTD cell. Much work has been reported in recent ten years. The lumped-element FDTD (LE-FDTD) is earlier proposed [2–5], which can include single passive elements such as resistors, inductors and capacitor, and also nonlinear or active devices such as diodes and transistor [9–11]. The main limitation of LE-FDTD is that it cannot easily and accurately account for two-terminal circuits consisting of the arbitrary connection of several lumped elements. Then the so-called lumped-network FDTD (LN-FDTD) is presented [12], which is an improvement of the LE-FDTD technique and allows a systematic and simple incorporation of arbitrary linear RLC one-port lumped networks into a single FDTD cell by using Z-transform signal-processing technique [9–11]. This approach can also be extended to model two-port lumped circuits, and it is often referred to as the TP-LN-FDTD method [12]. Based on Z-transform technique, the LN-FDTD or TP-LN-FDTD method can easily and accurately analyze arbitrary linear lumped networks.

The piecewise linear recursive convolution (PLRC) technique [13, 14], as an alternative technique, can also be applied to model arbitrary linear lumped networks. In [14], linear lumped loads are modeled using PLRC technique, and numerical results show that this technique need saving fewer field variables than Z-transform technique, but it can only simulate the lumped loads consisting of parallel combination of series RLC circuits, and cannot model arbitrary linear lumped networks and do not also have a general formula for arbitrary high-order linear network. In this paper, we apply the PLRC technique to simulate one-port high-order linear networks consisting of the arbitrary connection of several lumped elements, and obtain a general FDTD formula. Firstly, the admittance parameters in Laplace domain of lumped network are written as a summation form of several rational fractions; then the time domain admittance parameters can be obtained by means of inverse Fourier transform technique. Finally the time domain results are directly incorporated into the Maxwell-Ampere's difference equation using the PLRC technique. Then this technique is extended to arbitrary two-port lumped network. Though only one-port and two-port networks are discussed, this technique is also proper to multi-port linear networks. It is worth pointing out that this approach preserves the full explicit nature of the standard FDTD method. To validate this technique, two microwave circuits including lumped networks are considered and numerical results show that it have the same speed and accuracy as Z-transform technique.

2. THEORY ANALYSIS

2.1. One-port Network

Assuming that the lumped network is connected at the node E_z and along the z-direction, at time step $t = n + 1/2$, we write Maxwell's curl-H equation as:

$$\varepsilon\frac{E_z^{n+1} - E_z^n}{\Delta t} = [\nabla \times H]_z^{n+1/2} - J_z^{n+1/2} \tag{1}$$

J_z denotes the current density flowing through the lumped network.

Assuming the admittance of the one-port lumped network is Y, we can obtain the current-voltage relationship at time domain:

$$I_z(t) = Y(t) \otimes V_z(t) \tag{2}$$

where $\otimes$ denotes the convolution operation.

According to the PLRC technique, define two variables as:

$$\chi_m = \int_{m\Delta t}^{(m+1)\Delta t} Y(\tau)d\tau \tag{3}$$

$$\xi_m = \frac{1}{\Delta t}\int_{m\Delta t}^{(m+1)\Delta t} (\tau - m\Delta t)Y(\tau)d\tau \tag{4}$$

If the variables meet the following relationship

$$\rho = \frac{\chi_m}{\chi_{m-1}} = \frac{\xi_m}{\xi_{m-1}} \tag{5}$$

the time-domain current I_z^n can be updated using recursive equation:

$$I_z^{n+1} = (\chi_0 - \xi_0)\, V_z^{n+1} + \xi_0 V_z^n + \rho I_z^n \tag{6}$$

The details regarding PLRC formulation can be found in [13].

For a one-port lumped network consisting of the arbitrary connection of several linear lumped elements, in order to easily apply the PLRC technique, we can express its admittance in Laplace domain as:

$$Y(s) = \sum_{i=1}^{N} \frac{c_i}{s - a_i} + g + sh = \sum_{i=1}^{N} Y_i(s) + Y_o(s) \tag{7}$$

where the residues c_i and poles a_i are either real quantities or come in complex conjugate pairs, while g and h are real. According to (2) and (7), the discrete time-domain current can be expressed as:

$$I_z^{n+1/2} = \sum_{i=1}^{N} I_{z,i}^{n+1/2} + I_{z,o}^{n+1/2} \tag{8}$$

We discuss the two right terms of the above formulation in detail.
1. If the residues c_i and poles a_i are real quantities, $Y_i(s) = \frac{c_i}{s-a_i}$ is converted to the time-domain by inverse Fourier transform technique

$$Y_z(t) = c_i e^{a_i \Delta t} u(t) \tag{9}$$

$u(t)$ is the unit step function. By introducing (9) into (3) and (4), we can get:

$$\chi_{m,i} = -\frac{c_i}{a_i}\left(1 - e^{a_i \Delta t}\right) e^{m a_i \Delta t} \tag{10}$$

$$\xi_{m,i} = -\frac{c_i}{a_i^2 \Delta t}\left[\left(1 - a_i \Delta t\right) e^{a_i \Delta t} - 1\right] e^{m a_i \Delta t} \tag{11}$$

Obviously, (5) can be met

$$\rho_i = \frac{\chi_{m,i}}{\chi_{m-1,i}} = \frac{\xi_{m,i}}{\xi_{m-1,i}} = e^{a_i \Delta t} \tag{12}$$

In this case, the time-domain current I_{zi}^n can be updated using recursive equation:

$$I_{z,i}^{n+1} = (\chi_{0,i} - \xi_{0,i})\, V_z^{n+1} + \xi_{0,i} V_z^n + \rho_i I_{z,i}^n \tag{13}$$

2. If c_i and c_{i+1}, a_i and a_{i+1} are complex conjugate pairs, the Equations (9)–(13) are yet satisfied, but they can be simplified. From the definition of (3)–(5), we can prove that $\chi_{m,i}$ and $\chi_{m,i+1}$, $\xi_{m,i}$ and $\xi_{m,i+1}$, ρ_i and ρ_{i+1} are all complex conjugate pairs, so $I_{z,i}^{n+1}$ and $I_{z,i+1}^{n+1}$ are also complex conjugate pairs and their sum is a real quantity.

$$I_{z,i}^{n+1} + I_{z,i+1}^{n+1} = 2\mathrm{Re}\left(I_{z,i}^{n+1}\right) = 2\left\{\mathrm{Re}\left(\chi_{0,i} - \xi_{0,i}\right)\cdot V_z^{n+1} + \mathrm{Re}\left(\xi_{0,i}\right)\cdot V_z^n + \mathrm{Re}\left(\rho_i I_{z,i}^n\right)\right\} \tag{14}$$

So only one complex of a complex conjugate pair is needed. In other words, if c_i and a_i contain $2N_g$ conjugate complex, only N_g variables need to be saved.

3. Substituting $Y_o(s) = g + sh$ into (2), we can obtain $I_{z,o}(s) = (g + sh)V_z(s)$. Applied the transformation of $s \to \partial/\partial t$, the following central-difference formulation can be achieved

$$I_{z,o}^{n+1/2} = \left(g/2 + h/\Delta t\right) V_z^{n+1} + \left(g/2 - h/\Delta t\right) V_z^n \tag{15}$$

Finally, substituting (13)–(15) into (8) and assuming c_i and a_i are composed of N_r real quantities and N_g pairs of conjugate complex (total number $N = N_r + 2N_g$), we can write the total current as:

$$I_z^{n+1/2} = \left(\chi_{0,t} - \xi_{0,t} + g/2 + h/\Delta t\right) V_z^{n+1} + \left(\xi_{0,t} + g/2 - h/\Delta t\right) V_z^n + I_t^n \tag{16}$$

where

$$\begin{aligned}
\chi_{0,t} &= \frac{1}{2}\sum_{i=1}^{N_r} \chi_{0,i} + \sum_{i=N_r+1}^{N_r+N_g} \mathrm{Re}\left(\chi_{0,i}\right) \\
\xi_{0,t} &= \frac{1}{2}\sum_{i=1}^{N_r} \xi_{0,i} + \sum_{i=N_r+1}^{N_r+N_g} \mathrm{Re}\left(\xi_{0,i}\right) \\
I_t^n &= \frac{1}{2}\sum_{i=1}^{N_r} \left(\rho_i + 1\right) I_{z,i}^n + \sum_{i=N_r+1}^{N_r+N_g} \mathrm{Re}\left[\left(\rho_i + 1\right) I_{z,i}^n\right]
\end{aligned}$$

The relationships of the voltage with the electric field and the current intensity with the current density are approximately expressed as

$$V_z^n = \int E_z^n dz \approx E_z^n \Delta z \tag{17}$$

$$I_z^{n+1/2} = \iint J_d^{n+1/2} xdy \approx J_z^{n+1/2} \Delta x \Delta y \tag{18}$$

Replacing (17) and (18) into (1), we can obtain the FDTD iterative formulation of electric field at the loaded node as follows:

$$E_z^{n+1} = \frac{(\varepsilon/\alpha + h)/\Delta t - (\xi_{0,t} + g/2)}{(\varepsilon/\alpha + h)/\Delta t + (\chi_{0,t} - \xi_{0,t} + g/2)} E_z^n + \frac{1/\alpha}{(\varepsilon/\alpha+h)/\Delta t+(\chi_{0,t}-\xi_{0,t}+g/2)} \left\{ \left[\nabla\times\vec{H}\right]_z^{n+1/2} - \frac{1}{\Delta x \Delta y} I_t^n \right\} \tag{19}$$

where $\alpha = \Delta z/\Delta x \Delta y$. The rest electromagnetic fields can be calculated using the standard FDTD method.

2.2. Two-port Network

Assuming the two ports of lumped network are associated to the two electric field component E_{z1} and E_{z2}. At the two nodes, the Maxwell-Ampere's equation is expressed in following discrete form:

$$\begin{cases} \varepsilon \dfrac{E_{z1}^{n+1} - E_{z1}^n}{\Delta t} = \left[\nabla \times \vec{H}\right]_{z1}^{n+1/2} - J_1^{n+1/2} \\ \varepsilon \dfrac{E_{z2}^{n+1} - E_{z2}^n}{\Delta t} = \left[\nabla \times \vec{H}\right]_{z2}^{n+1/2} - J_2^{n+1/2} \end{cases} \tag{20}$$

The performance of two-port network can be characterized by its admittance matrix in Laplace domain

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} \tag{21}$$

where V_p and $I_p (p = 1, 2)$ denote the voltage and current at the two ports.

Introducing four auxiliary current intensities $I_{pq}(s)(p, q = 1, 2)$, we express (21) as

$$I_p(s) = \sum_{q=1,2} I_{pq}(s) \quad p = 1, 2 \tag{22}$$

where

$$I_{pq}(s) = Y_{pq}(s) V_q(s) \tag{23}$$

Similarly, to a two-port lumped network consisting of the arbitrary connection of several linear lumped elements, each entry of its admittance matrix can be expressed as follows:

$$Y_{pq}(s) = \sum_{i=1}^{N^{(p,q)}} \frac{c_i^{(p,q)}}{s - a_i^{(p,q)}} + g^{(p,q)} + sh^{(p,q)} \tag{24}$$

where the residues $c_i^{(p,q)}$, and poles $a_i^{(p,q)}$, are either real quantities or complex conjugate pairs, while $g^{(p,q)}$ and $h^{(p,q)}$ are real.

According to (16) in the above section, we can obtain the recursive equation of I_{pq} from (23)

$$I_{pq}^{n+1/2} = A_{pq} V_q^{n+1} + B_{pq} V_q^n + I_{pq,t}^n \tag{25}$$

where

$$\begin{aligned}
A_{pq} &= \left(\chi_{0,t}^{(p,q)} - \xi_{0,t}^{(p,q)} + g^{(p,q)}/2 + h^{(p,q)}/\Delta t\right) \\
B_{pq} &= \left(\xi_{0,t}^{(p,q)} + g^{(p,q)}/2 - h^{(p,q)}/\Delta t\right) \\
\chi_{0,t}^{(p,q)} &= \frac{1}{2} \sum_{i=1}^{N_r^{(p,q)}} \chi_{0,i}^{(p,q)} + \sum_{i=N_r^{(p,q)}+1}^{N_r^{(p,q)}+N_g^{(p,q)}} \mathrm{Re}\left(\chi_{0,i}^{(p,q)}\right) \\
\xi_{0,t}^{(p,q)} &= \frac{1}{2} \sum_{i=1}^{N_r^{(p,q)}} \xi_{0,i}^{(p,q)} + \sum_{i=N_r^{(p,q)}+1}^{N_r^{(p,q)}+N_g^{(p,q)}} \mathrm{Re}\left(\xi_{0,i}^{(p,q)}\right) \\
I_{pq,t}^n &= \frac{1}{2} \sum_{i=1}^{N_r^{(p,q)}} \left(\rho_i^{(p,q)}+1\right) I_{pq,i}^n + \sum_{i=N_r^{(p,q)}+1}^{N_r^{(p,q)}+N_g^{(p,q)}} \mathrm{Re}\left[\left(\rho_i^{(p,q)}+1\right) I_{pq,i}^n\right] \qquad (26) \\
\chi_{0,i}^{(p,q)} &= -\frac{c_i^{(p,q)}}{a_i^{(p,q)}} \left(1 - e^{a_i^{(p,q)}\Delta t}\right) \\
\xi_{0,i}^{(p,q)} &= -\frac{c_i^{(p,q)}}{\left(a_i^{(p,q)}\right)^2 \Delta t} \left[\left(1 - a_i^{(p,q)}\Delta t\right) e^{a_i^{(p,q)}\Delta t} - 1\right] \\
\rho_i^{(p,q)} &= e^{a_i^{(p,q)}\Delta t} \\
I_{pq,i}^{n+1} &= \left(\chi_{0,i}^{(p,q)} - \xi_{0,i}^{(p,q)}\right) V_q^{n+1} + \xi_{0,i}^{(p,q)} V_q^n + \rho_i^{(p,q)} I_{pq,i}^n \qquad (27)
\end{aligned}$$

At each port, the relationships of the voltage with the electric field and the current intensity with the current density are same as (17) and

(18), i.e.,

$$V_q^n = \int E_{zq}^n dz \approx E_{zq}^n \Delta z \tag{28}$$

$$I_p^{n+1/2} = \iint J_p^{n+1/2} dxdy \approx J_p^{n+1/2} \Delta x \Delta y \tag{29}$$

From (22), the time-domain current at each port can be expressed as

$$I_p^{n+1/2} = \sum_{q=1,2} I_{pq}^{n+1/2} \quad p = 1, 2 \tag{30}$$

Finally, substituting (28)–(30) and (25) into (20), we can obtain the FDTD formulation of the electric field at each port

$$\begin{bmatrix} E_{z1}^{n+1} \\ E_{z2}^{n+1} \end{bmatrix} = \frac{1}{\alpha} \begin{bmatrix} A_{11} + \varepsilon/\alpha\Delta t & A_{12} \\ A_{21} & A_{22} + \varepsilon/\alpha\Delta t \end{bmatrix}^{-1} \begin{bmatrix} T_1^n \\ T_2^n \end{bmatrix} \tag{31}$$

where

$$\alpha = \frac{\Delta z}{\Delta x \Delta y},$$

$$T_p^n = \frac{\varepsilon}{\Delta t} E_{zp}^n + \left[\nabla \times \vec{H}\right]_{zp}^{n+1/2} - \sum_{q=1,2} \left(\alpha B_{pq} E_{zq}^n + \frac{1}{\Delta x \Delta y} I_{pq,t}^n \right)$$

From (19) and (31), we can find that this approach preserves the full explicit nature of the standard FDTD method. For two-port lumped network, the resulting algorithm has the following steps in each time iteration.

Step 1: The magnetic field $\vec{H}^{n+1/2}$ is updated by using the standard FDTD method.

Step 2: The electric field at the two ports E_{z1}^{n+1} and E_{z2}^{n+1} are be updated by (31).

Step 3: Each of auxiliary current $I_{pq,i}^{n+1}$ and $I_{pq,t}^{n+1}$ is updated by using (26) and (27).

3. NUMERICAL RESULTS

To illustrate the validity of the proposed approach described above, we consider two microstrip circuits including one-port network and two-port network, respectively. In both cases, the terminal voltages are computed by the proposed approach and LN-FDTD or TP-LN-FDTD.

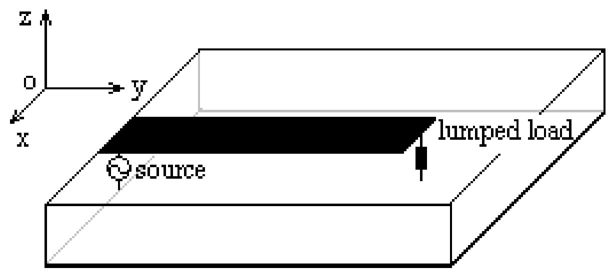

Figure 1. Structure of microstrip circuit including a lumped load.

3.1. One-port Network

Firstly, we analyze a microstrip circuit with lumped loads, which structure is showed in Figure 1. The parameters of the microstrip line used for our computations are thickness of the substrate ($H = 0.81\,\text{mm}$), width of the metal strip ($W = 2.4\,\text{mm}$), length of the metal strip ($L = 32\,\text{mm}$), dielectric constant of the substrate ($\varepsilon = 2.2$), and thickness of the metal strip (zero), which corresponds to the characteristic impedance of the microstrip line ($50\,\Omega$). In FDTD simulation, the space steps are $\Delta x = 0.4\,\text{mm}$, $\Delta y = 0.4\,\text{mm}$ and $\Delta z = 0.27\,\text{mm}$. The total mesh dimensions are $60\Delta x \times 90\Delta y \times 20\Delta z$. The first-order Mur's absorbing boundary condition is adopted to truncate the computing region, except the boundary of $z = 0$. A sinusoidal voltage source with frequency (20 GHz) and amplitude ($1v$) is used at one termination of the strip, and the other termination is connected with two different lumped loads which circuit diagrams are illustrated in Figure 2 and Figure 3. The time step is $\Delta t = 0.5\,\text{ps}$. The simulation is performed for 800 time steps.

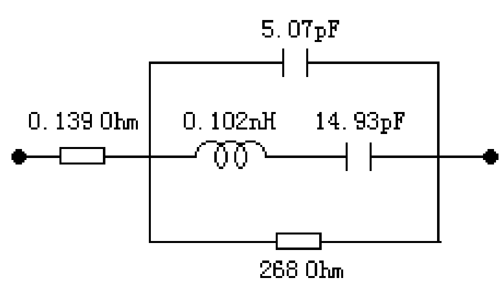

Figure 2. Circuit diagram of lumped load (a).

Applying the proposed approach and LN-FDTD to analyze this microstrip circuit, we obtain the results of the voltages across the two different lumped loads, which are showed in Figure 4 and Figure 5, respectively. The two approaches have the same speed, and the computation time is approximately 32 sec and 33 sec for the two lumped loads. Furthermore, it can be found from Figure 4 and Figure 5 that the results obtained by the two approaches have an unexceptionable agreement, so the two approaches have the same accuracy.

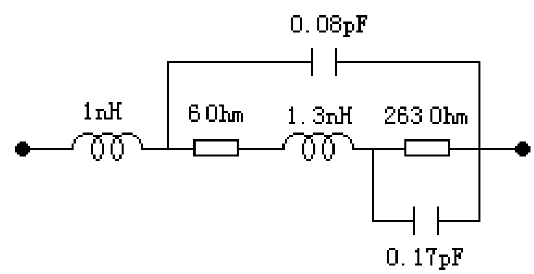

Figure 3. Circuit diagram of lumped load (b).

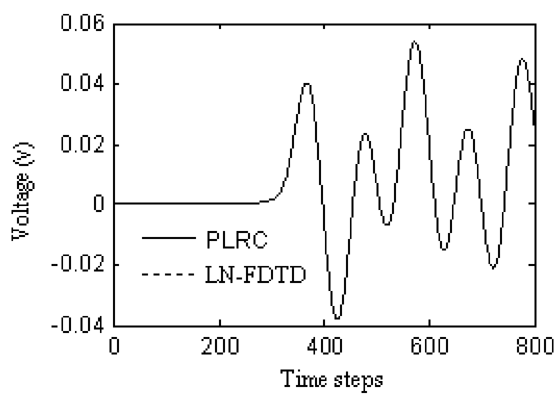

Figure 4. Voltage across lumped load (a).

3.2. Two-port Network

Secondly, we consider a microstrip circuit including two-port lumped networks, which structure is showed in Figure 6. The parameters of the microstrip line are thickness of the substrate ($H = 0.254$ mm), width of the metal strip ($W = 0.79$ mm), dielectric constant of the substrate ($\varepsilon = 2.17$), which corresponds to the characteristic impedance of the

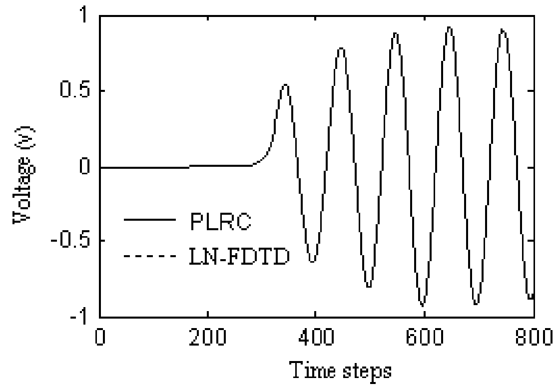

Figure 5. Voltage across lumped load (b).

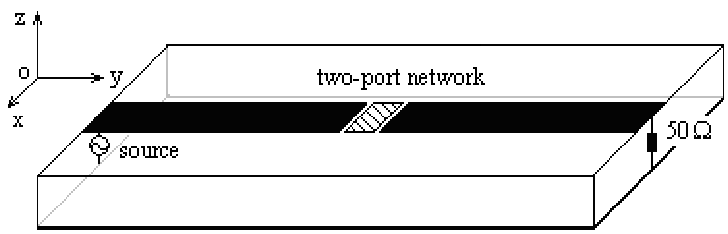

Figure 6. Structure of microstrip circuit including a two-port network.

microstrip line (50 Ω). Two metal strips have the same dimension with length ($L = 4$ mm) and thickness (zero). In FDTD simulation, the space steps are $\Delta x = 0.079$ mm, $\Delta y = 0.1$ mm and $\Delta z = 0.0846$ mm. The total mesh dimensions are $60\Delta x \times 105\Delta y \times 30\Delta z$. The first-order Mur's absorbing boundary condition is adopted to truncate the computing region, except the boundary of $z = 0$. A sinusoidal voltage source with an internal resistor (50 Ω), frequency (20 GHz) and amplitude ($1v$) is excited at one termination of the strip, and the other termination is truncated by a resistor load with a resistance (50 Ω). Two different lumped two-port networks are placed on the microstrip gap with span length (0.5 mm), as shown in Figure 6. The time step is $\Delta t = 0.16$ ps, and the simulation is performed for 1200 time steps.

The proposed approach and TP-LN-FDTD are used to simulate this microstrip circuit, respectively, and two terminal voltages, i.e., source and load termination, are obtained. The computation time of the two approaches is equal. The simulation results of two different

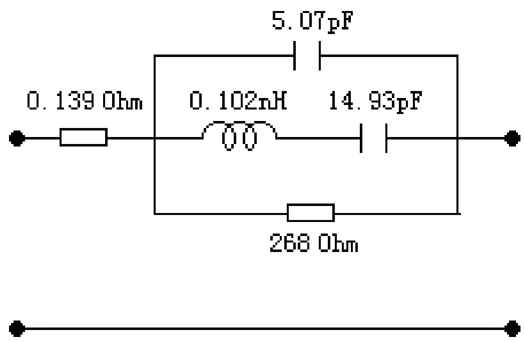

Figure 7. Circuit diagram of two-port network (a).

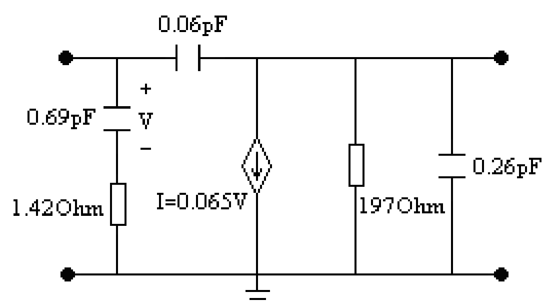

Figure 8. Circuit diagram of two-port network (b).

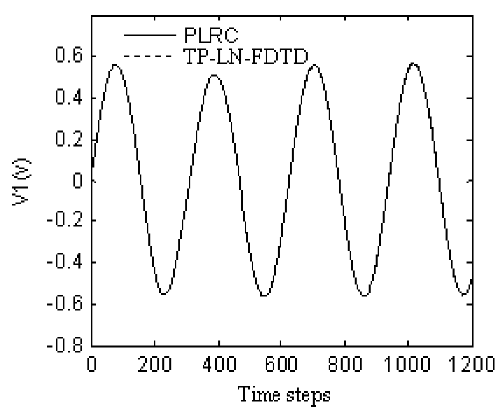

Figure 9. Voltage across source termination with network (a).

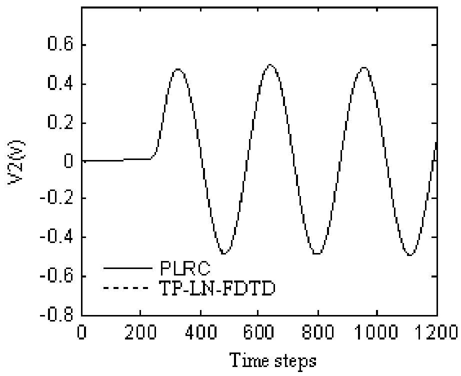

Figure 10. Voltage across load termination with network (a).

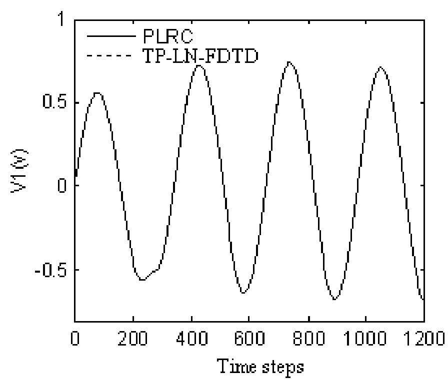

Figure 11. Voltage across source termination with network (b).

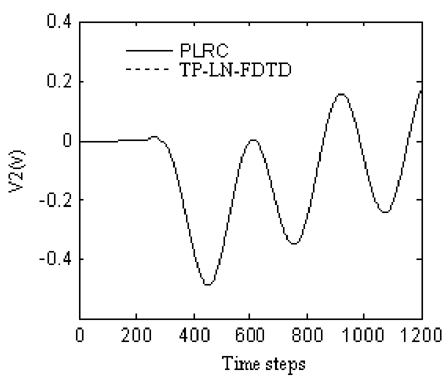

Figure 12. Voltage across load termination with network (b).

networks are showed in Figures 9–10 and Figures 11–12, respectively. From Figures 9–12, we can see that the two approaches also have the same computational accuracy.

4. CONCLUSION

The FDTD method is one of the best candidates to provide an efficient and powerful global electromagnetic tool, which can accurately predict the electromagnetic interaction among passive and active elements of microwave integrated circuits. In recent years, this technique is widely studied. How to introduce a lumped element or network into a FDTD grid is the key. In this paper, an approach of FDTD combining the PLRC technique is studied to model arbitrary linear lumped network, including one-port network and two-port network, and the general FDTD formulations are derived. This approach preserves the full explicit nature of the standard FDTD method. Compared with the accurately LN-FDTD and TP-LN-FDTD method, the proposed approach has the same speed and accuracy. It provides an alternative electromagnetic analysis tool. Furthermore, the proposed technique can also be extended to model multi-port linear lumped network.

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