

**AN EXPRESSION FOR THE RADAR CROSS SECTION
COMPUTATION OF AN ELECTRICALLY LARGE
PERFECT CONDUCTING CYLINDER LOCATED OVER
A DIELECTRIC HALF-SPACE**

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Abstract—A method is presented to calculate the monostatic Radar Cross Section (RCS) of an electrically large perfect conducting cylinder vertically located over a dielectric half-space using Physical Optics (PO) technique. The four-path modal method is used to approximate the influence of the half-space to the scattering mechanism. The comparison between the results calculated by this expression and that by Moment Method (MOM) show that the expression is effective and efficient.

1. INTRODUCTION

A cylinder may serve as generic model of several bodies, natural or artificial. Many methods have been proposed to calculate the scattering from a cylinder [1–5], but the cylinders in these references are all located in free space. For some objects located over a half-space, such as the ground or sea surface, the influence of the half-space to the scattering mechanism should be considered. An analysis of scattering of a perfect conducting cylinder located on a ground plane has been presented, but the ground is considered as an infinite conducting flat [6].

In this paper, an expression for the calculation of the Radar Cross Section (RCS) of an electrically large perfect conducting cylinder located vertically over a dielectric half-space is proposed. The scattered field is approximated by Physical Optics method, and the “four-path” modal method is used to approximate the influence of half-space [7–10].

2. MATHEMATICAL ANALYSIS

The geometry of a cylinder located over a half-space with a length l and a radius a is shown in Fig. 1. The distance from the bottom surface of the cylinder to the half-space surface is h . The half-space is filled with a dielectric of relative permittivity ε_r and conductivity σ . We introduce Cartesian coordinate (x, y, z) with associated orthogonal unit vectors $(\hat{x}, \hat{y}, \hat{z})$, and the standard sphere coordinate (r, θ, φ) and their associated orthogonal unit vectors $(\hat{r}, \hat{\theta}, \hat{\varphi})$. It is well known that

$$\hat{r} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \quad (1)$$

The surface of half-space is assumed to lie at the $z = 0$ plane, and under the approximation of the “four-path” modal and PO method, the expression of the monostatic RCS can be written as [10]:

$$\begin{aligned} \sqrt{\sigma\sigma} = j \frac{k}{\sqrt{\pi}} \iint_s & \left[\hat{r} \cdot \hat{n} \exp(-2jk\hat{r} \cdot \vec{r}') + (\hat{r}_r + \hat{r}) \cdot \hat{n} R \exp\{-jk(\hat{r} + \hat{r}_r)\vec{r}'\} \right. \\ & \left. + R^2 \hat{r}_r \cdot \hat{n} \exp\{-2jk\hat{r}_r \cdot \vec{r}'\} \right] dS' \end{aligned} \quad (2)$$

Where s is the illuminated portion of the scatterer, k is the wave number, \vec{r}' is the location of an arbitrary point on s , \hat{n} is the unit normal vector at \vec{r}' , \hat{r}_r is the reflected vector and is set as

$$\hat{r}_r = \hat{r} - 2\hat{z}(\hat{r} \cdot \hat{z}), \quad (3)$$

and R is the Fresnel reflection coefficients. For horizontal polarization,

$$R = \frac{\cos \theta - \sqrt{\varepsilon_r - j\sigma \frac{\eta_0}{k} - \sin^2 \theta}}{\cos \theta + \sqrt{\varepsilon_r - j\sigma \frac{\eta_0}{k} - \sin^2 \theta}}. \quad (4)$$

And for vertical polarization,

$$R = \frac{\left(\varepsilon_r - j\sigma \frac{\eta_0}{k}\right) \cos \theta - \sqrt{\varepsilon_r - j\sigma \frac{\eta_0}{k} - \sin^2 \theta}}{\left(\varepsilon_r - j\sigma \frac{\eta_0}{k}\right) \cos \theta + \sqrt{\varepsilon_r - j\sigma \frac{\eta_0}{k} - \sin^2 \theta}}. \quad (5)$$

Where, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the impedance in free space. In cylinder coordinate system,

$$\vec{r}' = (a \cos \varphi', a \sin \varphi', z') \quad (6)$$

$$\hat{n} = (\cos \varphi', \sin \varphi', 0) \quad (7)$$

$$ds' = a d\varphi' dz' \quad (8)$$

where φ' is the circumferential angle of the surface position vector from the plane containing the incident direction and the cylinder axis, and it can be valued form $\varphi - \frac{\pi}{2}$ to $\varphi + \frac{\pi}{2}$, which are the shadow boundaries due to the incident wave.

Take (1), (3), (6), (7) and (8) into (2), the result can be expressed as two integrals. One is expressed with a variable along the axial direction and the other is expressed with a variable in the circumferential direction. The result is:

$$\sqrt{\sigma\sigma} = j \frac{k}{\sqrt{\pi}} I_z I_\varphi \quad (9)$$

where,

$$I_z = \int_h^{h+l} \left(e^{-2jk \cos \theta z'} + 2R + R^2 e^{-2jk \cos \theta z'} \right) dz' \quad (10)$$

$$I_\varphi = a \sin \theta \int_{\varphi-\pi/2}^{\varphi+\pi/2} \cos(\varphi - \varphi') e^{-2jka \sin \theta \cos(\varphi-\varphi')} d\varphi' \quad (11)$$

The axis integral is

$$I_z = \frac{\sin(kl \cos \theta)}{kl \cos \theta} l [\exp\{-jk \cos \theta(2h + l)\} + R^2 \exp\{jk \cos \theta(2h + l)\}] + 2Rl \quad (12)$$

The circumferential integral (11) can be evaluated approximately by means of stationary phase method and it is easy to know that $\varphi' = \varphi$ is just its stationary phase point. So the contribution of circumferential integral can be expressed as:

$$I_\varphi = a \sin \theta e^{-2jka \sin \theta} \sqrt{\frac{j\pi}{ka \sin \theta}} \quad (13)$$

When (12) and (13) are taken into (9), the expression becomes

$$\sqrt{\sigma\sigma} = j\sqrt{ka \sin \theta} \left\{ \frac{\sin(kl \cos \theta)}{kl \cos \theta} [\exp\{-jk \cos \theta(2h + l)\} + R^2 \exp\{jk \cos \theta(2h + l)\}] + 2R \right\} e^{j\left(\frac{\pi}{4} - 2ka \sin \theta\right)} \quad (14)$$

3. RESULTS

The monostatic RCS of a cylinder shown in Fig. 1 is calculated, where, $a = 0.15$ m, $h = 0.5$ m, and $l = 0.8$ m. The frequency of the incident planar wave which is horizontal polarized is 3 GHz, and the relative permittivity and conductivity of the half-space is $\epsilon_r = 76.7 - j12$ and $\sigma = 4$ mho/m respectively. The comparison between the monostatic RCS calculated by the expression given in

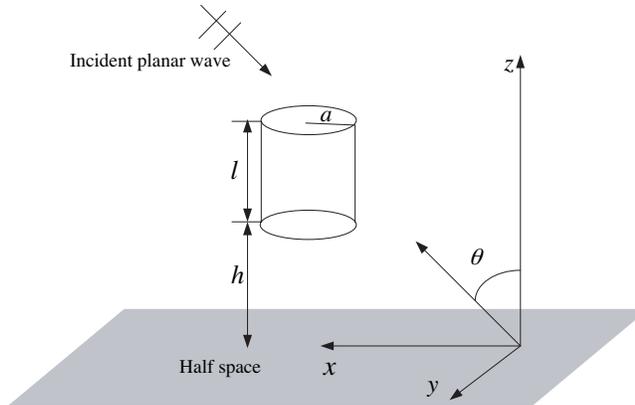


Figure 1. The geometry of a cylinder located over half-space.

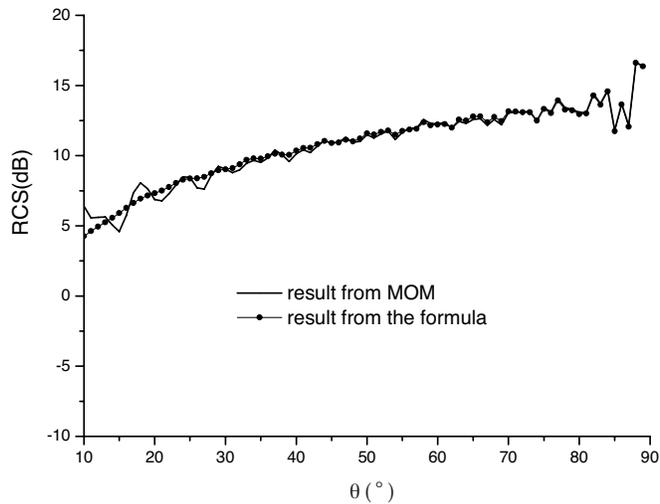


Figure 2. RCS of the cylinder for horizontal polarization.

this paper with that calculated by Moment Method (MOM) is shown in Fig. 2. It can be seen that the two results agree very well. The result from MOM includes the contributions from the two bottom surfaces whose influence to the total scattering is very small for low grazing angles. It costs about 23 minutes to get the result from MOM, while it just spends about 2 seconds to get the result from the expression presented in this paper.

4. CONCLUSION

In this paper, an expression for the computation of the RCS of an electrically large perfect conducting cylinder located over a dielectric half-space is presented using PO, in conjunction with the "four-path" modal approximation. Results show that the expression is accurate and timesaving.

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