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UNIFORM GAIN POWER-SPECTRUM ANTENNA-PATTERN THEOREM AND ITS POSSIBLE APPLICATIONS

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Abstract—For certain applications in radio astronomy, viz. radio spectrographs, spectrum monitoring etc., only the amplitude power spectrum coverage within an angle of observation could be of interest. Ideally, the antenna structures of such instruments should illuminate this covering angle with a fixed uniform gain. This might be achieved using a combination of dipole antennas, a single vertical dipole, a loop antenna etc., but are subjected to limited bandwidth. This limitation could be overcome if many electrically-identical wideband antennas are positioned across the perimeter of a circle lying in the horizontal plane such that the antennas' adjacent half power beam angles touch each other. It has been theoretically observed that if two identical antennas are positioned at an angle with respect to one-another in such a way that their adjacent half power beam angles coincide, then if the amplitude power spectrums of the two are added, the result is effectively an amplitude power spectrum obtained from a single antenna having an uniform gain and uniform signal to noise ratio within the angle subtended by them. This angle also happens to be equal to the half power beamwidth of the individual antennas. A proper design using frequency independent antennas might possibly result to an user specified uniform amplitude power spectrum gain coverage across any required angle, with a theoretically unlimited bandwidth. More number of identical antennas might be positioned in similar fashion for extending the angular coverage.

The power spectrums from these antennas could be directly added which effectively represent the power spectrum from a single antenna possessing uniform gain coverage within an angle equal to the product of individual half power beamwidth angle with one less the number of antennas, thus achieving user defined gain, wide bandwidth, and uniform signal to noise ratio across the angle. It is also possible to recover the time domain signal by applying Fourier Transform on the outputs of the antennas followed by an addition of their amplitudes while keeping the phase information identical to that of one antenna (taken as reference), and taking its inverse Fourier Transform.

1. INTRODUCTION

Scientific instrumentations for power spectrum radio data monitoring uses wideband antennas at its front end. In certain cases, wide angular coverage with uniform antenna illumination† is a requirement. Examples could be terrestrial spectrum monitoring stations, magnitude spectrographs, radio burst detectors for Sun and Jupiter, spectrum monitoring of satellites etc., where only the magnitude spectrum could be of prime interest [1–3]. Certain applications among the stated above might prefer a complete omnidirectional coverage from 0° to 360° in the azimuth plane. A vertical dipole above the ground [4], a monopole or a dipole in free space could act as an omnidirectional antenna in the plane perpendicular to its electric fields. Similar properties are also seen in loop, biconical or helical antennas [5] and in certain printed dipole arrays [11]. However, they might not be suitable if the required angular coverage is only a fraction of 360°‡ since they receive signals uniformly from all directions in one plane. Moreover, the gains of classical dipoles are fixed and possess a limited bandwidth. Fulfilling these requirements becomes easy if a basic cell antenna structure is devised which uniformly illuminate a small angle. Using a combination of few such cell antenna systems by placing them adjacently in angles, the effective covering angle might be increased to the desired value. Though it is possible to combine dipoles using phased array techniques to effectively represent a single antenna, but the bandwidths are limited [5–7, 12].

Certain antennas could be suitably positioned in the three

† This is in contrast to the majority of the RF equipments where antennas having narrow beamwidth are desirable.

‡ An example of such an application could be to monitor the radio spectrum emission from a nearby city which might require an angular coverage between 30° to 60°.

dimensional space and their outputs might be combined in phase and amplitude to obtain a semi-omnidirectional pattern based on requirements. The major difficulties lie in phase adjustments, especially if the individual antennas are broadband in nature. But if the requirements are restricted to amplitude power spectrum alone, the outputs from these antennas could be analyzed in frequency and the respective amplitudes could be added. This might be done using either swept frequency spectrum analyzers or discritizing the signal and taking its FFT (fast Fourier transform). In the later case, only the magnitude spectrum is of interest. However, if the time domain signal is required from the antenna combination, the FFT amplitudes could be added and combined with the phase information obtained from a single antenna (taken as reference) followed by an inverse FFT. It has been invented and theoretically verified by the first author, that a true uniform gain power spectrum antenna pattern across an angle, possessing uniform signal to noise ratio, could be obtained by using a combination of electrically identical antennas[§] positioned symmetrically in a circular/semi-circular arc, provided their adjacent half power angles touch each other. A minimum of two electrically identical antennas are required for uniformly illuminating an arc. The following Sections explain them in details. In Section 2 the basic theorem of power spectrum addition for producing uniform gain and signal to noise ratio within the angle subtended between two antennas are explained. It also discusses a possible method for combining both the amplitude and phase spectrums from the antennas and recovering the time domain signal from it. Section 3 focuses on the design aspects of a complete omnidirectional system.

2. TWO ANTENNA UNIFORM GAIN POWER SPECTRUM ANTENNA PATTERN THEOREM

As per the original invention of the first author, the theorem can be stated in two parts as:

If two electrically identical antennas possessing maximum individual gains G_0, positioned in free space in a plane, such that they subtend an angle α_v (equal to their half power beamwidths) with respect to each other, then effectively the antenna system produces a magnitude power spectrum with uniform antenna gain G_0 and a uniform signal to noise ratio across the angle α_v if the magnitude power spectrums of individual antennas are added.

[§] By electrically identical we mean that the antennas possess identical bandwidths and for every frequency, the radiation patterns, gains, polarizations and impedances are identical.

If the exponents of the imaginary phase values of every frequency channel obtained from one of the antennas are multiplied with the corresponding channels' sum of the amplitude spectrum and an inverse Fourier transform is applied, the time domain signal thus produced is effectively the signal that shall be obtained from a single antenna possessing a uniform gain G_0 across the angle α_v.

The above are valid under ideal conditions, i.e., when the antennas are positioned in free space as shown in Figure 1(a). Under actual conditions, the presence of ground might alter the individual radiation patterns. Thus the antennas might be positioned in a plane parallel to the ground as shown in Figure 1(b) such that the patterns remain symmetrical and identical in the horizontal plane. This type of setup is suitable for monitoring terrestrial spectrum. In case the antennas are to point towards the sky, the effect of ground might be reduced by placing the antenna over an electrically conductive spherical surface above the ground as shown in Figure 1(c) [8]. Theoretically, the antennas might pose any polarization, but care should be taken that the patterns of individual antennas within the plane of combination should be symmetrical and identical. It is equally important for the maximum side lobe level to be negligibly small [1] or much below the half power points of the main lobe, preferably by 10 dB or more which is generally true for a fairly designed antenna.

2.1. The Effective Gain Equation

The gain variations as a function of angle for the major lobes of the radiation pattern of any directional antennas could be mathematically modeled [1] and expressed as in equation (1). Here, G_0 is the maximum gain and κ is the angle scaling factor‖, θ and ϕ are the coordinates of a spherical coordinate system. With reference to Y-axis of Figure 1(a), the power gain pattern of Ant-1 in the X-Y plane might be expressed as in equation (2), where α increases in clockwise direction starting at zero from Y-axis. Similarly the power gain pattern for Ant-2 is expressed in equation (3). The amplitude power spectrum available at the spectrum combiner's output will effectively be the power spectrum obtained from a single antenna having a gain pattern equal to the summation of the gain patterns of Ant-1 and Ant-2 displaced by an angle α_v. We call this as the *Effective Gain Equation* and is expressed

‖ κ is to be used for theoretical modeling of low gain log periodic antennas having wide major lobes whose first nulls might appear after 90° [1]. For higher gain antennas, κ is taken as unity and the conventional antenna model could also be used [7].

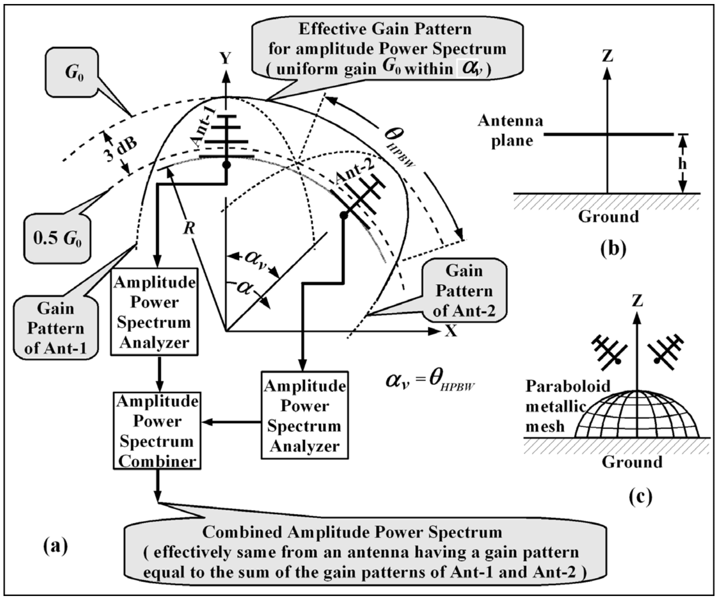

Figure 1. Antenna configuration and additional hardware for producing the amplitude power spectrum with uniform gain and uniform signal to noise ratio within the angle subtended between the two antennas. The polarizations could be vertical or horizontal. (a) Antenna configuration in free space. This configuration together with the additional electronic hardware might be treated as a single antenna-pair cell. (b) Antennas for monitoring terrestrial-signal's power spectrum. (c) Antennas for monitoring the sky-signal's power spectrum.

in equation (4).

$$G(\theta,\phi) = G_0\cos^n(\kappa\phi),\ \theta = \frac{\pi}{2},\ 0 \le \phi \le 2\pi,\ n \ge 1,\ 0 \le \kappa \le 1 \quad (1)$$

$$G_1(\alpha) = G_0\cos^n(\kappa\alpha),\ 0 \le \alpha \le \alpha_v,\ n \ge 1,\ 0 \le \kappa \le 1 \quad (2)$$

$$G_2(\alpha) = G_0\cos^n[\kappa(\alpha_v - \alpha)],\ 0 \le \alpha \le \alpha_v,\ n \ge 1,\ 0 \le \kappa \le 1 \quad (3)$$

$$\begin{aligned} G(\alpha) &= G_1(\alpha) + G_2(\alpha) = G_0[\cos^n(\kappa\alpha) + \cos^n\{\kappa(\alpha_v - \alpha)\}] \\ &\quad 0 \le \alpha \le \alpha_v,\ n \ge 1,\ \le 1 \end{aligned} \quad (4)$$

2.2. The Half Power Equation

In order to mathematically model any antenna pattern using equation (1), it is necessary to find the value of n from the measured pattern.

This could be obtained by substituting the left hand side of the equation (2) with 0.5 G_0 and on the right hand side substituting α with 0.5 θ_{HPBW} and then solving it. Here, θ_{HPBW} is the half power beam width angle. We call this equation as the *Half Power Equation* and is expressed in equation (5). Solving this, we obtain the value of n as expressed in equation (6). The logarithms in the numerator and denominator on the right hand side should be of identical base.

$$\cos^n (0.5\ \kappa\ \theta_{\mathrm{HPBW}}) = 0.5 \tag{5}$$

$$n = \frac{\log(0.5)}{\log\{\ \cos(0.5\ \kappa\ \theta_{\mathrm{HPBW}})\}} \tag{6}$$

We now evaluate the effective combined gain from equation (4) using a MATLAB routine, after having substituted n from equation (6). The behavior of equation (4) is found to be almost uniform, maintaining a nearly fixed gain G_0 within the angle α_v. The deviations of the actual effective gain from the fixed gain value of G_0 across the angle α_v in percentage are expressed in equation (7) and plotted in Figure 2. Table 1 lists the values of maximum and minimum gain deviations (percentage error) of the effective pattern constructed with antenna-pairs for various half power beamwidths. The maximum error is seen for a half power beamwidth of 90°, which is less than 8%. As the value of n increases, i.e., for further narrower beams, the error increases but tends to saturate with further increase in n.

$$\text{Percentage Error} = 100 \times \frac{G\,(\alpha) - G_0}{G_0} \tag{7}$$

2.3. The Directional Signal to Noise Ratio

For a fixed ambient temperature, the time-averaged noise power output from the antennas might be assumed to be constant. Since the noises from different antennas are incoherent, the total mean noise at the spectrum combiner's output shall be $\sqrt{2}\ \overline{N_0}$, where, $\overline{N_0}$ is the mean thermal noise power from the individual antennas over a significant period of time. The signal power is proportional to the gain and is a function of α. Since the signal to noise ratios change with direction (α) of incoming electromagnetic waves, we address them here as the *Directional Signal to Noise Ratio*. If S is the flux density in watt per square meter, λ is the wavelength in meters, η is the antenna efficiency and $\overline{N_0}$ is in watts, then the directional signal to noise ratios of Ant-1 and Ant-2 (as a functions of α) could be respectively expressed as

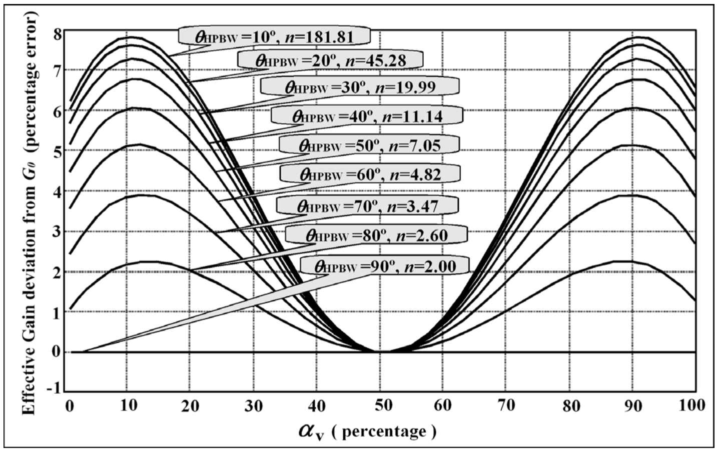

Figure 2. A simulated plot showing the percentage effective antenna gain deviations from G_0 (percentage error) as a function of percentage operating angle α_v. As a special case, if the elemental antennas posses $n = 2$, the errors vanish.

Table 1. Maximum and minimum percentage errors of the effective antenna gain pattern (for amplitude power spectrum) constructed with antennas having different half power beamwidths.

$\alpha_v = \theta_{\text{HPBW}}$ (Degrees)	n ($\kappa = 1$)	Min. Error (percentage)	Max. Error (percentage)
10	181.81	2.22×10^{-14}	7.79
20	45.28	2.22×10^{-14}	7.59
30	19.99	2.22×10^{-14}	7.25
40	11.14	2.22×10^{-14}	6.75
50	7.05	2.22×10^{-14}	6.05
60	4.82	2.22×10^{-14}	5.12
70	3.47	2.22×10^{-14}	3.88
80	2.60	0.00	2.24
90	2.00	-1.11×10^{-14}	0.00

in equations (8) and (9). Since the effective gain of the combined antenna remains almost constant within α_v, the signal to noise ratio is independent of α and is expressed in equation (10). The angular-mean signal to noise ratio (across the angle α_v) for a single antenna is expressed in equation (11). Comparing equations (10) with (11) we find that there is an improvement in the angular-mean signal to noise ratio by a factor of $\sqrt{2}$ if the amplitude power spectrums from the two antennas are combined.

$$SNR_1(\alpha) = \left(\frac{S\ \eta\ \lambda^2}{4\pi}\right)\left(\frac{G_1(\alpha)}{\overline{N_0}}\right) = \left(\frac{S\ \eta\ \lambda^2\ G_0}{4\pi\overline{N_0}}\right)\cos^n(\kappa\alpha) \tag{8}$$

$$SNR_2(\alpha) = \left(\frac{S\ \eta\ \lambda^2}{4\pi}\right)\left(\frac{G_2(\alpha)}{\overline{N_0}}\right) = \left(\frac{S\ \eta\ \lambda^2 G_0}{4\pi\overline{N_0}}\right)\cos^n[\kappa(\alpha_v - \alpha)] \tag{9}$$

$$SNR(\alpha) = \left(\frac{S\ \eta\ \lambda^2}{4\pi}\right)\left(\frac{G_1(\alpha) + G_2(\alpha)}{\sqrt{2}\ \overline{N_0}}\right) \approx \left(\frac{S\ \eta\ \lambda^2}{4\pi}\right)\left(\frac{G_0}{\sqrt{2}\ \overline{N_0}}\right),$$
$$0 \le \alpha \le \alpha_v,\ n \ge 1,\ 0 \le \kappa \le 1 \tag{10}$$

$$\overline{SNR_1} = \overline{SNR_2} \approx \left(\frac{S\ \eta\ \lambda^2}{4\pi}\right)\left(\frac{G_0}{2\overline{N_0}}\right) \tag{11}$$

2.4. Practical Problems While Using Ultra-wideband Frequency Independent Antennas

As mentioned earlier, in principle the amplitude power spectrums from identical frequency independent antennas might be combined to achieve wide bandwidth, flexibility in gain, wide beamwidth and uniform directional signal to noise ratio. But since the properties of the antennas do vary with frequency [5, 7], some complications might arise while designing. A good example of a frequency independent antenna is the log periodic antenna whose electrical characteristics (including the gain) repeat in steps of logarithm of frequency [5, 9, 10]. However, a fairly good design might restrict the ratio of maximum to minimum gain within 1.5 dB. Using such antennas, the antenna-pair cell shown in Figure 1(a) might be constructed with α_v considered equal to the half power beamwidth corresponding to its maximum gain pattern, such that the effective gain within this angular coverage remains greater than or equal to the individual maximum gain at any particular frequency. Thus even at those frequencies where the gains drop to minimum, the minimum directional signal to noise ratio of the effective antenna shall not reduce below that obtained from equation (10). However, calibration needs to be done for compensating the extra gain. Figure 3 shows the theoretical effective gain deviations (for the amplitude power spectrum) of antenna-cells constructed using frequency independent antennas, possessing variety of minimum gains, starting from 5 dB up to 12 dB, and a maximum gain deviation of

1.5 dB. The error increases as the gain of the constructor antennas increase.

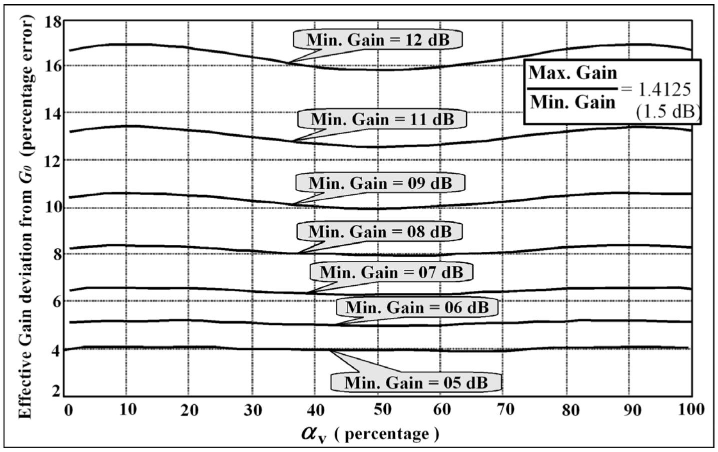

Figure 3. A simulated plot showing percentage deviation of the effective gain as a function of operating angle α_v using wide band antennas whose individual gain varies within a limited range as a function of frequency.

2.5. A possible Method for Combining both Amplitude and Phase Spectrums and Recovering the Time-domain Signal

As in the phased arrays, where the signals from various antenna elements are combined in phase and amplitude to form an effective antenna with high directivity at a required angle, the signals in the present case might be also be combined using additional hardware, and the time-domain signal might be recovered. Any signal might be expanded in a composite form consisting of sine and cosine components spread over various frequencies using the Fourier transform. For simplicity, we consider a sinusoidal signal $s(t)$ with amplitude A_0 and frequency f_0 being radiated from a distant location. With reference to Figure 1(a), let $s_1(t)$ and $s_2(t)$ be the signals obtained from antennas Ant-1 and Ant-2 respectively. These signals might be mathematically represented as in equation (12), where A_0 represents the amplitude of the transmitted signal, ϕ_1 and ϕ_2 represent the introduced phase delays in the received signals due to the distance of propagation and phase shifts occurring within the receiver antennas, and σ_1 and σ_2 are the constants of proportionality. The Fourier transforms of these are expressed in equation (13). The signals $s_1(t)$ and $s_2(t)$ could be directly combined provided $\phi_1 = \phi_2$. The other alternative way is to combine only the amplitudes (power spectrum) and associate it with the phase values of any particular antenna taken as the reference antenna. The phase difference of different Fourier components might be calculated as shown in equation (14). Considering Ant-1 as the reference antenna¶, its phase values should now be associated with the signal received by antenna Ant-2, and then both these signals should be added as expressed in equation (15), where $S_{total}(f)$ is the resultant spectrum. The effective time domain signal could be obtained by taking the inverse Fourier transform of the combined signal as shown in equation (16).

$$\begin{aligned} s(t) &= A_0 \ \exp(j2\pi f_0 \ t) \\ s_1(t) &= \sigma_1 \ A_0 \exp[j(2\pi f_0 \ t - \phi_1)] = \sigma_1 \, s(t) \exp[-j\phi_1] \\ s_2(t) &= \sigma_2 \ A_0 \exp[j(2\pi f_0 \ t - \phi_2)] = \sigma_2 \, s(t) \exp[-j\phi_2] \end{aligned} \tag{12}$$

$$\begin{aligned} S(f) &= A_0 \ \delta(f - f_0) \\ S_1(f) &= \sigma_1 \ A_0 \ \delta(f - f_0) \ \exp(-j\phi_1) = \sigma_1 \ S(f) \ \exp(-j\phi_1) \\ S_2(f) &= \sigma_2 \ A_0 \ \delta(f - f_0) \ \exp(-j\phi_2) = \sigma_2 \ S(f) \ \exp(-j\phi_2) \end{aligned} \tag{13}$$

$$\text{Phase Difference} = \angle S_1(f) - \angle S_2(f) = \phi_2 - \phi_1 \tag{14}$$

$$\begin{aligned} S_{\text{total}}(f) &= S_1(f) + S_2(f) \ \exp[j(\text{Phase Difference})] \\ &= [\sigma_1 + \sigma_2] \ S(f) \ \exp(-j\phi_1) \end{aligned} \tag{15}$$

$$\begin{aligned} s_{\text{total}}(t) &= [\sigma_1 + \sigma_2] A_0 \exp[j(2\pi f_0 t - \phi_1)] \\ &= [\sigma_1 + \sigma_2] s(t) \ \exp[-j\phi_1] \end{aligned} \tag{16}$$

In actual practice, the signals from the antennas could be sampled and discritized, following which the above operations might be performed in the discrete domain. The possible block diagram of the hardware is shown in Figure 4. The filters have been suggested for band-limiting the signals for sampling, i.e., for satisfying the Nyquist criterion. Both the filters should possess identical characteristics. After discritizing the signals in the sampler, fast Fourier transforms (FFT-1 and FFT-2) are applied and the resultant's magnitude parts are added. These are then multiplied with the exponent of the imaginary phase components produced by FFT-1. The inverse Fourier transform of this represents the combined signal in the time domain. We have simulated and tested these algorithms in MATLAB. The results are quite satisfactory. To be noted in Figure 4 that after the inverse

¶ Though we have taken antenna Ant-1 as the reference, it is not a mandatory choice. Antenna Ant-2 could also serve as the reference antenna.

Fourier transform, only the real values are supplied as the final output. Though theoretically the imaginary part should be zero, it is observed that some of the samples possess non-zero imaginary values too thereby introducing small error in phase values. This artifact could be due to limited number of samples. However the real part closely resembles the shape of the actual signal. It is later described in the next section, that if more number of antennas are present, their individual magnitude spectrums could be directly added and associated with a common phase derived from the signal corresponding to the reference antenna (shown in dotted lines in Figure 4).

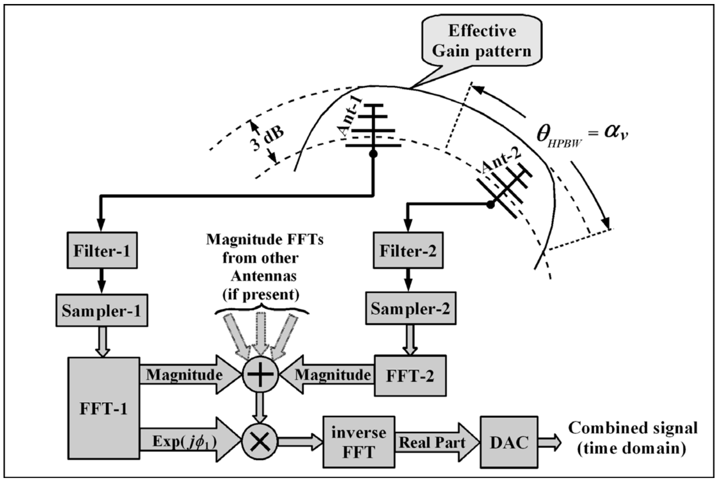

Figure 4. A possible scheme for combining the spectrums in magnitude and phase such that effectively the system behaves as a single antenna with flat angular coverage within α_v. Antenna Ant-1 is taken as reference antenna. If more number of antennas are present, all their magnitudes should be directly added in the adder.

3. ANTENNA SYSTEMS WITH COMPLETE OMNIDIRECTIONAL POWER SPECTRUM PATTERN

As stated earlier, it is possible to make a complete omnidirectional ultra wideband antenna system using more number of electrically identical frequency independent antennas based on the uniform gain power spectrum antenna pattern theorem. These antennas should be positioned pointing towards various directions on a plane parallel to the ground. They are to be equally separated in angle as shown in Figure 5. In order to cover the entire 360° of the omnidirectional plane, the design of the antennas should be carefully done. The gain of the antennas should be correctly chosen such that the ratio of 360° to the half power beamwidth θ_{HPBW} turns out to be an integer. This is expressed in equation (17) where N represents the number of antennas required. Though practically such an accurate design is difficult, especially when using frequency independent antennas, where the gain variations repeat as a function of frequency, but they might be fabricated with an acceptable error margin as discussed earlier.

$$N = \frac{360^\circ}{\theta_{\mathrm{HPBW}}} \text{ where, } N \text{ is an integer and } \theta_{\mathrm{HPBW}} \text{ is in degrees} \quad (17)$$

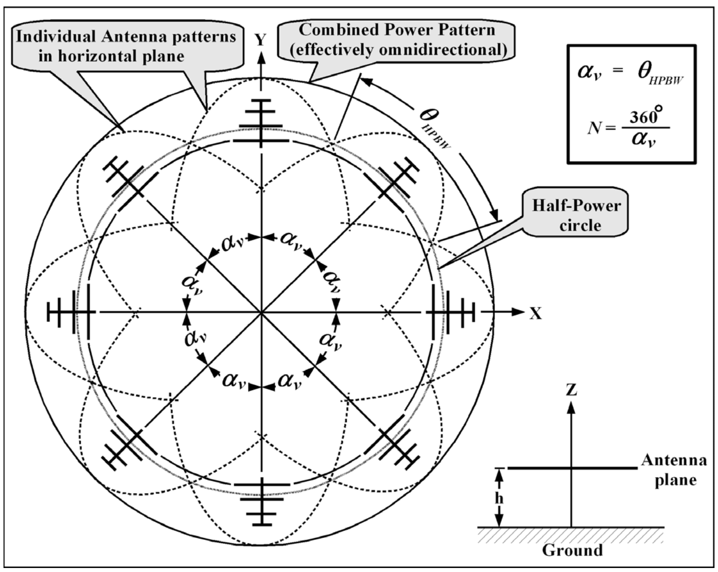

Figure 5. Multiple antennas of identical gain and characteristics forming a complete omnidirectional power spectrum antenna pattern.

4. REMARKS AND CONCLUSIONS

In this article a theorem was presented for achieving an effective uniform amplitude power spectrum gain composed out of two or more electrically identical antennas. The signal to noise ratio was shown to be more or less constant within the angle of observation. It was also shown that by the use of Fourier transform and its inverse transform, it is possible to recover the time-domain signals. For detecting the signals from a radio emissive object moving across the sky, it was suggested that effect of the ground might be reduced if antennas were mounted over inverted paraboloid convex ground surfaces. It has been noted that the interaction and mutual coupling among the antenna elements sometimes give rise to the strong electromagnetic oscillation phenomena [13]. For a multi-antenna system, careful study of related phenomena need to be done while designing.

It is speculated that the *uniform gain power spectrum antenna pattern theorem* shall find major applications in the future designs of power spectrum monitoring stations, ultra wideband omnidirectional systems, receiving radio signals from artificial satellites or astronomical objects like the Sun or the Jupiter (radio spectrographs) etc. Presently the mechanically static antenna based phased array techniques for tracking radio sources are gaining popularity which involves complicated electronics [14], but the need for increasing bandwidth is on demand. Hence it is speculated that the present theorem might provide an alternative method to replace the dynamically tracking antenna systems by static ones in the near future.

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