

**NOVEL MICROSTRIP TRIANGULAR RESONATOR  
BANDPASS FILTER WITH TRANSMISSION ZEROS  
AND WIDE BANDS USING FRACTAL-SHAPED  
DEFECTION**

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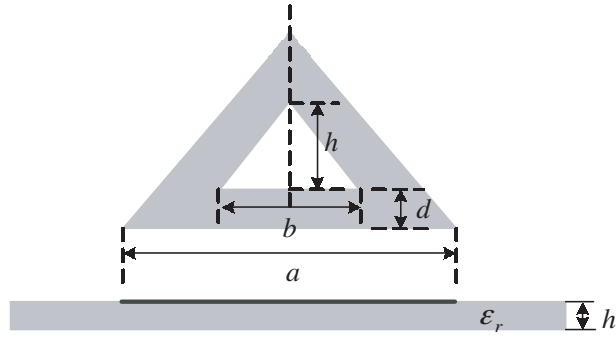
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**Abstract**—Fractal microwave passive circuits are simple and novel structures that attract much attention recently, however, the fractal technique is dominantly applied in antennas. In this paper, some new characteristics of microstrip equilateral triangular patch resonator with fractal deflection are analyzed, and novel fractal bandpass filters using equilateral triangular resonator are presented to implement high performances of multi-transmission zeros, wide passband and stopband, and low passband insertion loss as well as miniaturization. Using fractal deflection in patch, multi higher order modes are inspired for coupling a much wider passband, and parasitical harmonics are effectively suppressed. A new bandpass filter with a wide passband of about 0.92 GHz ( $\varepsilon_r = 9.8$ ) or 2.7 GHz ( $\varepsilon_r = 2.2$ ), maximum passband insertion loss of less than 0.5 dB, and multi-transmission zeros at both sides of passband, and a second bandpass filter with wide passband and stopbands of more than 2 GHz are implemented. Compared with some literatures, sizes of the new filters reduced and performances are greatly enhanced. The proposed filters have compact and simple structures, small sizes, high selectivity and so on, and all these features are the requirement of wireless communication circuits.

## 1. INTRODUCTION

Fractal [1–3] means broken or fractured which is derived from Latin word “fractus” dates back to the 19th century as a branch of classical mathematics. In a roughly words, fractal is in a sense of a total designation of self-similarity figures and structures without characteristic length. Fractal can generate almost any complex configurations in nature by iterating certain simple geometries. Basic fractal geometries can be divided into four types, they are Koch fractal, Minkowski fractal, Sierpinski and Hilbert fractals. All have advantages of miniaturization and wide band or multi-band operation [4], and these characteristics attribute to fractal’s two basic properties: self-similarity and space filling. Self-similarity means a portion of the fractal geometry always looks like that of the entire structure, and space filling means a fractal shape can be filled in a limited region as the order increases without increasing the whole area. The classical fractals that generated by certain mathematic method always have strict self-similarity, and can be called well-regulated fractals, and that most applied configurations are irregular fractals with rough self-similarity. Currently, fractal theory has been applied in many scientific research domain, and certainly attracts great interests of microwave engineering researchers’ for designing new microwave circuits and improving their performance as well as miniaturization, however, the application dominantly concentrates on antennas. Fractal can change the current pattern of filter, and make it distributes along the flexual conductor surface instead of the original simple patch surface, so the electric length increased. Fractal-shaped deflection can act as perturbation of filter and shift the resonant frequencies, induce dual-mode or multi-mode operation, and suppress the parasitical harmonics to implement multi-band or wide band as well as miniaturization for no other coupling resonator is required.

Microstrip bandpass filters have been widely used in a variety of microwave circuits and systems. Many kinds of microstrip patch resonator filter configurations such as square patch [5], circular patch [6, 7], triangular patch [8, 9], trapezoidal patch [10] and their ring patches, et al, have been reported, and some of them can be fabricated by direct coupling without coupling gaps [5]. Triangular resonator is studied by Ogasawara and Noguchi [11] in 1974 initially, and then, a microstrip one is analyzed by Helszajn and James [8], later, Hong et al. [9, 12] implement microstrip filters by using equilateral triangular resonators. In this paper, new microstrip bandpass filters with transmission zeros and wide bands(passband and stopband) by using equilateral triangular resonator with fractal-shaped deflection



**Figure 1.** Microstrip equilateral triangular resonator with fractal-shaped deflection.

are proposed, and filter performances are calculated, optimized and analyzed. Compared with the reported microstrip triangular patch resonator filters [9, 12, 13], and the line-based fractal bandpass filter [4], the proposed bandpass filters have better performances of wider passband, multi-transmission zeros, much wider stopbands, simple configuration and miniaturization which due to single patch operation, at the same time, the filter design complexity and manufacture uncertainty may be greatly reduced due to the simple structure.

## 2. CHARACTERISTICS OF MICROSTRIP EQUILATERAL TRIANGULAR RESONATOR

Configuration of the microstrip equilateral triangular resonator with fractal-shaped deflection is shown in Fig. 1, where,  $a$  is triangle side length,  $b$  is the length of triangular deflection hemline, and  $h$  is deflection height. When  $h = 0$ , it is an integrated equilateral triangular patch. For TM mode, the electromagnetic field patterns have no variation along the thickness of resonator, and the wave equation can be written as

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_{m,n,l}^2 \right) E_z = 0, \quad k_{m,n,l} = \frac{4\pi}{3a} \sqrt{m^2 + mn + n^2} \quad (1)$$

Where,  $E_z = A_{m,n,l} T(x, y)$ , and  $A_{m,n,l}$  is a constant, and based on the magnetic boundary conditions,  $T(x, y)$  can be expressed as [8]

$$T(x, y) = \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) l \right] \cdot \cos \left[ \frac{2\pi(m-n)y}{3a} \right]$$

$$\begin{aligned}
& + \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) m \right] \cdot \cos \left[ \frac{2\pi(n-l)y}{3a} \right] \\
& + \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) n \right] \cdot \cos \left[ \frac{2\pi(l-m)y}{3a} \right], \quad m + n + l = 0 \quad (2)
\end{aligned}$$

The electromagnetic field components of the resonator can be expressed as

$$\begin{aligned}
E_z = & A_{m,n,l} \cdot \left\{ \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) l \right] \cdot \cos \left[ \frac{2\pi(m-n)y}{3a} \right] \right. \\
& + \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) m \right] \cdot \cos \left[ \frac{2\pi(n-l)y}{3a} \right] \\
& \left. + \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) n \right] \cdot \cos \left[ \frac{2\pi(l-m)y}{3a} \right] \right\} \quad (3a)
\end{aligned}$$

$$\begin{aligned}
H_x = & \frac{j}{\omega\mu_0\mu_r} \frac{\partial E_z}{\partial y} \\
= & \frac{-jA_{m,n,l}}{\omega\mu_0\mu_r} \left\{ \frac{2\pi(m-n)}{3a} \cdot \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) l \right] \right. \\
& \cdot \sin \left[ \frac{2\pi(m-n)y}{3a} \right] + \frac{2\pi(n-l)}{3a} \cdot \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) m \right] \\
& \cdot \sin \left[ \frac{2\pi(n-l)y}{3a} \right] + \frac{2\pi(l-m)}{3a} \cdot \cos \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) n \right] \\
& \left. \cdot \sin \left[ \frac{2\pi(l-m)y}{3a} \right] \right\} \quad (3b)
\end{aligned}$$

$$\begin{aligned}
H_y = & \frac{-j}{\omega\mu_0\mu_r} \frac{\partial E_z}{\partial x} \\
= & \frac{jA_{m,n,l}}{\omega\mu_0\mu_r} \left\{ \frac{2\pi l}{\sqrt{3}a} \cdot \sin \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) l \right] \cdot \cos \left[ \frac{2\pi(m-n)y}{3a} \right] \right. \\
& + \frac{2\pi m}{\sqrt{3}a} \cdot \sin \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) m \right] \cdot \cos \left[ \frac{2\pi(n-l)y}{3a} \right] \\
& \left. + \frac{2\pi n}{\sqrt{3}a} \cdot \sin \left[ \left( \frac{2\pi x}{\sqrt{3}a} + \frac{2\pi}{3} \right) n \right] \cdot \cos \left[ \frac{2\pi(l-m)y}{3a} \right] \right\} \quad (3c)
\end{aligned}$$

$$E_x = E_y = H_z = 0 \quad (3d)$$

The dominant mode and the second order mode of microstrip equilateral triangular resonator are  $\text{TM}_{1,0,-1}$  and  $\text{TM}_{1,1,-2}$  [8], respectively, and for that  $m = 1$ ,  $n = 0$ ,  $l = -1$ , and  $m = 1$ ,  $n = 1$ ,  $l = -2$ , the electromagnetic field components of  $\text{TM}_{1,0,-1}$  mode and

TM<sub>1,1,-2</sub> mode can be obtained from expressions (3a)–(3d), and the corresponding cut-off wave-numbers can be got as

$$k_{1,0,-1} = \frac{4\pi}{3a}, \quad k_{1,1,-2} = \frac{4\pi}{\sqrt{3}a} \quad (4)$$

For an equilateral triangular patch resonator surrounded by perfect magnetic walls, the formula of resonant frequency for various modes can be expressed as

$$f_r = \frac{ck_{m,n,l}}{2\pi\sqrt{\varepsilon_r}} = \frac{2c}{3a\sqrt{\varepsilon_r}}\sqrt{m^2 + mn + n^2} \quad (5)$$

Where,  $c$  is the velocity of light,  $\varepsilon_r$  is the relative dielectric coefficient of substrate. In practice, there are no magnetic walls, and the accurate expressions of resonant frequency for the dominant mode TM<sub>1,0,-1</sub> and higher order modes can be expressed respectively as [14]

$$f_{1,0,-1} = \frac{2c}{3a_e\sqrt{\varepsilon_r}} \quad (6)$$

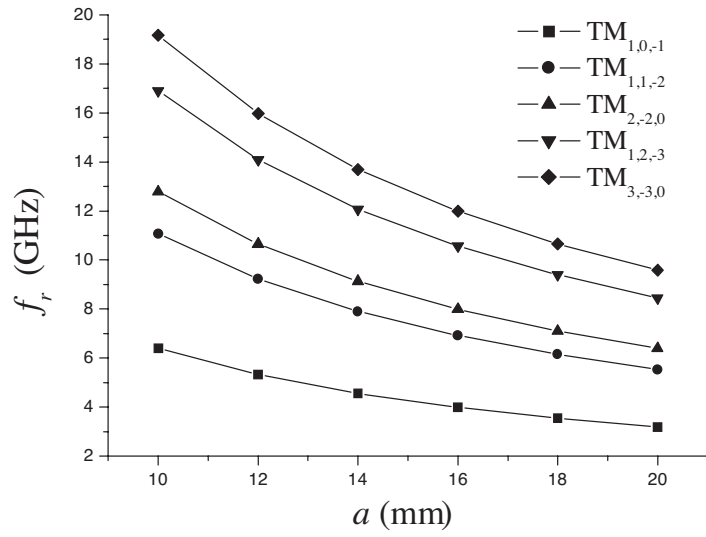
$$f_{m,n,l} = f_{1,0,-1}\sqrt{m^2 + mn + n^2} \quad (7)$$

Where,  $a_e$  is an effective value of equilateral triangular side length  $a$ , and it can be written as [14]

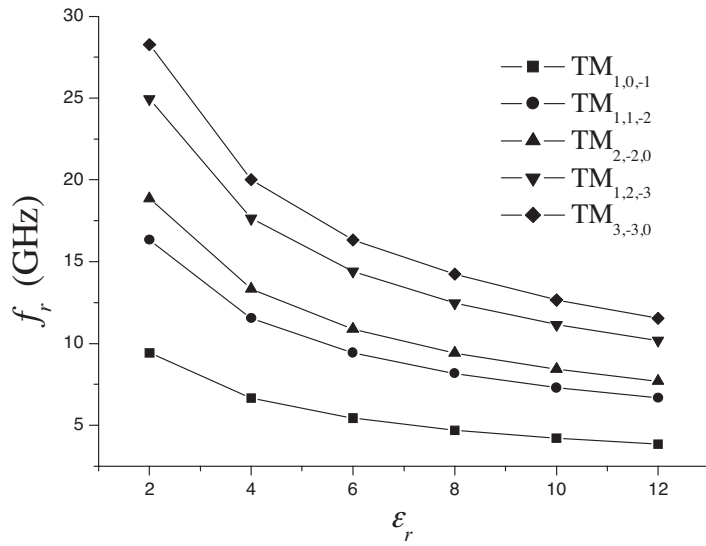
$$a_e = a \left[ 1 + 2.199\frac{h_1}{a} - 12.853\frac{h_1}{a\sqrt{\varepsilon_r}} + 16.436\frac{h_1}{a\varepsilon_r} + 6.182\left(\frac{h_1}{a}\right)^2 - 9.802\frac{1}{\sqrt{\varepsilon_r}}\left(\frac{h_1}{a}\right)^2 \right] \quad (8)$$

Here,  $h_1$  is the thickness of dielectric substrate.

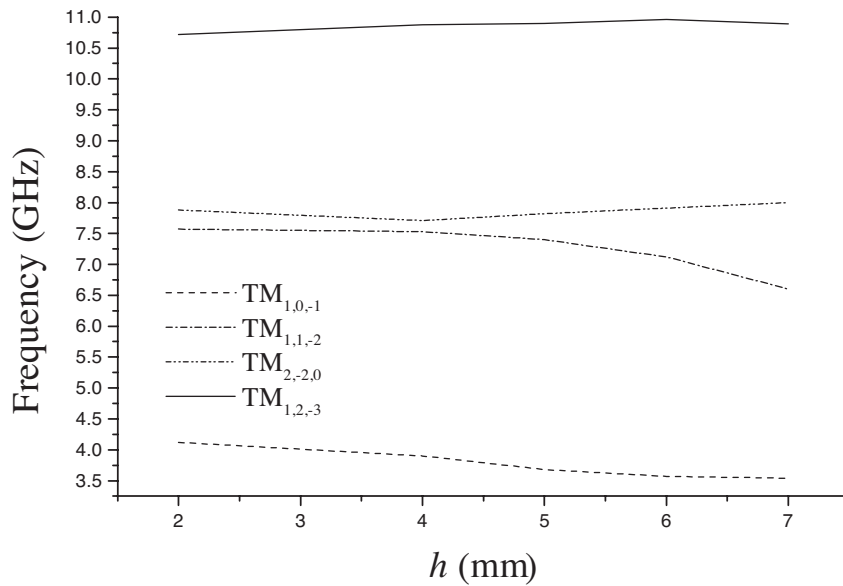
Reference [9] designed a dual-mode bandpass filter using equilateral triangular patch resonator, and gave the electromagnetic field components of this kind of resonator. Here, the resonant characteristics of equilateral triangular resonator with and without fractal-shaped deflection are calculated according to the above formulas and EM simulation, and magnetic field patterns of the resonator with fractal-shaped deflection are given, as shown in Fig. 2–Fig. 5. Fig. 2, Fig. 3 and Fig. 4 are resonant characteristics of microstrip equilateral triangular resonator without and with fractal-shaped deflection, respectively. Fig. 4 shows relationships of fractal deflection height and resonant frequencies for different resonant modes, it can be seen that for the dominant mode and the first higher order mode,



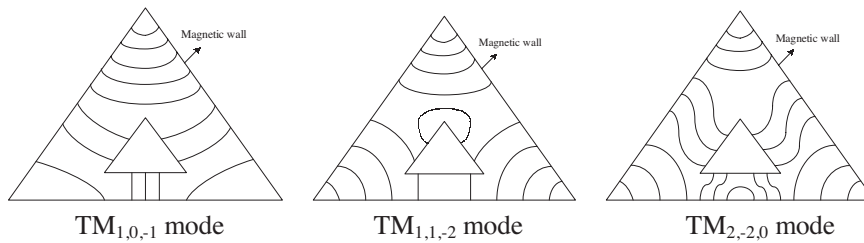
**Figure 2.** Relationships of resonant frequency and side length  $a$  ( $\epsilon_r = 9.8$ ,  $h = 0$ ).



**Figure 3.** Relationships of resonant frequency and dielectric constant ( $a = 15$  mm,  $h = 0$ ).



**Figure 4.** Relationships of deflection height and resonant frequency for different resonant modes ( $\epsilon_r = 9.8$ ,  $a = 15$  mm,  $b = 8$  mm,  $d = 2.0$  mm).

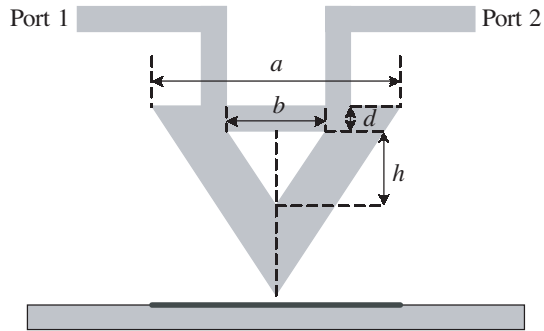


**Figure 5.** Magnetic field patterns of several resonant modes for an equilateral triangular patch resonator with one fractal-shaped deflection.

resonant frequencies decreased with deflection height increasing, while resonant frequencies of the second and third higher order modes have less variation. It is also noted that resonant frequencies of the first and second higher order modes are adjacent closely, simultaneously, they are far away from that of the dominant mode, and this is very useful to implement a wideband filter with multi-mode operation.

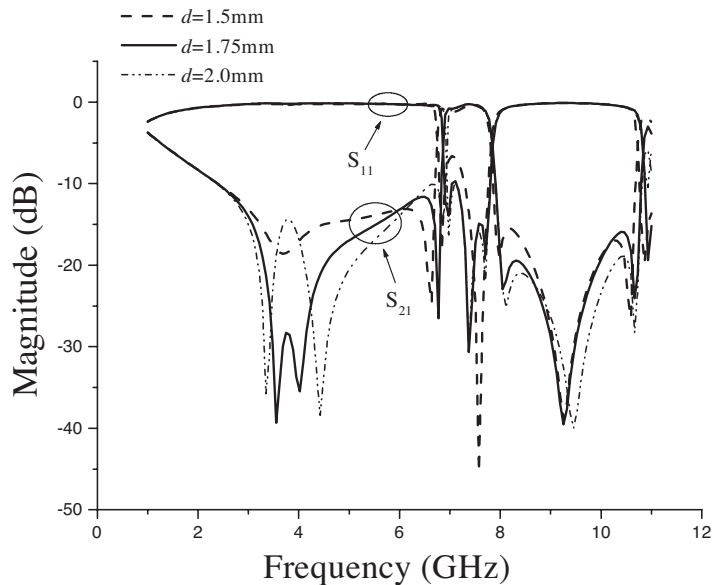
### 3. FRACTAL EQUILATERAL TRIANGULAR BANDPASS FILTERS WITH MULTI-MODE OPERATION

The basic principle for designing a microstrip filter is the selectivity and application of all sorts of resonant modes. Certain perturbation element and port feed method can change the patch resonator original resonant frequencies of different resonant modes, and some of the resonant frequencies may be shifted, if several resonances are adjacent, and simultaneously, the other resonances are shifted to far locations, the filter bandwidth can be greatly extended compared with a single mode or dual-mode operation. In this paper, for a bandpass filter with multi-mode operation implementation, fractal-shaped deflection acts as perturbation, and on one hand, the resonances of several required higher order modes should be adjacent each other for a close coupling, while on the other hand, resonance of the dominant mode should be possibly far away from these required resonances. According to the above principles and calculated results as shown in Fig. 4, we designed a multi-mode bandpass filter using equilateral triangular patch resonator, and with the help of EM simulator Ansoft Ensemble that using the method of moments(MoM), new filter configurations especially the fractal size can be optimized, here,  $a = 15$  mm,  $b = 8$  mm,  $h = 6$  mm, and  $f = 6.75$  mm. The filter is designed on an  $\text{Al}_3\text{O}_2$  ceramic substrate with a relative dielectric coefficient of 9.8 and a thickness of 1.27 mm, and I/O feed lines that set at the edge of hemline are parallel microstrip lines with characteristic impedance of  $50\ \Omega$ , and can be easily tuned to improve filter responses, as shown in Fig. 6.



**Figure 6.** Microstrip equilateral triangular bandpass filter with fractal-shaped deflection.

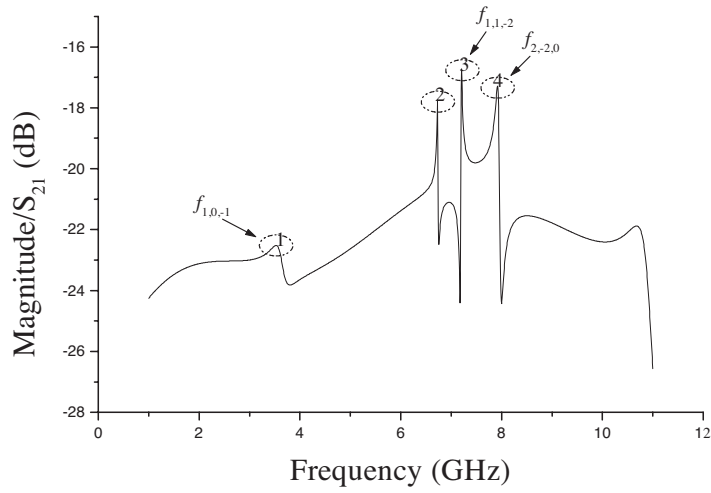




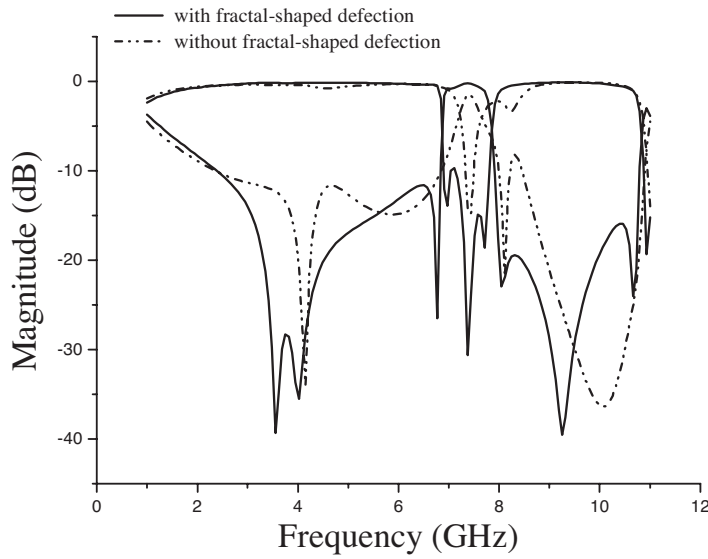
**Figure 7.** Simulated frequency responses.

Frequency responses of the first new multi-mode bandpass filter are shown in Fig. 7. It can be seen the filter is provided with wide stopbands, multi-transmission zeros at both sides of passband, and low passband insertion loss. For  $d = 1.75$  mm, it shows that centers at 7.33 GHz, the new bandpass filter possesses a 3 dB band width of 0.92 GHz, and a maximum passband insertion loss of  $-0.31$  dB, and return loss of  $-9.8$  dB, and multi-transmission zeros with maximum attenuation of more than 39 dB are implemented at both sides of passband. Fig. 8 shows multi-mode resonant property of the new filter using weak coupling, it shows three resonances including  $TM_{1,1,-2}$  mode and  $TM_{2,-2,0}$  mode involve in the operation, and resonance 2 can be seen as a degenerate mode of  $TM_{1,1,-2}$ . Fig. 9 shows comparison of S parameters with and without fractal deflection, and it can be seen that filter performances are greatly improved by the fractal perturbation. In order to show the extensively application of the proposed filter, a lower substrate dielectric coefficient is selected, and the filter frequency responses are shown in Fig. 10, and it shows that for  $\epsilon_r = 2.2$ , central frequency and the passband 3 dB bandwidth increase to 15.3 GHz and 2.75 GHz, respectively.

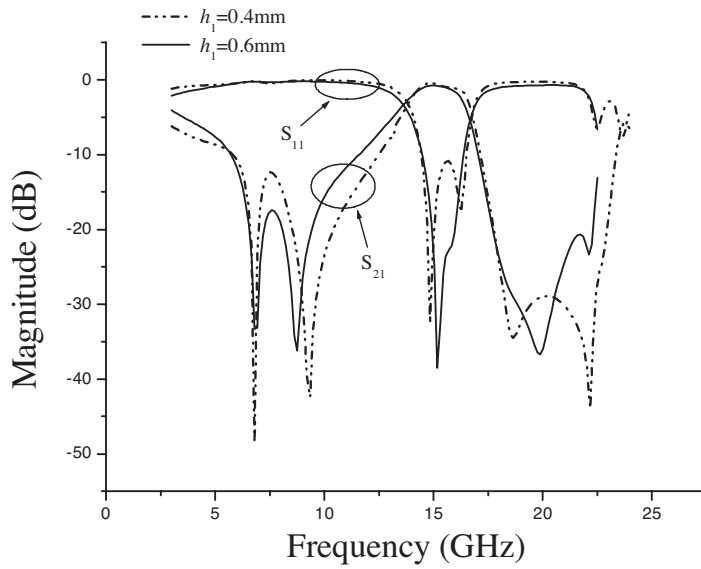
In order to obtain a bandpass filter with wider passband, more perturbation is required, so a new filter with the second fractal is



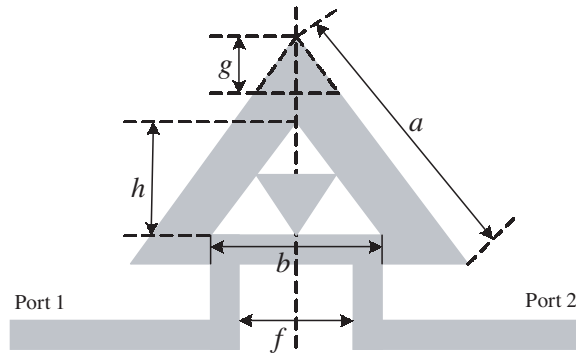
**Figure 8.** Multi-mode resonance of equilateral triangular resonator with fractal-shaped deflection ( $\epsilon_r = 9.8$ ,  $b = 8$  mm,  $h = 6$  mm,  $d = 1.75$  mm).



**Figure 9.** Comparison of frequency responses ( $\epsilon_r = 9.8$ ,  $d = 1.75$  mm).

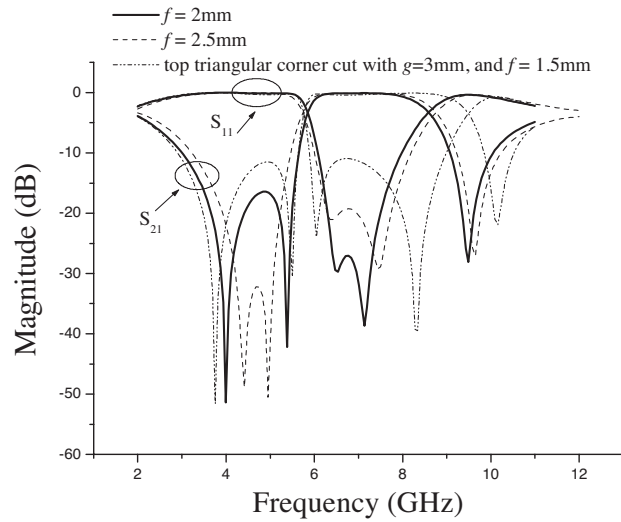


**Figure 10.** frequency responses with a lower substrate dielectric constant ( $\epsilon_r = 2.2$ ,  $d = 1.75$  mm).



**Figure 11.** Configuration of bandpass filter with the second fractal.

proposed, as shown in Fig. 11. Where, the grey parts denote metal, and the fractal-shaped defection with the first stage as shown in Fig. 6 is divided into four equal parts, and three of them are triangular defected cells, and  $a, b$  have the same value as the first filter scheme,  $h = 8$  mm. The new filter is designed with a dielectric constant of 2.56 and a substrate thickness of 0.775 mm. It can be seen from the simulated frequency responses as shown in Fig. 12 that transmission zeros with



**Figure 12.** Bandpass filter responses with the second fractal ( $\epsilon_r = 2.56$ ,  $h_1 = 0.775$  mm).

attenuation of more than 27.9 dB are implemented to both sides of passband, and bandwidth can be tuned by adjusting the distance of I/O feed lines. For  $f = 2$  mm, the filter centers at 7.215 GHz, and has nicer performance with a relative bandwidth of 37%, and a maximum insertion loss and return loss of  $-0.24$  dB and  $-26.9$  dB, respectively, and the bandwidth of rejectband at both sides of passband are more than 2 GHz. For  $f = 2.5$  mm, the 3 dB bandwidth of passband increases to about 3 GHz.

If the top triangle of equilateral triangular patch resonator is cut with height  $g = 3$  mm, the filter size can be reduced, simultaneously, the central frequency and 3 dB bandwidth of passband increased to 7.61 GHz and 3.74 GHz (relative bandwidth 48%), respectively, and the frequency responses comparison are shown in Fig. 12. The bandwidth increasing due to the top triangle cut brings more perturbation, and the space between neighboring resonant frequencies corresponding to the operating multi-mode are increased.

#### 4. CONCLUSION

Fractal planar circuits are novel fabrics with high performances and miniaturization, and can be quite useful in future wireless communication systems for fractal-shaped deflection can act as

perturbation which brings multi-mode or dual-mode operation. This paper presents two novel microstrip bandpass filters with multi-mode operation which implement multi transmission zeros, wide passbands and stopbands by using fractal equilateral triangular patch resonators. The designed filters have outstanding advantages such as novel and simple structures, compact size without coupling gaps, high selectivity, etc.

## ACKNOWLEDGMENT

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