ANALYSIS OF THE SURFACE MAGNETOPLASMON MODES IN THE SEMICONDUCTOR SLIT WAVEGUIDE AT TERAHERTZ FREQUENCIES

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Abstract—The propagation properties of surface plasmon polaritons (SPP) modes and surface magnetoplasmon polaritons (SMP) modes in a semiconductor slit waveguide are analyzed by the effective dielectric constant approach, and the interaction of the external magnetic field with the dispersion properties and field distributions of SMP modes in the Voigt configuration are emphasized in our analysis. Both the symmetric structure and the asymmetric structure are discussed in details. In contrast to the SPP modes which have one propagation band below the plasmon frequency only, the SMP modes have both the low-frequency propagation band below the plasmon frequency and the high-frequency propagation band above the plasmon frequency. When the external magnetic field increases, the two bands of the SMP modes will separate further in frequency, and the even symmetric distribution of the fundamental mode, which usually associates with the SPP mode, will be destroyed. These results can provide some guidance for the design of the tunable semiconductor waveguide in the terahertz regime.
1. INTRODUCTION

The surface plasmon polaritons (SPP) [1] are essentially light waves that are trapped on the metal surface due to their interaction with the electrons in the metal. The SPP on the metal-dielectric interface has led to a new route in nanophotonics [2,3]. Enhanced light transmission and light confinement have been found in subwavelength metallic structures [4–9], and in order to guide and confine the SPP in integrated photonic circuits, various plasmonic waveguides have been introduced, such as the V-groove structure [10–12], thin metal strip [13], metal nanoparticle chains [14,15], metal nanorods [16], and metallic photonic crystals [17]. However, at terahertz (THz) frequencies, the permittivity of the metal is very large, which causes a weak field confinement of SPP to the metal surface and a poor scattering of SPP from the defects in the metal surface [18,19]. Since the semiconductor has a much lower permittivity than the metal at THz frequencies, it is more suitable for the excitation and confinement of THz SPPs modes. Enhanced transmission at THz frequencies has been observed in hole arrays and groove gratings, which are made from doped silicon or indium antimonide (InSb) [20–25]. In an external magnetic field, due to the cyclotron motion of the electrons in semiconductor, the cyclotron frequency of the doped semiconductor is tunable by changing the magnitude of the external magnetic field. Since an electron has a smaller effective mass in the semiconductor, only a moderate external magnetic field is enough to generate cyclotron frequency in the THz region. The SPP modes under the external magnetic field are also called surface magnetoplasmon polaritons (SMP) modes. A comprehensive review on the dispersion of the SPP and the SMP modes in the semiconductor heterostructure has been presented by Kushwaha [26]. In the past decades, the physics of the SMP modes propagating in the semiconductor-air interface and the thin semiconductor film supported by two semi-infinite dielectrics has been extensively studied [26–33]. However, little work has been done on thin dielectric film bounded by two semi-infinite semiconductors. The semiconductor slit waveguide, similar to the metallic slit waveguide which has exhibited practical subwavelength confinement with moderate propagation loss at optical frequencies [34], has never been explored at THz frequencies.

In this paper, we present the analysis of the SPP and SMP modes in the semiconductor slit waveguide at THz frequencies by using the effective dielectric constant approach with the dielectric property of the semiconductor being characterized by the gyrodielectric tensor. Both symmetric structure and asymmetric structure are discussed in details,
and the effect of external magnetic field on the dispersion properties and field distributions of the SMP modes is focused on in our work.

2. ANALYSIS METHODOLOGY

The cross section of the semiconductor slit waveguide considered is shown in Figure 1. The dielectric core is bounded by two semiconductors, and two frequency independent dielectrics are used as the cladding and the substrate. The relative dielectric constants of the core, cladding, and substrate are assumed to be $\varepsilon_r1$, $\varepsilon_{rc}$, and $\varepsilon_{rs}$. The width and the height of the core are denoted as $w$ and $h$ respectively. When the external magnetic field $B_0$ is applied in the $y$-direction and the propagation direction of the guided wave is assumed in the $z$-direction (Voigt configuration), the relative dielectric constant of the semiconductor can be represented by the gyrodielectric tensor

$$
\bar{\varepsilon}_r2 = \begin{bmatrix}
\varepsilon_{xx} & 0 & -i\varepsilon_{xz} \\
0 & \varepsilon_{yy} & 0 \\
i\varepsilon_{zx} & 0 & \varepsilon_{zz}
\end{bmatrix}
$$

(1)

where the elements of this tensor are given by

$$
\varepsilon_{xx}(\omega) = \varepsilon_{zz}(\omega) = \varepsilon_\infty \left(1 - \frac{\omega_p^2}{(\omega^2 - \omega_b^2)}\right)
$$

(2)

$$
\varepsilon_{yy}(\omega) = \varepsilon_\infty \left(1 - \frac{\omega_p^2}{\omega^2}\right)
$$

(3)

$$
\varepsilon_{xz}(\omega) = \varepsilon_{zx}(\omega) = \varepsilon_\infty \left(\frac{\omega_p^2\omega_b}{\omega(\omega^2 - \omega_b^2)}\right)
$$

(4)

In these equations, $\varepsilon_\infty$ is the relative permittivity of the semiconductor at infinite frequency, $\omega_p = \sqrt{Ne^2/m^*\varepsilon_0\varepsilon_\infty}$ is the plasmon frequency, and $\omega_b = eB_0/m^*$ is the cyclotron frequency and is proportional to the static field $B_0$. Here, $N$ is the electron density, $m^*$ is the effective mass of the free charge carriers in the semiconductor, $e$ is the electronic charge, and $\varepsilon_0$ is the dielectric constant of air. The collision loss in the semiconductor is neglected. For instance, given that InSb has $m^* = 0.015m_0$ (where $m_0$ is the electron mass in vacuum), $\varepsilon_\infty = 15.68$, and $N \approx 1 \times 10^{22}/m^3$, then $\omega_p = 1.16 \times 10^{13} \text{ rad/s}$ and $\omega_b/\omega_p \approx B_0$.

In the narrow slit waveguide, the plasmon wave is polarized such that the primary component of magnetic field is tangential to the
Figure 1. The cross section of the semiconductor slit waveguide in the Voigt configuration.

Figure 2. The concept of the effective dielectric constant approach, the slit waveguide can be treated as a combination of two coupled one-dimensional slab waveguides in the \( y \) and \( x \) directions respectively. \( \varepsilon_{re} \) is the effective relative dielectric constant in the \( y \)-direction slab. (a) \( y \)-direction slab. (b) \( x \)-direction slab.

semiconductor-dielectric interfaces and the electric field is normal to semiconductor-dielectric interfaces, this is to say, \( E_x \) and \( H_y \) are the primary components in the slit waveguide. According to the effective dielectric constant approach [35, 36], the semiconductor slit waveguide could be treated as a combination of two coupled one-dimensional slab waveguides in the \( y \) and \( x \) directions respectively (see Figure 2). By deducing the guiding conditions for the TM modes in the \( y \)-direction slab and TE modes in the \( x \)-direction slab, we can get the dispersion
relation for the SMP modes in the slit waveguide as

\[
1 + \left( \frac{\varepsilon_{r1} \gamma_x}{\varepsilon_v k_x} \right)^2 - \left( \frac{\varepsilon_{r1} \varepsilon_{xz} k'_z}{\varepsilon_v \varepsilon_{xx} k_x} \right)^2 \right] \tanh(k_x w) = -2 \frac{\varepsilon_{r1} \gamma_x}{\varepsilon_v k_x} \tag{5}
\]

\[
\left( 1 - \frac{\gamma_{cy} \gamma_{sy}}{k_y^2} \right) \tan(k_y h) = \frac{\gamma_{cy} + \gamma_{sy}}{k_y} \tag{6}
\]

where \( \varepsilon_v = \varepsilon_{xx} - \frac{\varepsilon_{xz}^2}{\varepsilon_{xx}} \) is the Voigt relative dielectric constant of the semiconductor, \( k'_z = k_0^2 \varepsilon_{re} \) and \( \varepsilon_{re} = \varepsilon_{r1} + (k_x/k_0)^2 \) are propagation constant and the effective relative dielectric constant of the \( y \)-direction slab, and the transverse parameters in the two slabs are defined as

\[
\gamma_x^2 = k_0^2 (\varepsilon_{r1} - \varepsilon_v) + k_x^2 \tag{7}
\]

\[
\gamma_{cy}^2 = k_0^2 (\varepsilon_{re} - \varepsilon_{rc}) - k_y^2 \tag{8}
\]

\[
\gamma_{sy}^2 = k_0^2 (\varepsilon_{re} - \varepsilon_{rs}) - k_y^2 \tag{9}
\]

It should be pointed out that the transverse parameters \( \gamma_x, \gamma_{cy}, \) and \( \gamma_{sy} \), which indicate the radiation from the slit waveguide, and the condition that \( \gamma_x > 0, \gamma_{cy} > 0, \) and \( \gamma_{cs} > 0 \) are needed to satisfy the radiation boundary condition at infinity.

To solve \( k_x \) and \( k_y \) numerically, Eq. (5) can be rewritten as

\[
\begin{align*}
  k_x w &= \text{arctanh}(-\rho_{x1}) + \text{arctanh}(-\rho_{x2}) & (p = 1) \\
  k_x w &= \text{arccoth}(-\rho_{x1}) + \text{arccoth}(-\rho_{x2}) & (p = 2)
\end{align*}
\tag{10}
\]

and Eq. (6) can be rewritten as

\[
k_y h = \text{arctan}(\rho_{cy}) + \text{arctan}(\rho_{sy}) + (q - 1)\pi & \quad (q = 1, 2, \ldots) \tag{11}
\]

where \( \rho_{x1} = \frac{\varepsilon_{r1} k_x^2}{\varepsilon_v k_y^2} + \frac{\varepsilon_{xz} k'_z}{\varepsilon_{xx} k_x} \), \( \rho_{x2} = \frac{\varepsilon_{r1} (\gamma_{cy} k_x)}{\varepsilon_v k_y} + \frac{\varepsilon_{xz} k'_z}{\varepsilon_{xx} k_x} \), \( \rho_{cy} = \gamma_{cy} k_y \), and \( \rho_{sy} = \gamma_{sy} k_y \).

The hybrid modes associated with the slit waveguides are denoted as \( E_{pq}^x \), where the superscript \( x \) denotes the direction of the polarization, and the subscripts \( p = 1 \) and \( p = 2 \) correspond to the even and odd symmetries of the field distributions in the \( x \)-direction in the absence of the external magnetic field, and the subscript \( q \) is the number of the peaks in the field along the \( y \) direction. In the presence of external magnetic field, the field distribution will be distorted due to the interaction of the external magnetic field. In fact, the \( E_{2q}^x \) modes rarely exist in our proposed semiconductor slit waveguide geometry because they are suppressed by the small width and the large \( \varepsilon_{\infty} \) of the semiconductor slit waveguide.
Determining the precise mode solution of a semiconductor waveguide involves finding the \( k_x \) and \( k_y \) in the dispersion relations in Eqs. (10) and (11), the propagation constant \( k_z \) can be expressed as

\[
k_z = \sqrt{k_0^2 \varepsilon_r 1 + k_x^2 - k_y^2}
\]

(12)

3. RESULTS AND DISCUSSION

In this section, the magnetoplasmon dispersion of the semiconductor slit waveguide in the Voigt configuration is studied by using the effective dielectric constant approach, and both symmetric structure (\( \varepsilon_{rc} = \varepsilon_{rs} \)) and asymmetric structure (\( \varepsilon_{rc} \neq \varepsilon_{rs} \)) are considered. The dielectric material parameters in the symmetric structure are \( \varepsilon_{rc} = \varepsilon_{rs} = 1 \) and \( \varepsilon_{r1} = 1 \), and the dielectric material parameters in the asymmetric structure are \( \varepsilon_{rc} = 1 \) and \( \varepsilon_{rs} = \varepsilon_{r1} = 2.25 \), which correspond to the semiconductor slit being embedded in the glass substrate. The parameters \( \varepsilon_\infty = 15.68 \), \( \omega_p = 1.16 \times 10^{13} \text{ rad/s} \) (\( f_p = 1.85 \text{ THz} \)) are chosen to model the permittivity of InSb material. The dimensions of the slit waveguide are \( w = 0.2 \lambda_p \) and \( h = 0.4 \lambda_p \), where \( \lambda_p = 2\pi c/\omega_p \) is the plasmon wavelength.

Before discussing the effect of external magnetic field on the SMP modes, it is worthwhile to study the frequency dependence of the permittivity tensor and the corresponding Voigt relative dielectric constant in the InSb material, Figure 3 shows the results in the cases \( \omega_b/\omega_p = 0 \) and \( \omega_b/\omega_p = 1 \). We can see that the frequency dependence curve of Voigt relative dielectric constant in the absence of

![Figure 3](image-url)

**Figure 3.** The frequency dependence of the permittivity tensor and the corresponding Voigt dielectric constant for InSb material. (a) \( \omega_b/\omega_p = 0 \). (b) \( \omega_b/\omega_p = 1 \).
external magnetic field have one branch and monotonically increases with the frequency. When the external magnetic field is applied, the frequency dependence curve of Voigt relative dielectric constant becomes two branches, which are separated by the hybrid cyclotron plasmon frequency $\omega_H = \sqrt{\omega_b^2 + \omega_p^2}$. The corresponding effective plasmon frequencies for the two branches could then be expressed as

$$\omega_{p\pm} = 0.5 \left( \pm \omega_b + \sqrt{\omega_b^2 + 4 \omega_p^2} \right)$$ (13)

The Voigt dielectric constant could be negative in the case of $0 < \omega < \omega_{p-}$ and $\omega_H < \omega < \omega_{p+}$. This means that there could be two propagation bands for the SMP modes in the semiconductor slit waveguide when the external magnetic field is applied.

![Figure 4](image)

**Figure 4.** The dispersion of the SPP mode in the InSb slit waveguide. (a) Symmetric structure, $\varepsilon_{rc} = \varepsilon_{rs} = 1$, $\varepsilon_{r1} = 1$. (b) Asymmetric structure, $\varepsilon_{rc} = 1$, $\varepsilon_{rs} = \varepsilon_{r1} = 2.25$.

Next, we focus on the dispersion properties of the SPP modes and the SMP modes in the semiconductor slit waveguide. Figure 4 gives the dispersion properties of the SPP modes in the case of symmetric and asymmetric structures. It is shown that the SPP modes have one propagation band and they are located to the left side of the light line (slow wave region). As the operation frequency increases, all the SPP modes approach the asymptotic frequency $\omega_s$. The asymptotic frequency, which is specified by the solution of $\varepsilon_{xx}(\omega) + \varepsilon_{r1} = 0$, can be expressed as $\omega_s = \omega_p \sqrt{\varepsilon\infty/(\varepsilon\infty + \varepsilon_{r1})}$. In the case of the symmetric structure, the fundamental SPP mode $E_{11}^x$ does not have cutoff frequency, whereas the high-order SPP modes have cutoffs at the low-parts of the dispersion curves. In the case of the asymmetric structure, all the SPP modes have cut-off frequencies, which are related...
Figure 5. The dispersion of the SMP mode in the InSb slit waveguide in the Voigt configuration. (a) Symmetric structure, \( \varepsilon_{rc} = \varepsilon_{rs} = 1, \varepsilon_{r1} = 1 \). (b) Asymmetric structure, \( \varepsilon_{rc} = 1, \varepsilon_{rs} = \varepsilon_{r1} = 2.25 \).

to the non-symmetry of the cladding and the substrate. Typically, the higher order SPP modes would have higher cut-off frequencies. Due to the interaction of external magnetic field, the dispersion properties of the SMP modes become more complicated than the SPP modes. Figure 5 shows the dispersion properties of the SMP modes in the slit waveguide with \( \omega_b/\omega_p = 1 \). We can see that all the SMP modes have two propagation bands: the low-frequency band and the high-frequency band which locate below and above the plasmon frequency respectively. The SMP modes in the low-frequency band rise just to the left of the light line and approach the asymptotic frequency \( \omega_{s-} \), while the SMP modes in the high-frequency band start from the hybrid cyclotron plasmon frequency \( \omega_H \) and approach the asymptotic frequency \( \omega_{s+} \). The asymptotic frequency for SMP modes, which is specified by the solution of \( \varepsilon_{xx}(\omega) + \varepsilon_{xz}(\omega) + \varepsilon_{r1} = 0 \), can be expressed as

\[
\omega_{s\pm} = 0.5 \left( \pm \omega_b + \sqrt{\omega_b^2 + 4\varepsilon_\infty \omega_b^2/\left(\varepsilon_\infty + \varepsilon_{r1}\right)} \right)
\]  

(14)

It should be noted that the two asymptotic SMP modes are separated by \( |\omega_b| \). The SMP modes in the low-frequency band have the similar dispersion properties to the SPP modes. In the case of asymmetric structure, all the SMP modes have cut-off frequencies also, especially for the SMP modes in the low-frequency band.

In order to elucidate the effect of the external magnetic field on the dispersion properties of the SMP modes, the band map of the \( E_{11}^x \) mode in the slit waveguide is given in Figure 6 with \( \omega_b/\omega_p \) in the range of 0–2 Tesla. From Figure 6, we can see that the \( E_{11}^x \) mode has one
propagation band when no external magnetic field is applied, but has two propagation bands when the external magnetic field is greater than the critical value for the emergence of the high-frequency SMP modes. The critical cyclotron frequency, which could be specified by the condition $\omega_{s+} > \omega_H$, can be expressed as $\omega_{bc} = \omega_p / \sqrt{(\varepsilon_{\infty} / \varepsilon_{r1})^2 - 1}$. This means that the external magnetic field $B_0$ must be at least 0.064 Tesla and 0.145 Tesla to meet the condition for the high-frequency band emerging in our proposed symmetric and asymmetric structures, separately.

As the external magnetic field increases, the propagation band of the low-frequency modes moves to lower frequency, whereas the propagation band of the high-frequency modes shifts to higher frequency. The effect can be explained by the fact that the effective plasmon frequency $\omega_p$ decreases and $\omega_{s+}$ increases when the external magnetic field increases.

**Figure 6.** The band map of the SMP $E_{11}^x$ mode in the InSb slit waveguide in the Voigt configuration. (a) Symmetric structure, $\varepsilon_{rc} = \varepsilon_{rs} = 1$, $\varepsilon_{r1} = 1$. (b) Asymmetric structure, $\varepsilon_{rc} = 1$, $\varepsilon_{rs} = \varepsilon_{r1} = 2.25$.

Now, we turn our attention to the field distributions of the SPP modes and SMP modes. The normalized $H_y$ distribution of the SPP $E_{11}^{x}$ mode in the slit waveguide is illustrated in Figure 7. It is shown that the $H_y$ distribution of the SPP $E_{11}^{x}$ mode in the symmetric structure is symmetric in both width direction and height direction, while the $H_y$ distribution of the SPP $E_{11}^{x}$ mode in the asymmetric structure is symmetric only in the width direction. We can also observe that the evanescent fields of the SPP $E_{11}^{x}$ mode decay faster into the semiconductor than in the cladding and the substrate. As the operation frequency increases, there is more energy confined in the core of the slit waveguide. When the external magnetic field is applied, the field profiles of the SMP modes will be affected. Figure 8 gives the normalized $H_y$ distribution of the SMP $E_{11}^{x}$ mode in the case of
\( \omega_b/\omega_p = 1 \). It is shown that the symmetric distribution of the magnetic field \( H_y \) of the SMP \( E_{11}^x \) mode is distorted due to the interaction of the external magnetic field. When the operation frequency increases, there is more energy confined in the right semiconductor-dielectric interface when the SMP \( E_{11}^x \) mode is in the low-frequency band, while in the left semiconductor-dielectric interface in the high-frequency band.

**Figure 7.** The normalized \( H_y \) distribution of the SPP \( E_{11}^x \) mode in the InSb slit waveguide. (a) Symmetric structure, \( \varepsilon_{rc} = \varepsilon_{rs} = 1, \varepsilon_{r1} = 1 \). (b) Asymmetric structure, \( \varepsilon_{rc} = 1, \varepsilon_{rs} = \varepsilon_{r1} = 2.25 \).

**Figure 8.** The normalized \( H_y \) distribution of the SMP \( E_{11}^x \) mode in the InSb slit waveguide in the Voigt configuration. (a) Symmetric structure, \( \varepsilon_{rc} = \varepsilon_{rs} = 1, \varepsilon_{r1} = 1 \). (b) Asymmetric structure, \( \varepsilon_{rc} = 1, \varepsilon_{rs} = \varepsilon_{r1} = 2.25 \).
4. SUMMARY

We present a detail study on the properties of the SPP and SMP modes in the semiconductor slit waveguide by using the effective dielectric constant approach, especially emphasis on the interaction of external magnetic field with the dispersion properties and the field profiles of the SMP modes in the semiconductor slit waveguide in the Voigt configuration, and both the symmetric and asymmetric structures are discussed in details. In comparison with the single propagation band in the SPP modes, the SMP modes have two propagation bands: the low-frequency band and the high-frequency band, which will shift towards lower frequency and higher frequency respectively when the external magnetic field increases. In the presence of the external magnetic field, the symmetry of field distribution, which usually associated with the SPP fundamental mode, is distorted by the interaction of the external magnetic field. When the operation frequency increases, for the SMP mode in the low-frequency band, there is more energy confined in the right semiconductor-dielectric interface, while for the modes in the high-frequency band it confined in left semiconductor-dielectric interface. These results can provide some guidance for the design of the tunable semiconductor waveguide in THz frequencies.

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