

## **A NOTE ON SPHERICAL ELECTROMAGNETIC WAVE DIFFRACTION BY A PERFECTLY CONDUCTING STRIP IN A HOMOGENEOUS BI-ISOTROPIC MEDIUM**

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**Abstract**—We have studied the problem of diffraction of an electromagnetic spherical wave by a perfectly conducting finite strip in a homogeneous bi-isotropic medium and obtained some improved results. The problem was solved by using the Wiener-Hopf technique and Fourier transform. The scattered field in the far zone was determined by the method of steepest descent. The significance of the present analysis was that it recovered the results when a strip was widened into a half plane.

### **1. INTRODUCTION**

Beltrami flows were first introduced in the late 19th century [1, 2]. There was no significant work on Beltrami flows for next 60 years. However, in 1950s and onwards it gained wide application in fluid mechanics and other related areas. Chandrasekhar [3], re-introduced Beltrami flows and worked on force free magnetic fields. Lakhtakia [4] compiled a catalogue on contemporary works.

A Beltrami field is proportional to its own curl everywhere in a source-free region and can be either left-handed or right-handed. For the analysis of time-harmonic electromagnetic fields in isotropic chiral and bi-isotropic media, Bohren [5] was the pioneer and his work was enhanced by Lakhtakia [6]. Lakhtakia [7], and Lakhtakia and Weiglhofer [8] worked on the application of Beltrami field to time dependent electromagnetic field. On chiral wedges, Fisanov [9] and Przedziński [10] did exceptional job. Asghar and Lakhtakia [11] showed that the concept of Beltrami fields can be exploited to calculate the diffraction of only one scalar field and the rest can be obtained thereof.

A Beltrami magnetostatic field exerts no Lorentz force on an electrically charged particle, and for this reason the concept has been extensively used in astrophysics as well as magnetohydrodynamics [12, 13]. Beltrami fields also occur as the circularly polarized plane waves in electromagnetic theory [14]. Although circularly polarized plane waves in free space and natural, optically active media [15, 16] have been known since the time of Fresnel, their theoretical value is best expressed in biisotropic media [17–22]. In recent years, propagation of plane waves with negative phase velocity and its related applications in isotropic chiral materials can be found in [23–26]. It may be noted that scattering from strips, slits, half plane, impedance surfaces and study of high frequency diffraction are topics of current interest [37–48].

In this paper, the diffracted field due to a spherical electromagnetic wave by a perfectly conducting finite strip in a homogeneous bi-isotropic medium is obtained in an improved form by solving two uncoupled Wiener-Hopf equations. The significance of the present analysis is that the results of half plane [36] can be deduced by taking an appropriate limit  $l \rightarrow \infty$  whereas this is not possible in [32]. It is found that the two edges of the strip give rise to two diffracted fields (one from each edge) and an interaction field (double diffraction of two edges). A similar analysis [31] for the case of a cylindrical wave has recently been accepted in Journal of Modern Optics (article in press).

## 2. PROBLEM STATEMENT

Let all space is occupied by a homogeneous bi-isotropic medium with the exception of the perfectly conducting strip  $z = 0$ ,  $-l \leq x \leq 0$ . In the Fedorov representation [6, 35], the bi-isotropic medium is characterized by the following equations

$$\mathbf{D} = \varepsilon \mathbf{E} + \varepsilon \alpha \nabla \times \mathbf{E}, \quad (1)$$

$$\mathbf{B} = \mu \mathbf{H} + \mu \beta \nabla \times \mathbf{H}, \quad (2)$$

where  $\varepsilon$  and  $\mu$  are the permittivity and the permeability scalars, respectively, while  $\alpha$  and  $\beta$  are the bi-isotropy scalars.  $\mathbf{D}$  is the electric displacement,  $\mathbf{H}$  is the magnetic field strength,  $\mathbf{B}$  is the magnetic induction, and  $\mathbf{E}$  is the electric field strength. The bi-isotropic medium with  $\alpha = \beta$  is reciprocal and is then called a chiral medium. Recently, it has been proved [27] that non-reciprocal bi-isotropic media are not permitted by the structure of modern electromagnetic theory. Certainly in the MHz-PHz regime, this statement has not been experimentally challenged yet, although in the  $< 1$  kHz regime there is some experimental evidence to the contrary which has not been

independently confirmed [34]. However, in the mathematical study the case  $\alpha \neq \beta$  may also be considered for generality.

Let us assume the time dependence of Beltrami fields to be of the form  $\exp(-i\omega t)$ , where  $\omega$  is the angular frequency. The source free Maxwell curl postulates in the bi-isotropic medium can be set up as

$$\nabla \times \mathbf{Q}_1 = \gamma_1 \mathbf{Q}_1, \quad (3)$$

$$\nabla \times \mathbf{Q}_2 = -\gamma_2 \mathbf{Q}_2. \quad (4)$$

The two wave numbers  $\gamma_1$  and  $\gamma_2$  are given by

$$\gamma_1 = \frac{k}{(1 - k^2\alpha\beta)} \left\{ \sqrt{1 + \frac{k^2(\alpha - \beta)^2}{4}} + \frac{k(\alpha + \beta)}{2} \right\}, \quad (5)$$

and

$$\gamma_2 = \frac{k}{(1 - k^2\alpha\beta)} \left\{ \sqrt{1 + \frac{k^2(\alpha - \beta)^2}{4}} - \frac{k(\alpha + \beta)}{2} \right\}, \quad (6)$$

where Beltrami fields in terms of the electric field  $\mathbf{E}$  and the magnetic field  $\mathbf{H}$ , as given in [28], are :

$$\mathbf{Q}_1 = \frac{\eta_1}{\eta_1 + \eta_2} (\mathbf{E} + i\eta_2 \mathbf{H}), \quad (7)$$

and

$$\mathbf{Q}_2 = \frac{i}{\eta_1 + \eta_2} (\mathbf{E} - i\eta_1 \mathbf{H}), \quad (8)$$

where  $\mathbf{Q}_1$  is the left-handed Beltrami field and  $\mathbf{Q}_2$  is the right-handed Beltrami field. In Eqs. (7) and (8), the two impedances  $\eta_1$  and  $\eta_2$  are given by

$$\eta_1 = \frac{\eta}{\sqrt{1 + \frac{k^2(\alpha - \beta)^2}{4}} + \frac{k(\alpha - \beta)}{2}}, \quad (9)$$

and

$$\eta_2 = \eta \left\{ \sqrt{1 + \frac{k^2(\alpha - \beta)^2}{4}} - \frac{k(\alpha - \beta)}{2} \right\}, \quad (10)$$

where  $k = \omega \sqrt{\varepsilon \mu}$  and  $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ .

Since we are interested in scattering of electromagnetic waves with a prescribed  $y$ -variation, therefore, it is appropriate to decompose the Beltrami fields as [29].

$$\mathbf{Q}_1 = \mathbf{Q}_{1t} + yQ_{1y}, \quad (11)$$

with

$$\mathbf{Q}_{1t} = Q_{1x}\mathbf{i} + Q_{1z}\mathbf{k}. \quad (12)$$

and

$$\mathbf{Q}_2 = \mathbf{Q}_{2t} + yQ_{2y}. \quad (13)$$

where the fields  $\mathbf{Q}_{1t}$  and  $\mathbf{Q}_{2t}$  lie in the  $xz$ -plane and  $\mathbf{j}$  is a unit vector along the  $y$ -axis such that  $\mathbf{j} \cdot \mathbf{Q}_{1t} = 0$  and  $\mathbf{j} \cdot \mathbf{Q}_{2t} = 0$ . Now, from Eqs. (3) and (4)

$$Q_{1x} = \frac{1}{\gamma_1^2} \left[ \frac{\partial^2 Q_{1y}}{\partial y \partial x} - \frac{\partial^2 Q_{1x}}{\partial y^2} \right] - \frac{1}{\gamma_1} \frac{\partial Q_{1y}}{\partial z}, \quad (14)$$

$$Q_{1z} = \frac{1}{\gamma_1^2} \left[ \frac{\partial^2 Q_{1y}}{\partial y \partial z} - \frac{\partial^2 Q_{1z}}{\partial y^2} \right] + \frac{1}{\gamma_1} \frac{\partial Q_{1y}}{\partial x}, \quad (15)$$

$$Q_{2x} = \frac{1}{\gamma_2^2} \left[ \frac{\partial^2 Q_{2y}}{\partial y \partial x} - \frac{\partial^2 Q_{2x}}{\partial y^2} \right] + \frac{1}{\gamma_2} \frac{\partial Q_{2y}}{\partial z}, \quad (16)$$

$$Q_{2z} = \frac{1}{\gamma_2^2} \left[ \frac{\partial^2 Q_{2y}}{\partial y \partial z} - \frac{\partial^2 Q_{2z}}{\partial y^2} \right] - \frac{1}{\gamma_2} \frac{\partial Q_{2y}}{\partial x}. \quad (17)$$

It is sufficient to explore the scattering of the scalar field  $Q_{1y}$  and  $Q_{2y}$  because the other components of  $\mathbf{Q}_1$  and  $\mathbf{Q}_2$  can then be completely determined by using Eqs. (14)–(17).

Now using the constitutive relations (1) and (2), the Maxwell curl postulates  $\nabla \times \mathbf{E} = i\omega\mathbf{B} - \mathbf{K}$  and  $\nabla \times \mathbf{H} = -i\omega\mathbf{D} + \mathbf{J}$  may be written as:

$$\nabla \times \mathbf{Q}_1 - \gamma_1 \mathbf{Q}_1 = \mathbf{S}_1, \quad (18)$$

$$\nabla \times \mathbf{Q}_2 - \gamma_2 \mathbf{Q}_2 = \mathbf{S}_2, \quad (19)$$

where  $\mathbf{S}_1$  and  $\mathbf{S}_2$  are the corresponding source densities and are given by

$$\mathbf{S}_1 = \frac{\eta_1}{\eta_1 + \eta_2} \left( \frac{i\gamma_1}{\omega\epsilon} \mathbf{J} - (1 + \alpha\gamma_1) \mathbf{K} \right), \quad (20)$$

$$\mathbf{S}_2 = \frac{\eta_1}{\eta_1 + \eta_2} \left( -\frac{i\gamma_2}{\omega\mu} \mathbf{K} - (1 + \beta\gamma_2) \mathbf{J} \right). \quad (21)$$

In deriving Eqs. (20) and (21), we have used the following relations

$$1 + \omega\varepsilon\alpha\eta_2 = (1 - k^2\alpha\beta)(1 + \alpha\gamma), \quad (22)$$

$$1 - \omega\varepsilon\alpha\eta_1 = (1 - k^2\alpha\beta)\eta_1 \frac{\gamma_2}{\omega\mu}, \quad (23)$$

$$\eta_2 + \omega\mu\beta = (1 - k^2\alpha\beta) \frac{\gamma_1}{\omega\varepsilon}, \quad (24)$$

$$\eta_1 - \omega\mu\beta = (1 - k^2\alpha\beta)\eta_1(1 - \beta\gamma_2). \quad (25)$$

Furthermore,  $\mathbf{Q}_1$  is  $\mathbf{E}$  like and  $\mathbf{Q}_2$  is  $\mathbf{H}$  like. Similarly  $\mathbf{S}_1$  is  $\mathbf{K}$  like and  $\mathbf{S}_2$  is  $\mathbf{J}$  like where  $\mathbf{J}$  and  $\mathbf{K}$  are the electric and magnetic source current densities, respectively. Since electric field vanishes on a perfectly conducting surface. Therefore, the boundary conditions on a perfectly conducting finite-plane in terms of the electric field components take the form  $E_x = E_y = 0$ , for  $z = 0$ ,  $-l \leq x \leq 0$ . Using this fact in Eqs. (7) and (8), the boundary conditions on the finite plane take the form

$$Q_{1y} - i\eta_2 Q_{2y} = 0, \quad z = 0, \quad -l \leq x \leq 0, \quad (26a)$$

and

$$Q_{1x} - i\eta_2 Q_{2x} = 0, \quad z = 0, \quad -l \leq x \leq 0. \quad (26b)$$

With the help of Eqs. (14) and (15), Eq. (26b) becomes

$$\begin{aligned} & \frac{1}{\left(\gamma_1^2 + \frac{\partial^2}{\partial y^2}\right)} \left[ \frac{\partial^2 Q_{1y}}{\partial y \partial x} - \gamma_1 \frac{\partial Q_{1y}}{\partial z} \right] \\ & - i\eta_2 \frac{1}{\left(\gamma_2^2 + \frac{\partial^2}{\partial y^2}\right)} \left[ \frac{\partial^2 Q_{2y}}{\partial y \partial x} + \gamma_2 \frac{\partial Q_{2y}}{\partial z} \right] = 0, \quad z = 0, \quad -l \leq x \leq 0. \end{aligned} \quad (27)$$

Thus the scalar fields  $Q_{1y}$  and  $Q_{2y}$  satisfy the boundary conditions (26a) and (27). Now, eliminating  $Q_{2y}$  from Eqs. (26a) and (27), we obtain

$$\frac{\partial^2 Q_{1y}}{\partial y \partial x} \mp \frac{1}{(\gamma_2 - \gamma_1)} \left[ \gamma_1 \gamma_2 + \frac{\partial^2}{\partial y^2} \right] \frac{\partial Q_{1y}}{\partial z} = 0, \quad z = 0^\pm, \quad -l \leq x \leq 0, \quad (28a)$$

It is worthwhile to note that the boundary conditions (28a) are of the same form as impedance boundary conditions [30]. We observe that there is no boundary for  $-\infty < x < -l$ ,  $x > 0$ ,  $z = 0$ . Therefore the continuity conditions are given by

$$Q_{1y}(x, y, z^+) = Q_{1y}(x, y, z^-); \quad -\infty < x < -l, \quad x > 0, \quad z = 0, \quad (28b)$$

$$\frac{\partial Q_{1y}(x, y, z^+)}{\partial z} = \frac{\partial Q_{1y}(x, y, z^-)}{\partial z}; \quad -\infty < x < -l, \quad x > 0, \quad z = 0. \quad (28c)$$

The edge conditions (local properties) on the field that invoke the appropriate physical constraint of finite energy near the edges of the boundary discontinuities require that

$$Q_{1y}(x, y, 0) = O(1) \text{ and } \frac{\partial Q_{1y}(x, y, 0)}{\partial z} = O\left(x^{-\frac{1}{2}}\right) \text{ as } x \rightarrow 0^+, \quad (29a)$$

$$Q_{1y}(x, y, 0) = O(1) \text{ and } \frac{\partial Q_{1y}(x, y, 0)}{\partial z} = O(x+l)^{-\frac{1}{2}} \text{ as } x \rightarrow -l. \quad (29b)$$

Finally, the scattered field must satisfy the radiation conditions in the limit  $(x^2 + y^2 + z^2)^{1/2} \rightarrow \infty$ . We must also observe at this juncture that, in effect, we need to consider the diffraction of only one scalar field, that is either  $Q_{1y}$  or  $Q_{2y}$ , at a time, but the presence of the other scalar field is reflected in the complicated nature of the boundary condition (28a).

Now we consider a point source which is located at  $(x_0, y_0, z_0)$ . Then, the scalar fields  $Q_{1y}$  and  $Q_{2y}$  in the presence of a point source of strength  $S_{01}$  and  $S_{02}$ , respectively, satisfy the reduced scalar equations [27]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)Q_{1y} + \gamma_1^2 Q_{1y} = S_{01}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0), \quad (30a)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)Q_{2y} + \gamma_2^2 Q_{2y} = S_{02}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0). \quad (30b)$$

### 3. THE WIENER-HOPF EQUATIONS

The Fourier transform and its inverse over the variable  $y$  are defined respectively as:

$$\tilde{Q}_{1y}(x, s, z) = \int_{-\infty}^{\infty} Q_{1y}(x, y, z)e^{-i\gamma_1 sy} dy, \quad (31a)$$

$$Q_{1y}(x, y, z) = \frac{\gamma_1}{2\pi} \int_{-\infty}^{\infty} \tilde{Q}_{1y}(x, s, z)e^{i\gamma_1 sy} ds. \quad (31b)$$

In Eqs. (31a) and (31b), the transform parameter is taken conveniently to be  $\gamma_1 s$ ,  $s$  is non-dimensional. In addition  $\gamma_1$  must be complex for causality, as remarked earlier and has a small positive imaginary part. Transforming Eq. (30a) and the boundary conditions (28a), (28b) and (28c) with respect to  $y$  by using Eq. (31a), we obtain

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \chi_1^2 \gamma_1^2\right)\tilde{Q}_{1y}(x, s, z) = \tilde{a}\delta(x-x_0)\delta(z-z_0), \quad (32)$$

$$\frac{\partial}{\partial x} \tilde{Q}_{1y}(x, s, z) \mp \lambda \frac{\partial}{\partial z} \tilde{Q}_{1y}(x, s, z) = 0, \quad z = 0^\pm, \quad -l \leq x \leq 0, \quad (33a)$$

$$\tilde{Q}_{1y}(x, s, z^+) = \tilde{Q}_{1y}(x, s, z^-); \quad -\infty < x < -l, \quad x > 0, \quad z = 0, \quad (33b)$$

$$\frac{\partial}{\partial z} \tilde{Q}_{1y}(x, s, z^+) = \frac{\partial}{\partial z} \tilde{Q}_{1y}(x, s, z^-); \quad -\infty < x < -l, \quad x > 0, \quad z = 0. \quad (33c)$$

where

$$\tilde{a} = S_{01} e^{-i\gamma_1 s y_0}, \quad \chi_1^2 = 1 - s^2, \quad \lambda = \frac{(\gamma_2 - \gamma_1 s^2)}{is((\gamma_2 - \gamma_1))} \quad (34)$$

A solution of Eq. (32) can be written in the form

$$Q_{1y}(x, s, z) = \tilde{Q}_{1y}^{inc}(x, s, z) + \tilde{Q}_{1y}^{sca}(x, s, z), \quad (35)$$

where  $\tilde{Q}_{1y}^{inc}(x, s, z)$  is the solution of inhomogeneous wave Eq. (32), that corresponds to the incident wave and  $\tilde{Q}_{1y}^{sca}(x, s, z)$  is diffracted field corresponding to the solution of the homogeneous Eq. (32). Thus,  $\tilde{Q}_{1y}^{inc}(x, s, z)$  and  $\tilde{Q}_{1y}^{sca}(x, s, z)$  satisfy the following equations:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \chi_1^2 \gamma_1^2 \right) \tilde{Q}_{1y}^{inc}(x, s, z) = \tilde{a} \delta(x - x_0) \delta(z - z_0), \quad (36)$$

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \chi_1^2 \gamma_1^2 \right) \tilde{Q}_{1y}^{sca} = 0, \quad (37)$$

$$\begin{aligned} & \left( \frac{\partial}{\partial x} \mp \lambda \frac{\partial}{\partial z} \right) \tilde{Q}_{1y}^{inc}(x, s, 0^\pm) \\ & + \left( \frac{\partial}{\partial x} \mp \lambda \frac{\partial}{\partial z} \right) \tilde{Q}_{1y}^{sca}(x, s, 0^\pm) = 0, \quad -l \leq x \leq 0, \end{aligned} \quad (38a)$$

$$\tilde{Q}_{1y}^{sca}(x, s, z^+) = \tilde{Q}_{1y}^{sca}(x, s, z^-); \quad -\infty < x < -l, \quad x > 0, \quad z = 0, \quad (38b)$$

$$\frac{\partial}{\partial z} \tilde{Q}_{1y}^{sca}(x, s, z^+) = \frac{\partial}{\partial z} \tilde{Q}_{1y}^{sca}(x, s, z^-); \quad -\infty < x < -l, \quad x > 0, \quad z = 0. \quad (38c)$$

Now, we define the Fourier transform  $\bar{\Psi}(v, s, z)$  of  $\hat{Q}_{1y}^{sca}(x, s, z)$  as

$$\begin{aligned} \bar{\Psi}(v, s, z) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{Q}_{1y}^{sca}(x, s, z) e^{ivx} dx \\ &= \bar{\Psi}_+(v, s, z) + e^{-ivl} \bar{\Psi}_-(v, s, z) + \bar{\Psi}_1(v, s, z), \end{aligned} \quad (39)$$

where

$$\begin{aligned}\bar{\Psi}_+(v, s, z) &= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \hat{Q}_{1y}^{sca}(x, s, z) e^{ivx} dx, \\ \bar{\Psi}_-(v, s, z) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-l} \hat{Q}_{1y}^{sca}(x, s, z) e^{iv(x+l)} dx, \\ \bar{\Psi}_1(v, s, z) &= \frac{1}{\sqrt{2\pi}} \int_{-l}^0 \hat{Q}_{1y}^{sca}(x, s, z) e^{ivx} dx,\end{aligned}\quad (40)$$

where  $\bar{\Psi}_-(v, s, z)$  is regular for  $\text{Im}v < \text{Im}(\chi_1\gamma_1)$ , and  $\bar{\Psi}_+(v, s, z)$  is regular for  $\text{Im}v > -\text{Im}(\chi_1\gamma_1)$  and  $\bar{\Psi}_1(v, s, z)$  is an integral function and therefore,  $\bar{\Psi}_1(v, s, z)$  is analytic in the common region  $-\text{Im}(\chi_1\gamma_1) < \text{Im}v < \text{Im}(\chi_1\gamma_1)$ . The solution of Eq. (32) can be written as

$$\begin{aligned}\hat{Q}_{1y}^{inc}(x, s, z) &= -\frac{\hat{a}}{4i} H_0^{(1)} \left[ \chi_1\gamma_1 \sqrt{(x-x_0)^2 + (z-z_0)^2} \right], \\ &= \frac{\hat{a}}{4\pi i} \int_{-\infty}^{\infty} \frac{e^{-iv(x-x_0)+i\kappa|z-z_0|}}{\kappa} dv,\end{aligned}\quad (41)$$

where

$$\kappa^2 = (\chi_1^2\gamma_1^2 - v^2), \quad \text{Im}\kappa > 0, \quad (41a)$$

and  $H_0^{(1)}(\cdot)$  is the Hankel function of order zero and first type and its asymptotic expansion is given by

$$H_0^{(1)}(M) = \left( \frac{2}{\pi M} \right)^{\frac{1}{2}} e^{i[M - (\frac{\pi}{4})]}. \quad (41b)$$

The incident wave in the cartesian coordinates can now be written by taking an inverse Fourier transform of Eq. (41) over the parameter  $\gamma_1 s$  as

$$Q_{1y}^{inc}(x, y, z) = S_{01} \frac{e^{i\gamma_1 r_1}}{4\pi r_1}, \quad (42)$$

where

$$r_1 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}. \quad (42a)$$

Taking the Fourier transform of Eq. (37) with respect to  $x$ , we obtain

$$\left(\frac{d^2}{dz^2} + \kappa^2\right)\bar{\Psi}(v, s, z) = 0, \quad (43)$$

The solution of Eq. (43) satisfying radiation conditions is given by

$$\bar{\Psi}(v, s, z) = \begin{cases} A(v)e^{i\kappa z} & \text{if } z > 0, \\ C(v)e^{-i\kappa z} & \text{if } z < 0. \end{cases} \quad (44)$$

Transforming the boundary conditions (38b) and (38c), we have

$$\left. \begin{aligned} \bar{\Psi}_{\pm}(v, s, 0^+) &= \bar{\Psi}_{\pm}(v, s, 0^-) = \bar{\Psi}_{\pm}(v, s, 0), \\ \bar{\Psi}'_{\pm}(v, s, 0^+) &= \bar{\Psi}'_{\pm}(v, s, 0^-) = \bar{\Psi}'_{\pm}(v, s, 0), \end{aligned} \right\} \quad (45)$$

where the primes denote differentiation with respect to the variable  $z$ . Now making change of variables

$$z_0 = r_0 \sin \phi, \quad x_0 = r_0 \cos \phi, \quad \pi \leq \phi \leq \frac{3\pi}{2},$$

in Eq. (41) and when  $r_0 \rightarrow \infty$ , the source recedes to infinity, we obtain, using the asymptotic expansion of Hankel function

$$\widehat{Q}_{1y}^{inc}(x, s, z) = b(s)e^{i(k_{1x}x + k_{1z}z)}, \quad (46a)$$

where

$$b(s) = -\frac{e^{-i\gamma_1 s y_0}}{4i} \sqrt{\frac{2}{\pi \chi_1 \gamma_1 r_0}} \exp\left[i\left(\chi_1 \gamma_1 r_0 - \frac{\pi}{4}\right)\right], \quad (46b)$$

$$k_{1x} = -\chi_1 \gamma_1 \cos \phi, \quad k_{1z} = -\chi_1 \gamma_1 \sin \phi. \quad (46c)$$

The Fourier transform of the incident wave (46a) in the region  $-l \leq x \leq 0$  gives

$$\bar{\Psi}_1^{inc}(v, s, 0) = \frac{ib(s)}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]]. \quad (47)$$

Taking the Fourier transform of Eq. (38a) and using Eq. (47), we get

$$\begin{aligned} & -iv\bar{\Psi}_1(v, s, 0^+) - \lambda\bar{\Psi}'_1(v, s, 0^+) \\ & - \frac{(k_{1x} - \lambda k_{1z})b(s)}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] = 0, \end{aligned} \quad (48)$$

$$\begin{aligned} & -iv\bar{\Psi}_1(v, s, 0^-) + \lambda\bar{\Psi}'_1(v, s, 0^-) \\ & - \frac{(k_{1x} + \lambda k_{1z})b(s)}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] = 0. \end{aligned} \quad (49)$$

Using Eq. (39) and (45) in Eq. (44), we have

$$\left. \begin{aligned} \bar{\Psi}_+(v, s, 0) + e^{-ivl}\bar{\Psi}_-(v, s, 0) + \bar{\Psi}_1(v, s, 0^+) &= A(v), \\ \bar{\Psi}_+(v, s, 0) + e^{-ivl}\bar{\Psi}_-(v, s, 0) + \bar{\Psi}_1(v, s, 0^-) &= C(v), \\ \bar{\Psi}'_+(v, s, 0) + e^{-ivl}\bar{\Psi}'_-(v, s, 0) + \bar{\Psi}'_1(v, s, 0^+) &= i\kappa A(v), \\ \bar{\Psi}'_+(v, s, 0) + e^{-ivl}\bar{\Psi}'_-(v, s, 0) + \bar{\Psi}'_1(v, s, 0^-) &= -i\kappa C(v). \end{aligned} \right\} \quad (50)$$

The unknown function  $A(v)$  and  $C(v)$  can be obtained from Eq. (50) as

$$A(v) = J_1(v, s, 0) + \frac{J'_1(v, s, 0)}{i\kappa}, \quad (51)$$

and

$$C(v) = -J_1(v, s, 0) + \frac{J'_1(v, s, 0)}{i\kappa}, \quad (52)$$

where

$$J_1(v, s, 0) = \frac{1}{2} [\bar{\Psi}_1(v, s, 0^+) - \bar{\Psi}_1(v, s, 0^-)], \quad (53)$$

and

$$J'_1(v, s, 0) = \frac{1}{2} [\bar{\Psi}'_1(v, s, 0^+) - \bar{\Psi}'_1(v, s, 0^-)]. \quad (54)$$

From Eqs. (48)–(50), we obtain

$$\begin{aligned} &\bar{\Psi}'_+(v, s, 0) + e^{-ivl}\bar{\Psi}'_-(v, s, 0) - i\kappa L(v)J_1(v, s, 0) \\ &+ \frac{b(s)k_{1z}}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] = 0, \end{aligned} \quad (55)$$

and

$$\begin{aligned} &-iv\bar{\Psi}_+(v, s, 0) - ive^{-ivl}\bar{\Psi}_-(v, s, 0) \\ &+ \lambda L(v)J'_1(v, s, 0) + \frac{b(s)k_{1x}}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] = 0. \end{aligned} \quad (56)$$

#### 4. SOLUTION OF WIENER-HOPF EQUATIONS

In Eqs. (55) and (56)

$$L(v) = \left(1 + \frac{v}{\lambda\kappa}\right). \quad (57)$$

Eqs. (55) and (56) are the standard Wiener-Hopf equations. Let us proceed to find the solution for these equations. For the solution of Wiener-Hopf functional equations, the functions  $L(v)$  and  $\kappa(v)$  can be factorized as

$$L(v) = \left(1 + \frac{v}{\lambda\kappa}\right) = L_+(v)L_-(v), \quad (58a)$$

also

$$\kappa(v) = \kappa_+(v)\kappa_-(v) = (\chi_1\gamma_1 + v)^{\frac{1}{2}}(\chi_1\gamma_1 - v)^{\frac{1}{2}}, \quad (58b)$$

where  $L_+(v)$  and  $\kappa_+(v)$  are regular for  $\text{Im}v > -\text{Im}\chi_1\gamma_1$ , i.e., for upper half plane and  $L_-(v)$  and  $\kappa_-(v)$  are regular for  $\text{Im}v < \text{Im}\chi_1\gamma_1$  i.e.; lower half plane. This factorization has been done in [30]. By using the values of  $J_1(v, s, 0)$  and  $J_1'(v, s, 0)$  from Eqs. (55) and (56) respectively in Eqs. (51) and (52), we get

$$\begin{aligned} A(v) = \frac{1}{i\kappa L(v)} & \left[ \bar{\Psi}'_+(v, s, 0) + e^{-ivl}\bar{\Psi}'_-(v, s, 0) + \frac{b(s)k_{1z}}{\sqrt{2\pi}(k_{1x} + v)} \right. \\ & \left. [-1 + \exp[-i(k_{1x} + v)l]] \right] + \frac{v\lambda_1}{\kappa L(v)} \left[ \bar{\Psi}_+(v, s, 0) + e^{-ivl}\bar{\Psi}_-(v, s, 0) \right. \\ & \left. - \frac{b(s)k_{1x}}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] \right], \quad (59) \end{aligned}$$

and

$$\begin{aligned} C(v) = \frac{-1}{i\kappa L(v)} & \left[ \bar{\Psi}'_+(v, s, 0) + e^{-ivl}\bar{\Psi}'_-(v, s, 0) \right. \\ & \left. + \frac{b(s)k_{1z}}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] \right] \\ & + \frac{v\lambda_1}{\kappa L(v)} \left[ \bar{\Psi}_+(v, s, 0) + e^{-ivl}\bar{\Psi}_-(v, s, 0) \right. \\ & \left. - \frac{b(s)k_{1x}}{\sqrt{2\pi}(k_{1x} + v)} [-1 + \exp[-i(k_{1x} + v)l]] \right], \quad (60) \end{aligned}$$

where  $\lambda_1 = \frac{1}{\lambda}$ . In [32], the terms of  $O(\lambda_1)$  are neglected while in the present analysis the  $\lambda_1$  parameter is taken up to order one so that the results due to semi infinite barrier [36] can be recovered by taking

an appropriate limit. To accomplish this, we have to solve both the Wiener-Hopf equations to find the values of unknown functions  $A(v)$  and  $C(v)$ . For this we use Eqs. (58a) and (58b) in Eqs. (55) and (56), which gives

$$\begin{aligned} & \bar{\Psi}'_+(v, s, 0) + e^{-ivl}\bar{\Psi}'_-(v, s, 0) + \mathbf{s}(v)J_1(v, s, 0) \\ &= \frac{-\chi_1\gamma_1 \sin \varphi b(s)}{\sqrt{2\pi}(v - \chi_1\gamma_1 \cos \varphi)} \times [1 - \exp[-i(v - \chi_1\gamma_1 \cos \varphi)l]], \end{aligned} \quad (61)$$

and

$$\begin{aligned} & -iv\bar{\Psi}'_+(v, 0) - ive^{-ivl}\bar{\Psi}'_-(v, 0) + \lambda L_+(v)L_-(v)J'_1(v, s, 0) \\ &= \frac{-\chi_1\gamma_1 \cos \varphi b(s)}{\sqrt{2\pi}(v - \chi_1\gamma_1 \cos \varphi)} \times [1 - \exp[-i(v - \chi_1\gamma_1 \cos \varphi)l]], \end{aligned} \quad (62)$$

where

$$S(v) = -i\kappa(v)L(v) = S_+(v)S_-(v), \quad (63)$$

and  $S_+(v)$  and  $S_-(v)$  are regular in upper and lower half plane respectively. Equations of types (61) and (62) have been considered by Noble [30] and a similar analysis may be employed to obtain an approximate solution for large  $\chi_1\gamma_1 r$ . So, we follow the procedure given in [30] (Sec. 5.5, p. 196) and deduce that

$$\bar{\Psi}'_+(v, s, 0) = \frac{-\chi_1\gamma_1 \sin \varphi b(s)S_+(v)}{\sqrt{2\pi}} [G_1(v) + T(v)C_1(\chi_1\gamma_1)], \quad (64)$$

$$\bar{\Psi}'_-(v, s, 0) = \frac{-\chi_1\gamma_1 \sin \varphi b(s)S_-(v)}{\sqrt{2\pi}} [G_2(-v) + T(-v)C_2(\chi_1\gamma_1)], \quad (65)$$

$$\bar{\Psi}_+(v, s, 0) = \frac{ib(s)L_+(v)}{\sqrt{2\pi v}} [G'_1(v) + T(v)C'_1(\chi_1\gamma_1)], \quad (66)$$

and

$$\bar{\Psi}_-(v, s, 0) = \frac{-ib(s)L_-(v)}{\sqrt{2\pi v}} [G'_2(-v) - T(-v)C'_2(\chi_1\gamma_1)], \quad (67)$$

where

$$S_+(v) = (v + \chi_1\gamma_1)^{\frac{1}{2}}L_+(v), \quad (68a)$$

and

$$S_-(v) = (v - \chi_1\gamma_1)^{\frac{1}{2}}L_-(v), \quad (68b)$$

$$G_1(v) = \frac{1}{(v - \chi_1 \gamma_1 \cos \varphi)} \left[ \frac{1}{S_+(v)} - \frac{1}{S_+(\chi_1 \gamma_1 \cos \varphi)} \right] - e^{-i l \chi_1 \gamma_1 \cos \varphi} R_1(v), \quad (69)$$

$$G_2(v) = \frac{e^{-i l \chi_1 \gamma_1 \cos \varphi}}{(v + \chi_1 \gamma_1 \cos \varphi)} \left[ \frac{1}{S_+(v)} - \frac{1}{S_+(-\chi_1 \gamma_1 \cos \varphi)} \right] - R_2(v), \quad (70)$$

$$C_1 = S_+(\chi_1 \gamma_1) \left[ \frac{G_2(\chi_1 \gamma_1) + S_+(\chi_1 \gamma_1) G_1(\chi_1 \gamma_1) T(\chi_1 \gamma_1)}{1 - S_+^2(\chi_1 \gamma_1) T^2(\chi_1 \gamma_1)} \right], \quad (71)$$

$$C_2 = S_+(\chi_1 \gamma_1) \left[ \frac{G_1(\chi_1 \gamma_1) + S_+(\chi_1 \gamma_1) G_2(\chi_1 \gamma_1) T(\chi_1 \gamma_1)}{1 - S_+^2(\chi_1 \gamma_1) T^2(\chi_1 \gamma_1)} \right], \quad (72)$$

$$G'_1(v) = \frac{v}{(v - \chi_1 \gamma_1 \cos \varphi)} \left[ \frac{1}{L_+(v)} - \frac{1}{L_+(\chi_1 \gamma_1 \cos \varphi)} \right] - e^{-i l \chi_1 \gamma_1 \cos \varphi} R_1(v), \quad (73)$$

$$G'_2(v) = \frac{e^{-i l \chi_1 \gamma_1 \cos \varphi}}{(v + \chi_1 \gamma_1 \cos \varphi)} \left[ \frac{v}{L_+(v)} - \frac{\chi_1 \gamma_1 \cos \varphi}{L_+(-\chi_1 \gamma_1 \cos \varphi)} \right] - R_2(v), \quad (74)$$

$$C'_1 = L_+(\chi_1 \gamma_1) \left[ \frac{G'_2(\chi_1 \gamma_1) + L_+(\chi_1 \gamma_1) G'_1(\chi_1 \gamma_1) T(\chi_1 \gamma_1)}{1 - L_+^2(\chi_1 \gamma_1) T^2(\chi_1 \gamma_1)} \right], \quad (75)$$

$$C'_2 = L_+(\chi_1 \gamma_1) \left[ \frac{G'_1(\chi_1 \gamma_1) + L_+(\chi_1 \gamma_1) G'_2(\chi_1 \gamma_1) T(\chi_1 \gamma_1)}{1 - L_+^2(\chi_1 \gamma_1) T^2(\chi_1 \gamma_1)} \right], \quad (76)$$

$$R_{1,2}(v) = \frac{E_{-1} [W_{-1}\{-i(\chi_1 \gamma_1 \mp \chi_1 \gamma_1 \cos \varphi)l\} - W_{-1}\{-i(\chi_1 \gamma_1 + v)l\}]}{2\pi i (v \pm \chi_1 \gamma_1 \cos \varphi)}, \quad (77)$$

$$T(v) = \frac{1}{2\pi i} E_{-1} W_{-1}\{-i(\chi_1 \gamma_1 + v)l\}, \quad (78)$$

$$E_{-1} = 2e^{i\frac{\pi}{4}} e^{i\chi_1 \gamma_1 \cos \varphi l} (l)^{\frac{1}{2}} (i)^{-1} h_{-1}, \quad (79)$$

and

$$W_{n-\frac{1}{2}}(p) = \int_0^\infty \frac{u^n e^{-u}}{u+p} du = \Gamma(n+1) e^{\frac{p}{2}} p^{\frac{1}{2}n-\frac{1}{2}} W_{-\frac{1}{2}(n+1), \frac{1}{2}n}(p), \quad (80)$$

where  $p = -i(\chi_1 \gamma_1 + v)l$  and  $n = \frac{-1}{2}$ .  $W_{m,n}$  is known as a Whittaker function.

Now, making use of Eqs. (64)–(67) in Eqs. (59) and (60), we get

$$\left. \begin{array}{l} A(v) \\ C(v) \end{array} \right\} = \frac{-\chi_1 \gamma_1 \sin \varphi b(s) \operatorname{sgn}(z)}{\sqrt{2\pi} i \kappa L(v)} \left\{ \begin{array}{l} S_+(v) G_1(v) + S_+(v) T(v) C_1(\chi_1 \gamma_1) + e^{-ivl} S_-(v) \\ [G_2(-v) - T(-v) C_2(\chi_1 \gamma_1)] - \frac{(1 - e^{-i(v - \chi_1 \gamma_1 \cos \varphi)l}}{(v - \chi_1 \gamma_1 \cos \varphi)} \end{array} \right\} \\ + \frac{b(s)v\lambda_1}{\sqrt{2\pi\kappa L(v)}} \left\{ \begin{array}{l} L_+(v) G'_1(v) + T(v) L_+(v) C'_1(\chi_1 \gamma_1) \\ + e^{-ivl} [(L_-(v) G'_2(-v) + T(-v) \\ L_+(v) C'_2(\chi_1 \gamma_1))] - \frac{(1 - e^{-il(k_{1x} + v)})}{(v - \chi_1 \gamma_1 \cos \varphi)} \end{array} \right\} \quad (81)$$

where  $A(v)$  correspond to  $z > 0$  and  $C(v)$  correspond to  $z < 0$ . We can see that the second term in the above equation was altogether missing in Eq. (67) of [32]. This term included the effect of  $\lambda_1$  parameter in it which can be seen from the solution also. Now,  $\tilde{Q}_{1y}^{sca}(x, s, z)$  can be obtained by taking the inverse Fourier transform of Eq. (44). Thus

$$\tilde{Q}_{1y}^{sca}(x, s, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left\{ \begin{array}{l} A(v) \\ C(v) \end{array} \right\} \exp(i\kappa |z| - ivx) dv, \quad (82)$$

where  $A(v)$  and  $C(v)$  are given by Eq. (81). Substituting the value of  $A(v)$  and  $C(v)$  from Eq. (81) into Eq. (82) and using the approximations (69)–(76), one can break up the field  $\tilde{Q}_{1y}^{sca}(x, s, z)$  into two parts

$$\tilde{Q}_{1y}^{sca}(x, s, z) = \tilde{Q}_{1y}^{sca(sep)}(x, s, z) + \tilde{Q}_{1y}^{sca(int)}(x, s, z), \quad (83)$$

where

$$\tilde{Q}_{1y}^{sca(sep)}(x, s, z) = \frac{\chi_1 \gamma_1 \sin \varphi b(s) \operatorname{sgn}(z)}{2\pi i} \int_{-\infty}^{\infty} \frac{S_+(v) \exp(i\kappa |z| - ivx)}{i\kappa L(v) S_+(\chi_1 \gamma_1 \cos \varphi) (v - \chi_1 \gamma_1 \cos \varphi)} dv \\ + \frac{\chi_1 \gamma_1 \sin \varphi b(s) \operatorname{sgn}(z)}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{-il(k_{1x} + v)} S_-(v) \exp(i\kappa |z| - ivx)}{i\kappa L(v) S_+(-\chi_1 \gamma_1 \cos \varphi) (v - \chi_1 \gamma_1 \cos \varphi)} dv$$

$$\begin{aligned}
& -\frac{b(s)}{2\pi i} \int_{-\infty}^{\infty} \frac{\lambda_1 e^{-il(k_{1x}+v)} \exp(i\kappa |z| - ivx)}{\kappa L(v)(v - \chi_1 \gamma_1 \cos \varphi)} dv \\
& + \frac{b(s)}{2\pi i} \int_{-\infty}^{\infty} \frac{L_-(v) e^{-il(k_{1x}+v)} \exp(i\kappa |z| - ivx)}{\kappa L(v)(v - \chi_1 \gamma_1 \cos \varphi) L_+(-\chi_1 \gamma_1 \cos \varphi)} dv \\
& + \frac{b(s)}{2\pi i} \int_{-\infty}^{\infty} \frac{\lambda_1 \exp(i\kappa |z| - ivx)}{\kappa L(v)(v - \chi_1 \gamma_1 \cos \varphi)} dv, \tag{84}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{Q}_{1y}^{sca(int)}(x, s, z) &= \frac{\chi_1 \gamma_1 \sin \varphi b(s) \operatorname{sgn}(z)}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{i\kappa L(v)} \\
& \left[ S_+(v) R_1(v) e^{-il\chi_1 \gamma_1 \cos \varphi} - C_1(\chi_1 \gamma_1) S_+(v) T(v) \right. \\
& \left. + S_+(-v) e^{-ilv} R_2(-v) - C_2(\chi_1 \gamma_1) T(-v) S_+(-v) e^{-ilv} \right] \\
& \exp(i\kappa |z| - ivx) dv - \frac{b(s)}{2\pi i} \int_{-\infty}^{\infty} \frac{\lambda_1}{\kappa L(v)} \\
& \left[ T(v) L_+(v) C_1'(\chi_1 \gamma_1) + T(-v) L_-(v) C_2'(\chi_1 \gamma_1) \right. \\
& \left. - L_+(v) R_1(v) e^{-il\chi_1 \gamma_1 \cos \varphi} - L_-(v) R_2(-v) e^{-ilv} \right] \\
& \exp(i\kappa |z| - ivx) dv. \tag{85}
\end{aligned}$$

Here,  $\tilde{Q}_{1y}^{sca(sep)}(x, s, z)$  consists of two parts each representing the diffracted field produced by the edges at  $x = 0$  and  $x = -l$ , respectively, as though the other edges were absent while  $\tilde{Q}_{1y}^{sca(int)}(x, s, z)$  gives the interaction of one edge upon the other.

## 5. FAR FIELD SOLUTION

The far field may now be calculated by evaluating the integrals appearing in Eqs. (82), (84) and (85), asymptotically [33]. For that put  $x = r \cos \vartheta$ ,  $|z| = r \sin \vartheta$  and deform the contour by the transformation  $v = -\chi_1 \gamma_1 \cos(\vartheta + ip)$ , ( $0 < \vartheta < \pi$ ,  $-\infty < p < \infty$ ).

Hence, for large  $\chi_1\gamma_1 r$ , Eqs. (82), (84) and (85) become

$$\begin{aligned} \tilde{Q}_{1y}^{sca(sep)}(x, s, z) = & b(s) [i \cdot sgn(z) \sin \varphi \cdot f_1(-\chi_1\gamma_1 \cos \vartheta) + g_1(-\chi_1\gamma_1 \cos \vartheta)] \\ & \times \frac{1}{\sqrt{2\pi\chi_1\gamma_1 r_0}} \exp\left(i\chi_1\gamma_1 r - i\frac{\pi}{4}\right), \end{aligned} \quad (86)$$

and

$$\begin{aligned} \tilde{Q}_{1y}^{sca(int)}(x, s, z) = & b(s) [i \cdot sgn(z) \sin \varphi \cdot f_2(-\chi_1\gamma_1 \cos \vartheta) + g_2(-\chi_1\gamma_1 \cos \vartheta)] \\ & \times \frac{1}{\sqrt{2\pi\chi_1\gamma_1 r_0}} \exp\left(i\chi_1\gamma_1 r - i\frac{\pi}{4}\right), \end{aligned} \quad (87)$$

where  $A(-\chi_1\gamma_1 \cos \vartheta)$  and  $C(-\chi_1\gamma_1 \cos \vartheta)$  can be found from Eq. (81), while

$$\begin{aligned} & f_1(-\chi_1\gamma_1 \cos \vartheta) \\ = & \frac{S_+(-\chi_1\gamma_1 \cos \vartheta)}{L(-\chi_1\gamma_1 \cos \vartheta)S_+(\chi_1\gamma_1 \cos \phi)(-\chi_1\gamma_1 \cos \phi - \chi_1\gamma_1 \cos \vartheta)} \\ & - \frac{e^{-il(-\chi_1\gamma_1 \cos \phi - \chi_1\gamma_1 \cos \vartheta)}S_+(\chi_1\gamma_1 \cos \vartheta)}{1 \cos \phi)(-\chi_1\gamma_1 \cos \phi - \chi_1\gamma_1 \cos \vartheta)}, \end{aligned} \quad (88)$$

$$\begin{aligned} & g_1(-\chi_1\gamma_1 \cos \vartheta) \\ = & \frac{1}{(-\chi_1\gamma_1 \cos \phi - k_{1xz} \cos \vartheta)} \\ & \times \left[ \begin{array}{c} \frac{\lambda_1 e^{-il(-\chi_1\gamma_1 \cos \phi - \chi_1\gamma_1 \cos \vartheta)}}{L(-\chi_1\gamma_1 \cos \vartheta)} \\ - \frac{L_+(\chi_1\gamma_1 \cos \vartheta)e^{-il(-\chi_1\gamma_1 \cos \phi - \chi_1\gamma_1 \cos \vartheta)}}{L(-\chi_1\gamma_1 \cos \vartheta)L_+(-\chi_1\gamma_1 \cos \phi)} \\ - \frac{\lambda_1}{L(-\chi_1\gamma_1 \cos \vartheta)} \end{array} \right], \end{aligned} \quad (89)$$

$$\begin{aligned} & f_2(-\chi_1\gamma_1 \cos \vartheta) \\ = & \frac{1}{L(-\chi_1\gamma_1 \cos \vartheta)} \\ & \times \left[ \begin{array}{c} S_+(-\chi_1\gamma_1 \cos \vartheta)R_1(-\chi_1\gamma_1 \cos \vartheta)e^{il\chi_1\gamma_1 \cos \phi} \\ + S_+(\chi_1\gamma_1 \cos \vartheta)e^{il\chi_1\gamma_1 \cos \vartheta}R_2(\chi_1\gamma_1 \cos \vartheta) \\ - C_1(\chi_1\gamma_1)S_+(-\chi_1\gamma_1 \cos \vartheta)T(-\chi_1\gamma_1 \cos \vartheta) \\ - C_2(\chi_1\gamma_1)T(\chi_1\gamma_1 \cos \vartheta)S_+(\chi_1\gamma_1 \cos \vartheta)e^{il\chi_1\gamma_1 \cos \vartheta} \end{array} \right], \end{aligned} \quad (90)$$

and

$$\begin{aligned}
 & g_2(-\chi_1\gamma_1 \cos \vartheta) \\
 = & \frac{1}{L(-\chi_1\gamma_1 \cos \vartheta)} \\
 & \times \begin{bmatrix} L_+(-\chi_1\gamma_1 \cos \vartheta)R_1(-\chi_1\gamma_1 \cos \vartheta) e^{i\chi_1\gamma_1 \cos \phi} \\ + L_+(\chi_1\gamma_1 \cos \vartheta)R_2(\chi_1\gamma_1 \cos \vartheta) e^{i\chi_1\gamma_1 \cos \vartheta} \\ - T(-\chi_1\gamma_1 \cos \vartheta)L_+(-\chi_1\gamma_1 \cos \vartheta)C'_1(\chi_1\gamma_1) \\ - T(\chi_1\gamma_1 \cos \vartheta)L_+(\chi_1\gamma_1 \cos \vartheta)C'_2(\chi_1\gamma_1) \end{bmatrix}. \quad (91)
 \end{aligned}$$

Now, substituting Eq. (46b) in Eqs. (86) and (87) and then taking the inverse Fourier transform w.r.t “ $y$ ” and using Eq. (31b), we obtain

$$\begin{aligned}
 Q_{1y}^{sca(sep)}(x, y, z) = & \frac{S_{01}i \cdot sgn(z) \sin \varphi}{8\pi^2 \sqrt{rr_0}} \int_{-\infty}^{\infty} f_1(-\chi_1\gamma_1 \cos \vartheta) \\
 & \exp i\gamma_1[\chi_1(r+r_0) + s(y-y_0)]ds + \frac{S_{01}i}{8\pi^2 \sqrt{rr_0}} \\
 & \int_{-\infty}^{\infty} g_1(-\chi_1\gamma_1 \cos \vartheta) \exp i\gamma_1[\chi_1(r+r_0) + s(y-y_0)]ds, \quad (92) \\
 Q_{1y}^{sca(int)}(x, y, z) = & -\frac{S_{01}i \cdot sgn(z) \sin \varphi}{8\pi^2 \sqrt{rr_0}} \int_{-\infty}^{\infty} f_2(-\chi_1\gamma_1 \cos \vartheta) \\
 & \exp i\gamma_1[\chi_1(r+r_0) + s(y-y_0)]ds - \frac{S_{01}i\gamma_1}{8\pi^2 \sqrt{rr_0}} \\
 & \int_{-\infty}^{\infty} g_2(-\chi_1\gamma_1 \cos \vartheta) \exp i\gamma_1[\chi_1(r+r_0) + s(y-y_0)]ds. \quad (93)
 \end{aligned}$$

For the evaluation of integrals in Eqs. (92) and (93), we introduce  $r+r_0 = r_{12} \sin \sigma$ ,  $(y-y_0) = r_{12} \cos \sigma$  and the transformation  $s = \cos(\sigma + iq)$ , which changes the contour of integration over into a hyperbola passing through the point  $\cos \sigma$ . The integrals are then solved asymptotically by using the steepest decent method and the

resulting expressions are given by

$$Q_{1y}^{sca(sep)}(x, y, z) = -\frac{i \cdot \operatorname{sgn}(z) S_{01}}{4\pi\sqrt{2\pi\gamma_1 r r_0 r_{12}}} \left[ \begin{array}{l} \sin \varphi f_1(-\gamma_1 \sin \sigma \cos \vartheta) \\ + g_1(-\gamma_1 \sin \sigma \cos \vartheta) \end{array} \right] \exp i \left( \gamma_1 r_{12} - \frac{\pi}{4} \right), \quad (94)$$

$$Q_{1y}^{sca(int)}(x, y, z) = -\frac{S_{01} i \gamma_1 \sin \sigma}{4\pi\sqrt{2\pi\gamma_1 r r_0 r_{12}}} \left[ \begin{array}{l} \sin \varphi f_2(-\gamma_1 \sin \sigma \cos \vartheta) \\ + g_2(-\gamma_1 \sin \sigma \cos \vartheta) \end{array} \right] \exp i \left( \gamma_1 r_{12} - \frac{\pi}{4} \right), \quad (95)$$

where  $f_1(-\gamma_1 \sin \sigma \cos \vartheta)$ ,  $g_1(-\gamma_1 \sin \sigma \cos \vartheta)$ ,  $f_2(-\gamma_1 \sin \sigma \cos \vartheta)$  and  $g_2(-\gamma_1 \sin \sigma \cos \vartheta)$  given by Eqs. (88)–(91) respectively. Thus, the complete solution of the system is given by

$$Q_{1y}(x, y, z) = Q_{1y}^{inc}(x, y, z) + Q_{1y}^{sca}(x, y, z). \quad (96)$$

Substituting the values of  $Q_{1y}^{inc}(x, y, z)$  from Eq. (42) and of  $Q_{1y}^{sca(sep)}(x, y, z)$  and  $Q_{1y}^{sca(int)}(x, y, z)$  from Eqs. (94) and (95) into Eq. (96), we get the desired result of the system as

$$Q_{1y}(x, y, z) = S_{01} \frac{e^{i\gamma_1 r_1}}{4\pi r_1} - \frac{i \cdot \operatorname{sgn}(z) S_{01}}{4\pi\sqrt{2\pi\gamma_1 r r_0 r_{12}}} \left[ \begin{array}{l} \sin \varphi f_1(-\gamma_1 \sin \sigma \cos \vartheta) \\ + g_1(-\gamma_1 \sin \sigma \cos \vartheta) \end{array} \right] \exp i \left( \gamma_1 r_{12} - \frac{\pi}{4} \right) - \frac{S_{01} i \gamma_1 \sin \sigma}{4\pi\sqrt{2\pi\gamma_1 r r_0 r_{12}}} \left[ \begin{array}{l} \sin \varphi f_2(-\gamma_1 \sin \sigma \cos \vartheta) \\ + g_2(-\gamma_1 \sin \sigma \cos \vartheta) \end{array} \right] \exp i \left( \gamma_1 r_{12} - \frac{\pi}{4} \right). \quad (97)$$

### Remarks:

Mathematically, we can derive the results of the half plane problem a follows:

For the analysis purpose, in Eq. (81), we take the wave number  $\chi_1 \gamma_1$  to be pure imaginary and using the  $L$  Hopital rule successively, the value of  $E_{-1}$ , reduces to  $Lt_{l \rightarrow \infty} \left( \frac{e^{ikt}}{\sqrt{kt}} \right)$  which becomes zero and in turn result the quantities  $T(v)$ ,  $R_{1,2}(v)$ ,  $G_2'(v)$ ,  $C_1'$  and  $G_2(v)$  in zero. The third term in Eqs. (69), (70) and (73) also becomes zero as

$l \rightarrow \infty$ . The Eq. (81), after these eliminations reduces to

$$A(v) = \frac{1}{\sqrt{2\pi}} \left[ \frac{-k_{1z}\kappa_+(v)L_+(v)}{i\kappa(v)L(v)(k_{1x} + v)\kappa_+(-k_{1x})L_+(-k_{1x})} + \frac{\delta_1 v L_+(v)}{\kappa(v)L(v)(k_{1x} + v)L_+(-k_{1x})} \right].$$

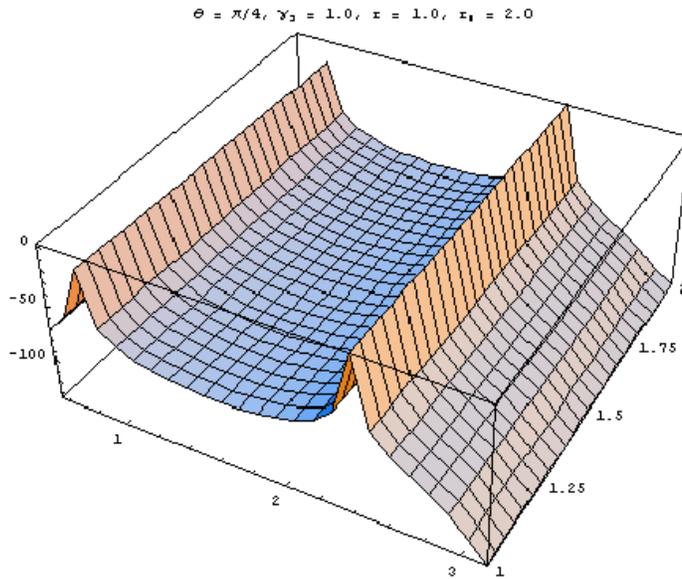
Using the factorization

$$L(v) = L_+(v)L_-(v),$$

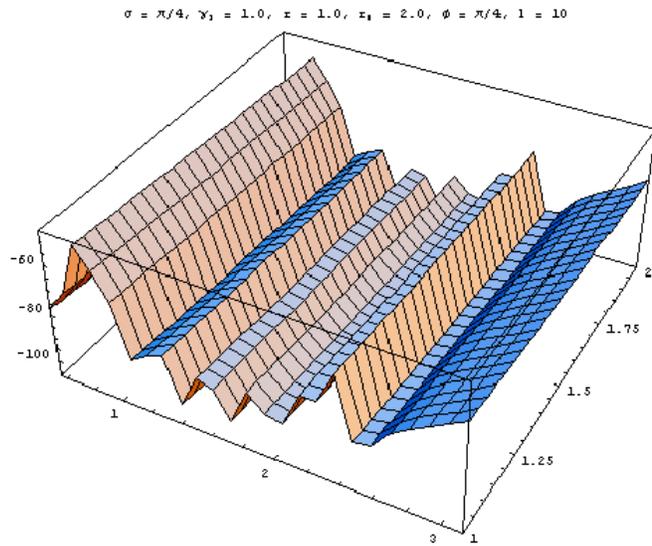
and

$$\kappa(v) = \kappa_+(v)\kappa_-(v).$$

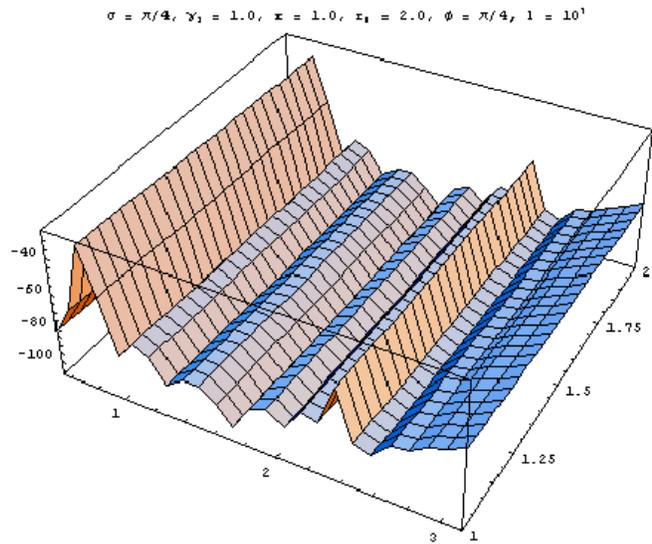
and substituting the pole contribution  $v = -k_{1x}$ , the above result reduces to Eq. (72) of the Half Plane [36] which in turn results the separated field of the strip into the diffracted field [36] as the strip is widened to half plane by taking the limit  $l \rightarrow \infty$  which can be considered as check of the validity of the analysis in this paper.



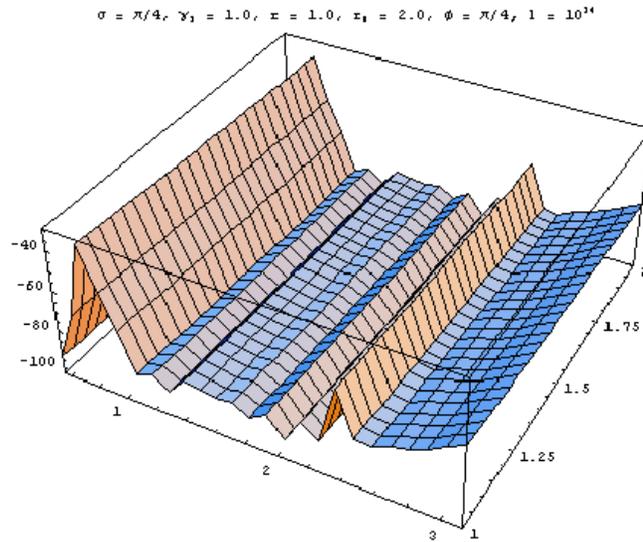
**Figure 1.** Variation of the amplitude of diffracted field (db) versus observation angle (radians) for values from 0 to 3.2, and against  $r_{12}$  for values from 1 to 2.



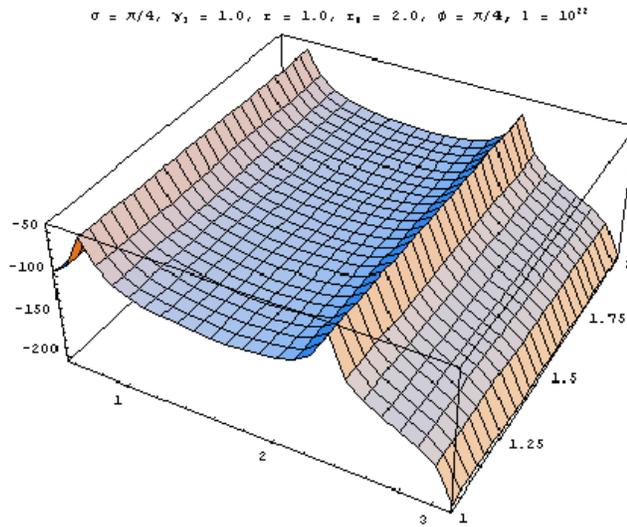
**Figure 2.** Variation of the amplitude of separated field (db) versus observation angle (radians) for values from 0 to 3.2, and against  $r_{12}$  for values from 1 to 2, for  $l = 10$ .



**Figure 3.** Variation of the amplitude of separated field (db) versus observation angle (radians) for values from 0 to 3.2, and against  $r_{12}$  for values from 1 to 2, for  $l = 10^7$ .



**Figure 4.** Variation of the amplitude of separated field (db) versus observation angle (radians) for values from 0 to 3.2, and against  $r_{12}$  for values from 1 to 2, for  $l = 10^{14}$ .



**Figure 5.** Variation of the amplitude of separated field (db) versus observation angle (radians) for values from 0 to 3.2, and against  $r_{12}$  for values from 1 to 2, for  $l = 10^{22}$ .

## 6. GRAPHICAL RESULTS

A computer program MATHEMATICA has been used for graphical plotting. The main features of the graphical results can be seen in graphs (1), (2), (3), (4) and (5) are as follows:

(a) The graphs of the diffracted field corresponding to the half plane is given in Fig. 1. It is observed that the separated field of the strip for different values of  $l$ , Figs. 2–5, are in comparison with Fig. 1.

(b) It is observed that as  $l = 10^{22}$ , which can be considered as an infinity in case of strip in Fig. 5 is in close comparison with half plane Fig. 1, verifying our claim that strip is widened into the half plane as  $l \rightarrow \infty$ .

## 7. CONCLUSION

The diffracted field due to a spherical electromagnetic wave by a perfectly conducting finite strip in a homogeneous bi-isotropic medium is obtained in an improved form. It is found that the two edges of the strip give rise to two diffracted fields (one from each edge) and an interaction field (double diffraction of two edges). This seems to be the first attempt in this direction as we can deduce the results of half plane [36] by taking an appropriate limit. In [32], the  $\lambda$  parameter was not taken into account which ends up in an equation from which one cannot deduce the results for semi infinite barrier [36]. This can be considered as check of the validity of the analysis in this paper. Thus, the new solution can be regarded as a correct solution for a perfectly conducting barrier.

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