ANALYSIS OF RADIATION PATTERNS OF COMPOUND BOX-HORN ANTENNA

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Abstract—A new type of antenna named as compound box-horn antenna is designed and analyzed for its radiation pattern. The present analysis is based on plane wave spectra for three-dimensional fields. The compound box-horn antenna is obtained by combining modified box-horn and pyramidal horn antennas, in which modified box-horn is coupled to pyramidal horn to excite TE_{10} and TE_{30} modes at the input of pyramidal horn. Thus, the compound box-horn antenna has properties and advantages of both the modified box-horn and pyramidal horn antennas. The radiation patterns and corresponding half-power beam widths (HPBWs) of compound box-horn antenna in free-space are computed at 10 GHz and compared for different flare angles in E- and H-planes of larger size pyramidal horn section of the compound box-horn. The results for HPBWs in E- and H-planes demonstrate that the radiation patterns in E- and H-planes for compound box-horn can be made narrower by decreasing the flare angles in both E- and H-planes of larger size pyramidal horn section of the compound box-horn. The radiation patterns of compound box-horn are also compared with those for TE_{10}-mode pyramidal horn of same aperture size and it found that the former horn is narrower in E- as well as H-plane than the latter. The analysis has been validated against the experimental results available in the literature. The work presented here can provide useful design
1. INTRODUCTION

The type of antenna selected for communication depends on its radiation properties. Emerging application of microwave antenna in modern wireless communication demands challenging characteristics of the antenna. The aperture field distribution of the antenna should be uniform to increase its directivity. The antenna must possess relatively larger aperture size at a given frequency to sharpen the beam and facilitate its application for wireless communication. The antenna must provide desired half-power beam widths (HPBWs) in $E$- and $H$-planes. These requirements, put together provide a challenging list of specifications that demand innovation in antenna design. The theoretical investigations of open-ended waveguide for communication purpose appear to have been initiated by Chu [1] and the theory was extended by many researchers [2–16]. Several types of antennas have been investigated by numerous researchers and are described in the literature including open-ended waveguide [1, 3], circular aperture horn [4], Conical horn [25, 26], biconical horn [5], modified horn [6], pyramidal horn [7, 8], TEM horn [9], E-sectoral horn [10], Gaussian profiled horn [11, 23], double-ridged horn [12, 24], woodpile EBG sectoral horn [13, 14], modified box-horn [15, 16; 21], corrugated horn [22], slotted antenna [27], dielectric resonator antenna [28], lens antennas [29, 30] etc.

In this paper, the authors have designed a new type of antenna named as compound box-horn antenna and analyzed its radiation pattern using plane wave spectra for three-dimensional fields. The compound box-horn antenna is obtained by combining a modified box-horn with a pyramidal horn, in which the modified box-horn is coupled to pyramidal horn to excite TE_{10} and TE_{30} modes at the input of larger size pyramidal horn section. Thus, the compound box-horn antenna possesses the properties and advantages of both the modified box-horn and pyramidal horn antenna. Modified box-horn [15, 16] is a novel and improved version of box-horn (i.e., conventional box-horn) [17] in which the horn exciting the ‘box’ is flared in both $E$- as well as $H$-planes to increase its aperture size. The modified box-horn [15, 16] consists primarily of a piece of waveguide of length $L$, frequently referred to as a ‘box’, whose magnetic plane dimension
is large enough to support TE$_{10}$- and TE$_{30}$-modes. The resultant electric field distribution over the modified box-horn aperture along H-plane thus becomes relatively uniform which improves the directivity. The pyramidal horn is one whose opening is flared (tapered) in both directions of the E- and H-fields [18]. The amplitude distribution at the aperture of modified box-horn will be almost uniform when ratio of the amplitudes of TE$_{30}$ and TE$_{10}$ mode electric fields is chosen 0.3 and phase difference between TE$_{30}$ and TE$_{10}$ mode electric fields is kept 180°. The axial length of larger size pyramidal horn section of the compound box-horn is such chosen that the phase error at the aperture of compound box-horn for both modes is approximately same and the phase difference between two modes remains approximately 180°. Thus, the aperture electric field of compound box-horn is nearly uniform, which gives higher directivity in comparison of same aperture sized and same flare angles TE$_{10}$ modes pyramidal horn. The radiation patterns and the corresponding half-power beam widths (HPBWs) of compound box-horn antenna in free-space are computed at 10GHz and compared for different flare angles in E- and H-planes of larger size pyramidal horn section of the compound box-horn. The radiation patterns and HPBWs of compound box-horn are also compared with those for TE$_{10}$-mode pyramidal horn of same aperture size. For validity of the analysis, the results for TE$_{10}$-mode pyramidal horn obtained using the present analysis have been compared with the experimental results available in the literature [7].

2. ANALYSIS OF RADIATION PATTERN OF COMPOUND BOX-HORN ANTENNA

A compound box-horn antenna is shown in Fig. 1. The narrow and broad dimensions of the aperture of modified box-horn section of the compound box-horn are denoted as $a$ and $b$ respectively. The length

![Figure 1. Compound box-horn.](image-url)
of box of the modified box-horn section along z-direction is denoted as \( L \). \( \theta_{EM} \) and \( \theta_{HM} \) are respectively the flare angles of smaller pyramidal horn exciting the box in \( E \)- and \( H \)-planes of modified box-horn section of the compound box-horn. The narrow and broad dimensions of the aperture of larger size pyramidal horn section of the compound box-horn are denoted as \( A \) and \( B \) respectively. \( \theta_{EP} \) and \( \theta_{HP} \) are the flare angles of the larger size pyramidal horn section in \( E \)- and \( H \)-planes respectively. The modified box-horn section of the compound box-horn may be excited by inserting a coaxial probe in the middle of the broader wall of the waveguide on the left hand side at \( \lambda_g/4 \) distance from the short-circuited end, as shown in Fig. 1, where \( \lambda_g \) is guide wavelength in the waveguide. Thus the probe excites \( TE_{10} \)-mode in the waveguide. \( TE_{30} \)-mode is generated at the discontinuity plane between the smaller pyramidal horn and the box. Thus, modified box-horn section as well as compound box-horn supports \( TE_{10} \)- and \( TE_{30} \)-modes.

The electric field at the aperture of modified box-horn \([15,16]\) carrying \( TE_{10} \)- and \( TE_{30} \)-modes is represented by

\[
E_{x\ MBH \ Aperture} = a_{10} \cos\left(\frac{\pi y}{b}\right) e^{-j\beta_{10} L} + a_{30} \cos\left(\frac{3\pi y}{b}\right) e^{-j\beta_{30} L} \tag{1}
\]

where \( a_{10} \) and \( a_{30} \) are amplitude coefficients and \( \beta_{10} \) and \( \beta_{30} \) are the phase constants for \( TE_{10} \)- and \( TE_{30} \)-modes respectively.

The fields are polarized in the \( x-z \) plane of Fig. 1. There is no \( y \)-component of electric field. \( k \), the propagation constant in free-space, is given by \( k = \omega \sqrt{\mu_0 \varepsilon_0} \), where \( \varepsilon_0 \) and \( \mu_0 \) are permittivity and permeability of free-space respectively.

The electric field at the aperture of compound box-horn carrying \( TE_{10} \)- and \( TE_{30} \)-modes can be found as follows:

\[
E_x(x, y, 0) = \left[ a_{10} \cos\left(\frac{\pi y}{B}\right) e^{-j\beta_{10} L} + a_{30} \cos\left(\frac{3\pi y}{B}\right) e^{-j\beta_{30} L} \right] e^{-j\delta} \tag{2}
\]

where \( \delta \) is the phase error due to flaring in \( E \)- and \( H \)-planes of the larger size pyramidal horn section of the compound box-horn. Assuming two modes propagate beyond the second transition with propagation constant \( k \), the phase error \( \delta \) can be calculated in a manner similar to the pyramidal horn \([18]\) and is given below.

\[
\delta = \frac{k}{2} \left( \frac{x^2}{R_1} - \frac{y^2}{R_2} \right) \tag{3}
\]

where \( R_1 \) and \( R_2 \) are the larger size pyramidal horn lengths from the centre of mouth to the equivalent apex points of larger size pyramidal horn in \( E \)- and \( H \)-planes respectively.
Hence, the electric field at the aperture of compound box-horn can be written as

$$E_x(x, y, 0) = \left[ a_{10} \cos \left( \frac{\pi y}{B} \right) e^{-j\beta_{10} L} + a_{30} \cos \left( \frac{3\pi y}{B} \right) e^{-j\beta_{30} L} \right] e^{-\frac{j k}{2} \left( \frac{x^2}{R_1^2} - \frac{y^2}{R_2^2} \right)} e^{-j k x \sin \theta \cos \phi} e^{-j k y \sin \theta \sin \phi}$$

(4)

The radiation pattern of the compound box-horn is derived by plane wave spectra approach for three dimensional fields [19]. The radiation fields for the aperture antenna [19] is given by

$$E(r, \theta, \phi) = \frac{j e^{-j k r}}{2 \lambda r} (1 + \cos \theta) (\hat{\theta} \cos \phi - \hat{\phi} \sin \phi) F(k \sin \theta \cos \phi, k \sin \theta \sin \phi)$$

(5)

where $r$ is distance between center of compound box-horn aperture, $o(0, 0, 0)$ and field point, $P(r, \theta, \phi)$. $\theta$ and $\phi$ are angles from $z$- and $x$-axes, respectively. $\lambda$ is wavelength in free-space. $\hat{\theta}$ and $\hat{\phi}$ are unit vectors in directions of $\theta$ and $\phi$, respectively. $F(k_x, k_y)$ is the Fourier transform of the tangential electric field in the aperture [19] and can be evaluated as given below.

$$F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x, y, 0) e^{j k_x x} e^{j k_y y} dx dy$$

(6)

Therefore,

$$F(k \sin \theta \cos \phi, k \sin \theta \sin \phi) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x, y, 0) e^{j k_x x \sin \theta \cos \phi} e^{j k_y y \sin \theta \sin \phi} dx dy$$

(7)

Since $E_x(x, y, 0) = 0$ for $|x| > A/2$ and $|y| > B/2$ (i.e., the electric field outside the aperture of the compound box-horn is considered to be zero). Therefore,

$$F(k \sin \theta \cos \phi, k \sin \theta \sin \phi) = \int_{-A/2}^{A/2} \int_{-B/2}^{B/2} E_x(x, y, 0) e^{j k_x x \sin \theta \cos \phi} e^{j k_y y \sin \theta \sin \phi} dx dy$$

(8)

or

$$F(k \sin \theta \cos \phi, k \sin \theta \sin \phi) = \int_{-A/2}^{A/2} \int_{-B/2}^{B/2} \left[ a_{10} \cos \left( \frac{\pi y}{B} \right) e^{-j\beta_{10} L} + a_{30} \cos \left( \frac{3\pi y}{B} \right) e^{-j\beta_{30} L} \right]$$

$$\times e^{-\frac{j k}{2} \left( \frac{x^2}{R_1^2} - \frac{y^2}{R_2^2} \right)} e^{j k x \sin \theta \cos \phi} e^{j k y \sin \theta \sin \phi} dx dy$$

(9)
Hence, the radiation field for compound box-horn antenna, obtained with the help of Eqns. (5) and (9) is given below.

\[
E(r, \theta, \phi) = \frac{je^{-jkr}}{2\lambda r} (1 + \cos \theta)(\hat{\theta} \cos \phi - \hat{\phi} \sin \phi)
\]

\[
= \int_{-A/2}^{A/2} \int_{-B/2}^{B/2} \left[ a_{10} \cos \left( \frac{\pi y}{B} \right) e^{-j\beta_{10} L} + a_{30} \cos \left( \frac{3\pi y}{B} \right) e^{-j\beta_{30} L} \right]
\]

\[
\times e^{-j\frac{k}{2} \left( \frac{x^2}{R_1^2} - \frac{x^2}{R_2^2} \right)} e^{jkr \sin \theta \cos \phi} e^{jky \sin \theta \sin \phi} dxdy
\]

The definite double integral of Eqn. (10) is numerically evaluated synchronously by MATLAB® software using ‘dblquad’ function; which uses quadrature function ‘quadl’. The function ‘quadl’ is based on ‘high order recursive adaptive Lobatto quadrature algorithm’.

The radiation field in E-plane (x-z plane), \( E_\theta \) at a fixed radial distance r can be found by putting \( \phi = 0 \) in Eqn. (10). In this case, \( E_\phi = 0 \). Similarly, the radiation field in H-plane (y-z plane), \( E_\phi \) at a fixed radial distance r is obtained by putting \( \phi = \pi/2 \) in Eqn. (10). In this case, \( E_\theta = 0 \).

3. DESIGN OF COMPOUND BOX-HORN ANTENNA

The compound box-horn is designed to operate at 10GHz. The larger size pyramidal horn section of the compound box-horn is designed for optimum gain as discussed by Terman [20]. For modified box-horn section of the compound box-horn, the E- and H-plane flared-horn (smaller size pyramidal horn) exciting the box is designed for optimum gain as per Terman [20] and box is designed as per design guidelines given by Silver [17]. For the sake of brevity, design procedure is not given here. Flare angle in H-plane (\( \theta_{HM} \)) of smaller size pyramidal horn exciting the box, determines the H-plane dimension of the mouth of smaller size pyramidal horn and the ratio of the H-plane dimensions of the mouth of smaller size pyramidal horn and box waveguide determines the ratio of the coefficients \( a_{30} \) to \( a_{10} \). Therefore, flare angle in H-plane (\( \theta_{HM} \)) governs the ratio of \( a_{30} \) to \( a_{10} \). The modified box-horn is designed for the ratio \( a_{30}/a_{10} \) equal to 0.3 (for which \( \theta_{HM} = 15^\circ \) [17]), which is a fairly good approximation to uniform aperture field distribution, optimum efficiency, maximum gain and minimum amplitude taper loss as discussed by Silver [17]. Flare angle in E-plane (\( \theta_{EM} \)) governs the narrow dimension of the aperture of modified box-horn, a [15,16]. The smaller size pyramidal horn exciting the box is fed with a EIA WR-90 waveguide with aperture size \( 1.016 \text{cm} \times 2.286 \text{cm} \). The aperture size of smaller size pyramidal horn...
Table 1. HPBW of the compound box-horn antennas with different Θ_{EP} and constant Θ_{EP}.

<table>
<thead>
<tr>
<th>Antenna Type</th>
<th>Aperture size of compound box-horn (A×B cm²)</th>
<th>HPBW in E-Plane (degree)</th>
<th>HPBW in H-Plane (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compound box-horn</td>
<td>11.39×8.96</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>(θ_{EP}=15°, θ_{HP}=30°)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compound box-horn</td>
<td>8.51×8.96</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>(θ_{EP}=20°, θ_{HP}=30°)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compound box-horn</td>
<td>6.77×8.96</td>
<td>24</td>
<td>22</td>
</tr>
<tr>
<td>(θ_{EP}=25°, θ_{HP}=30°)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(with θ_{EM} = 15° and θ_{HM} = 15°) exciting the box is 3.45 cm×3.65 cm. The computed dimensions of modified box-horn section designed for optimum gain and uniform aperture field at 10 GHz are a = 3.45 cm, b = 4.80 cm, and L = 2.49 cm.

To observe the effect of different flare angles in E- and H-planes of larger size pyramidal horn section of the compound box-horn on the radiation patterns of compound box-horn in free-space, three compound box-horn antennas with different flare angles in E-plane (θ_{EP}) and constant flare angle in H-plane (θ_{HP}) of larger size pyramidal horn section are considered and designed at 10 GHz. The aperture size of the three compound box-horns is listed in Table 1. Similarly, three other compound box-horn antennas with different flare angles in H-plane (θ_{HP}) and constant flare angle in E-plane (θ_{EP}) of larger size pyramidal horn section are also considered and designed at 10 GHz. The aperture size of these compound box-horns is given in Table 2.

4. VALIDATION OF THE ANALYSIS

The analysis is validated by calculating E- and H-plane radiation patterns of a pyramidal horn carrying TE_{10}-mode in free-space at 10 GHz using present analysis and comparing it with experimental results [7] and is shown in Fig. 2. The pyramidal horn is fed with a EIA WR-90 waveguide, and the rectangular aperture of the pyramidal horn has the dimensions A = 8.1 cm (3.19 inch), B = 11.1 cm (4.36 inch), R₁ = 19.1 cm (7.52 inch), and R₂ = 20.8 cm (8.18 inch). The flare angles of the pyramidal horn are 11.98° and 14.92° in E- and H-planes. The theoretically computed radiation patterns (E- and H-
Table 2. HPBW of the compound box-horn antennas with constant θ_{EP} and different θ_{EP}.

<table>
<thead>
<tr>
<th>Antenna Type</th>
<th>Aperture size of compound box-horn (AxB cm²)</th>
<th>HPBW in E-Plane (degree)</th>
<th>HPBW in H-Plane (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compound box-horn (θ_{EP}=15°, θ_{HP}=30°)</td>
<td>11.39 x 8.96</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>Compound box-horn (θ_{EP}=15°, θ_{HP}=35°)</td>
<td>11.39 x 7.61</td>
<td>13</td>
<td>28</td>
</tr>
<tr>
<td>Compound box-horn (θ_{EP}=15°, θ_{HP}=40°)</td>
<td>11.39 x 6.59</td>
<td>13</td>
<td>32</td>
</tr>
</tbody>
</table>

Figure 2. Radiation patterns of pyramidal horn in free-space at 10GHz in (a) E-plane and (b) H-plane: validation of the analysis against experimental results [7].

planes) are in agreement with the experimental results available in the literature [7]. Inaccuracies in the sidelobe structure may be caused by the limitation of the measurements’ precision, and because the diffracted fields were neglected in the analytical model.

5. NUMERICAL RESULTS AND DISCUSSION

The radiation field patterns in E- and H-planes for compound box-horn are computed at 10GHz using MATLAB® software and the results are presented in Figs. 3–5.

The radiation patterns of compound box-horn antenna in free-space at 10GHz for different flare angles in E-plane (θ_{EP}) while
Figure 3. Radiation patterns of compound box horn in free-space at 10 GHz in (a) E-plane and (b) H-plane with different flare angles in E-plane for larger size pyramidal horn section.

Figure 4. Radiation patterns of compound box horn in free-space at 10 GHz in (a) E-plane and (b) H-plane with different flare angles in H-plane for larger size pyramidal horn section.

keeping the flare angle in H-plane ($\theta_{HP}$) of larger size pyramidal horn section, constant at 30° are computed and presented in Fig. 3. The HPBW in both the E- and H-planes is evaluated for the compound box-horn and given in Table 1. It can be observed from Fig. 3 and Table 1 that the radiation pattern in E-plane for compound box-horn can be made narrower by decreasing the flare angle in E-plane ($\theta_{EP}$) of larger size pyramidal horn section. Variation of $\theta_{EP}$ has no effect on HPBWs in H-plane. The decrease in $\theta_{EP}$ causes larger narrow dimension of compound box-horn $A$ to sustain optimum gain, therefore narrower
beam (low HPBW) is obtained in $E$-plane and no effect in $H$-plane due to unchanged broad dimension of compound box-horn $B$.

The radiation patterns of compound box-horn antenna in free-space at 10GHz for constant flare angle at $15^\circ$ in $E$-plane ($\theta_{EP}$) with different flare angles in $H$-plane ($\theta_{HP}$) of larger size pyramidal horn section are computed and presented in Fig. 4. The HPBW in both the $E$- and $H$-planes is evaluated for the compound box-horn and given in Table 2. It is investigated from Fig. 4 and Table 2 that the radiation pattern in $H$-plane for compound box-horn can be made narrower by decreasing the flare angle in $H$-plane ($\theta_{HP}$) of larger size pyramidal horn section. Variation of $\theta_{HP}$ has no effect on HPBWs in $E$-plane. The decrease in $\theta_{HP}$ causes larger broad dimension of compound box-horn $B$ to retain optimum gain, therefore narrower beam (low HPBW) is obtained in $H$-plane and no effect in $E$-plane due to unchanged narrow dimension of compound box-horn $A$.

In Fig. 5, the radiation patterns of compound box-horn are compared with those for TE10-mode pyramidal horn of same aperture size and at 10GHz. The flare angles of the pyramidal horn are equal to respective flare angles of the outer pyramidal horn section of the compound box-horn. The HPBW in both the $E$- and $H$-planes evaluated for the compound box-horn and pyramidal horn is listed in Table 3. It can be seen from Fig. 5 and Table 3 that the former horn is narrower in both $E$- and $H$-planes than the latter, since compound box-horn supports TE$_{10}$- and TE$_{30}$-modes giving more uniform field distribution along the long side of its aperture, while pyramidal horn supports only TE$_{10}$-mode with half-sinusoidal aperture field distribution.
Table 3. HPBW of the compound box-horn and pyramidal horn antennas.

<table>
<thead>
<tr>
<th>Antenna Type</th>
<th>Aperture size of compound box-horn (A×B cm²)</th>
<th>HPBW in E-Plane (degree)</th>
<th>HPBW in H-Plane (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compound box-horn</td>
<td>11.39 × 8.96</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>(θβ=15°, θη=30°)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pyramidal horn</td>
<td>11.39 × 8.96</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>(θβ=15°, θη=30°)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. CONCLUSION

A new compound box-horn has been designed at 10 GHz and analyzed for its radiation pattern and HPBW. It is shown that the radiation pattern in E- and H-plane for compound box-horn can be made narrower by decreasing the flare angles in both E- and H-planes of larger size pyramidal horn section. The compound box-horn is narrower in E- as well as H-planes in comparison to TE_{10}-mode pyramidal horn of same aperture size. Thus, compound box-horn exhibits the basic characteristics and benefits of both the modified box-horn and the pyramidal horn. In short, we can say that compound box-horn provides larger aperture size, relatively uniform field at its aperture, and desired HPBW in E- and H-planes.

The analytical model and design guidelines presented in this paper can be useful for development of prototypes of compound box-horn, which may find potential application as a high-directivity transmitting horn for antenna measurements in the laboratory or as a range illuminator or in microwave communication etc.

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