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ELECTROMAGNETIC SCATTERING FROM A PERFECT ELECTROMAGNETIC CONDUCTOR CYLINDER BURIED IN A DIELECTRIC HALF-SPACE

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Abstract—An analytical solution is presented for the electromagnetic scattering from a perfect electromagnetic conducting circular cylinder, embedded in the dielectric half-space. The solution utilizes the spectral (plane wave) representations of the fields and accounts for all the multiple interactions between the buried circular cylinder and the dielectric interface separating the two half spaces.

1. INTRODUCTION

Over the past few decades, a significant amount of research effort has been spent in the study of buried object detection. The results of such research can be applied to the buried land mines, pipes, and other buried objects of interest. Various analytical and numerical contributions are available in this direction. The analysis of a buried object below flat surface has been done analytically by D'Yakonov [1], and subsequently has been explained by Howard [2] and Ogunade [3], where the solution was obtained by the eigenfunction expansion of the total fields. A number of other analytical studies involving the scattering from buried objects below a flat interface have also been done [4–15]. Problem of a buried conducting cylinder of arbitrary geometry below a flat surface using the method of moments (MoM) is treated in [16, 17]. Scattering of electromagnetic waves by multiple cylinders had been extensively studied in published literature [18–27]. Perfectly conducting, dielectric, impedance and bi-isotropic/chiral cylinders were considered for the analysis using different methods. Most commonly used methods are the integral equation formulation, partial differential equation formulation and hybrid techniques which combine the partial differential equation method with a surface integral equation or with an eigenfunction expansion.

Recently concept of perfect electromagnetic conductor (PEMC) as generalization of the perfect electric conductor (PEC) and perfect magnetic conductor (PMC) [28] has been introduced and has attracted the attention of many researchers [29–39]. It is well known that PEC boundary may be defined by the conditions

$$\mathbf{n} \times \mathbf{E} = \mathbf{0}, \qquad \mathbf{n}.\mathbf{B} = \mathbf{0}$$

While PMC boundary may be defined by the boundary conditions

$$\mathbf{n} \times \mathbf{H} = \mathbf{0}, \qquad \mathbf{n}.\mathbf{D} = \mathbf{0}$$

The PEMC boundary conditions are of the more general form

$$\mathbf{n} \times (\mathbf{H} + \mathbf{M}\mathbf{E}) = \mathbf{0}, \qquad \mathbf{n}.(\mathbf{D}.\mathbf{M}\mathbf{B}) = \mathbf{0}$$

where M denotes the admittance of the PEMC boundary. It is obvious that PMC corresponds to $M = 0$, while PEC corresponds to $M = \pm\infty$.

The purpose of this work is to derive an analytical solution for a perfect electromagnetic conducting cylinder (PEMC), buried beneath a dielectric half space. The solution is obtained by employing the plane wave representation of the fields and adding the successive reflections from the interface and the scattered fields from the buried cylinder. All the multiple interactions are accounted for in this analysis. Details of the analytical work are presented in the following sections.

2. ANALYTICAL DEVELOPMENT

2.1. Initial Reflected and Transmitted Fields

Consider a perfect electromagnetic conducting (PEMC) circular cylinder of infinite extent and buried in a dielectric half space geometry as shown in Figure 1. It is assumed that axis of the cylinder is coincident with z-axis of the coordinate system. Radius of the cylinder is a and depth of the cylinder from dielectric interface is d. It is assumed that both half spaces are lossless, isotropic, homogeneous, linear and nondispersive. Region above the dielectric interface has been termed as region 0 and has wavenumber $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ while region between the dielectric interface and cylinder is termed as region 1 and has wavenumber $k_1 = \omega\sqrt{\mu_0\epsilon_1}$. Time harmonic $[\exp(-j\omega t)]$ electromagnetic plane wave has been considered as source of excitation and time dependency has been suppressed through out the discussion. Incident field is given as

$$E_z^i = e^{j(k_x^i x - k_{0y}^i y)} \tag{1}$$

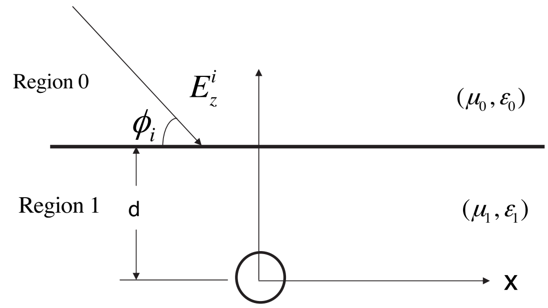


Figure 1. Geometry of buried PEMC cylinder inside a dielectric half space.

The reflected and transmitted fields from the dielectric interface, as if no cylinder is not present, are

$$E_z^r = R_{01}(k_x^i)e^{j(k_x^i x + k_{0y}^i y)} \tag{2}$$

$$E_z^t = T_{01}(k_x^i)e^{j(k_x^i x - k_{1y}^i y)} \tag{3}$$

where R_{01} and T_{01} are the coefficients of the reflected and transmitted fields respectively and are given as

$$R_{01}(k_x^i) = \left[\frac{k_{0y}^i - k_{1y}^i}{k_{0y}^i + k_{1y}^i}\right] e^{j2k_{0y}^i d} \tag{4a}$$

$$T_{01}(k_x^i) = \left[\frac{2k_{0y}^i}{k_{0y}^i + k_{1y}^i}\right] e^{j(k_{0y}^i - k_{1y}^i)d} \tag{4b}$$

Incident field in terms of cylindrical coordinates (ρ, ϕ) is given as

$$E_z^i = e^{jk_0 \rho \cos(\phi - \phi_i)} \tag{5}$$

ϕ_i is the angle of incidence with respect to the horizontal axis. The transmitted field into the region 1 is given as

$$E_z^t = T_{01}(k_x^i)e^{jk_1 \rho \cos(\phi - \phi_i)} \tag{6}$$

2.2. Scattered Fields in Region 1

The incident field (TM_z) for the PEMC cylinder may be written in terms of cylindrical wave functions as

$$E_z^i = \sum_{n=-\infty}^{\infty} j^n J_n(k_1 \rho) e^{jn(\phi - \phi_i)} \tag{7}$$

$$H_\phi^i = \frac{1}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n J_n'(k_1\rho) e^{jn(\phi-\phi_i)} \tag{8}$$

$J_n(.)$ is the Bessel function and prime represents the derivative with respect to the argument. The scattered fields by the PEMC cylinder may be assume in terms of unknown coefficients as

$$E_z^s = \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn(\phi-\phi_i)} \tag{9}$$

$$H_\phi^s = \frac{1}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)'}(k_1\rho) e^{jn(\phi-\phi_i)} \tag{10}$$

$$H_z^s = -\frac{j}{\eta_0} \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)}(k_1\rho) e^{jn(\phi-\phi_i)} \tag{11}$$

$$E_\phi^s = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)'}(k_1\rho) e^{jn(\phi-\phi_i)} \tag{12}$$

where b_n and c_n are the unknown scattering coefficients of the co-polarized and cross-polarized scattered fields respectively.

The scattered field from the an isolated PEMC cylinder may be obtained by considering field given in Equations (7) and (8) as being incident upon a PEMC cylinder and by assuming that PEMC cylinder is in a homogeneous medium. It may be noted that unlike for PEC cylinder or PMC cylinder, the scattered field from a PEMC contains TE_z fields in addition to the TM_z fields for TM_z excitation [37].

The unknown scattering coefficients may be calculated by applying the boundary conditions at the surface of the PEMC cylinder. The boundary conditions for the tangential components and radial components, at the surface of PEMC cylinder are given as [28, 37]

$$H_t^i + ME_t^i + H_t^s + ME_t^s = 0 \tag{13}$$

$$\epsilon_0 E_r^i - M\mu_0 H_r^i + \epsilon_0 E_r^s - M\mu_0 H_r^s = 0 \tag{14}$$

where M is the admittance of the PEMC. Subscript t in equation (13) stands for tangential component while subscript r in equation (14) stands for radial component. Putting (7) through (12), in (13) and solving we get

$$b_n = -\frac{H_n^{(1)}(k_1a)J_n'(k_1a) + M^2\eta_0^2 J_n(k_1a)H_n^{(1)'}(k_1a)}{(1+M^2\eta_0^2)H_n^{(1)}(k_1a)H_n^{(1)'}(k_1a)} \tag{15}$$

$$c_n = \frac{2M\eta_0}{\pi k_1 a(1+M^2\eta_0^2)H_n^{(1)}(k_1a)H_n^{(1)'}(k_1a)} \tag{16}$$

Thus by using b_n and c_n, in equations (9) and (12), we can get the co-polarized and cross-polarized scattered fields from an isolated PEMC cylinder respectively. The scattered fields given in equations (9) to (12) may easily be written to account for the spectrum of incident plane waves by integration over k_x. Integrating the eigenfunction solutions in (9) and (12), over k_x and using the appropriate weighting from (6), the initial scattered co-polarized field, for half space geometry, becomes

$$\begin{aligned} E_z^{s1} &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn[\phi - \tan^{-1}(-k_{1y}^i/k_x^i)]} [T_{01}\delta(k_x - k_x^i)] dk_x \\ &= \sum_{n=-\infty}^{\infty} j^n b_n C_n^{(1)} H_n^{(1)}(k_1\rho) e^{jn\phi} \end{aligned} \tag{17}$$

Similarly the initially scattered cross-polarized field becomes

$$\begin{aligned} E_\phi^{s1} &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho) e^{jn[\phi - \tan^{-1}(-k_{1y}^i/k_x^i)]} [T_{01}\delta(k_x - k_x^i)] dk_x \\ &= \sum_{n=-\infty}^{\infty} j^n c_n C_n^{(1)} H_n^{(1)\prime}(k_1\rho) e^{jn\phi} \end{aligned} \tag{18}$$

where

$$C_n^{(1)} = T_{01}(k_x^i) e^{-jn\tan^{-1}(-k_{1y}^i/k_x^i)} \tag{19}$$

equations (17) and (18) are the scattered co- and cross-polarized fields resulting from the first interaction of the incident field with the buried PEMC cylinder.

Subsequent discussion takes into account multiple interaction between the cylinder and dielectric interface separating the two half spaces. Using the integral representation of $H_n^{(1)}(k_1\rho)e^{jn(\phi)}$, Equations (17) and (18) may be expanded into their spectral representation as [11, 40]

$$E_z^{s1} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{k_{1y}} e^{j(k_x x + k_{1y} y)} \sum_{n=-\infty}^{\infty} b_n C_n^{(1)} e^{jn\ \tan^{-1}(k_{1y}/k_x)} dk_x \tag{20}$$

$$E_\phi^{s1} = \frac{j}{\pi} \int_{-\infty}^{\infty} \frac{1}{k_{1y}} e^{j(k_x x + k_{1y} y)} \sum_{n=-\infty}^{\infty} c_n C_n^{(1)} e^{jn\ \tan^{-1}(k_{1y}/k_x)} dk_x \tag{21}$$

Above equations show the linear combination of co- or cross-polarized plane waves propagating in $+y$ direction, specifically determined by k_x and incident upon the interface from below. Downward traveling waves which are reflected by the interface can be obtained by incorporating the reflection coefficient and are given as

$$\tilde{E}_z^{s1} = \frac{1}{\pi}\int_{k_x}\frac{1}{k_{1y}}\sum_{n=-\infty}^{\infty} b_n C_n^{(1)} e^{jn\ \tan^{-1}(k1y/k_x)}[R_{10}(k_x)e^{j(k_x x - k_{1y}y)}]dk_x \tag{22}$$

$$\tilde{E}_\phi^{s1} = \frac{j}{\pi}\int_{k_x}\frac{1}{k_{1y}}\sum_{n=-\infty}^{\infty} c_n C_n^{(1)} e^{jn\ \tan^{-1}(k1y/k_x)}[R_{10}(k_x)e^{j(k_x x - k_{1y}y)}]dk_x \tag{23}$$

The reflection coefficient $R_{10}(k_x)$ is given as

$$R_{10}(k_x) = \frac{k_{1y} - k_{0y}}{k_{1y} + k_{0y}} e^{-2jk_{1y}d}$$

where $k_{0y} = \sqrt{k_0^2 - k_x^2}$ and $k_{1y} = \sqrt{k_1^2 - k_x^2}$. The superposition of downward traveling plane waves given in (22) and (23), become incident upon the cylinder. Once again, the known eigenfunction solutions for a PEMC, cylinder with plane waves incident are employed. When the superposition of downward traveling waves described by (22), become incident upon the cylinder, it radiates both co- and cross-polarized fields. Thus we have to use (9) and (12), for the scattered fields. Again integrating (9) and (12), over k_x and using the appropriate weighting from (22), the second order scattered fields by the cylinder may be written as

$$E_z^{s2} = \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho)e^{jn(\phi)}\left[\frac{1}{\pi}\sum_{m=-\infty}^{\infty} b_m C_m^{(1)} I_{(m,n)}\right] \tag{24}$$

$$E_\phi^{s2} = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho)e^{jn(\phi)}\left[\frac{1}{\pi}\sum_{m=-\infty}^{\infty} b_m C_m^{(1)} I_{(m,n)}\right] \tag{25}$$

where

$$I_{(m,n)} = \int_{k_x}\frac{1}{k_{1y}} R_{10}(k_x)e^{jm\ \tan^{-1}(k1y/k_x)}e^{-jn\ \tan^{-1}(-k_{1y}/k_x)} \tag{26}$$

We may write (24) and (25) as

$$E_z^{s2} = \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho)e^{jn\phi}\left[C_n^{(2)}\right] \tag{27}$$

$$E_\phi^{s2} = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho)e^{jn\phi}\left[C_n^{(2)}\right] \tag{28}$$

where

$$C_n^{(2)} = \frac{1}{\pi} \sum_{m=-\infty}^{\infty} b_m C_m^{(1)} I_{(m,n)} \tag{29}$$

Similarly for the downward traveling plane wave spectrum described by (23), the scattered co- and cross-polarized fields are obtained following the same procedure adopted to obtain equations (9) to (12) and (24) to (29), we get

$$E_z^{s2} = \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn\phi} \left[D_n^{(2)}\right] \tag{30}$$

$$E_\phi^{s2} = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho) e^{jn\phi} \left[D_n^{(2)}\right] \tag{31}$$

where

$$D_n^{(2)} = \frac{j}{\pi} \sum_{m=-\infty}^{\infty} c_m D_m^{(1)} I_{(m,n)} \tag{32}$$

and

$$D_n^{(1)} = C_n^{(1)}$$

Hence the second order total scattered co-polarized and cross-polarized fields are given as

$$E_z^{s2} = \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn\phi} \left[C_n^{(2)} + D_n^{(2)}\right] \tag{33}$$

$$E_\phi^{s2} = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho) e^{jn\phi} \left[C_n^{(2)} + D_n^{(2)}\right] \tag{34}$$

In general the qth order total scattered co-polarized and cross-polarized fields may be written as

$$E_z^{s(q)} = \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn\phi} \left[C_n^{(q)} + D_n^{(q)}\right] \tag{35}$$

$$E_\phi^{s(q)} = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho) e^{jn\phi} \left[C_n^{(q)} + D_n^{(q)}\right] \tag{36}$$

where

$$C_n^{(q)} = \frac{1}{\pi} \sum_{m=-\infty}^{\infty} b_m C_m^{(q-1)} I_{(m,n)} \tag{37}$$

$$D_n^{(q)} = \frac{j}{\pi} \sum_{m=-\infty}^{\infty} c_m D_m^{(q-1)} I_{(m,n)} \tag{38}$$

Equations (37) and (38), show that the qth coefficients are found in terms of $(q-1)$th coefficients, indicating the recursive nature of this solution for the cylinder. Thus the total scattered co-polarized field by the PEMC cylinder, inside the dielectric half space is written as

$$\begin{aligned} E_z^{(\text{tot})} &= \sum_{q=1}^{\infty} E_z^{s(q)} \\ &= \sum_{q=1}^{\infty} \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn\phi} \left[C_n^{(q)} + D_n^{(q)} \right] \qquad (39) \\ E_z^{(\text{tot})} &= \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(1)}(k_1\rho) e^{jn\phi} \left[C_n + D_n \right] \qquad (40) \end{aligned}$$

In a similar way, we can get the total scattered cross-polarized field by PEMC cylinder inside the dielectric half space as

$$E_\phi^{(\text{tot})} = \sum_{n=-\infty}^{\infty} j^n c_n H_n^{(1)\prime}(k_1\rho) e^{jn\phi} \left[C_n + D_n \right] \qquad (41)$$

where in equations (40) and (41) we have

$$C_n = \sum_{q=1}^{\infty} C_n^{(q)}$$

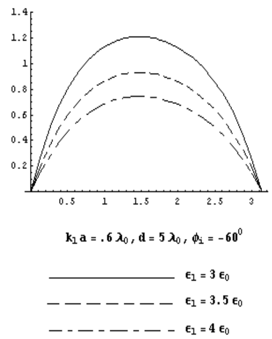
$$D_n = \sum_{q=1}^{\infty} D_n^{(q)}$$

2.3. Scattered Fields in Region 0

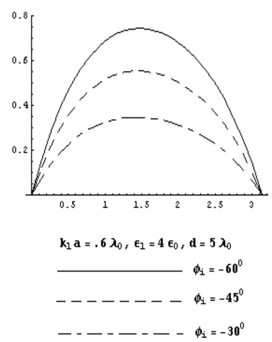
In order to compute the the scattered fields in region 0, the scattered field in region 1 is first expanded into spectral representation. Using the integral expansion of $H_n^{(1)}(k_1\rho)e^{jn\phi}$, the scattered co-polarized and cross-polarized fields of equations (40) and (41), after simplification become

$$E_z^0 = \frac{1}{\pi} \int_{k_x} \frac{1}{k_{1y}} e^{j(k_x x + k_{0y} y)} \sum_{n=-\infty}^{\infty} b_n [C_n + D_n] e^{jn \tan^{-1}(k_{1y}/k_x)} dk_x \quad (42)$$

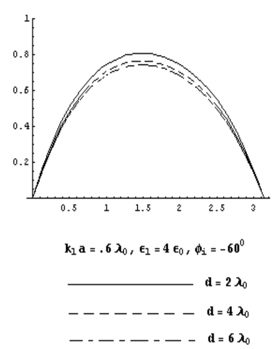
$$E_\phi^0 = \frac{j}{\pi} \int_{k_x} \frac{1}{k_{1y}} e^{j(k_x x + k_{0y} y)} \sum_{n=-\infty}^{\infty} c_n [C_n + D_n] e^{jn \tan^{-1}(k_{1y}/k_x)} dk_x \quad (43)$$



Variation with different values of permitivity of half-space



Variation with the different values of angle of incidence



Variation with the different values of depth

Figure 2. Scattered co-polarized field.

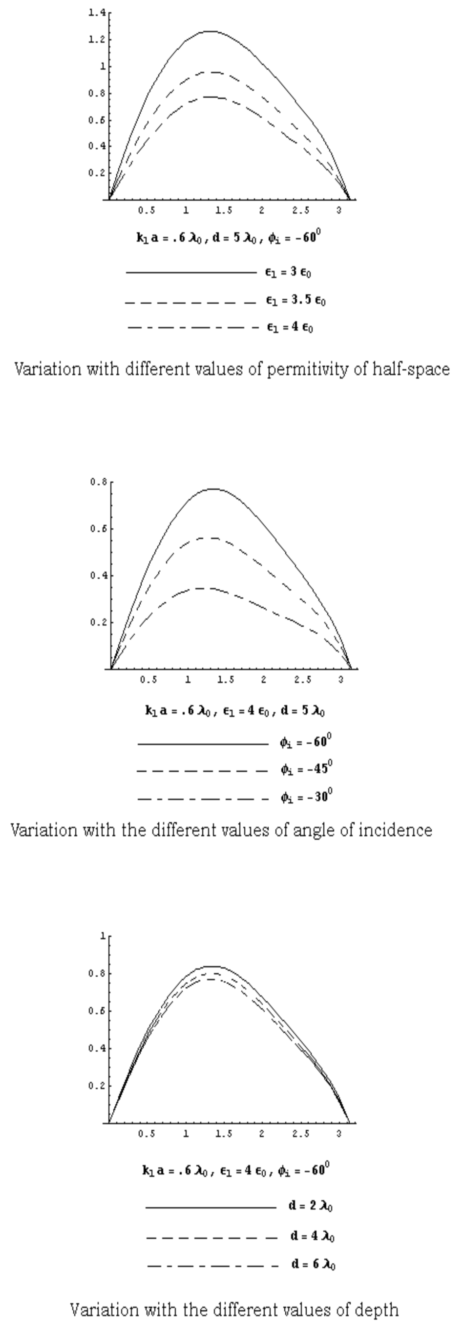


Figure 3. Scattered cross-polarized field.

The spectrum of upward traveling waves described by equations (42) and (43), are transmitted through the interface and scaled by the transmission coefficients to produce the scattered co- and cross-polarized fields in Region 0. Once the transmission coefficients are included, the scattered fields in Region 0 may be expressed as

$$E_z^0 = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} b_n [C_n + D_n] I_n^t \tag{44}$$

$$E_\phi^0 = \frac{j}{\pi} \sum_{n=-\infty}^{\infty} c_n [C_n + D_n] I_n^t \tag{45}$$

where

$$I_n^t = \int_{k_x} \frac{1}{k_{1y}} T_{10}(k_x) e^{jn\tan^{-1}(\frac{k_{1y}}{k_x})} e^{j(k_x x + k_{0y} y)} dk_x$$

$$T_{10}(k_x) = \frac{2k_{1y}}{k_{0y} + k_{1y}} e^{j(k_{0y} - k_{1y})d}$$

3. SIMULATIONS

Buried PEMC circular cylinder with $M\eta_0 = -1$ has been considered for analysis. We have considered TM plane wave as source of excitation. Figure 2 contains the co-polarized far zone scattered field for different values of permittivities of host medium, angle of excitation and depth of the buried cylinder. Figure 3 contains the cross-polarized far zone scattered field for different values of permittivities of host medium, angle of excitation and depth of the buried cylinder. To obtain the plots, the specular component of the initial reflected wave from the interface has not been included. In order to calculate the far-field scattering pattern, the highly oscillatory integrals I_n^t, has been evaluated using the saddle point method of integration [41].

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