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TRAPPED SURFACE WAVE AND LATERAL WAVE IN THE PRESENCE OF A FOUR-LAYERED REGION

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Abstract—In this paper, propagation model considers the region as a perfect conductor, covered by the two layer dielectrics, and air above. Propagation of the electromagnetic field in the presence of a four-layered region is examined in detail when a vertical electric dipole and observation point are located in the air. Similar to the three-layered case, analytical results are found for the electromagnetic field, which includes four wave modes: a direct wave, an ideal reflected wave, trapped surface waves, and lateral waves. The wave number of the trapped surface wave, which is contributed by the sums of residues of the poles, is between the wave numbers k_0 in the air and k_2 in the lower dielectric layer. The lateral wave is evaluated by the integrations along the branch cut. Analysis and computations shows that the trapped surface wave play a major role in communication at large distance when both the source point and observation point are on or close to the boundary between the air and the upper dielectric layer.

1. INTRODUCTION

The problem of the electromagnetic field generated by a dipole source in a layered region has been studied by many investigator in the past decades because of its wide useful applications [1–36]. In the pioneering works by Wait [1–5], the electromagnetic field of a dipole source in the layered region was examined by using surface-impedance technique. Lately, further developments on the electromagnetic field of a dipole source in a three-layered have been carried out, especially, the lateral wave is addressed carefully [8–13]. Lately, in the works by Li et al. [17–19], the dyadic Green's function technique is applied to examine the electromagnetic field in a four-layered forest environment.

In 1990's, Wait and Mahmoud wrote comments on the 1994 paper by King and Sandle and regarded that the terms of the trapped surface wave, which varies as $\rho^{-1/2}$ in the far region, should be considered for the three-layered case [14, 16]. The debates rekindled several investigators to re-visit the problem again [20–28]. It has been demonstrated that the trapped surface wave, which is contributed by the sums of residues of the poles, should be considered in the three-layered case. It is concluded, naturally, that the trapped surface wave can also be excited efficiently by a dipole source in the four-layered region.

In this paper, the propagation model considers the region as a perfect conductor, coated with the two layer dielectrics, and air above. We will attempt to outline the trapped surface wave and lateral wave excited by a vertical electric dipole in the presence of the four-layered region when both the source point and observation point are located in the air.

2. FORMULATION OF THE PROBLEM

Consider the four-layered region as shown in Fig. 1. From (11.5.1)–(11.5.3) in the chapter 11 of the monograph by King, Owens, and Wu [8], use is made of the time dependence $e^{-i\omega t}$, the integrated representations for the electromagnetic field of a vertical electric dipole in the four-layered region can be obtained readily. They are

$$\begin{aligned} B_{0\phi}(\rho, z) \;=\; & \frac{i\mu_0}{4\pi}\int_0^\infty \gamma_0^{-1}\Big[e^{i\gamma_0|z-d|}+e^{i\gamma_0(z+d)} \\ & -(Q+1)e^{i\gamma_0(z+d)}\Big]J_1(\lambda\rho)\lambda^2 d\lambda \end{aligned} \tag{1}$$

$$E_{0\rho}(\rho, z) \;=\; \frac{i\omega\mu_0}{4\pi k_0^2}\int_0^\infty \Big[\pm e^{i\gamma_0|z-d|}+e^{i\gamma_0(z+d)}$$

$$-(Q+1)e^{i\gamma_0(z+d)}\Big]J_1(\lambda\rho)\lambda^2 d\lambda$$

$$\begin{array}{c} d < z \\ 0 \le z \le d \end{array} \tag{2}$$

$$\begin{aligned} E_{0z}(\rho,z) &= -\frac{\omega\mu_0}{4\pi k_0^2}\int_0^\infty \gamma_0^{-1}\Big[e^{i\gamma_0|z-d|}+e^{i\gamma_0(z+d)} \\ &\quad -(Q+1)e^{i\gamma_0(z+d)}\Big]J_0(\lambda\rho)\lambda^3 d\lambda \end{aligned} \tag{3}$$

where

$$\gamma_j = \sqrt{k_j^2-\lambda^2}, \qquad j=0,1,2 \tag{4}$$

$$k_j = \omega\sqrt{\mu_0\epsilon_j}, \qquad j=0,1,2 \tag{5}$$

In this paper, we consider the situation that,

$$k_3 \to \infty \tag{6}$$

so,

$$Q = -\frac{\gamma_0 + \dfrac{k_0^2}{\omega\mu_0}\dfrac{i\omega\mu_0\gamma_1}{k_1^2}\dfrac{\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2}{\gamma_1 k_2^2-\gamma_2 k_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2}}{\gamma_0 - \dfrac{k_0^2}{\omega\mu_0}\dfrac{i\omega\mu_0\gamma_1}{k_1^2}\dfrac{\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2}{\gamma_1 k_2^2-\gamma_2 k_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2}} \tag{7}$$

It is now convenient to rewrite (1) to (3) in the following forms.

$$B_{0\phi}(\rho,z) = B_{0\phi}^{(1)}+B_{0\phi}^{(2)}+B_{0\phi}^{(3)} \tag{8}$$

$$E_{0\rho}(\rho,z) = E_{0\rho}^{(1)}+E_{0\rho}^{(2)}+E_{0\rho}^{(3)} \tag{9}$$

$$E_{0z}(\rho,z) = E_{0z}^{(1)}+E_{0z}^{(2)}+E_{0z}^{(3)}. \tag{10}$$

where

$$B_{0\phi}^{(1)} = \frac{i\mu_0}{4\pi}\int_0^\infty \gamma_0^{-1}e^{i\gamma_0|z-d|}J_1(\lambda\rho)\lambda^2 d\lambda \tag{11}$$

$$B_{0\phi}^{(2)} = \frac{i\mu_0}{4\pi}\int_0^\infty \gamma_0^{-1}e^{i\gamma_0(z+d)}J_1(\lambda\rho)\lambda^2 d\lambda \tag{12}$$

$$E_{0\rho}^{(1)} = \frac{i\omega\mu_0}{4\pi k_0^2}\int_0^\infty \pm e^{i\gamma_0|z-d|}J_1(\lambda\rho)\lambda^2 d\lambda \tag{13}$$

$$E_{0\rho}^{(2)} = \frac{i\omega\mu_0}{4\pi k_0^2}\int_0^\infty e^{i\gamma_0(z+d)}J_1(\lambda\rho)\lambda^2 d\lambda \tag{14}$$

$$E_{0z}^{(1)} = -\frac{\omega\mu_0}{4\pi k_0^2}\int_0^\infty \gamma_0^{-1}e^{i\gamma_0|z-d|}J_0(\lambda\rho)\lambda^3 d\lambda \tag{15}$$

$$E_{0z}^{(2)} = -\frac{\omega\mu_0}{4\pi k_0^2}\int_0^\infty \gamma_0^{-1}e^{i\gamma_0(z+d)}J_0(\lambda\rho)\lambda^3 d\lambda. \tag{16}$$

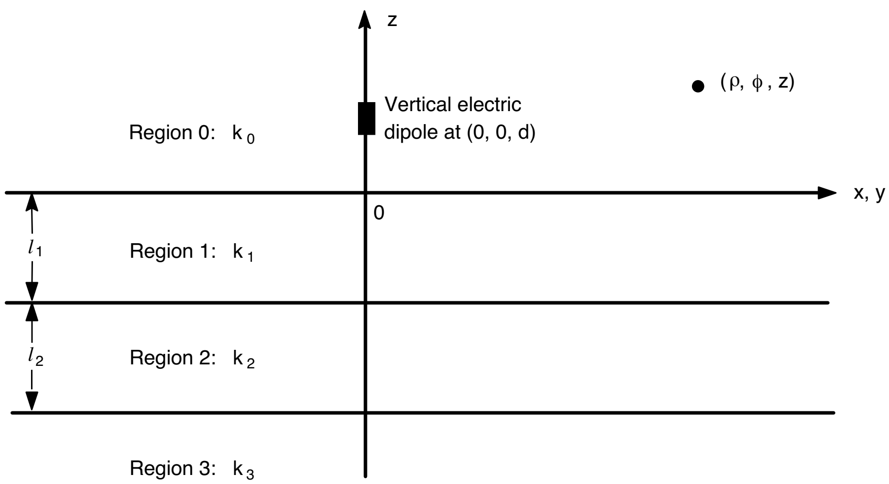

Figure 1. Geometry of a vertical electric dipole in the four-layered region.

The first and second terms in (1)–(3) represent the direct wave and ideal reflected wave, respectively. The two terms have already been evaluated many years ago [8]. The next main tasks are to evaluate the third terms in (8)–(10). Since γ_0, γ_1, and γ_2 are even functions, and use is made of the relations between Bessel function and Hankel function

$$J_n(\lambda\rho) = \frac{1}{2}\left[H_n^{(1)}(\lambda\rho) + H_n^{(2)}(\lambda\rho)\right] \tag{17}$$

$$H_n^{(1)}(-\lambda\rho) = H_n^{(2)}(\lambda\rho)(-1)^{n+1} \tag{18}$$

the third terms in (8)–(10) are expressed in the following forms, respectively.

$$B_{0\phi}^{(3)} = -\frac{\mu_0 k_0^2}{4\pi}\int_{-\infty}^\infty \frac{\gamma_1 k_2^2 \tan\gamma_1 l_1 + \gamma_2 k_1^2 \tan\gamma_2 l_2}{q(\lambda)\gamma_0} \cdot\gamma_1 e^{i\gamma_0(z+d)}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \tag{19}$$

$$E_{0\rho}^{(3)} = -\frac{\omega\mu_0}{4\pi}\int_{-\infty}^{\infty}\frac{\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2}{q(\lambda)} \cdot\gamma_1 e^{i\gamma_0(z+d)}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \quad (20)$$

$$E_{0z}^{(3)} = -\frac{i\omega\mu_0}{4\pi}\int_{-\infty}^{\infty}\frac{\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2}{q(\lambda)\gamma_0} \cdot\gamma_1 e^{i\gamma_0(z+d)}H_0^{(1)}(\lambda\rho)\lambda^3 d\lambda \quad (21)$$

where

$$q(\lambda) = k_1^2\gamma_0\left(\gamma_1 k_2^2-\gamma_2 k_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2\right) -ik_0^2\gamma_1\cdot\left(\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2\right). \quad (22)$$

Because of the high oscillatory of the Bessel functions $J_i(\lambda\rho)$ or $H_i^{(1)}(\lambda\rho)$ $(j = 0,\ 1)$, the above three integrals converge very slowly. It is necessary to evaluate the above three integrals by using analytical techniques.

3. EVALUATIONS OF THE TRAPPED SURFACE WAVE AND LATERAL WAVE

In this section, we will attempt to evaluate the integrals in (19)–(21). We rewrite $B_{0\phi}^{(3)}$ in the form

$$B_{0\phi}^{(3)} = B_{0\phi}^{(3,1)} + B_{0\phi}^{(3,2)} \quad (23)$$

where

$$B_{0\phi}^{(3,1)} = -\frac{\mu_0 k_0^2}{4\pi}\int_{-\infty}^{\infty}\frac{\gamma_1\gamma_2 k_1^2\tan\gamma_2 l_2}{q(\lambda)\gamma_0}\cdot e^{i\gamma_0(z+d)}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \quad (24)$$

$$B_{0\phi}^{(3,2)} = -\frac{\mu_0 k_0^2}{4\pi}\int_{-\infty}^{\infty}\frac{\gamma_1^2 k_2^2\tan\gamma_1 l_1}{q(\lambda)\gamma_0}\cdot e^{i\gamma_0(z+d)}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda. \quad (25)$$

In order to evaluate the integral $B_{0\phi}^{(3)}$, it is necessary to shift the contour around the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The configuration of the poles and the branch cuts is shown in Fig. 2. The next main tasks are to determine the poles and to evaluate the integrations along the branch cuts Γ_0, Γ_1, and Γ_2. First we consider the following pole equation.

$$q(\lambda) = k_1^2\gamma_0\left(\gamma_1 k_2^2-\gamma_2 k_1^2\tan\gamma_1 l_1\tan\gamma_2 l_2\right) -ik_0^2\gamma_1\cdot\left(\gamma_1 k_2^2\tan\gamma_1 l_1+\gamma_2 k_1^2\tan\gamma_2 l_2\right) = 0. \quad (26)$$

Comparing with the corresponding three-layered case as addressed in the work by [20], the pole equation becomes more complex. In this paper the conditions $k_0 \leq k_1 \leq k_2$ and $k_3 \to 0$ are satisfied. Obviously, it is seen that no pole exists in the intervals $\lambda < k_0$ and $\lambda > k_2$. In the intervals $k_0 \leq \lambda \leq k_1$ and $k_1 \leq \lambda \leq k_2$, the poles may exist. In Figs. 3 and 4, the poles were determined by using Newton method and it is found that the poles appear in the intervals $k_0 \leq \lambda \leq k_1$ and $k_1 \leq \lambda \leq k_2$. It is noted that k_1 is a removable pole.

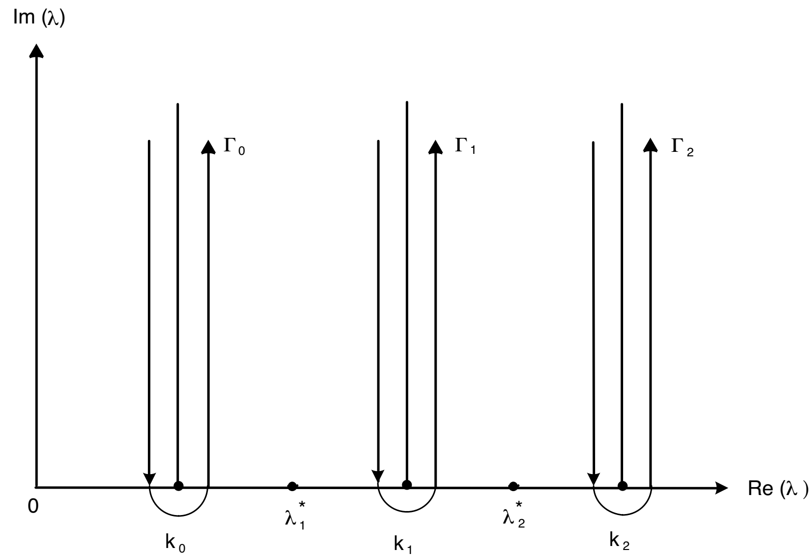

Figure 2. The configuration of the poles and the branch cuts.

On the two sides of the branch cut Γ_0, the phase of γ_0 will change by π, and γ_1 and γ_2 will have the same values on the two sides of Γ_1 and Γ_2. It is easily verified that the evaluations of the integrals in (24) and (25) along the branch cuts Γ_1 and Γ_2 are zero. Therefore, the integral (24) can be written as

$$\begin{aligned} B_{0\phi}^{(3,1)} &= 2\pi i \frac{-\mu_0 k_0^2}{4\pi} \sum_j \frac{k_1^2 \gamma_1(\lambda_j^*)\gamma_2(\lambda_j^*)\tan(\gamma_2(\lambda_j^*) l_2)}{q'(\lambda_j^*)\gamma_0(\lambda_j^*)} \\ &\quad \cdot e^{i\gamma_0(\lambda_j^*)(z+d)} H_1^{(1)}(\lambda_j^* \rho) \lambda_j^{*2} \\ &\quad - \frac{\mu_0 k_0^2}{4\pi} \int_{\Gamma_0} \frac{\gamma_1 \gamma_2 k_1^2 \tan \gamma_2 l_2}{q(\lambda)\gamma_0} \cdot e^{i\gamma_0(z+d)} H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \qquad (27) \end{aligned}$$

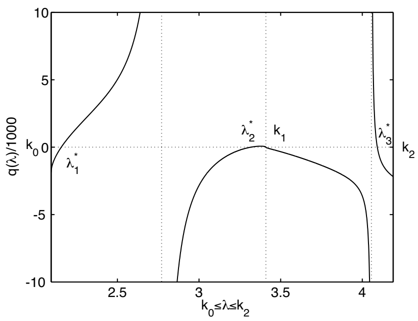

Figure 3. Roots of (26) for $f = 100\,\mathrm{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4.0$, $l_1 = l_2 = 0.75\,\mathrm{m}$.

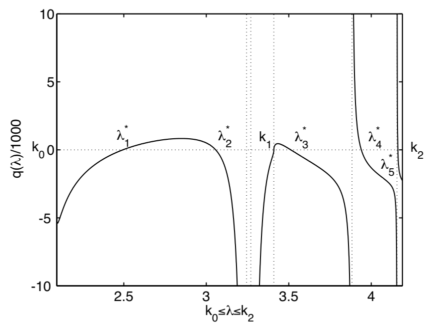

Figure 4. Roots of (26) for $f = 100\,\mathrm{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4.0$, $l_1 = 2.0\,\mathrm{m}$, and $l_2 = 0.5\,\mathrm{m}$.

where

$$\begin{aligned} q'(\lambda) \;=\; & -k_1^2 \frac{\lambda}{\gamma_0}\left(\gamma_1 k_2^2 - \gamma_2 k_1^2 \tan\gamma_1 l_1 \tan\gamma_2 l_2\right) \\ & + i k_0^2 \frac{\lambda}{\gamma_1}\left(\gamma_1 k_2^2 \tan\gamma_1 l_1 + \gamma_2 k_1^2 \tan\gamma_2 l_2\right) \\ & + k_1^2 \gamma_0 \left[-\frac{\lambda}{\gamma_1} k_2^2 + \frac{\lambda}{\gamma_2} k_1^2 \tan\gamma_1 l_1 \tan\gamma_2 l_2 \right. \\ & \left. + \, k_1^2 \gamma_2 \lambda \left(\frac{l_1}{\gamma_1}\sec^2\gamma_1 l_1 \tan\gamma_2 l_2 + \frac{l_2}{\gamma_2}\tan\gamma_1 l_1 \sec^2\gamma_2 l_2\right)\right] \\ & + i k_0^2 \gamma_1 \lambda \cdot \left(\frac{k_2^2 \tan\gamma_1 l_1}{\gamma_1} + \frac{k_1^2 \tan\gamma_2 l_2}{\gamma_2}\right. \\ & \left. + k_2^2 l_1 \sec^2\gamma_1 l_1 + k_1^2 l_2 \sec^2\gamma_2 l_2\right). \end{aligned} \tag{28}$$

Considering the far-field condition $k_0\rho \gg 1$ and $(z + d) \ll \rho$, the dominant contribution of the integral along the branch line Γ_0 comes from the vicinity of k_0. Let $\lambda = k_0(1+i\tau^2)$, The following approximated formulas can be obtained readily.

$$\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx k_0 e^{i\frac{3}{4}\pi}\sqrt{2}\tau, \tag{29}$$

$$\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx \sqrt{k_1^2 - k_0^2} = \gamma_{10}, \tag{30}$$

$$\gamma_2 = \sqrt{k_2^2 - \lambda^2} \approx \sqrt{k_2^2 - k_0^2} = \gamma_{20}, \tag{31}$$

and

$$H_1^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi k_0\rho}} e^{i(k_0\rho - \frac{3}{4}\pi)} \cdot e^{-k_0\rho\tau^2}. \tag{32}$$

Then, we have

$$\begin{aligned} B_{0\phi}^{(3,1)} &= -\frac{\mu_0 k_0^2}{4\pi} \frac{\gamma_{10}\gamma_{20}\sqrt{\frac{2k_0}{\pi\rho}}\tan(\gamma_{20}l_2)}{\gamma_{10}k_2^2 - \gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)} \\ &\quad \cdot \exp\left[i\left(k_0\rho + \frac{\pi}{4}\right) + i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}\right)^2\right] \\ &\quad \cdot \int_{-\infty}^{\infty} \frac{\exp\left[-k_0\rho\left(\tau + \frac{e^{i\pi/4}}{\sqrt{2}}\frac{z+d}{\rho}\right)^2\right]}{\tau - \triangle} d\tau, \end{aligned} \tag{33}$$

where

$$\triangle = -e^{-i\frac{\pi}{4}} \frac{k_0}{k_1^2} \frac{\gamma_{10}}{\sqrt{2}} \cdot \frac{\gamma_{10}k_2^2\tan(\gamma_{10}l_1) + \gamma_{20}k_1^2\tan(\gamma_{20}l_2)}{\gamma_{10}k_2^2 - \gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}. \tag{34}$$

In terms of the variable $t = \tau + \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}$, the result becomes

$$\int_{-\infty}^{\infty} \frac{e^{-k_0\rho t^2}}{t - e^{i\frac{\pi}{4}}\triangle'} dt = i\pi e^{-i\triangle'^2 k_0\rho} \operatorname{erfc}\left(\sqrt{-i\triangle'^2 k_0\rho}\right) \tag{35}$$

where

$$\triangle' = \frac{z+d}{\sqrt{2}\rho} + i\frac{k_0}{k_1^2}\frac{\gamma_{10}}{\sqrt{2}} \cdot \frac{\gamma_{10}k_2^2\tan(\gamma_{10}l_1) + \gamma_{20}k_1^2\tan(\gamma_{20}l_2)}{\gamma_{10}k_2^2 - \gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}. \tag{36}$$

Here, the phase of $\sqrt{-i\triangle'^2 k_0\rho}$ in (35) requires to be

$$\left|\operatorname{Arg}\sqrt{-i\triangle'^2 k_0\rho}\right| \le \frac{\pi}{4}. \tag{37}$$

The error function and Fresnel integral are defined by

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt, \tag{38}$$

$$F(p^*) = \frac{1}{2}(1+i) - \int_0^{p^*} \frac{e^{it}}{\sqrt{2\pi t}} dt \tag{39}$$

where

$$p^* = \frac{k_0\rho}{2}\left[\frac{z+d}{\rho} + i\frac{k_0}{k_1^2}\gamma_{10}\cdot\frac{\gamma_{10}k_2^2\tan(\gamma_{10}l_1)+\gamma_{20}k_1^2\tan(\gamma_{20}l_2)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}\right]^2 \quad (40)$$

and considering the relation between the error function and Fresnel integral

$$\operatorname{erfc}\left(\sqrt{-ip^*}\right) = \sqrt{2}e^{-i\frac{\pi}{4}}F(p^*). \quad (41)$$

Then, we can obtain

$$\begin{aligned} B_{0\phi}^{(3,1)} &= \mu_0 k_0^2 k_1^2 \sqrt{\frac{1}{2\pi\rho}} e^{i\frac{3\pi}{4}} \\ &\quad \sum_j \left[\frac{\tan\gamma_2(\lambda_j^*)l_2}{q'(\lambda_j^*)\gamma_0(\lambda_j^*)}\cdot\gamma_1(\lambda_j^*)\gamma_2(\lambda_j^*)(\lambda_j^*)^{3/2}e^{i\gamma_0(\lambda_j^*)(z+d)+i\lambda_j^*\rho}\right] \\ &\quad -\frac{i\mu_0 k_0^3}{2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}e^{-ip^*}F(p^*) \\ &\quad \cdot\frac{\gamma_{10}\gamma_{20}\tan(\gamma_{20}l_2)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)} \end{aligned} \quad (42)$$

where

$$r_2 = \sqrt{\rho^2+(z+d)^2} \approx \rho\left[1+\frac{1}{2}\left(\frac{z+d}{\rho}\right)^2\right]. \quad (43)$$

Similarly, we have

$$\begin{aligned} B_{0\phi}^{(3,2)} &= k_0^2 k_2^2 \mu_0 \sqrt{\frac{1}{2\pi\rho}} e^{i\frac{3\pi}{4}} \\ &\quad \sum_j \left[\frac{\tan\gamma_1(\lambda_j^*)l_1}{q'(\lambda_j^*)\gamma_0(\lambda_j^*)}\cdot\gamma_1^2(\lambda_j^*)(\lambda_j^*)^{3/2}e^{i\gamma_0(\lambda_j^*)(z+d)+i\lambda_j^*\rho}\right] \\ &\quad -\frac{i\mu_0 k_0^3}{2}\frac{k_2^2}{k_1^2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}e^{-ip^*}F(p^*) \\ &\quad \cdot\frac{\gamma_{10}^2\tan(\gamma_{10}l_1)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}. \end{aligned} \quad (44)$$

Using the above derivations and the results for the direct wave and ideal reflected addressed in the monograph by King, Owen, and Wu [8] and the final expressions of the field component $B_{0\phi}$ can be obtained readily. It is

$$\begin{aligned}
B_{0\phi} &= -\frac{\mu_0}{4\pi}e^{ik_0r_1}\left(\frac{\rho}{r_1}\right)\left(\frac{ik_0}{r_1}-\frac{1}{r_1^2}\right)-\frac{\mu_0}{4\pi}e^{ik_0r_2}\cdot\left(\frac{\rho}{r_2}\right)\left(\frac{ik_0}{r_2}-\frac{1}{r_1^2}\right)\\
&\quad+\mu_0k_0^2\sqrt{\frac{1}{2\pi\rho}}e^{i\frac{3\pi}{4}}\cdot\sum_j\frac{(\lambda_j^*)^{3/2}e^{i\gamma_0(\lambda_j^*)(z+d)+i\lambda_j^*\rho}}{q'(\lambda_j^*)\gamma_0(\lambda_j^*)}\\
&\quad\cdot\left[k_2^2\gamma_1^2(\lambda_j^*)\cdot\tan\gamma_1(\lambda_j^*)l_1+k_1^2\gamma_1(\lambda_j^*)\gamma_2(\lambda_j^*)\tan\gamma_2(\lambda_j^*)l_2\right]\\
&\quad-\frac{i\mu_0k_0^3}{2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}e^{-ip^*}F(p^*)\\
&\quad\cdot\frac{\gamma_{10}\gamma_{20}\tan(\gamma_{20}l_2)+\dfrac{k_2^2}{k_1^2}\gamma_{10}^2\tan(\gamma_{10}l_1)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}. \qquad (45)
\end{aligned}$$

With the similar steps, the components $E_{0\rho}$ and E_{0z} are expressed as follows:

$$\begin{aligned}
E_{0\rho} &= -\frac{\omega\mu_0}{4\pi k_0}e^{ik_0r_1}\left(\frac{\rho}{r_1}\right)\left(\frac{z-d}{r_1}\right)\left(\frac{ik_0}{r_1}-\frac{3}{r_1^2}-\frac{3i}{k_0r_1^3}\right)\\
&\quad-\frac{\omega\mu_0}{4\pi k_0}e^{ik_0r_2}\left(\frac{\rho}{r_2}\right)\left(\frac{z+d}{r_2}\right)\cdot\left(\frac{ik_0}{r_2}-\frac{3}{r_2^2}-\frac{3i}{k_0r_2^3}\right)\\
&\quad-\omega\mu_0\sqrt{\frac{1}{2\pi\rho}}e^{-i\frac{\pi}{4}}\cdot\sum_j\frac{(\lambda_j^*)^{3/2}e^{i\gamma_0(\lambda_j^*)(z+d)+i\lambda_j^*\rho}}{q'(\lambda_j^*)}\\
&\quad\cdot\left[k_1^2\gamma_1(\lambda_j^*)\gamma_2(\lambda_j^*)\tan\gamma_2(\lambda_j^*)l_2+k_2^2\gamma_1^2(\lambda_j^*)\cdot\tan\gamma_1(\lambda_j^*)l_1\right]\\
&\quad+\frac{\omega\mu_0k_0^2}{2\pi}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}\cdot\frac{\gamma_{10}\gamma_{20}\tan(\gamma_{20}l_2)+\dfrac{k_2^2}{k_1^2}\gamma_{10}^2\tan(\gamma_{10}l_1)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}\\
&\quad\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi\frac{k_0\gamma_{10}}{k_1^2}e^{-ip^*}F(p^*)\right.\\
&\quad\left.\cdot\frac{\gamma_{10}k_2^2\tan(\gamma_{10}l_1)+\gamma_{20}k_1^2\tan(\gamma_{20}l_2)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}\right] \qquad (46)
\end{aligned}$$

$$\begin{aligned}
E_{0z} &= \frac{\omega\mu_0}{4\pi k_0}e^{ik_0r_1}\left[\frac{ik_0}{r_1}-\frac{1}{r_1^2}-\frac{i}{k_0r_1^3}-\left(\frac{z-d}{r_1}\right)^2\right.\\
&\quad\left.\cdot\left(\frac{ik_0}{r_1}-\frac{3}{r_1^2}-\frac{3i}{k_0r_1^3}\right)\right]+\frac{\omega\mu_0}{4\pi k_0}e^{ik_0r_2}
\end{aligned}$$

$$\cdot\left[\frac{ik_0}{r_2}-\frac{1}{r_2^2}-\frac{i}{k_0r_2^3}-\left(\frac{z+d}{r_2}\right)^2\cdot\left(\frac{ik_0}{r_2}-\frac{3}{r_2^2}-\frac{3i}{k_0r_2^3}\right)\right]$$
$$+\omega\mu_0\sqrt{\frac{1}{2\pi\rho}}e^{-i\frac{\pi}{4}}\sum_j\frac{(\lambda_j^*)^{5/2}e^{i\gamma_0(\lambda_j^*)(z+d)+i\lambda_j^*\rho}}{q'(\lambda_j^*)\gamma_0(\lambda_j^*)}$$
$$\cdot\left[k_1^2\gamma_1(\lambda_j^*)\gamma_2(\lambda_j^*)\tan\gamma_2(\lambda_j^*)l_2+k_2^2\gamma_1^2(\lambda_j^*)\tan\gamma_1(\lambda_j^*)l_1\right]$$
$$+\frac{i\omega\mu_0k_0^2}{2}\sqrt{\frac{1}{\pi k_0\rho}}e^{ik_0r_2}e^{-ip^*}F(p^*)$$
$$\cdot\frac{\gamma_{10}\gamma_{20}\tan(\gamma_{20}l_2)+\frac{k_2^2}{k_1^2}\gamma_{10}^2\tan(\gamma_{10}l_1)}{\gamma_{10}k_2^2-\gamma_{20}k_1^2\tan(\gamma_{10}l_1)\tan(\gamma_{20}l_2)}. \quad (47)$$

If Region 1 is made the air by setting $k_1 = k_0$ or both Regions 2 and 3 are made the same dielectric by setting $k_2 = k_1$, the above results reduce to the three-layered results addressed [20].

4. COMPUTATIONS AND CONCLUSIONS

Similar to the three-layered case, the electromagnetic field of a vertical electric dipole in the presence of a four-layered region, which both the source point and observation point are located in the air, includes the following four modes: the direct wave, the ideal reflected wave, the trapped surface wave, and the lateral wave. From the expressions of the three components $B_{0\phi}$, $E_{0\rho}$, and E_{0z} in (45)–(47), it is found that

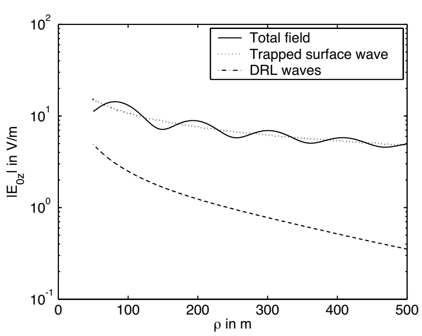

Figure 5. The electric field $|E_z|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{1r} = 4$, $k_0l_1 = k_0l_2 = 0.3$, and $z = d = 0$.

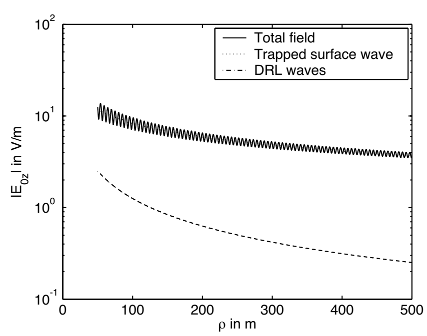

Figure 6. The electric field $|E_z|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{1r} = 4$, $k_0l_1 = k_0l_2 = 1.3$, and $z = d = 0$.

the term of the trapped surface is determined by the sums of residues of the poles λ_j^*. In the interval $k_0 \leq \lambda_j^* \leq k_2$, $\gamma_{0j}^* = i\sqrt{\lambda_j^{*2} - k_0^2}$ is always a positive imaginary number, the term including the factor $e^{i\gamma_{0j}^* z}$ will attenuates exponentially as $e^{-\sqrt{\lambda_j^{*2}-k_0^2}z}$ in the $\hat{z}$ direction. The amplitude of the trapped surface wave attenuates as $\rho^{-1/2}$ in the $\hat{\rho}$ when both the source point and observation point are on or close to the interface between the air and the upper dielectric layer. The wave numbers of the trapped surface wave are the poles λ_j^*, which are between k_0 and k_2, can be evaluated by using Newton method. The lateral wave, which can be determined by the integrations along the branch cuts, is also important in the layered region.

With $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, and $\epsilon_{2r} = 4.0$, the total field, the trapped surface wave, and the DRL waves, which include the direct wave, the reflected wave, and the lateral wave for the component E_{0z}, are computed for $k_0l_1 = k_0l_2 = 0.3$ and $k_0l_1 = k_0l_2 = 1.3$ at $z = d = 0$, as shown in Figs. 5 and 6, respectively. Similarly, in the same four-layered region, the total field, the trapped surface wave, and the DRL waves are computed for $k_0l_1 = k_0l_2 = 0.3$ and $k_0l_1 = k_0l_2 = 1.3$ at $z = d = 3\,\text{m}$ and shown in Figs. 7 and 8, respectively. From Figs. 5 and 6, it is concluded that the far field is determined primarily by the trapped surface wave in the four-layered region when both the dipole point and the observation point are on or near the boundary. From Figs. 7 and 8, it is seen that the trapped surface wave can be neglected when the source point or observation point is away to the boundary between the air and the upper dielectric layer. These characteristics are similar with those in the three-layered case.

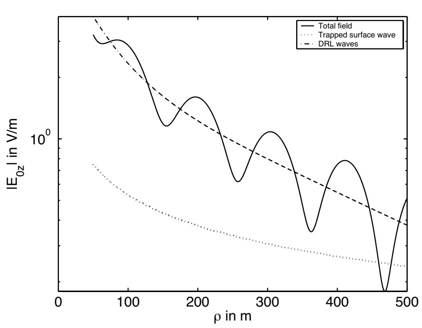

Figure 7. The electric field $|E_z|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{1r} = 4$, $k_0l_1 = k_0l_2 = 0.3$, and $z = d = 3\,\text{m}$.

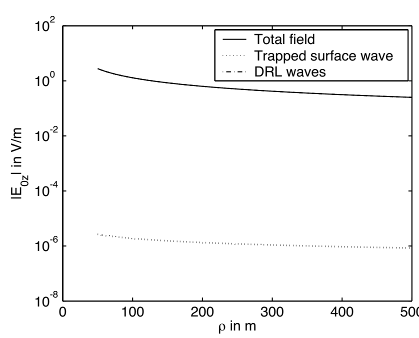

Figure 8. The electric field $|E_z|$ in V/m with $f = 100\,\text{MHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{1r} = 4$, $k_0l_1 = k_0l_2 = 1.3$, and $z = d = 3\,\text{m}$.

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