TIME DOMAIN INVERSE SCATTERING OF A
TWO-DIMENSIONAL HOMOGENEOUS DIELECTRIC
OBJECT WITH ARBITRARY SHAPE BY PARTICLE
SWARM OPTIMIZATION

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Abstract—This paper presents a computational approach to the two-
dimensional time domain inverse scattering problem of a dielectric
cylinder based on the finite difference time domain (FDTD) method
and the particle swarm optimization (PSO) to determine the shape,
location and permittivity of a dielectric cylinder. A pulse is incident
upon a homogeneous dielectric cylinder with unknown shape and
dielectric constant in free space and the scattered field is recorded
outside. By using the scattered field, the shape and permittivity of
the dielectric cylinder are reconstructed. The subgridding technique is
implemented in the FDTD code for modeling the shape of the cylinder
more closely. In order to describe an unknown cylinder with arbitrary
shape more effectively, the shape function is expanded by closed cubic-
spline function instead of frequently used trigonometric series. The
inverse problem is resolved by an optimization approach, and the global
searching scheme PSO is then employed to search the parameter space.
Numerical results demonstrate that, even when the initial guess is far
away from the exact one, good reconstruction can be obtained. In
addition, the effects of Gaussian noise on the reconstruction results are
investigated. Numerical results show that even the measured scattered
E fields are contaminated with some Gaussian noise, PSO can still yield
good reconstructed quality.

1. INTRODUCTION

The objective of the inverse scattering is to determine the
electromagnetic properties of the scatterer from scattering field
measured outside. Inverse scattering problems have attracted much
attention in the past few years. This kind of problem has several
important applications such as medical imaging, microwave remote
sensing, geophysical exploration, and nondestructive testing.

In the past twenty years, the inversion techniques are developed
intensively for the microwave imaging both in frequency domain and
time domain [1–25]. Although intrinsic ill-posedness and nonlinearity
of these problems appear consequentially in the inverse scattering
problems [26–29], the study can be applied in widespread use. Most of
the inversion techniques are investigated for the inverse problem using
only single frequency scattering data (monochromatic source) [1–11].
However, the time domain scattering data is important for the inverse
problem because the available information content about scatterer is
more than the only single frequency scattering data. Therefore, various
time domain inversion approaches are proposed [12–25] that could
be briefly classified as the layer-stripping approach [12], the iterative
approach: Born iterative method (BIM) [13–15], the distorted Born
iterative method (DBIM) [16], Local Shape Function (LSF) [17] and
optimization approach [18–21]. Traditional iterative inverse algorithms
are founded on a functional minimization via some gradient-type
scheme. In general, during the search of the global minimum, they
tend to get trapped in local minima when the initial guess is far from
the exact one. Some global optimal searching method such as genetic
algorithm [22–24], neural network [25], have be proposed to search the
global extreme of the nonlinear functional problem. In the 1995, the
Kennedy and Eberhart first proposed the particle swarm optimization
(PSO) [30]. The particle swarm optimization is a population based
stochastic optimization algorithm. It is a kind of swarm intelligence
that is based on social behavior. In recent year, some researchers
have focused on applying PSO in the inverse problem [31–35]. To the
best of our knowledge, there is still no investigation on using the PSO
to reconstruct the electromagnetic imaging of homogeneous dielectric
cylinders with arbitrary shape in free space under time domain.

In this paper, the computational methods combining the FDTD
method [36] and the PSO algorithm are presented. The forward
problem is solved by the FDTD method, for which the subgridding
technique [37] is implemented to closely describe the fine structure of
the cylinder. The inverse problem is formulated into an optimization
one and then the global searching scheme PSO is used to search
the parameter space. Cubic spline Interpolation techniques [38] are
employed to reduce the number of parameters needed to closely
describe a cylinder of arbitrary shape as compared to the Fourier
series expansion. In Section 2, the theoretical formulation for the
electromagnetic imaging is presented. The general principle of the PSO
and the way we applied them to the imaging problem are described. Numerical results for various objects of different shapes are given in Section 3. Section 4 is the conclusions.

Figure 1. Geometry for the inverse scattering of an arbitrary shape dielectric cylinder in free space.

2. THEORETICAL FORMULATION

Consider a 2-D homogeneous dielectric cylinder in a free space as shown in Figure 1. The cylinder is assumed infinite long in $z$ direction, while the cross-section of the cylinder is arbitrary. The object is illuminated by line source with Gaussian pulse located at these points denoted by $Tx$ around the scatterer. The incident waves of $TM_z$ polarization are generated by a home made FDTD code with fine grid to mimic the experimental data, and only scattered waves are recorded at those points denoted by $Rx$. The computational domain is discretized by the Yee’s cell. It should be mentioned that the computational domain is surrounded by the optimized PML absorber [39] to reduce the reflection from the air-PML interface.

2.1. Forward Problem

The direct scattering problem is to calculate the scattered electric fields while the shape, location and permittivity of the scatterer is given. The shape function $F(\theta)$ of the scatterer is approximated by
the trigonometric series in the direct scattering problem

\[ F(\theta) = \sum_{n=0}^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta) \]

In order to closely describe the shape of the cylinder for the forward scattering procedure, the subgridding technique is implemented in the FDTD code, the details are presented as follows.

2.1.1. Subgrid FDTD

A subgridding scheme is employed to divide the problem space into regions with different grid sizes. The grid size in coarse region is about \( \frac{1}{20} \sim \frac{1}{100} \lambda_{\text{max}} \) as in normal FDTD, while in the fine region the grid size is scaled by an integer ratio. As an example, the Yee cells with subgridding structure are shown in Figure 2, of which the scaling ratio is 1:3. For the time domain scattering and/or inverse scattering problem, the scatterers can be assigned with the fine region such that the fine structure can be easily described. If higher resolution is needed, only the fine region needs to be rescaled using a higher ratio for subgridding. This can avoid gridding the whole problem space using the finest resolution such that the computational resources are utilized in a more efficient way, which is quite important for the computational intensive inverse scattering problems.

![Figure 2. The structure of the TMz FDTD major grids and local grids for the scaling ratio (1:3): H fields are aligned with the MG-LG boundary.](image-url)
Because non-magnetic material is used in this study, the interfaces of the major grid and local grid (MG-LG) are set along with those lines containing the H fields as shown in Figure 2.

Since the permeability and thus the magnetic fields are continues across the MG-LG interfaces, no special treatment is required for evaluating the magnetic fields at those interfaces.

In Figure 2, E and H stand for the electric and magnetic fields on the major grids, respectively, while e and h denote the electric and magnetic fields on the local grids. If the scaling ratio is set at odd-ratio, for example 1 : 3, then the E and H fields coincide with e and h fields in the fine region and in the time domain as shown in Figure 2 and Figure 3, respectively. Figure 3 shows the corresponding update sequence for the E, H, e and h fields in the fine region of Figure 2. Since the local grid size is one third of the main grid size, the time stepping interval \( \Delta t' \) for the e and h fields on the local grids is also one third of that for the E and H fields on the main grids.

Figure 3. The update sequence for the (E, H) fields and (e, h) fields in the fine region of Figure 2.

Note that the e and h fields inside the fine region can be updated through the normal Yee-cell algorithm except those at the MG-LG boundary, such as \( h_1 \), \( h_2 \) and \( h_3 \) in Figure 2, for example.

The h fields at the MG-LG interface can be linearly interpolated as follows:

\[
\begin{align*}
h_1^{n+v} &= H_1^{n+v} + 2/3 \left( H_2^{n+v} - H_1^{n+v} \right) \\
h_2^{n+v} &= H_2^{n+v}, \quad \text{for} \quad v = \frac{1}{3}, \frac{2}{3} \text{ and } \frac{3}{3} \\
h_3^{n+v} &= H_2^{n+v} + 1/3 \left( H_3^{n+v} - H_2^{n+v} \right)
\end{align*}
\]
Note that for (1) the $H^{n+v}$ fields don’t exist on the main grids actually for $v = \frac{1}{3}$ and $\frac{2}{3}$ and need extra parabolic interpolation calculation by

$$H^{n+v} = H^n + Av + \frac{Bv^2}{2} \quad (2)$$

with

$$A = \frac{H^{n+1} - H^{n-1}}{2}$$

$$B = H^{n+1} - H^{n-1} - 2H^n$$

The flow chart associated with Figure 3 to update the fields in the fine region is shown in Figure 4. Note that at the time step $n + \frac{3}{6}$ the $E^{n+\frac{1}{2}}$ fields on the main grids should be updated by the coincided $e^{n+\frac{1}{2}}$ fields on the local grids. Similarly, at the time step $n + \frac{2}{6}$ the $H^{n+1}$ fields are updated by the coincided $h^{n+\frac{1}{2}}$ fields.

Finally, in order to avoid the unstability due to the mismatch of grid size at MG-LG interface, the $h_2$ fields right next to the $H_1$ fields of the MG-LG boundary as shown in Figure 5 are updated by

$$h_2 = \alpha h_2 + (1 - \alpha) \left( \frac{H_1 + h_3}{2} \right) \quad (3)$$

while the coincided $E_2$ and $e_2$ fields right closest to the MG-LG boundary are updated by

$$E_2 = \beta E_2 + (1 - \beta) e_2$$
$$e_2 = (1 - \beta) E_2 + \beta e_2 \quad (4)$$

where $\alpha = 0.95$ and $\beta = 0.8$ are adopted in this paper.

2.2. Inverse Problem

For the inverse scattering problem, the shape, location and permittivity of the dielectric cylinder are reconstructed by the given scattered electric field obtained at the receivers. This problem is resolved by an optimization approach, for which the global searching scheme PSO is employed to minimize the following cost function (CF):

$$CF = \sum_{n=1}^{N_n} \sum_{m=1}^{M} \sum_{t=0}^{T} \left| E_{z,exp}^{n,m}(n,m,t) - E_{z,cal}^{n,m}(n,m,t) \right|$$
$$\sum_{n=1}^{N_n} \sum_{m=1}^{M} \sum_{t=0}^{T} \left| E_{z,exp}^{n,m}(n,m,t) \right|$$
Calculation of $E$ fields and $H$ fields in main grid

Calculation of $H$ fields on the MG-LG boundary at each local-grid time step

To obtain $h$ fields of the MG-LG boundary using linear interpolation

Calculation of $e$ fields and $h$ fields in local grid

E fields and H fields at the MG-LG boundary are obtained.

When time step is equal $n+3/6$, let $E(n+1/2)=e(n+3/6)$.

When time step is equal $n+6/6$, let $H(n+1)=h(n+6/6)$.

Figure 4. The flowchart to update the $(E, H)$ fields on the major grids and $(e, h)$ fields on local grids.

where $E^{\text{exp}}_z$ and $E^{\text{cal}}_z$ are experimental electric fields and the calculated electric fields, respectively. The $N_t$ and $M$ are the total number of the transmitters and receivers, respectively. $T$ is the time duration of the recorded electric fields.

2.2.1. Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) was proposed by Kennedy and Eberhart in 1995 which is a population-based, self-adaptive search optimization technique. It is a kind of swarm intelligence that is based on social behavior. The social behavior in PSO is a population of particles moving towards the most promising region of the search space.

The PSO is initialized with a population of random solutions which assigns a randomized velocity to each potential solution, called the particle. Thus, each particle has a position and velocity vector, and moves through the problem space. In each generation, the particle changes its velocity by its best experience, called $pbest$, and that of the
best particle in the swarm, called \( g_{best} \). Assume there are \( N_{p} \) particles in the swarm that is in a search space in \( D \) dimensions, the position and velocity could be determine according to the following equations:

\[
\begin{align*}
    v_{id}^{k} &= w \cdot v_{id}^{k-1} + c_{1} \cdot \varphi_{1} \left( p_{best_id} - x_{id}^{k-1} \right) + c_{2} \cdot \varphi_{2} \left( g_{best_d} - x_{id}^{k-1} \right) \\
    x_{id}^{k} &= x_{id}^{k-1} + v_{id}^{k}
\end{align*}
\]

where \( v_{id}^{k} \) and \( x_{id}^{k} \) are the velocity and position of the \( i \)-th particle in the \( d \)-th dimension at \( k \)-th generation, \( \varphi_{1} \) and \( \varphi_{2} \) are both the random number between 0 and 1, \( c_{1} \) and \( c_{2} \) are learning coefficients and \( w \) is the inertial weighting factor that can avoid the particle trapped into the local minimized solution. After generations, the PSO can find the best solution according to the best solution experience.

\subsection*{2.2.2. Cubic Spline Interpolation Technique}

In order to reduce the unknowns required to describe the arbitrary cylinder, the shape function of the cylinder is expressed in terms of a cubic spline. As shown in Figure 6, the cubic spline consists of the polynomials of degree 3 \( P_{i}(\theta) \), \( i = 1, 2, \ldots, N \), which satisfy the following smooth conditions:

\[
\begin{align*}
    P_{i}(\theta_{i}) &= P_{i+1}(\theta_{i}) = \rho_{i} \\
    P'_{i}(\theta_{i}) &= P'_{i+1}(\theta_{i}) \\
    P''_{i}(\theta_{i}) &= P''_{i+1}(\theta_{i})
\end{align*}
\]
and

\[ P_1(\theta_0) = P_N(\theta_N) \]
\[ P'_1(\theta_0) = P'_N(\theta_N) = \rho'_N \]
\[ P''_N(\theta_i) = P''_N(\theta_N) \]

Through the interpolation of the cubic spline, an arbitrary smooth cylinder can be easily described through a few parameters \( \rho_1, \rho_2, \ldots, \rho_N \) and the slope \( \rho'_N \), the details are referred to [38]. By combining the PSO and the cubic spline interpolation technique, we are able to reconstruct the microwave image efficiently.

3. NUMERICAL RESULTS

As shown in Figure 1, the problem space is divided in \( 68 \times 68 \) grids with the grid size \( \Delta x = \Delta y = 1.47 \) cm. The homogeneous dielectric cylinder is located in free space. The cylindrical object is illuminated by a transmitter at four different positions, \( N_i = 4 \). The scattered E fields for each illumination are collected at the eight receivers, \( M = 8 \). The transmitters and receivers are collocated at a distance of 24 grids from the origin. The excitation waveform \( I_z(t) \) of the transmitter is the Gaussian pulse, given by:

\[
I_z(t) = \begin{cases} 
A e^{-\alpha (t - \beta \Delta t)^2}, & t \leq T_w \\
0, & t > T_w 
\end{cases}
\]
where $\beta = 24$, $A = 1000$, $\Delta t = 34.685\, \text{ps}$, $T_w = 2\beta \Delta t$, and $\alpha = \left(\frac{1}{4\beta \Delta t}\right)^2$.

The time duration $T$ is set to $300\Delta t$. Note that in order to accurately describe the shape of the cylinder, the subgridding FDTD technique is used both in the forward scattering (1:9) and the inverse scattering (1:5) parts — but with different scaling ratios as indicated in the parentheses. For the forward scattering, the E fields generated by the FDTD with fine subgrids are used to mimic the experimental data in (5).

Three examples are investigated for the inverse scattering of the proposed structure by using the PSO. There are twelve unknown parameters to retrieve, which include the center position ($X_0, Y_0$), the radius $\rho_i$, $i = 1, 2, \ldots, 8$ of the shape function and the slope $\rho'_N$ plus the relative permittivity of the object, $\varepsilon_r = \varepsilon_2/\varepsilon_0$. Very wide searching ranges are used for the PSO to optimize the cost function given by (5). The parameters and the corresponding searching ranges are listed as follow: $-30.88\, \text{cm} \leq X_0 \leq 30.88\, \text{cm}$, $-30.88\, \text{cm} \leq Y_0 \leq 30.88\, \text{cm}$, $0\, \text{cm} \leq \rho_i \leq 11.8\, \text{cm}$, $i = 1, 2, \ldots, 8$, $-1 \leq \rho'_N \leq 1$ and $1 \leq \varepsilon_r \leq 15$. The relative coefficient of the PSO are set as below: The learning coefficients, $c_1$ and $c_2$, are both set to 2. The inertial weighting factor is set to 0.4 and the population size set to 120.

**Figure 7.** The average fitness value versus generation for example 1 using the Gaussian pulse illumination as the PSO executed five times.
The first example, a simple circular cylinder is tested, of which the shape function \( F(\theta) \) is chosen to be \( F(\theta) = 7.352 \text{ cm} \), and the relative permittivity of the object is \( \varepsilon_r = 3 \). The average convergence curve of cost function versus generation as the PSO executed five times is shown in Figure 7. The reconstructed shape function of the best population member (particle) is plotted in Figure 8 for different generation. The final reconstructed shape and the cylinder position \((X_0^c, Y_0^c)\) are compares to the exact shape and the position \((X_0, Y_0)\) in Figure 9. It is observed that even reconstructed cylinder position \((X_0^c, Y_0^c)\) is far away from exact one, the cubic spline interpolation technique can still recover it well. The r.m.s. error \((DF)\) of the reconstructed shape \( F^{cal}(\theta) \) and the relative error \((DIPE)\) of \( \varepsilon_r^{cal} \) with respect to the exact values versus generation are shown in Figure 10. Here, DF and DIPE are defined as

\[
DF = \left\{ \frac{1}{N'} \sum_{i=1}^{N'} \left[ F^{cal}(\theta_i) - F(\theta_i) \right]^2 / F^2(\theta_i) \right\}^{1/2} \tag{11}
\]

\[
DIPE = \frac{\left| \varepsilon_r^{cal} - \varepsilon_r \right|}{\varepsilon_r} \tag{12}
\]

where the \( N' \) is set to 160. The r.m.s. error DF is about 1.77\% and DIPE= 0.97\% in final. It is seen that the reconstruction is good.
Figure 9. The final reconstructed shape and center position of the cylinder compared to the exact ones for example 1.

Figure 10. Shape-function error and permittivity error at each generation of example 1.
In the second example, the dielectric cylinder with shape function 
\[ F(\theta) = 5.88 - 1.47 \sin(2\theta) \text{ cm} \] and relative permittivity \( \varepsilon_r = 3.5 \) is considered. The reconstructed images for different generations and the relative error of this object are shown in Figure 11 and Figure 12, respectively. The r.m.s. error DF is about 4.8% and DIPE=1.8% in final generation. From the reconstructed result of this object, we conclude the proposed method can be used to reconstruct dielectric cylinder successfully when the dielectric object with high-contrast permittivity.

![Figure 11](image.png)

**Figure 11.** The reconstructed cross section of the cylinder of example 2 at sequential generations.

The reconstructed result of the final example is shown in Figure 13, where the shape is 
\[ F(\theta) = 5.88 + 1.47 \cos(\theta) + 2.94 \cos(3\theta) \text{ cm} \] and the relative permittivity of the object is \( \varepsilon_r = 3.2 \). Figure 14 shows that the relative errors of the shape and the permittivity decrease quickly by generations. The r.m.s. error DF is about 4.45% and DIPE = 1.5%. In order to investigate the sensitivity of the imaging algorithm against random noise, the additive white Gaussian noise of zero mean is added into the experimental electric fields. The signal to noise ratio (SNR)
Figure 12. Shape-function error and permittivity error at different generations of example 2.

Figure 13. The reconstructed cross section of the cylinder of example 3 at sequential generations.
Figure 14. Shape-function error and permittivity error at different generations of example 3.

Figure 15. Shape error and the relative permittivity errors as functions of SNR (dB).
is defined as:

\[
SNR = 20 \log_{10} \frac{\sum_{n=1}^{N_i} \sum_{m=1}^{M} \sum_{t=0}^{T} |E_{\text{exp}}(n, m, t)|}{\sum_{n=1}^{N_i} \sum_{m=1}^{M} \sum_{t=0}^{T} |\text{noise}(n, m, t)|}
\] (13)

Figure 15 shows the reconstructed results under the condition that the experimental scattered field is contaminated by noise. The SNR include 40 dB, 30 dB, 20 dB, 10 dB, 7 dB and 3 dB for simulation purpose. It could be observed that good reconstruction has been obtained for both the relative permittivity and shape of the dielectric cylinder when the SNR is above 10 dB.

4. CONCLUSION

We present a study of the time domain inverse scattering of an arbitrary cross section dielectric cylinder in free space. By combining the FDTD method and the PSO, good reconstructed results are obtained by using Gaussian pulse illuminations. The subgridding scheme is employed to closely describe the shape of the cylinder for the FDTD method. Some stabilization techniques to avoid the mismatch at the MG-LG interface are adopted. The forward problem is solved by using the subgridding FDTD method and the shape function of the cylinder is approximated by Fourier series expansion. The inverse problem is reformulated into an optimization one, and then the global searching scheme PSO is employed to search the parameter space. Interpolation technique through cubic spline is utilized to reduce the number of parameters needed to describe an arbitrary shape. By using the PSO, the shape, location and dielectric constant of the object can be successfully reconstructed even when the dielectric constant is fairly large. In our study, even when the initial guess is far from the exact one, the PSO can still yield a good solution for the properties of the object, while the gradient-based methods often get stuck in a local extreme. Numerical results have been carried out and good reconstruction has been obtained even in the presence of white Gaussian noise in experimental data.
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