

## **A NEW STRUCTURE FOR LOCALIZING ELECTROMAGNETIC ENERGY USING TWO SEMI-INFINITE LEFT-HANDED-MEDIUM SLABS**

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**Abstract**—It is well known that a left-handed-medium (LHM) slab with negative permittivity  $-\varepsilon_0$  and negative permeability  $-\mu_0$  can be made as a perfect lens for its negative refraction. In this paper, we show that such two semi-infinite lossless LHM slabs can realize electromagnetic energy localization completely through an accurate analysis. If two current sources with the same amplitude and opposite direction are placed at the right edge of the left LHM slab and the perfect-imaging point of the right LHM slab separately, we have demonstrated that all electromagnetic waves are completely confined in a region between the two sources and there is no power radiating outwards the region.

### **1. INTRODUCTION**

In recent years, artificial materials with negative permittivity and permeability simultaneously, named as the left-handed medium (LHM) or metamaterials, have attracted much experimental and theoretical interest [1–5]. The concept of negative refractive index was first indicated by Veselago in 1968, who predicted that a LHM slab could refocus the electromagnetic waves emitted from a point source at an image point [6]. Later, Pendry proposed that a LHM slab with negative permittivity and negative permeability in the free space can be made as a perfect lens [7].

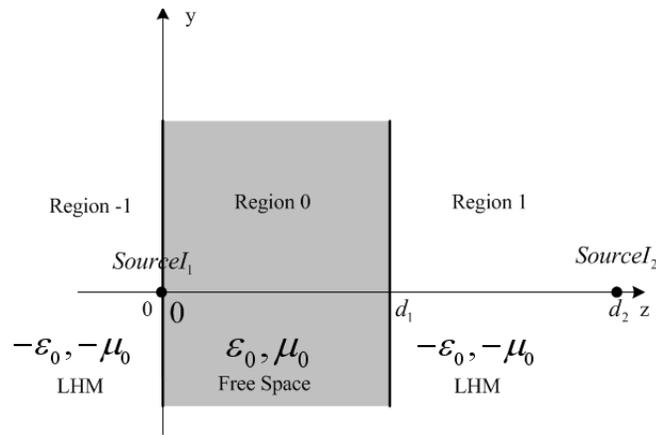
The localization of electromagnetic waves and energy is applied widely in science and engineering. For example, it can be used to design narrow-band filters and low-threshold lasers [8]. Traditionally, some structure defects introduced to the regular structure of photonic crystals or a three-dimensional fractal cavity were used to realize the

localization of electromagnetic waves [9, 10]. In recent years, it has been shown that a left-handed medium (LHM) slab can be used to localize electromagnetic waves and energy [11].

In this paper, we propose a new structure for localizing electromagnetic waves using two semi-infinite LHM slabs instead of using a thin LHM slabs. We place a line current source at the right edge of the left LHM slab, and another at the perfect-imaging point of the right LHM slab, which could confine the electromagnetic waves completely between the two currents and the electromagnetic fields outside the region are zero. Therefore, the LHM slabs can be used to realize the localization of electromagnetic waves in another way. In fact, the localization of electromagnetic could be applied in microwave and optical devices.

## 2. ELECTRONIC AND MAGNETIC FIELDS OF THE STRUCTURE

Consider a case when a line current source  $I_1$  is located at the right edge of the left LHM slab, and another source  $I_2$  is placed at the perfect-imaging point of the right LHM slab with negative permittivity  $-\epsilon_0$  and negative permeability  $-\mu_0$ . Under the Cartesian coordinate system shown in Fig. 1, there is an image point at  $z = d_2 = 2d_1$ . From EM theory we have the electric fields in different region produced by  $I_1$  as [12]



**Figure 1.** A line current source located at the right edge of the left LHM slab and another at the perfect-imaging point of the right LHM slab. The two currents have the same amplitude and opposite direction.

$$E_x^{(1)} = \begin{cases} \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} (1+R) e^{-ik_{-1z}z} e^{ik_y y} & z \leq 0 \\ \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} dk_y (E_0^+ e^{ik_{0z}z} + E_0^- e^{-ik_{0z}z}) e^{ik_y y} & 0 < z \leq d_1 \\ \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} dk_y E_1^+ e^{ik_{1z}z} e^{ik_y y} & z > d_1 \end{cases} \quad (1)$$

where,  $k_{0z} = \sqrt{k_0^2 - k_y^2}$  ( $i = 0, 1$ ), so for all propagating waves ( $|k_y| < k_0$ ), we have  $k_{0z} = -k_{1z} = -k_{-1z}$ .  $R$  is the reflection coefficient of the left slab, and  $E_1^+$ ,  $E_1^-$  and  $E_2^+$  are transmission coefficients. Such coefficients are given by

$$R = \frac{R_{-10} e^{-i2k_{0z}d_1} + R_{01}}{e^{-i2k_{0z}d_1} + R_{-10}R_{01}} \quad (2)$$

$$E_0^+ = \frac{1}{k_{0z}} \left( \frac{1 + p_{0,-1}}{2} + \frac{1 - p_{0,-1}}{2} R \right) \quad (3)$$

$$E_0^- = \frac{1}{k_{0z}} \left( \frac{1 - p_{0,-1}}{2} + \frac{1 + p_{0,-1}}{2} R \right) \quad (4)$$

$$E_1^+ = \frac{2}{1 + p_{01}} E_0^+ e^{i2k_{0z}d_1} \quad (5)$$

in which  $p_{ij} = \mu_i k_{jz} / \mu_j k_{iz}$ , and  $R_{ij} = (1 - p_{ij}) / (1 + p_{ij})$  is the Fresnel coefficient on the slab boundaries. Since  $k_{0z} = -k_{1z} = -k_{-1z}$ , we have  $p_{0,-1} = 1$ ,  $p_{0,1} = 1$  and  $R_{-10} = R_{01} = 0$ . In such case,  $R = 0$ ,  $E_0^+ = \frac{1}{k_{0z}}$ ,  $E_0^- = 0$ ,  $E_1^+ = \frac{1}{k_{0z}} e^{i2k_{0z}d_1}$ .

Finally, the fields can be written as

$$E_x^{(1)} = \begin{cases} \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} e^{ik_{0z}z} e^{ik_y y} & z \leq d_1 \\ \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} e^{ik_{0z}(2d_1-z)} e^{ik_y y} & z > d_1 \end{cases} \quad (6)$$

Similarly, the electric fields radiated by  $I_2$  can also be obtained as

$$E_x^{(2)} = \begin{cases} \frac{\omega\mu_0 I_2}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} e^{ik_{0z}z} e^{ik_y y} & z \leq 0 \\ \frac{\omega\mu_0 I_2}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} e^{-ik_{0z}z} e^{ik_y y} & 0 < z \leq d_1 \\ \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} e^{-ik_{0z}(2d_1-z)} e^{ik_y y} & d_1 \leq z < d_2 \\ \frac{\omega\mu_0 I_1}{4\pi} \int_{-k_0}^{k_0} \frac{dk_y}{k_{0z}} e^{ik_{0z}(2d_1-z)} e^{ik_y y} & z > d_2 \end{cases} \quad (7)$$

Then considering a special case when  $I_1 = -I_2 = I$ , we can calculate the total fields produced by  $I_1$  and  $I_2$  easily through the summation of Eqs. (1) and (4), we have

$$E_x(y, z) = \begin{cases} \frac{i\omega\mu_0}{\pi} I \int_0^{k_0} \frac{dk_y}{k_{0z}} \sin(k_{0z}z) \cos(k_y y) & 0 < z \leq d_1 \\ \frac{i\omega\mu_0}{\pi} I \int_0^{k_0} \frac{dk_y}{k_{0z}} \sin[k_{0z}(2d_1 - z)] \cos(k_y y) & d_1 < z \leq d_2 \end{cases} \quad (8)$$

From above formulations, we observe that: (1) all electric fields are confined in a region between the two currents; (2) the waves behave like standing waves; (3) all electric fields are imaginary; (4) the field is continuous in all position along the  $z$  axis.

Based on the Maxwell's equation, we know that the magnetic field has  $z$  and  $y$  component as

$$H_y(y, z) = \begin{cases} -\frac{I}{\pi} \int_0^{k_0} \cos(k_{0z}z) \cos(k_y y) dk_y & 0 < z \leq d_1 \\ -\frac{I}{\pi} \int_0^{k_0} \cos[k_{0z}(2d_1 - z)] \cos(k_y y) dk_y & d_1 < z \leq d_2 \end{cases} \quad (9)$$

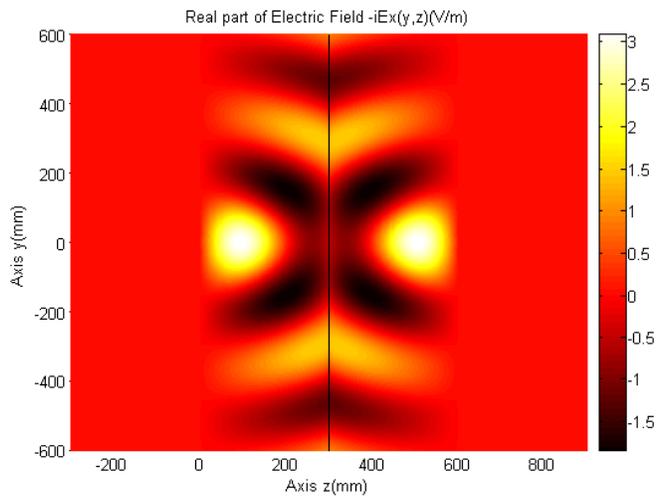
$$H_z(y, z) = \begin{cases} \frac{I}{\pi} \int_0^{k_0} \frac{k_y}{k_{0z}} \sin(k_{0z}z) \sin(k_y y) dk_y & 0 < z \leq d_1 \\ -\frac{I}{\pi} \int_0^{k_0} \frac{k_y}{k_{0z}} \sin[k_{0z}(2d_1 - z)] \sin(k_y y) dk_y & d_1 < z \leq d_2 \end{cases} \quad (10)$$

Similar to the electric field, we could see clearly that: (1) all magnetic fields are confined in a region between the two currents; (2) the waves behave like standing waves; (3) all magnetic fields are real; (4) the component of  $H_y$  are discontinuous at  $z = 0$  and  $z = d_2$  because of the existence of line sources; (5) the component of  $H_z$  are discontinuous at  $z = d_1$  but  $B_z$  are continuous which satisfies the boundary condition.

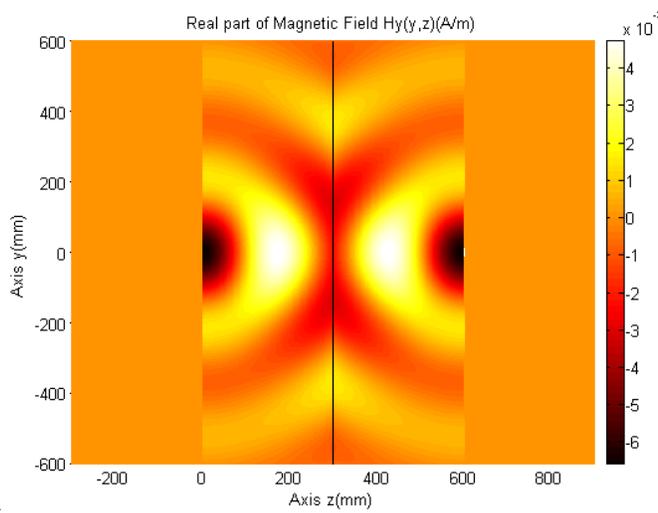
### 3. LOCALIZATION OF ELECTROMAGNETIC ENERGY

As the total field shown above, we find that the complex Poynting vectors which defined by  $S = \frac{1}{2}E \times H^*$  are always imaginary. Therefore, the time-averaged power density  $\text{Re}(S)$  is always zero, which denotes that there is no power radiating outside and no power transmitting inside. All electromagnetic energies are located within the region between the two sources.

Physically speaking, the field produced by  $I_2$  goes through the LHM slab and experiences a time reversal at the image point that

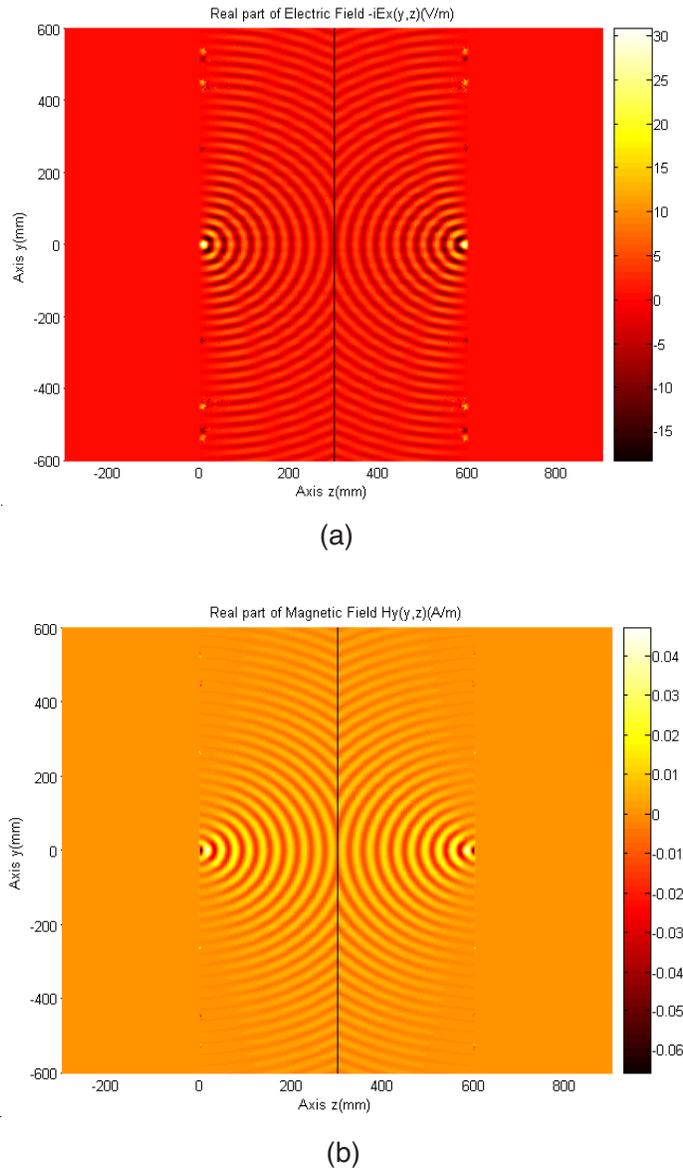


(a)



(b)

**Figure 2.** The field patterns of waves within and outside the LHM slab when the frequency is 1 GHz and  $d_1 = 300$  mm and  $d_2 = 600$  mm. (a) Electric field  $E_x$ . (b) Magnetic field  $H_y$ .



**Figure 3.** The field patterns of waves within and outside the LHM slab when the frequency is 10 GHz and  $d_1 = 300$  mm and  $d_2 = 600$  mm. (a) Electric field  $E_x$ . (b) Magnetic field  $H_y$ .

forms a perfect-imaging point. In the left region of the source  $I_1$ , the field radiated by  $I_1$  has the same amplitude but opposite direction as that by  $I_2$  because of  $I_1 = -I_2 = I$ . These two fields cancel each other, and the total field in this region is zero. So it is in the right region of the source  $I_2$ .

For the sake of observing the field distribution between the two sources, we have computed the wave patterns for a LHM slab when  $d_1 = 300$  mm and  $d_2 = 600$  mm. At the frequency of 1 GHz, the electric and magnetic field patterns are shown in Fig. 2 when the source  $I_1 = 1$  mA is placed at  $z = 0$  and the source  $I_2 = -1$  mA is placed at  $z = 600$  mm. Obviously, both electric and magnetic fields are confined between the two sources and no power radiating outside the region. From Fig. 2, we clearly see that both the electric and magnetic fields are symmetrical along the axes  $y = 0$  and  $z = 300$  mm.

In above example, the thickness of LHM slab is only one wavelength. If the frequency increases to 10 GHz, the same LHM slab has a thickness of ten wavelengths. In such a case, the electric and magnetic field patterns of waves are computed in Fig. 3. From these figures, we clearly see that the symmetrical property of the patterns is completely the same as that in Fig. 2.

#### 4. CONCLUSIONS

We have proposed a new structure in this paper for localizing electromagnetic waves and energies using two semi-infinite LHM slabs. Through an exact analysis, we have shown that these slabs with negative permittivity  $-\epsilon_0$  and negative permeability  $-\mu_0$  can localize the electromagnetic wave in a small region completely. If one source is located at the edge of the LHM slab and another is located at the perfect image of the right slab, all electromagnetic waves are confined in the region between two sources and there is no power radiation outside and no power transmitting inside. Such localization of electromagnetic waves may be found important application in new optical and microwave devices.

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