LOCALIZATION OF OMNI-DIRECTIONAL MOBILE DEVICE IN MULTIPATH ENVIRONMENTS

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Abstract—This paper presents a comprehensive Non Line of Sight (NLOS) localization scheme in a multipath environment where the scatterers with smooth surfaces are aligned parallel or perpendicular to each other. It leverages on the estimation of Angle of Arrival (AOA) and Time of Arrival (TOA) of the omni-directional mobile device’s signal received at the reference devices. Unlike the conventional Line of Sight (LOS) localization schemes that rely on the various mitigation techniques to mitigate the multipaths that are mistaken as the LOS signal, our proposed two step localization scheme not only utilizes the LOS path but also any one bound scattering NLOS multipath arriving at the reference devices for localization. Channel experiment coupled with simulation results in a typical multipath environment has demonstrated that our proposed localization scheme outperforms the conventional localization schemes that are coupled with their own mitigation techniques. Robustness in performance of our proposed localization scheme towards different scatterers’ orientation where they are not aligned parallel or perpendicular to each other are also investigated.

1. INTRODUCTION

There is a proliferating demand for both commercial and government applications of wireless localization services that ascertain the position of a mobile device in a cellular or sensor network [1–4]. Besides facilitating emergency safety systems to allow 911 subscriber calls, the outdoor cellular systems also accommodate proximity advertising, location sensitive billing and intelligent transport tracking systems. For the indoor channel environments, dedicated localization sensor systems have been developed that either leverage on existing indoor
Wireless LAN 802.11x infrastructure or on specified Radio Frequency technology such as Ultra Wideband (UWB) [5–9]. These systems have been designed to provide localization information for applications such as the tracking of assets and personnel.

There is numerous number of wireless localization estimation schemes and they can be broadly classified according into whether they take a conventional LOS or the bi-directional NLOS approach. Under the LOS approach, the LOS geometric relationship between the mobile device and its reference is exploited to establish the Euclidean distance between them and to identify the physical location of the device. The information that is used to determine the location can be the measured Time of Arrival (TOA) [10–12], the Time Difference of Arrival (TDOA) [13–15], the Angle of Arrival (AOA) [16] or the Received Signal Strength (RSS) [17] of the mobile device’s signal at the reference devices. There are also hybrid localization techniques that use a combination of the above metrics such as fusion of TOA and AOA data to achieve the same ends [18–20]. In the bi-directional NLOS approaches [21–24] which are the emerging novel techniques, both the TOA and AOA measurement data for either the LOS or one bound scattering (multipath that undergoes one bound scattering phenomenon) paths or both paths at both reference and mobile devices are leveraged on for location estimation. Fig. 1 depicts the above two approaches.

In general, for the conventional LOS approach, at least two reference devices are required for AOA localization scheme and three reference devices for TOA localization scheme to produce a two dimensional estimate of the mobile device under the Line of Sight (LOS) condition. However, these conventional localization schemes pose a challenge in a rich (or heavy) multipath channel environment with numerous scatterers [25–28]. In such environments, if one or more reference devices are not in the LOS range of the mobile device, the LOS localization scheme renders erroneous estimate of the mobile location. Various NLOS mitigation techniques have emerged [12,19,29–34] to overcome this problem. The two emerged mitigation techniques are classified as follows: the first is called residual weighting [12,29–31]; the second is the LOS reference devices identification methodology [19,32–34]. The first mitigation technique attempts to minimize the contribution of the NLOS multipaths, leaving the unanswered questions of about its overall mitigation technique reliability. The second methodology focuses on the identification of NLOS reference devices and discards them for localization. However, a rich multipath environment has abundant scatterers in the proximity of both the reference and mobile devices. This may result in almost all
Figure 1. (a) Conventional LOS localization schemes that use only LOS path measurement data at the reference devices. (b) NLOS localization schemes that use both LOS and one bound scattering NLOS paths’ measurement data at both reference and mobile devices.
reference devices to be in NLOS region except for the one that is the closest to the mobile device. As a result, the number of devices may not be sufficient for localization. The two NLOS mitigation techniques mentioned above will not perform satisfactorily as they require that the number of LOS reference devices that are available be greater than the number of NLOS reference devices deployed in the environment.

In recent years, Multiple Input and Multiple Output (MIMO) systems have emerged as key technologies in providing high bandwidth communications services for the next generation cellular network (4G) and the wireless LAN 802.11n. MIMO exploits antenna arrays [35–46] that are coupled with smart antenna technology at both reference and mobile devices which facilitate the AOA estimation at both ends (referred to as bi-directional estimation). Localization using such principle has been explored in [21–24]. In our earlier papers [21,22], comprehensive bi-directional least square localization schemes have been designed that are based on the TOA and AOA measurement data of the LOS [21,22] and one bound scattering NLOS [21] paths at both the reference and mobile devices. Multiple bound scattering NLOS multipaths have been successfully rejected through the designed proximity and multipath rejection scheme [21]. Although the bi-directional approaches [21–24] with just one reference device has been able to localize the mobile device much better than the conventional LOS approach that use three reference devices in multipath environment, the cost of using antenna array at the mobile device in term of computation and physical implementation for localization is expensive.

In this paper, we will propose a novel NLOS localization scheme that removes the above mentioned limitation pertaining to the NLOS approach. It just uses the TOA and AOA measurement data of the mobile device’s signal arriving at the reference devices and yet able to localize the non-directional mobile device using not only the LOS path but also utilizing the one bound scattering NLOS multipath. The mobile device does not need to be equipped with antenna array (omni-directional). Furthermore, the proposed NLOS localization scheme which is a two step Determination and Selection (two step DS) scheme, is robust to the detrimental effect of the multiple bound scattering multipaths on the localization accuracy without leveraging on any mitigation or multipath rejection scheme. The proposed two step DS localization scheme comprises of determining the centroid (the likely mobile device location) among the cluster of Line of Possible Mobile Device Location (LPMD). These LPMDs are the lines that contain the possible mobile device location. They are constructed using the LOS and the one bound scattering NLOS paths’ TOA and
AOA measurement data at the reference devices. The one bound scattering NLOS propagation paths arise from the specular reflection at the scatterers that are aligned parallel and perpendicular to each other. This centroid is found to be situated near the LPMDs of the LOS and one bound scattering NLOS paths. It segregates from the LPMDs of the multiple bound scattering paths. We will delve into the construction of these LPMDs in next section. The second step of the two step DS localization scheme is to find the appropriate pair of LPMDs that has the shortest Euclidean distance from the centroid and select it as the mobile device location. Another novel concept that is conceived in our proposed localization scheme is the adjustment of the TOA and the AOA measurement data of the LOS paths arriving at the reference devices so that these measurement data are close to the actual values once the centroid is determined. This renders more accurate location estimation. Section 2 will show the formulation of the proposed two step DS localization scheme while Section 3 will delve into the derivation of analytical localization error of our proposed localization scheme. Section 4 analyses the performance of our proposed scheme through the experimental and simulation results in a typical environment at the Nanyang Technological University, School of Electrical and Electronics Engineering (EEE). It has been demonstrated that our proposed two step DS localization scheme outperforms the conventional LOS localization schemes that are coupled with their own mitigation schemes in all cases. Conclusions on our research efforts are drawn in Section 5.

2. THEORY AND FORMULATION

2.1. Concept of Line of Possible Mobile Device Location (LPMD)

Consider a typical multipath environment [21,47–49] where the scatterers are smooth surfaces and are aligned parallel and perpendicular to each other as shown in Fig. 2. Three reference devices (RDs) were placed at $RD_1$ (25 m, 9 m), $RD_2$ (18 m, 4 m) and $RD_3$ (3 m, 14 m) with $MD$ at (20 m, 10.8 m). The first dominant signal path arriving at each $RD$ are shown. $RD_1$ and $RD_3$ are in LOS with the $MD$ while $RD_2$ is in NLOS with the $MD$. To illustrate the concept of LPMD, consider the signal path that is arriving at $RD_2$ that has undergone one bound scattering (reflection) phenomenon at the scatterer $S_{2,1}$. The subscript “2” and “1” in the notation $S_{2,1}$ indicate the scatterer associated with $RD_2$ and the first dominant signal path received at $RD_2$ respectively. With the measured TOA $t_{2,1}$, AOA $\alpha_{2,1}$ of the one bound scattering signal path at $RD_2$ and
Figure 2. Line of Possible Mobile Device Location (LPMD) for MD at (20m, 10.8m) due to NLOS signal received at RD2 at Nanyang Technological University, School of EEE, Block S1, Level 3 (S1-B3).

no prior knowledge of the location of the scatterer S2,1, there will be two possible lines of mobile device (MD) locations. These two lines arise from the reflection at the horizontal and vertical planes of S2,1. The first line is a straight horizontal line that extends from the point MD$_{LOS}$ and passes through the actual MD location (MD). This horizontal line arises from the signal path that undergoes one bound scattering at the vertical plane of S2,1 and is referred as the LPMD of RD2 for the vertical plane of the scatterer, LPMD$_{v2,1}$. MD$_{LOS}$ is the perceived MD location if the signal path is a LOS path that arrives at RD2 with a TOA $t_{2,1}$ and AOA $\alpha_{2,1}$. The second line is a vertical line that extends from MD$_{LOS}$ and passes through the point MD$'$. This vertical line arises from the signal path that undergoes one bound scattering at the horizontal plane of S2,1 and is referred as the LPMD of RD2 for the horizontal plane of the scatterer, LPMD$_{h2,1}$. Therefore, both LPMD$_{v2,1}$ and LPMD$_{h2,1}$ which contain all possible MD locations, form a set of LPMDs for each signal path with a given TOA and AOA measurement data. As shown, for RD2 with TOA $t_{2,1}$, AOA $\alpha_{2,1}$ measurement data and an unknown scatterer S2,1, the actual MD location will lie on the LPMD$_{v2,1}$ while MD$'$ which is one of the possible MD location will lie on the LPMD$_{h2,1}$. If the path is a LOS path, the two LPMDs will intersect and give rise to the
LOS point, $MD_{LOS}$. The exact location of $MD$ can be determined by constructing another set of LPMDs (one $LPMD_v$ and one $LPMD_h$) using another signal path between $RD_2$ (or another reference device) and $MD$, and by locating the intersection of these two sets of LPMDs. For $N$ reference devices with $M$ paths each, the exact location of $MD$ can be resolved using $2NM$ lines of LPMDs. This conceptual principle forms the working basis of our NLOS localization scheme. Fig. 3 illustrates the plot of the LPMDs for all the $RD$s each with a single dominant path with $MD$ at $(20\,\text{m}, 10.8\,\text{m})$.

**Figure 3.** Plot of LPMDs for all the $RD$s for MD at $(20\,\text{m}, 10.8\,\text{m})$ at Nanyang Technological University, School of EEE, Block S1, Level 3 (S1-B3). The TOA standard deviation (in terms of distance) is $1\,\text{m}$ while AOA standard deviation is $2^\circ$.

### 2.2. Two Step DS Localization Scheme

#### 2.2.1. Determination of the Centriod

With reference to Figs. 2 and 3, the $x$ axis of $LPMD_{h_{n,m}}$ and $y$ axis of $LPMD_{v_{n,m}}$ of the $RD$s are utilized to determine the centriod $C$, which is the first estimation of the true $MD$ location. $n = 1 \ldots N$ and $m = 1 \ldots M$ where $N$ is the number of $RD$s and $M$ is the number of dominant paths associated with each $RD$. In general, for $N$ reference
devices with \( M \) paths each, the centroid \( \mathbf{C} \) can be found as

\[
\mathbf{C} = \left[ \begin{array}{c}
\sum_{n=1}^{N} \sum_{m=1}^{M} w_{n,m} x_{h_{n,m}} \\
\sum_{n=1}^{N} \sum_{m=1}^{M} w_{n,m} y_{v_{n,m}}
\end{array} \right] T
\]

\[
\sum_{n=1}^{N} \sum_{m=1}^{M} w_{n,m}
\]

where \( x_{h_{n,m}} \) and \( y_{v_{n,m}} \) are the \( x \) coordinate of the \( LPMD_{h_{n,m}} \) and \( y \) coordinate of the \( LPMD_{v_{n,m}} \) of the \( m \)th path of \( RD_n \) respectively. They are given as

\[
x_{h_{n,m}} = x_n + d_{n,m} \cos \alpha_{n,m}, \quad y_{v_{n,m}} = y_n + d_{n,m} \sin \alpha_{n,m}
\]

with \( \mathbf{Z}_n = [x_n, y_n]^T \) as the position coordinate of \( RD_n \) and \( d_{n,m} = c t_{n,m} \). \( t_{n,m} \) and \( \alpha_{n,m} \) are the TOA and AOA of the \( m \)th signal path of \( RD_n \) respectively. \( c \) is the speed of propagation and \( w_{n,m} \) is the path weight associated with the \( m \)th path of \( RD_n \). It is given by

\[
w_{n,m} = \frac{1}{d_{n,m}^2} e^{-\left(\sigma_{d_{n,m}} + d_{n,m} \sigma_{\alpha_{n,m}}\right)}
\]

This path weight is an inverse function of the square of the propagation distance \( d_{n,m} \). It can be also correlated to the power of the \( m \)th signal path at \( RD_n \) where it is inversely proportional to the square of the propagation distance \( d_{n,m} \). The longer the propagation distance the path has, the lesser the weight is being assigned to this path. The rationale is that in general, the multiple bound scattering path will have longer propagation distance [21, 47–49]. By assigning lesser weight, we are able to segregate our centroid from the LPMDs of the multiple bound scattering path. \( \sigma_{d_{n,m}}, \sigma_{d_{n,m}} \sigma_{\alpha_{n,m}} \) are the standard deviation of TOA and AOA measurement noises (in terms of distances) respectively where \( \sigma_{d_{n,m}} = c \sigma_{t_{n,m}}, \sigma_{t_{n,m}}, \sigma_{\alpha_{n,m}} \) are the actual standard deviation of TOA and AOA measurement noises respectively. Similarly, the higher the standard deviations are, the lower the weight is assigned to the associated path. Fig. 3 depicts the LPMDs of all the paths arriving at the \( RDs \) where \( N = 3, M = 1 \) for \( MD \) located at (20 m, 10.8 m) with \( \sigma_{d_{n,m}} = 1 \) m and \( \sigma_{\alpha_{n,m}} = 2^\circ \). As shown, the centroid is estimated close to the actual \( MD \) location.

2.2.2. Selection of the Intersection Point

The next step is to determine the \( MD \) location by selecting a pair of LPMDs (a \( LPMD_{h} \) and a \( LPMD_{v} \)) that has the smallest
Euclidean distance between the intersection point of such pair of LPMDs and the centroid $C$. For $2NM$ number of LPMDs (one $LPMD_h$ and one $LPMD_v$ for each path), there will be $(NM)^2$ number of intersection points. The Euclidean distance between the centroid and the intersection point of a pair of LPMDs can be calculated as

$$\xi_{(h_{n,m}), (v_{j,k})} = \| C - Z_{(h_{n,m}), (v_{j,k})} \|$$  \hspace{1cm} (4)

where $Z_{(h_{n,m}), (v_{j,k})} = [x_{h_{n,m}}, y_{v_{j,k}}]^T$ is the intersection point of a pair of $LPMD_{h_{n,m}}$ and $LPMD_{v_{j,k}}$. $n = 1 \ldots N$, $j = 1 \ldots N$, $m = 1 \ldots M$, $k = 1 \ldots M$. Therefore the $MD$ location $Z = [x, y]^T$ can be found through the intersection point of a pair of LPMDs that has the minimum $\xi$ and is given as

$$Z = [x, y]^T = Z_{(h_{n,m}), (v_{j,k}) \text{min} \xi} = [x_{h_{n,m}}, y_{v_{j,k}}]^T \text{min} \xi$$  \hspace{1cm} (5)

To illustrate the concept, consider Fig. 3 where $n, j = 1 \ldots 3$ and $m, k = 1$. As shown in the diagram, the pair of LPMDs that has the minimum $\xi$ is the $LPMD_h$ and $LPMD_v$ that arises both from $RD_1$. As the result, the calculated $MD$ is given as $Z = Z_{(h_{1,1}), (v_{1,1})} = [x_{h_{1,1}}, y_{v_{1,1}}]^T$. This is very close to the actual $MD$ located at $(20m, 10.8m)$ as shown.

### 2.3. LOS Path Identification and Enhancement Technique

With the calculated centroid $C$ in (1), we can obtain an estimated LOS TOA and AOA measurement data between the centroid and the reference device which is calculated as

$$r_{n,1} = \| C - Z_n \| \hspace{1cm} \beta_{n,1} = \tan^{-1}\left(\frac{[C]_2 - [Z_n]_2}{[C]_1 - [Z_n]_1}\right)$$  \hspace{1cm} (6)

where $r_{n,1}, \beta_{n,1}$ are the calculated LOS TOA (in terms of distance) and AOA at the centroid respectively. $[C]_1$ and $[Z_n]_1$ are the $x$ or first component of the column vector $[C]$ and $[Z_n]$ respectively. Similarly, $[C]_2$ and $[Z_n]_2$ are the $y$ or second component of the column vector $[C]$ and $[Z_n]$ respectively. Therefore, a measurement path can be considered a LOS path if

$$|\alpha_{n,1} - \beta_{n,1}| \leq 180^\circ \pm 3\sigma_{\alpha_n} \hspace{1cm} |d_{n,1} - r_{n,1}| \leq 3\sigma_{d_n}$$  \hspace{1cm} (7)

where $\sigma_{d_n}$ and $\sigma_{\alpha_n}$ represent the maximum standard deviation of the TOA (in terms of distance) and AOA measurement noise at $RD_n$.
respectively. We will use the notation that \( m = 1 \) is the LOS path since it is always the shortest path.

In relation to a LOS path that has been confirmed by satisfying the criteria in (7), we can ingeniously improve the accuracy of \( \alpha_{n,1} \) because of the geometrical relationship [21] that exists between \( \alpha_{n,1} \) and \( \beta_{n,1} \) as shown in Fig. 4. As shown, \( \alpha'_{n,1} \) and \( \beta'_{n,1} \) has the following relationship with \( \alpha_{n,1} \) and \( \beta_{n,1} \):

\[
\alpha'_{n,1} = \alpha_{n,1}, \quad \beta'_{n,1} = 360^\circ - \beta_{n,1}
\]

with

\[
\alpha_{n,1} = \alpha^o_{n,1} + n_{\alpha_{n,1}}, \quad \beta_{n,1} = \beta^o_{n,1} + n_{\beta_{n,1}}
\]

where \( \alpha^o_{n,1} \) and \( \beta^o_{n,1} \) denote the true noise free AOA values of the LOS path for \( RD_n \) and the centriod \( C \) respectively. \( n_{\alpha_{n,1}} \) and \( n_{\beta_{n,1}} \) represent the Gaussian noise associated with each. Under ideal circumstance where the noises are absent, the following angle relationship will always hold:

\[
\alpha'_{n,1} + \beta'_{n,1} = \alpha^o_{n,1} + \beta^o_{n,1} = 180^\circ
\]

The impact of measurement noise on \( \alpha_{n,1} \) and \( \beta_{n,1} \) will render (10) an inequality. However, we can minimize the noise error in \( \alpha_{n,1} \) and \( \beta_{n,1} \) and reestablish the equality relationship by adjusting the values of \( \alpha_{n,1} \) and \( \beta_{n,1} \) using the following criteria:

\[
e_n = \alpha'_{n,1} + \beta'_{n,1} - 180^\circ
\]

\[
\alpha'_{n,1} = \alpha'_{n,1} - e_n f_{\alpha_n}, \quad \beta'_{n,1} = \beta'_{n,1} - e_n f_{\beta_n}
\]

where \( e_n \) is defined as the noise angle difference for \( RD_n \) and \( f_{\alpha_n}, f_{\beta_n} \) are the respective error weighting factors which are calculated as

\[
f_{\alpha_n} = \frac{\sigma_{\alpha_n}}{\sigma_{\alpha_n} + \sigma_{\beta_n}}, \quad f_{\beta_n} = \frac{\sigma_{\beta_n}}{\sigma_{\alpha_n} + \sigma_{\beta_n}}
\]

\( \sigma_{\beta_n} \) is the standard deviation of the calculated AOA of the LOS path at the centriod. As for the TOA of the LOS path, the measured \( d_{n,1} \) is replaced by the calculated \( r_{n,1} \) in (6). Therefore by using (6), (7) and (12), the accuracy of both the measured TOA and AOA of the LOS path can be improved. By recalculating the centriod \( C \) in (1) and the new intersection point \( \xi \) in (4) once again with the adjusted LOS path’s TOA and AOA, the \( MD \) location accuracy is further improved.
3. ANALYTICAL PERFORMANCE BOUND

In all estimator design problems, it is important to determine the performance of the proposed estimator analytically so that we can understand the performance of the estimator at a glance without resorting to tedious simulation. One of the main criteria on the performance of the estimator is the Mean Square Error (MSE) variance of the estimator [21, 50]. In the following sections, we will derive both the LOS and NLOS path localization error variance for our two step DS localization scheme.

3.1. Proposed NLOS Path Localization Error Variance

Assume that the unidirectional measurement data \( d_{n,m}, \alpha_{n,m} \) are independent Gaussian random variables, such that \( d_{n,m} \sim N(d_{0,n,m}, \sigma_{d_{n,m}}^2) \) and \( \alpha_{n,m} \sim N(\alpha_{0,n,m}, \sigma_{\alpha_{n,m}}^2) \) [18, 19, 21–24]. \( d_{0,n,m} \) and \( \alpha_{0,n,m} \) are the actual values of the TOA and AOA of the \( m \)th path of \( RD_n \) respectively. \( \sigma_{d_{n,m}}^2 \) and \( \sigma_{\alpha_{n,m}}^2 \) are the noise variances of the TOA and AOA of the \( m \)th path of \( RD_n \) respectively. Defining \( \theta_o = [d_{0}^T, \alpha_{0}^T]^T \) and \( \theta = [d^T, \alpha^T]^T \) where \( d = [d_{1,1} \ldots d_{N,M}]^T \), \( \alpha^o = [\alpha_{0,1} \ldots \alpha_{0,N,M}]^T \), \( \alpha = [\alpha_{1,1} \ldots \alpha_{N,M}]^T \). We also let the variance \( \sigma_{d}^2 = [\sigma_{d_{1,1}}^2 \ldots \sigma_{d_{N,M}}^2]^T \) and \( \sigma_{\alpha}^2 = [\sigma_{\alpha_{1,1}}^2 \ldots \sigma_{\alpha_{N,M}}^2]^T \).

For small variance of the measurement data, we can adopt the Taylor series first order expansion [50] and approximate the estimated coordinates of the MD from the minimum \( \xi (\theta) \) as

\[
x_{h_{n,m}}(\theta) \approx x_{h_{n,m}}(\theta^o) + \nabla x_{h_{n,m}}(\theta^o)(\theta - \theta^o)
\]

\[
y_{v_{j,k}}(\theta) \approx y_{v_{j,k}}(\theta^o) + \nabla y_{v_{j,k}}(\theta^o)(\theta - \theta^o)
\]

where

\[
\nabla x_{h_{n,m}}(\theta^o) = \frac{\partial x_{h_{n,m}}(\theta^o)}{\partial \theta^T} \in \mathbb{R}^{1 \times 2NM}
\]

\[
\nabla y_{v_{j,k}}(\theta^o) = \frac{\partial y_{v_{j,k}}(\theta^o)}{\partial \theta^T} \in \mathbb{R}^{1 \times 2NM}
\]

and subject to the constraint that

\[
\xi(h_{n,m}, v_{j,k}) (\theta^o) + \sqrt{E \left[ \left( \xi(h_{n,m}, v_{j,k}) (\theta) - \xi(h_{n,m}, v_{j,k}) (\theta^o) \right)^2 \right]} \]

(16)
is minimum. (16) is the sum of the mean and Root Mean Square (RMS) of \( \xi_{(h_{n,m}), (v_{j,k})} (\theta) \). This implies that we would always choose a set of \( LPMD_h \) and \( LPMD_v \) that ensures the minimum of (16) for each varying instantaneous \( \theta \) before computing (15). The \( E \left[ (\cdot)^2 \right] \) in (16) can be derived by using Taylor series expansion as follows where

\[
\xi_{(h_{n,m}), (v_{j,k})} (\theta) \approx \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) + \nabla \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) (\theta - \theta^o) \tag{17}
\]

with

\[
\nabla \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) = \frac{\partial \xi_{(h_{n,m}), (v_{j,k})} (\theta^o)}{\partial \theta^T} \in \mathbb{R}^{1 \times 2NM} \tag{18}
\]

However, only four elements in the row vector in (18) which correspond to the set of measurement variables that arises from an associated pair of \( LPMD_h \) and \( LPMD_v \) are not null. In other words,

\[
\frac{\partial \xi_{(h_{n,m}), (v_{j,k})} (\theta^o)}{\partial d_{n,m}}, \frac{\partial \xi_{(h_{n,m}), (v_{j,k})} (\theta^o)}{\partial d_{v,k}}, \frac{\partial \xi_{(h_{n,m}), (v_{j,k})} (\theta^o)}{\partial \alpha_{n,m}}, \frac{\partial \xi_{(h_{n,m}), (v_{j,k})} (\theta^o)}{\partial \alpha_{v,k}} \neq 0
\]

where \( d_{n,m}, d_{v,k}, \alpha_{n,m}, \alpha_{v,k} \) arise from a particular pair of \( LPMD_{h_{n,m}} \) and \( LPMD_{v_{j,k}} \) which generates the \( \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) \) that is in consideration. Next, by rearranging (17) and taking the square and Expectation on both sides,

\[
E \left[ \left( \xi_{(h_{n,m}), (v_{j,k})} (\theta) - \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) \right)^2 \right] \approx \nabla \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) E \left[ (\theta - \theta^o) (\theta - \theta^o)^T \right] \nabla \xi_{(h_{n,m}), (v_{j,k})} (\theta^o) \tag{19}
\]

with

\[
E \left[ (\theta - \theta^o) (\theta - \theta^o)^T \right] = diag \left( \sigma_d^2, \sigma_\alpha^2 \right) \tag{20}
\]
Similarly, rearranging (14) and taking the square and expectation on both sides,

\[
E\left[(x_{h,n,m}(\theta) - x_{h,n,m}(\theta^o))^2\right]
\approx \nabla x_{h,n,m}(\theta^o) E\left[(\theta - \theta^o)(\theta - \theta^o)^T\right] \nabla x_{h,n,m}(\theta^o)
\]

\[
E\left[(y_{v,j,k}(\theta) - y_{v,j,k}(\theta^o))^2\right]
\approx \nabla y_{v,j,k}(\theta^o) E\left[(\theta - \theta^o)(\theta - \theta^o)^T\right] \nabla y_{v,j,k}(\theta^o)
\]

Finally, the variance of the localization error for our proposed NLOS localization scheme in (5) can be derived as

\[
E\left[(x - x^o)^2\right] 
\approx E\left[(x_{h,n,m}(\theta^o) - x^o)^2\right] + E\left[(x_{h,n,m}(\theta) - x_{h,n,m}(\theta^o))^2\right]
\]

\[
E\left[(y - y^o)^2\right] 
\approx E\left[(y_{v,j,k}(\theta^o) - y^o)^2\right] + E\left[(y_{v,j,k}(\theta) - y_{v,j,k}(\theta^o))^2\right]
\]

where \([x^o, y^o]^T\) is the true MD location coordinate. From (22), the RMS localization error (RMSE) for our proposed NLOS localization scheme will then be given as \(\sqrt{E\left[(x - x^o)^2\right] + E\left[(y - y^o)^2\right]}\).

### 3.2. Proposed LOS Path Localization Error Variance

If there is a LOS path, the variance in (20) pertaining to the LOS path need to be altered due to the readjustment of the measurement data \(t_{n,1}\) and \(\alpha_{n,1}\) for the LOS path of \(RD_n\). The modified noise error arising out of the AOA measurement has to be computed before we are able to derive the variance in the localization error in the LOS path. By substituting (9) into (8) and then (11), the noise angle difference \(e_n\) can be computed as follows:

\[
e_n = \begin{cases} 
 n_{\alpha,1} - n_{\beta,1} & \text{if } 0^\circ < \alpha_{n,1} \leq 90^\circ, \ 270^\circ < \alpha_{n,1} \leq 360^\circ \\
 n_{\beta,1} - n_{\alpha,1} & \text{if } 90^\circ < \alpha_{n,1} \leq 180^\circ, \ 180^\circ < \alpha_{n,1} \leq 270^\circ 
\end{cases}
\]

Putting (23) into (12) and subsequently into (8), it can be found that

\[
\alpha_{n,1} = \alpha_{n,1}^o + n'_{\alpha,1}, \quad n'_{\alpha,1} = n_{\alpha,1} - (n_{\alpha,1} - n_{\beta,1}) f_{\alpha_n}
\]
where \( n'_{\alpha,n,1} \) is the modified noise error arising from the \( \alpha_{n,1} \). From (24), it is clear that modification in (12) serves to minimize the measurement noise and seeks to provide the true AOA value of \( \alpha_{n,1} \). Therefore, by putting (24) into (20) for the AOA of the LOS path, the new \( \sigma_{\alpha,n,1}^2 \) in (20) is given as

\[
E \left[ (\alpha_{n,1} - \alpha_{n,1}^0) (\alpha_{n,1} - \alpha_{n,1}^0)^T \right] = E \left[ n'_{\alpha,n,1} n'_{\alpha,n,1}^T \right] = E \left[ (f_{\alpha,n} - 1)^2 n_{\alpha,n,1}^2 + f_{\alpha,n}^2 \bar{\beta}_{n,1} \right] = (f_{\alpha,n} - 1)^2 \sigma_{\alpha,n,1}^2 + f_{\alpha,n}^2 \sigma_{\bar{\beta},n,1}^2 \tag{25}
\]

where \( \sigma_{\bar{\beta},n,1}^2 \) can be obtained similarly from Taylor series expansion of \( \beta_{n,1} (\alpha_{n,1,1}, d_{n,1}) \) and is given as:

\[
\sigma_{\bar{\beta},n,1}^2 = E \left[ (\beta_{n,1} (\alpha_{n,1,1}, d_{n,1}) - \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0))^2 \right] \approx \nabla \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) \text{diag} \left( \sigma_{\alpha,n,1}^2, \sigma_{\bar{\beta},n,1}^2 \right) \nabla \beta_{n,1}^T (\alpha_{n,1,1}, d_{n,1}) \tag{26}
\]

with

\[
\beta_{n,1} (\alpha_{n,1,1}, d_{n,1}) \approx \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) + \nabla \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) \text{diag} \left( \alpha_{n,1,1} - \alpha_{n,1,1}^0, d_{n,1} - d_{n,1}^0 \right)
\]

\[
\nabla \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) = \left[ \frac{\partial \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0)}{\partial \alpha_{n,1}}, \frac{\partial \beta_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0)}{\partial d_{n,1}} \right] \in \mathbb{R}^{1 \times 2}
\]

To find out the new TOA variance of the LOS path that has undergone readjustment of the measurement data, we can adopt the same approach in which the new \( \sigma_{\bar{\tau},n,1}^2 \) (= \( \sigma_{\tau,n,1}^2 \) after LOS adjustment) can be recalculated as

\[
\sigma_{\bar{\tau},n,1}^2 = E \left[ (r_{n,1} (\alpha_{n,1,1}, d_{n,1}) - d_{n,1}^o)^2 \right] \approx \nabla r_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) \text{diag} \left( \sigma_{\alpha,n,1}^2, \sigma_{\bar{\tau},n,1}^2 \right) \nabla r_{n,1}^T (\alpha_{n,1,1}, d_{n,1}) \tag{27}
\]

with

\[
r_{n,1} (\alpha_{n,1,1}, d_{n,1}) \approx r_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) + \nabla r_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) \text{diag} \left( \alpha_{n,1,1} - \alpha_{n,1,1}^0, d_{n,1} - d_{n,1}^0 \right)
\]

\[
\nabla r_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0) = \left[ \frac{\partial r_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0)}{\partial \alpha_{n,1}}, \frac{\partial r_{n,1} (\alpha_{n,1,1}^0, d_{n,1}^0)}{\partial d_{n,1}} \right] \in \mathbb{R}^{1 \times 2}
\]
Therefore, the new calculated $\sigma^2_{\alpha_{n,1}}$ in (25) and $\sigma^2_{\alpha_{n,1}}$ in (27) for the LOS path are substituted into (20) to form the variance matrix to calculate RMS localization error.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

To evaluate the performance of our proposed localization scheme, a typical environment at Nanyang Technological University, School of Electrical and Electronic Engineering (EEE), Block S1, Level B3 (S1-B3) [21] is exploited as shown in Fig. 5. Channel measurements were taken and the measured data metrics (TOAs and AOAs) were verified accordingly to the traditional ray tracing methodology [47–49]. As mentioned earlier, the three RDs were located at $RD_1$ (25m, 9m), $RD_2$ (18m, 4m), $RD_3$ (3m, 14m) with the concrete walls served as the scatterers as shown in the plot. For simplicity, the dominant path ($M = 1$) are extracted from each RD and used for performance analysis. In total, there will be three paths from the RDs for each performance analysis. Fig. 5 traces the actual rays between the three RDs and MD. The following scenarios will be illustrated to examine the performance of our proposed NLOS localization scheme:

a) Case A — all RDs are in LOS with MD;

b) Case B — two RDs are in LOS with MD; and

c) Case C — one RD is in LOS with MD.

Figure 4. Geometrical relationship between AOAs for the LOS path measurement data adjustment.
The actual data metrics \((t^o_{n,m}, \alpha^o_{n,m})\) are obtained by correlating ray tracing methodology with the measurement data. For thorough comparison with existing conventional localization schemes, each actual data metrics will be subjected to noise effect using Gaussian random variable noise with a mean of zero, and unknown variance (variances of all devices are identical unless otherwise specified) \([18,19,21–24]\). The RMS localization error \(\sigma_{\text{RMS}}\) that is related to the true \(MD\) location is calculated as \(\sqrt{(x - x^o)^2 + (y - y^o)^2}\). It is computed based on 10,000 independent simulation runs. \(MD\) location \((x, y)\) is obtained from (5).

The conventional LOS localization schemes used for performance analysis are the TOA localization scheme \([12]\), and the TOA/AOA localization scheme that is brought about by the modification of the TDOA/AOA localization scheme in \([18]\). The rationale for modifying \([18]\) into TOA/AOA localization by extending the AOA formulation in \([18]\) into TOA formulation in \([12]\) is to allow stricter comparison. The TOA/AOA localization scheme will have lower localization error variance than the TDOA/AOA localization scheme because the latter sacrifices one time observation for the sake of synchronization. The conventional TOA localization scheme \([12]\) has its own mitigation technique while the conventional TOA/AOA localization scheme is coupled with the LOS reference devices detection technique \([19]\). Furthermore as the \(\sigma_{\beta_n}\) in (13) is unknown, we equate it to \(\sigma_{\alpha_n}\) which would render sufficient improvement in the LOS path measurement.

**Figure 5.** Ray tracing between the \(RDs\) and various \(MD\) locations at Nanyang Technological University, School of EEE, Block S1, level B3 (S1-B3).
4.1. Performance Probability Distribution Due to TOA, AOA Inaccuracy

Figure 6 depicts the location accuracy of the proposed NLOS localization scheme and compares with the existing localization schemes via their cumulative probability distribution (CDF). The AOA standard deviations $\sigma_\alpha$ for all devices are $2^\circ$ [21, 24, 35, 51] while the distance standard deviations $\sigma_d$ are 1 m [17, 21]. In this scenario, MD is located at $A$ (15 m, 13 m) where all RDs are in LOS with it. As shown in the plot, our proposed localization scheme’s performance is equivalent to the conventional TOA/AOA localization scheme.

![Figure 6](image.png)

**Figure 6.** Comparison of the CDF performance using distance standard deviation $\sigma_d = 1$ m and AOA standard deviation $\sigma_\alpha = 2^\circ$ for MD located at (15 m, 13 m). Case A — All RDs are in LOS with MD.

Figure 7 depicts the CDF performance for MD location at $B$ (20 m, 10.8 m). In this scenario, $RD_2$ is in NLOS with MD. The dominant path for $RD_2$ is a one bound scattering path arising out of the specular reflection at the wall (13 m, 6.83 m). Both the conventional LOS TOA and TOA/AOA localization schemes are coupled with their own NLOS mitigation schemes. For the LOS TOA/AOA localization scheme, this NLOS path is successfully detected and discarded in all the 10,000 runs. However, our proposed localization scheme still superiorly outperforms these conventional schemes, attaining $\sigma_{rms} \leq 2$ m for about 70% of the time.

Figure 8 illustrates the case where only $RD_2$ is in LOS with MD that is located at $C$ (16 m, 8 m). $RD_1$ has a dominant one bound scattering path that arises from the specular reflection at
Figure 7. Comparison of the CDF performance using distance standard deviation $\sigma_d = 1$ m and AOA standard deviation $\sigma_\alpha = 2^\circ$ for $MD$ located at $(20 \text{ m}, 10.8 \text{ m})$. Case B — Two RDs are in LOS with $MD$.

Figure 8. Comparison of the CDF performance using distance standard deviation $\sigma_d = 1$ m and AOA standard deviation $\sigma_\alpha = 2^\circ$ for $MD$ located at $(16 \text{ m}, 8 \text{ m})$. Case C — One RD is in LOS with $MD$.

the wall $(21.14 \text{ m}, 12 \text{ m})$. For $RD_3$, the dominant path is a triple bound scattering path that undergoes three reflection at the walls $(8 \text{ m}, 16.5 \text{ m})$, $(18 \text{ m}, 12.5 \text{ m})$ and $(13 \text{ m}, 9.5 \text{ m})$. As shown, both the conventional LOS schemes’ NLOS mitigation schemes could not
function well as the number of NLOS paths is greater than the number of LOS paths. As depicted, our proposed localization scheme outperforms the conventional schemes and is able to attain $\sigma_{rms} \leq 2\text{m}$ for more than 90% of the time.

4.2. Performance Bound Comparison

Figures 9 and 10 illustrate the Average Location Error (ALE) performance of all the schemes with 10,000 simulated $MD$ locations. These locations are uniformly distributed in the environment to demonstrate the robustness of our proposed localization scheme. Furthermore, at these uniformly distributed $MD$ locations, the dominant propagation paths arriving at the $RDs$ include all possible propagation paths. These propagation paths include not only the LOS, single and multiple specular reflection paths but also single edge diffraction path and path that comprises of a combination of multiple specular reflection and a single edge diffraction. The ALE is obtained by averaging the sum of the RMS location error obtained for the 10,000 $MD$ location. Fig. 9 illustrates the ALE performance for distance standard deviation of all $RDs$, $\sigma_d = 1\text{m}$ with varying AOA standard deviation, $\sigma_\alpha$ while Fig. 10 depicts the ALE performance for AOA standard deviation of all $RDs$, $\sigma_\alpha = 2^\circ$ with varying distance standard deviation, $\sigma_d$. As shown in both plots, our proposed localization scheme not only outperforms the conventional localization schemes throughout the 10,000 different $MD$ locations, but also robust to the ascending AOA standard deviation. Therefore, our proposed localization scheme demonstrates performance stability in relation to variations in both the location of $MD$ and the AOA standard deviation.

Figures 9 and 10 also illustrate the performance comparison between the RMS localization error of our proposed localization scheme and that of the derived analytical RMS expression (square root of the sum in (22)) for our proposed localization scheme. As shown in both plots, the difference between the analytical RMS expression and the actual localization scheme’s RMS error is subtle, thus supporting the accuracy of our derived analytical expression.

4.3. Performance Due to Different Degrees of Titled Scatterer

Finally, to illustrate the robustness in performance for our proposed NLOS localization scheme in environment where some of the scatterers are not parallel or perpendicular to each other, one of the scatterers near $MD$ in the environment is simulated to be titled to different
degrees, $\phi$ as shown in Fig. 11. The scatterer is rotated in clockwise direction at an angle $\phi$ from $0^\circ$ to $30^\circ$. As shown, the $RDs'$ received signals can transit from a LOS to a one bound scattering path as the scatterer is rotated. Fig. 12 depicts the performance of our proposed NLOS localization scheme for the titled scatterer at different degrees of clockwise rotation. As shown, our proposed localization scheme
Figure 11. Ray tracing between the RDs and MD for different degrees of titled scatterer for distance standard deviation $\sigma_d = 1$ m and AOA standard deviation $\sigma_\alpha = 2^\circ$.

Figure 12. Performance comparison due to different degrees of titled scatterer for distance standard deviation $\sigma_d = 1$ m and AOA standard deviation $\sigma_\alpha = 2^\circ$. 
still outperforms the conventional localization schemes as the titled scatterer is rotated, demonstrating our proposed scheme’s performance stability in relation to variations in the scatterers’ orientation and rotation.

5. CONCLUSION

A novel approach of two step NLOS localization scheme using estimation of TOA/AOA with the LOS and one bound scattering paths has been proposed with the key advantage of being able to work robustly in multipath environment. It has been demonstrated experimentally and coupled with simulations in a typical environment that our proposed NLOS localization scheme not only outperforms the conventional TOA and TOA/AOA localization schemes in all cases but also sustains performance stability in relation to variations in the location of \( MD \), the AOA standard deviation and the scatterers’ orientation. An average of \( \sigma_{rms} \leq 2 \text{m} \) is obtained for 90% of time for our proposed 2 step DS localization scheme without any mitigation scheme.

REFERENCES


