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A THEORETICAL STUDY OF ELECTROMAGNETIC TRANSIENTS IN A LARGE PLATE DUE TO VOLTAGE IMPACT EXCITATION

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Abstract—Maxwell's equations are solved to determine transient electromagnetic fields inside as well as outside of a large conducting plate of an arbitrary thickness. The plate is carrying a uniformly distributed excitation winding on its surfaces. Transient fields are produced due to sudden application of a d.c. voltage at the terminals of the excitation winding. On the basis of a linear treatment of this initial value problem it is concluded that the transient fields may decay at a faster rate for conducting plates with smaller values of relaxation time. It is also shown that the growth of flux in a perfectly nonconducting plate is a piecewise linear function of time and the current in its excitation winding is a series of step function of time.

1. INTRODUCTION

If a step d.c. voltage is applied to a coil, the resulting current in the coil is time-dependent. At any instant the coil terminal voltage must equal the resistance drop minus the induced voltage due to the growth of coil flux. This leads to an ordinary differential equation. The magnetic field in the core of the coil varies with time as well as space coordinates. Therefore, the system involves both, ordinary as well as partial differential equations.

As an example of voltage impact excitation, consider a large plate carrying a uniformly distributed excitation winding on its surfaces. When a step d.c. voltage is impressed across the terminals of this winding an electric field, E_o, which is equal to the applied voltage per unit conductor length, appears instantly along the winding conductor. Consequently, a d.c. current flows in the winding. If the plate is made of a conducting material, this current does not appear instantly. It grows from zero to the steady-state value, which is the ratio of the applied voltage to the winding resistance. The rate of growth of this excitation current is influenced by eddy currents induced in the conducting plate. The excitation current together with eddy currents in the plate, produces magnetic flux in the plate. In the case of a perfectly nonconducting plate, the magnetic flux is produced by the excitation current alone. The rate of change of this magnetic flux induces electric field opposing the applied electric field E_o, along the excitation winding on the plate surface.

Many technical papers have been published on electromagnetic transients in solid blocks of steel. Weber [1], Wagner [2], Concordia and Poritsky [3], and also Pohl [4] are amongst the early contributors. A number of research papers on electromagnetic transients [5–13] appeared subsequently. Recently, research papers on transient analyses of grounding systems, transmission lines and impulsive sources appeared [14–18], indicating the importance of the study.

Study of transient fields due to current impact excitation has been reported [19–21] by various authors. In the present treatment of voltage impact excitation, Maxwell's equations are solved for transient fields in a large conducting plate with constant values for permeability, μ, permittivity, ϵ, and conductivity, σ. Transient fields for nonconducting plates are also studied. The analysis presented here is a sequel to an earlier work [21] on impact excitation, published in this journal. This paper, however, is readable independently.

2. FIELD EQUATIONS

Consider Fig. 1, showing a large conducting plate of thickness W, carrying uniform current sheets of density $\pm K_y$ on its surfaces located at $x = \pm W/2$. These current sheets simulate the excitation winding carrying the transient current produced due to sudden application of a d.c. voltage across its terminals at $t = 0$.

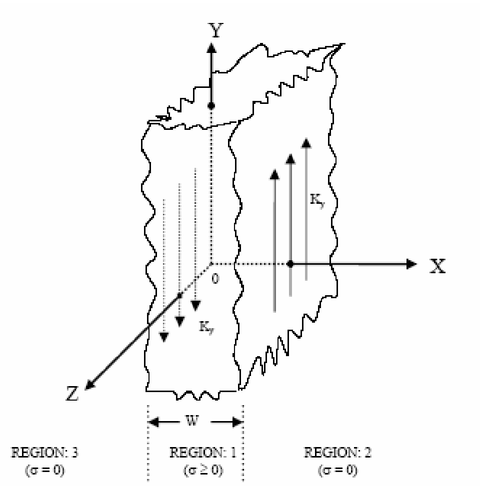

Figure 1. Large conducting plate with surface current sheets.

Because of symmetry only y-component of electric field and only z-component of magnetic field exist. The former is an odd function and the latter is an even function of x. Further, both transient fields vanish as t tends to infinity. However, a steady magnetic field is established in the plate. These fields satisfy Maxwell's equations in one dimension:

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \tag{1}$$

and

$$-\frac{\partial H_z}{\partial x} = \sigma E_y + \epsilon \frac{\partial E_y}{\partial t} \tag{2}$$

Therefore, electromagnetic fields obey the following equations:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu\sigma \frac{\partial E_y}{\partial t} + \mu\epsilon \frac{\partial^2 E_y}{\partial t^2} \tag{3}$$

and

$$\frac{\partial^2 H_z}{\partial x^2} = \mu\sigma \frac{\partial H_z}{\partial t} + \mu\epsilon \frac{\partial^2 H_z}{\partial t^2} \tag{4}$$

For free space:

$$\sigma = 0, \tag{5}$$

$$\mu = \mu_0, \tag{6}$$

and

$$\epsilon = \epsilon_0 \tag{7}$$

3. STEADY STATE DISTRIBUTION OF ELECTROMAGNETIC FIELDS

As shown in Fig. 1, let the region occupied by the plate, $-W/2 < x < W/2$, be indicated as region 1, while the regions $x > W/2$ and $x < -W/2$, be indicated as regions 2 and 3, respectively. In view of the symmetry, it will be sufficient to consider the field distributions in regions 1 and 2 only. At the boundary between these two regions, i.e., at $x = W/2$, we have

$$H_{1z} = H_{2z} + K_y \tag{8}$$

and

$$E_{1y} = E_{2y} \tag{9}$$

where suffix 1 indicates fields in region 1, and suffix 2 indicates fields in region 2. The current sheet with a time varying surface current density K_y, simulates the excitation winding carrying the transient current. Its value is zero until the d.c. voltage is applied to the winding, i.e., for $t \leq 0$.

Before the onset of transient, i.e., for $t < 0$, the initial fields are:

$$H_{1z} = 0 \tag{10a}$$

and

$$E_{1y} = 0 \tag{10b}$$

over $-W/2 < x < W/2$, while,

$$H_{2z} = 0 \tag{11a}$$

and

$$E_{2y} = 0 \tag{11b}$$

for $W/2 < x$.

Further, for $t = \infty$:

$$H_{1z} = K_\infty, \text{ (say)} \tag{12a}$$

and

$$E_{1y} = 0, \tag{12b}$$

over $-W/2 < x < W/2$, while

$$H_{2z} = E_{2y} = 0 \tag{13}$$

for $W/2 < x$.

Lastly, for $t = 0$ and $x = W/2$:

$$H_{1z} = H_{2z} = 0 \tag{14}$$

and

$$E_{1y} = E_{2y} = -E_o \tag{15}$$

These solutions are consistent with boundary conditions defined by Eqs. (8) and (9) as K_y is zero for $t < 0$. The value of the surface current density K_y becomes a constant, say K_∞, once the excitation current is stabilized to a steady state value. A relation between K_∞ and E_o can be given as:

$$E_o = \rho_s K_\infty \tag{16}$$

where the surface resistivity ρ_s, depends on the excitation winding resistance and is independent of the plate conductivity σ.

4. TRANSIENT FIELDS FOR CONDUCTING PLATES

4.1. Fields inside the Plate

Consider the electromagnetic fields inside the plate. These fields must satisfy the initial conditions given by Eqs. (10a) and (10b), for $-W/2 < x < W/2$. Further, inside the plate, the magnetic field must converge to K_∞ and the electric field must vanish as t tends to infinity. Also, on the plate surface $x = W/2$, these fields at $t = 0$ must satisfy Eqs. (14) and (15). Therefore, field expressions satisfying Eqs. (1)–(4), can be given as:

$$\begin{aligned} E_{1y} \;=\; & -E_o.\zeta.\frac{2x}{W}.e^{-\frac{t}{\tau}} - E_o.\left(1-\zeta\right).\frac{\sin\left(\theta\frac{2x}{W}\right)}{\sin\left(\theta\right)}.\left[1-\frac{t}{2\tau}\right].e^{-\frac{t}{2\tau}} \\ & + \sum_{m=1}^{\infty}\frac{a_m}{(\alpha_m-\beta_m)}\sin\left(m\pi\frac{2x}{W}\right).\left[\alpha_m.e^{-\alpha_m t}-\beta_m.e^{-\beta_m t}\right] \end{aligned} \tag{17}$$

and

$$H_{1z} = K_\infty - K_\infty.e^{-\frac{t}{\tau}} + K_\infty.(1-\zeta).\frac{\theta}{\tau.\zeta}.\frac{\cos\left(\theta\frac{2x}{W}\right)}{\sin(\theta)}.t.e^{-\frac{t}{2\tau}} + \sum_{m=1}^{\infty} b_m.\cos\left(m\pi\frac{2x}{W}\right).\left[e^{-\alpha_m t} - e^{-\beta_m t}\right] \tag{18}$$

where,

$$\theta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}.\frac{W}{2} \tag{19}$$

$$\zeta = \frac{\sigma}{\epsilon}.\frac{\mu}{\rho_s}.\frac{W}{2} \tag{20}$$

$$\tau = \frac{\epsilon}{\sigma} \tag{21}$$

$$\alpha_m, \beta_m = \frac{1}{2\tau} \pm \frac{1}{2\tau}.\frac{\pi}{\theta}.\sqrt{(\theta/\pi)^2 - m^2} \tag{22}$$

$$a_m = -E_o.\frac{2}{\pi}.\cos(m\pi)\left[\frac{m^2 - (\sqrt{\zeta}.\theta/\pi)^2}{m.\{m^2 - (\theta/\pi)^2\}}\right] \tag{23a}$$

and

$$b_m = a_m.\frac{1}{\mu}.\frac{\left(m\frac{2\pi}{W}\right)}{(\alpha_m - \beta_m)} \tag{23b}$$

4.2. Transient Current Sheet

The voltage applied to the excitation winding is equal to the sum of the voltage drop in the winding resistance and the rate of change of flux linking the winding. The magnetic flux ϕ, per unit length in the y-direction is:

$$\phi = \mu \int_{-W/2}^{W/2} H_{1z}.dx \tag{24}$$

Using the expression for H_{1z} from Eq. (18), one gets:

$$\phi = \mu.WK_\infty.\left[\left(1 - e^{-\frac{t}{\tau}}\right) + \left(\frac{1-\zeta}{\zeta}\right).\frac{t}{\tau}.e^{-\frac{t}{2\tau}}\right] \tag{24a}$$

Therefore,

$$E_o = \rho_s.K_y + \frac{d\phi}{dt} \tag{24b}$$

in view of Eqs. (16), (24a) and (24b) one gets:

$$K_y = K_\infty.\left[1 - \zeta.e^{-\frac{t}{\tau}} - (1-\zeta)\,.\left\{1 - \frac{t}{2\tau}\right\}.e^{-\frac{t}{2\tau}}\right] \tag{25}$$

From this equation it may be seen that:

$$K_y = 0, \text{ for } t = 0 \tag{26a}$$

and

$$K_y = K_\infty, \text{ for } t = \infty \tag{26b}$$

4.3. Fields outside the Plate

Both electric and magnetic fields, as shown by Eqs. (11a) and (11b) are zero until the instant $t = 0$, when the step d.c. voltage is applied to the excitation winding. At this instant the applied electric field on the plate surface at $x = W/2$, being E_o, the induced electric field suddenly changes from its original zero value to $-E_o$, vide Eq. (15). This causes electromagnetic transients. A sudden change in the magnitude of electromagnetic field also occurs for $x > W/2$, though at a later instant of time as the electromagnetic disturbance propagates in free space with a finite velocity c.

The sudden appearance of electric field on the plate surface initiates traveling waves in free space that vanish as t tends to infinity. Wave-front of each wave moves away from the plate surface with the velocity c.

The electromagnetic fields in region-2 satisfy wave equation, thus

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E_y}{\partial t^2} \tag{27a}$$

and

$$\frac{\partial^2 H_z}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 H_z}{\partial t^2} \tag{27b}$$

Further, we have

$$H_z\left(t \pm \sqrt{\mu_0\epsilon_0}.x'\right) = \mp\sqrt{\frac{\epsilon_0}{\mu_0}}.E_y\left(t \pm \sqrt{\mu_0\epsilon_0}.x'\right) \tag{28}$$

where,

$$x' = x - W/2 \tag{29}$$

Therefore, in view of boundary conditions, vide Eqs. (8) and (9), the expressions for electric and magnetic fields in region 2, found using Eqs. (17), (18), (25), and (28), are as follows:

$$\begin{aligned}
E_{2y} \; = \; & -\frac{1}{2}E_o.\left(1-\zeta\right).\left[\left(1-\frac{t_-}{2\tau}\right).e^{-\frac{t_-}{2\tau}} + \left(1-\frac{t_+}{2\tau}\right).e^{-\frac{t_+}{2\tau}}\right] \\
& -\frac{1}{2}E_o.\zeta.\left[e^{-\frac{t_-}{\tau}} + e^{-\frac{t_+}{\tau}}\right] - \sqrt{\frac{\mu_0}{\epsilon_0}}.\frac{1}{2}K_\infty.\left(1-\zeta\right).\left[e^{-\frac{t_-}{\tau}} - e^{-\frac{t_+}{\tau}}\right] \\
& +\sqrt{\frac{\mu_0}{\epsilon_0}}.\frac{1}{2}K_\infty.\left(1-\zeta\right).\left[1-\left(1-\frac{2\theta}{\zeta}.\cot\theta\right).\frac{t_-}{2\tau}\right].e^{-\frac{t_-}{2\tau}} \\
& -\sqrt{\frac{\mu_0}{\epsilon_0}}.\frac{1}{2}K_\infty.\left(1-\zeta\right).\left[1-\left(1-\frac{2\theta}{\zeta}.\cot\theta\right).\frac{t_+}{2\tau}\right].e^{-\frac{t_+}{2\tau}} \\
& +\sqrt{\frac{\mu_0}{\epsilon_0}}.\frac{1}{2}\sum_{m=1}^{\infty} b_m.\cos\left(m\pi\right).\left[\left(e^{-\alpha_m.t_-} - e^{-\beta_m.t_-}\right)\right. \\
& \left. -\left(e^{-\alpha_m.t_+} - e^{-\beta_m.t_+}\right)\right]
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
H_{2z} \; = \; & \frac{1}{2}K_\infty.\left(1-\zeta\right).\left[1-\left(1-\frac{2\theta}{\zeta}.\cot\theta\right).\frac{t_-}{2\tau}\right].e^{-\frac{t_-}{2\tau}} \\
& +\frac{1}{2}K_\infty.\left(1-\zeta\right).\left[1-\left(1-\frac{2\theta}{\zeta}.\cot\theta\right).\frac{t_+}{2\tau}\right].e^{-\frac{t_+}{2\tau}} \\
& +\frac{1}{2}\sum_{m=1}^{\infty} b_m.\cos\left(m\pi\right).\left[\left(e^{-\alpha_m.t_-} - e^{-\beta_m.t_-}\right)\right. \\
& \left. +\left(e^{-\alpha_m.t_+} - e^{-\beta_m.t_+}\right)\right] - \frac{1}{2}K_\infty.\left(1-\zeta\right).\left[e^{-\frac{t_-}{\tau}} + e^{-\frac{t_+}{\tau}}\right] \\
& -\sqrt{\frac{\epsilon_0}{\mu_0}}.\frac{1}{2}E_o.\zeta.\left[e^{-\frac{t_-}{\tau}} - e^{-\frac{t_+}{\tau}}\right] \\
& -\sqrt{\frac{\epsilon_0}{\mu_0}}.\frac{1}{2}E_o.\left(1-\zeta\right).\left[\left(1-\frac{t_-}{2\tau}\right).e^{-\frac{t_-}{2\tau}} - \left(1-\frac{t_+}{2\tau}\right).e^{-\frac{t_+}{2\tau}}\right]
\end{aligned} \tag{31}$$

where, the retarded time t_-, and the accelerated time t_+ are defined as:

$$t_- = t - \sqrt{\mu_0\epsilon_0}.x' \tag{32a}$$

and

$$t_+ = t + \sqrt{\mu_0 \epsilon_0}.x' \tag{32b}$$

In order to satisfy the initial conditions, vide Eqs. (11a) and (11b), we multiply the R.H.S. of Eqs. (30) and (31) by the unit step function $u\{t_-\}$. Field expressions thus modified are consistent with the observation made at the beginning of this section.

5. TRANSIENT FIELDS FOR NONCONDUCTING PLATES

Consider a perfectly nonconducting large plate, carrying a uniformly distributed excitation winding on its surfaces. If at $t = 0$, a step d.c. voltage is applied to the terminals of this winding, an electric field E_o appears instantly along the winding conductor. There being no eddy currents in the plate, an initial current immediately sets up in the winding. Let this current be represented as a uniform surface current density $\pm K_o$, on the plate surface at $x = \pm W/2$. The sudden appearance of surface currents will initiate traveling-waves in the three regions shown in Fig. 1. These waves, for region-1 and -2 can be expressed in terms of unit step functions as:

$$H_{1z} = K_{\langle 1|1\rangle}.u\left\{t + \sqrt{\mu\epsilon}.\left(x - W/2\right)\right\} + K_{\langle 1|1\rangle}.u\left\{t - \sqrt{\mu\epsilon}.\left(x + W/2\right)\right\} \tag{33a}$$

$$\begin{aligned} E_{1y} \;=\; & -\sqrt{\mu/\epsilon}.K_{\langle 1|1\rangle}.u\left\{t + \sqrt{\mu\epsilon}.\left(x - W/2\right)\right\} \\ & +\sqrt{\mu/\epsilon}.K_{\langle 1|1\rangle}.u\left\{t - \sqrt{\mu\epsilon}.\left(x + W/2\right)\right\} \end{aligned} \tag{33b}$$

$$H_{2z} = -K_{\langle 2|1\rangle}.u\left\{t - \sqrt{\mu_0\epsilon_0}.\left(x - W/2\right)\right\} \tag{34a}$$

$$E_{2y} = -\sqrt{\mu_0/\epsilon_0}.K_{\langle 2|1\rangle}.u\left\{t - \sqrt{\mu_0\epsilon_0}.\left(x - W/2\right)\right\} \tag{34b}$$

for $0 < t < T$, where,

$$T = W.\sqrt{\mu\epsilon} \tag{35}$$

while, $K_{\langle 1|1\rangle}$ and $K_{\langle 2|1\rangle}$ indicate arbitrary constants and $u\{.\}$ indicates unit step function.

In view of Eqs. (8) and (9):

$$K_{\langle 1|1\rangle} + K_{\langle 2|1\rangle} = K_o \tag{36}$$

and

$$\sqrt{\mu/\epsilon}.K_{\langle 1|1\rangle} = \sqrt{\mu_0/\epsilon_0}.K_{\langle 2|1\rangle} \tag{37}$$

Consider Eq. (33a). At the instant $t = 0$, an H-wave initiating from each plate surface travels across the plate towards the opposite plate surface, with a constant velocity, $1/\sqrt{\mu\epsilon}$. While the two H-waves travel in the opposite directions, a time varying magnetic flux, ϕ_o, is established in the plate. This flux, per unit plate length in the y-direction, is given as:

$$\phi_o = 2\mu.K_{\langle 1|1\rangle}.W.\left(\frac{t}{T}\right) \tag{38}$$

for $0 < t < T$.

Therefore, in view of Eqs. (24b) and (35):

$$E_o = \rho_s.K_o + \sqrt{\mu/\epsilon}.K_{\langle 1|1\rangle} \tag{39}$$

Solving Eqs. (36), (37) and (39), one gets:

$$K_{\langle 1|1\rangle} = \frac{E_o.\sqrt{\epsilon/\mu}}{F_0} \tag{40a}$$

$$K_{\langle 2|1\rangle} = \frac{E_o.\sqrt{\epsilon_0/\mu_0}}{F_0} \tag{40b}$$

and

$$K_o = \frac{E_o.\left(\sqrt{\epsilon/\mu} + \sqrt{\epsilon_0/\mu_0}\right)}{F_0} \tag{40c}$$

where,

$$F_0 = 1 + \rho_S.\left(\sqrt{\epsilon/\mu} + \sqrt{\epsilon_0/\mu_0}\right) \tag{41}$$

At the instant $t = T$, the two H-waves in the plate region, traveling in opposite directions across the plate thickness, reach air regions. At this instant the growth of the flux ϕ_o, due to the surface current K_o, completes and the rate of growth of this flux ceases abruptly. Therefore, at this instant in view of Eq. (24b), the surface current density increases instantly, by an amount, say K_1. The resulting surface current density is the sum of K_o and K_1. This sudden change in the surface currents initiates a new set of traveling-waves in the three regions shown in Fig. 1. The resulting electromagnetic fields in the plate region can now be given as follows:

$$\begin{aligned} H_{1z} &= 2K_{\langle 1|1\rangle} + K_{\langle 1|2\rangle}.u\left\{(t-T)+\sqrt{\mu\epsilon}.(x-W/2)\right\} \\ &\quad +K_{\langle 1|2\rangle}.u\left\{(t-T)-\sqrt{\mu\epsilon}.(x+W/2)\right\} \end{aligned} \tag{42a}$$

and

$$\begin{aligned} E_{1y} &= -\sqrt{\mu/\epsilon}.K_{\langle 1|2\rangle}.u\left\{(t-T)+\sqrt{\mu\epsilon}.(x-W/2)\right\} \\ &\quad +\sqrt{\mu/\epsilon}.K_{\langle 1|2\rangle}.u\left\{(t-T)-\sqrt{\mu\epsilon}.(x+W/2)\right\} \end{aligned} \tag{42b}$$

for $T < t < 2T$, where, $K_{\langle 1|2\rangle}$ indicates an arbitrary constant.

While, electromagnetic fields in region-2 can be given as:

$$\begin{aligned} H_{2z} &= K_{\langle 1|1\rangle}.u\left\{(t-T)-\sqrt{\mu_0\epsilon_0}.(x-W/2)\right\} \\ &\quad -K_{\langle 2|1\rangle}.u\left\{t-\sqrt{\mu_0\epsilon_0}.(x-W/2)\right\} \\ &\quad -K_{\langle 2|2\rangle}.u\left\{(t-T)-\sqrt{\mu_0\epsilon_0}.(x-W/2)\right\} \end{aligned} \tag{43a}$$

and

$$\begin{aligned} E_{2y} &= \sqrt{\mu_0/\epsilon_0}.K_{\langle 1|1\rangle}.u\left\{(t-T)-\sqrt{\mu_0\epsilon_0}.(x-W/2)\right\} \\ &\quad -\sqrt{\mu_0/\epsilon_0}.K_{\langle 2|1\rangle}.u\left\{t-\sqrt{\mu_o\epsilon_0}.(x-W/2)\right\} \\ &\quad -\sqrt{\mu_o/\epsilon_0}.K_{\langle 2|2\rangle}.u\left\{(t-T)-\sqrt{\mu_0\epsilon_0}.(x-W/2)\right\} \end{aligned} \tag{43b}$$

for $T < t < 2T$, where, $K_{\langle 2|2\rangle}$ indicates an arbitrary constant.

Now, in view of Eqs. (8), (9) and (35):

$$K_{\langle 1|2\rangle} + K_{\langle 2|2\rangle} = K_1 \tag{44}$$

and

$$\sqrt{\mu/\epsilon}.K_{\langle 1|2\rangle} = \sqrt{\mu_0/\epsilon_0}.\left(K_{\langle 2|2\rangle} + K_{\langle 2|1\rangle} - K_{\langle 1|1\rangle}\right) \tag{45}$$

While the new H-waves travel in the opposite directions in the plate, due to the surface current density $(K_o + K_1)$ a time varying magnetic flux ϕ_1, is established in the plate. This flux, per unit plate length in the y-direction, is given as:

$$\phi_1 = 2\mu.W.K_{\langle 1|1\rangle} + 2\mu.W.K_{\langle 1|2\rangle}.\frac{(t-T)}{T} \tag{46}$$

for $T < t < 2T$.

Therefore, Eqs. (24b) and (35) give:

$$E_o = \rho_S.(K_o + K_1) + \sqrt{\mu/\epsilon}.K_{\langle 1|2\rangle} \tag{47}$$

On solving Eqs. (44), (45) and (47), one gets in view of Eqs. (40a)–(40c):

$$K_{\langle 1|2\rangle} \;=\; E_o.\frac{F_1}{F_0^2}.\sqrt{\frac{\epsilon}{\mu}} \tag{48a}$$

$$K_{\langle 2|2\rangle} \;=\; E_o.\frac{F_2}{F_0^2}.\sqrt{\frac{\epsilon}{\mu}} \tag{48b}$$

and

$$K_1 = E_o.\frac{2}{F_0}.\sqrt{\frac{\epsilon}{\mu}} \tag{48c}$$

where,

$$F_1 \;=\; 1-\rho_S.(\sqrt{\epsilon/\mu}-\sqrt{\epsilon_0/\mu_0}) \tag{49a}$$

$$F_2 \;=\; 1+\rho_S.(\sqrt{\epsilon/\mu}-\sqrt{\epsilon_0/\mu_0}) \tag{49b}$$

At the instant $t = 2T$, the rate of growth of the flux ϕ_1, ceases abruptly, resulting a further increment in the surface current density. This sudden increase in the surface current density triggers fresh electromagnetic waves inside and outside of the plate. This process is repeated periodically with time-period T, resulting progressively diminishing increments in the surface current density. Therefore, the excitation current is an almost periodic function of time.

Consider the instant $t = m.T$, (for $m = 1, 2, 3, \dots \infty$.). At this instant, let the increment in the surface current density be K_m, and the amplitude of new H-waves inside and outside of the plate, respectively be $K_{\langle 1|m+1\rangle}$ and $K_{\langle 2|m+1\rangle}$. The generalized form for Eqs. (44) and (45) are:

$$K_{\langle 1|m+1\rangle} + K_{\langle 2|m+1\rangle} \;=\; K_m \tag{50}$$

$$\sqrt{\frac{\mu}{\epsilon}}.K_{\langle 1|m+1\rangle} \;=\; \sqrt{\frac{\mu_0}{\epsilon_0}}.\sum_{n=1}^{m}\left[K_{\langle 2|n+1\rangle} - K_{\langle 1|n\rangle}\right] \tag{51}$$

for $m = 1, 2, 3, \dots, \infty$.

Further, the generalized form for the Eq. (47) is:

$$E_o = \rho_S.\left[K_o + \sum_{n=1}^{m} K_n\right] + \sqrt{\frac{\mu}{\epsilon}}.K_{\langle 1|m+1\rangle} \tag{52}$$

where, the expression for K_o is given by Eq. (40c).

Solution for Eqs. (50)–(52), results:

$$K_{\langle 1|m+1\rangle} = E_o.\frac{\sqrt{\epsilon/\mu}}{F_0}.\left[\frac{F_1}{F_0}\right]^m \tag{53}$$

$$K_{\langle 2|m+1\rangle} = E_o.\left[\frac{\sqrt{\epsilon_0/\mu_0}}{F_0} + \frac{\sqrt{\epsilon/\mu} - \sqrt{\epsilon_0/\mu_0}}{F_1}\right].\left[\frac{F_1}{F_0}\right]^m \tag{54}$$

$$K_m = E_o.\left[\frac{\sqrt{\epsilon/\mu}+\sqrt{\epsilon_0/\mu_0}}{F_0} + \frac{\sqrt{\epsilon/\mu}-\sqrt{\epsilon_0/\mu_0}}{F_1}\right].\left[\frac{F_1}{F_0}\right]^m \tag{55}$$

for $m = 1, 2, 3, \ldots, \infty$.

Using Eqs. (16), (40c) and (55), it may be seen that the steady state surface current density K_∞, satisfies the following equation:

$$K_\infty = K_o + \sum_{m=1}^{\infty} K_m \tag{56}$$

5.1. Growth of the Magnetic Flux in the Plate

The generalized form for Eq. (46), giving the magnetic flux in the plate per unit plate length in the y-direction, is as follows:

$$\phi_m = 2.W\mu\sum_{n=1}^{m} K_{\langle 1|n\rangle} + 2.W\mu.K_{\langle 1|m+1\rangle}.\frac{(t - m.T)}{T} \tag{57}$$

over $m.T < t < (m + 1).T$, for $m = 1, 2, 3, \ldots, \infty$.

Therefore the expression for the steady state value of this flux is:

$$\phi_\infty = 2.W\mu\sum_{n=1}^{\infty} K_{\langle 1|n\rangle} = W\mu K_\infty \tag{58}$$

6. CONCLUSION

The treatment for the conducting plate assumes a finite, but nonzero value for the plate conductivity, σ.

(1) Eqs. (17) and (18) show that the transient electromagnetic fields in the conducting plate can be expressed as continuous functions of time and space coordinates. Transient fields, for a plate with small values of relaxation time, τ, decay at a faster rate.

(2) Eq. (24a) shows that the magnetic flux in the plate can be expressed as a continuous nonlinear function of space and time. This flux grows smoothly from zero value to the steady state value. Its rate of growth depends on the relaxation time τ and the plate parameter ζ, which includes winding resistance as well as constants of the plate, such as σ, ϵ, μ and W.

(3) Eq. (25) indicates that the excitation current does not appear instantly. It grows smoothly from zero to the steady state value. Its rate of growth depends on the same two parameters, viz. τ and ζ.

(4) Transient electromagnetic waves are found outside the plate, vide Eqs. (30) and (31). Wave-front of these nonuniform (modulated) traveling-waves move away from the two surfaces of the plate. These waves are discontinuous functions of space and time.

The treatment for the nonconducting plate leads to the following conclusions:

(1) As shown by Eqs. (33a) and (33b), the transient electromagnetic fields in a nonconducting plate can be expressed as a pair of uniform traveling-waves moving in the opposite directions, from one plate surface to the other. Thus electromagnetic fields in the plate are discontinuous functions of space and time.

(2) Eq. (57) shows that the flux in the plate is a continuous function of time. It grows from zero to a steady state value, as a piece-wise linear function of time.

(3) In view of Eq. (55), the excitation current is a discontinuous function of time. An initial non-zero value of this current instantly sets in at the very moment a d.c. voltage source is connected to the terminals of the excitation winding. The excitation current consists of an infinite series of equally delayed step functions of time, with progressively diminishing amplitude.

(4) from Eqs. (43a) and (43b), one may infer that electromagnetic fields outside the plate consist of a series of uniform traveling-waves with equally delayed wave-fronts, each moving away from the plate surfaces. Thus, electromagnetic fields outside the plate are discontinuous functions of space and time.

The differences in the two sets of conclusions drawn above, highlight the effects of eddy currents induced in a conducting plate. Lastly, it may be pointed out that the treatment presented in this paper can be readily adapted for plates made of left-handed materials with simultaneously negative permittivity and permeability [22–24].

The treatment considers a hypothetical situation involving a plate with infinite surface area and uniformly distributed current sheets on its surfaces. In a practical situation, every time a circuit is switched on, a part of the power supplied by the d.c. source is used to establish magnetic field, another part is dissipated as eddy current loss in conducting parts and ohmic loss in current carrying resisters. The rest of the power from the d.c. source is lost in radiation.

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