Progress In Electromagnetics Research, PIER 87, 233–244, 2008

MODAL SOLUTIONS FOR JUNCTION PARAMETERS OF DISCONTINUITY PROBLEMS IN DIELECTRIC RECTANGULAR WAVEGUIDES

S. Jigyasa

Applied Physics Department, Delhi College of Engineering
Delhi University
Bawana Road, Delhi 110042, India

A. De

Electronics and Communication Department
Delhi College of Engineering
Delhi University
Bawana Road, Delhi 110042, India

Abstract—In this paper the discontinuity problems of the junction of two different dielectric rectangular waveguides has been studied, both for two dimensional (2D) and three dimensional (3D) cases. The technique used, is to obtain expressions for the z-directed complex power due to all modes (propagating and non propagating) present at the step junction for a normalized incident field. The expressions for junction parameters like admittance and susceptance have been derived for the structures with step junctions in x direction, in y direction and for the 3D case where the step is both x and y directed. The numerical results have then been computed for different step ratios of these three cases.

1. INTRODUCTION

Discontinuity problems in Dielectric waveguides of both closed and open types play an important role in practical applications like filter design [10, 14, 16] and in integrated circuits ranging from microwave to optical wavelengths. The cascade of steps in a rectangular dielectric waveguide occurs in active and passive components for integrated optics and optical communication such as grating coupler, transformer and distributed feedback laser [1, 3, 7, 13].

Analysis of dielectric slab waveguides have been carried out by various authors [12, 15] by using techniques such as analytical continuity method [11].

The isolated step in a dielectric waveguide is a basic discontinuity problem occurring in various optical and millimeter wave components. Various techniques [4, 5, 8] have been used in obtaining solutions for different parameters at the step. The scattering parameters to observe the scattering behavior of the step discontinuity have been discussed in [6, 8].

The main objective of the proposed work is to obtain the junction admittance for a single step in a dielectric waveguide for both 2D and 3D problems. The junction susceptance is then derived which is useful for the microwave, millimeter wave networks and designing of components like filters, couplers, etc. The analysis presented in this paper is based on a very general concept of modal expansion of fields and hence can be extended for the problems of dielectric image lines or dielectric loaded metallic waveguides.

The analysis has been carried out for dielectric guide with high dielectric constant and thus no substantial leakage of field takes place in the waveguide. Hence radiation modes have been neglected for the analysis.

In the structures shown in Figures 1, 2, and 3 wave propagates from one dielectric waveguide to another with a different cross section where the two structures are connected to create abrupt discontinuities. The modes existing in the waveguide depend on the excitation of the guide. The non propagating modes are of appreciable magnitude only in the vicinity of sources or the discontinuity. These non propagating modes present at the step junction of our interest are nothing but the localization of energy at that discontinuity. These then give rise to the circuit parameters of capacitance and inductance. This reactance (or susceptance) is the basis for designing of any passive component used in the microwave and millimeter wave. In order to obtain the capacitance and inductance the reactive part of the power has to be separated from the total power present at the junction. This has been accomplished in this paper by exploring the propagation possibilities in all the modes after the arbitrary field inside a section of waveguide is expanded as a sum over all possible modes. Then the non propagating and propagating modes for a particular waveguide dimension are separated. In a lossless guide, the power for a propagating mode is a real quantity and that for a non propagating mode is imaginary. This imaginary power is the basis for the junction susceptance which has been computed for different cases in this paper.

2. ANALYSIS OF DISCONTINUITY

The structures of Figures 1 and 2 are two different problems which have been handled independently in this paper. Figure 1(c) represents the 3D discontinuity in both the transverse x and y directions.

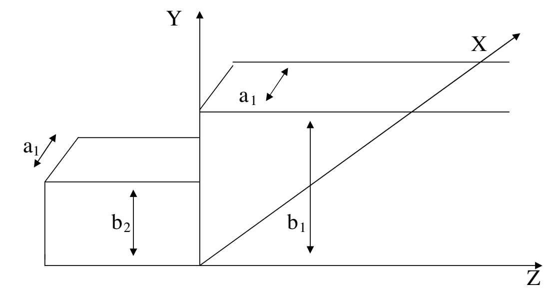

Figure 1. Y-directed step.

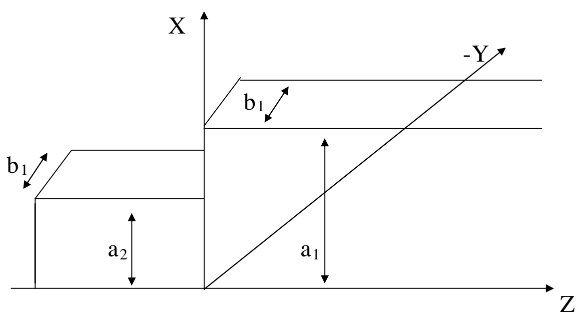

Figure 2. X-directed step.

The TM_x excitation for even modes has been considered and for all the structures we assume that almost all the fields are well confined to the waveguide.

For a guide of dimensions $a_1 \times b_1$ the wave functions for the bound mode (even) would be [9]

$$\psi = A\cos(ux)\cos(u_1 y)e^{jk_z z} \tag{1}$$

where u, u_1 and k_z are the wave numbers of the dielectric in the x and y transverse directions and z direction respectively.

2.1. Structure 1 (y Directed Step)

In the structure shown in Figure 1, let $H_y = f(x, y)$ be known over the $z = 0$ cross section. The field at $z > 0$ is to be determined assuming that the guide is matched. Considering the superposition of all TM_x modes we can write the ψ function as [8]

$$\psi = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos(m\pi x/2a_1) \cos(n\pi y/2b_1) e^{-\gamma_{mn} z} \tag{2}$$

where A_{mn} are the mode amplitudes and γ_{mn} the mode propagation constants. The value of H_y at $z = 0$ can be obtained as

$$H_{y|z=0} = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} -\gamma_{mn} A_{mn} \cdot \cos(m\pi x/2a_1) \cdot \cos(n\pi y/2b_1) \tag{3}$$

the above equation is in the form of a double Fourier series, $\gamma_{mn} A_{mn}$ are thus the Fourier coefficients of H_y and can be evaluated as

$$\begin{aligned} -\gamma_{mn} A_{mn} &= H_{mn} = (2\varepsilon_n/a_1 \cdot b_1) \\ &\quad \cdot \int_0^{a} \int_0^{b_1} H_{y|z=0} \cdot \cos(m\pi x/2a_1) \cdot \cos(n\pi y/2b_1) dx dy \end{aligned} \tag{4}$$

where $\varepsilon_n = 1$ for $n = 0$ and is 2 otherwise.

2.1.1. Modal Expansions of Fields

(i) Propagation in the Smaller Guide [$z < 0$]:

Assuming that only the normalized dominant mode (TM_{10}) propagates in the Waveguide

$$\begin{aligned} H_y &= \cos(\pi x/2a) \cdot e^{-\gamma_{10} z} & y < b_2 \\ &= 0 & y > b_2 \end{aligned} \tag{5}$$

H_y at the junction is then given as

$$\begin{aligned} H_{y|z=0} &= \cos(\pi x/2a_1) & y < b_2 \\ &= 0 & y > b_2 \end{aligned} \tag{6}$$

using Equations (4) and (6) the value of H_{10} at the junction can be obtained as

$$H_{10} = (2/a_1 \cdot b_1) \int_0^{a_1} \int_0^{b_2} \cos(\pi x/2a_1) \cdot \cos(\pi x/2a_1) dx dy \tag{7}$$

which reduces to

$$H_{10} = b_2/b_1 \tag{8}$$

(ii) Propagation in the Bigger Guide [$z > 0$]:

The mode amplitude derived in Equation (8) is that of the incident field for the bigger guide.

Due to the discontinuity at $z = 0$ spurious modes are generated in the second guide. Hence apart from the dominant mode of guide 2 other modes are also present at the junction which can be expressed as TM_{mn} in general. For them the modal amplitudes H_{mn} can be calculated using eqn. (4). Imposing the boundary condition on H_y that the tangential component of H vanishes along the discontinuity outside, we have for any general TM_{mn} mode

$$H_{mn} = (2\varepsilon_n/a_1 \cdot b_1) \cdot \int_0^{a_1} \int_0^{b_1} H_{y|z=0} \cdot \cos(m\pi x/2a_1) \cdot \cos(n\pi y/2b_1)\, dxdy \tag{9}$$

For the calculation of the above equation, taking different values of m and n three different cases arise. For $m \neq 1$, $n \neq 0$

$$\begin{aligned} H_{mn} &= 8\pi^2 \cdot \left(m^2 - 1\right) \cdot n\,[(m-1) \cdot \sin\{(\pi/2)(m+1)\} \\ &\quad + (m+1) \cdot \sin\{(\pi/2)(m-1)\}] \cdot \sin(n \cdot \pi \cdot b_2/2 \cdot b_1) \end{aligned} \tag{10}$$

For $m = 1$, $n \neq 0$

$$H_{1n} = (4/n \cdot \pi) \cdot \sin(n \cdot \pi \cdot b_2/2 \cdot b_1) \tag{11}$$

For $n = 0$, $m \neq 1$

$$\begin{aligned} H_{m0} &= 4b_2/\pi \left(m^2 - 1\right) \cdot [(m-1) \cdot \sin\{(\pi/2)(m+1)\} \\ &\quad + (m+1) \cdot \sin\{(\pi/2)(m-1)\}] \end{aligned} \tag{12}$$

2.1.2. Derivation of Power

For the wave function given by Equation (2) using the field equation

$$E_x = (1/j\omega\varepsilon_d) * (\delta^2/\delta x^2 + k^2)\psi \tag{13a}$$

$$H_y = \delta\psi/\delta z \tag{13b}$$

the z directed complex power at $z = 0$ is given by

$$P = \int_0^{a_1}\int_0^{b_1}\left(E_x X H^*_{y|z=0}\right) dxdy \tag{14}$$

The TM_x characteristic wave admittances are given by

$$(Y_0)_{mn} = -\left[(\gamma_{mn}) / \left(k_d^2 - (m\pi/2 \cdot a_1)^2\right)\right] .j\omega\varepsilon_d \tag{15}$$

Using (13), (14) and (15) and because of the orthogonality of the wave functions the z directed complex power reduces to

$$P = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} |H_{mn}|^2 \cdot (a_1 \cdot (b_1) / 2 \cdot \varepsilon_n) \cdot (1/(Y_0)_{mn}) \tag{16}$$

Equation (16) is simply a summation of powers for the individual modes which can be divided into 2 parts

$$P_{total} = P_{dominant} + P_{mn} \qquad (m \neq 1,\ n \neq 0) \tag{17}$$

where P_{mn} is the power due to the modes other than the dominant mode of guide 2 which comprise of the modal amplitudes given by Equations (10), (11) and (12).

The power for the dominant mode $P_{dominant}$ for the particular geometry is real and P_{mn} imaginary as k_z is real for the dominant mode and imaginary otherwise. The imaginary portion of the total power indicates no time-average power transmitted in these modes and hence the entire energy is localized to the junction.

2.1.3. Derivation of the Junction Parameters

To obtain the junction admittance, we consider the voltage across the centre of the junction ($z = 0$) as V. The junction admittance will then be given by

$$Y_j = P^* / |V|^2 \tag{18}$$

As the component of electric field contributing to z-directed complex power is E_x the voltage across the junction will be directed along the x direction and hence for the structure in Figure 1 the value would be $V = a$. ($V = E * d$)

$$Y_j = P^* / {a_1}^2 \tag{19}$$

Imaginary part of this equation is the junction susceptance B_j.

2.2. Structure 2 (x Directed Step)

2.2.1. Modal Expansion of Fields

(i) Propagation in the Smaller Guide [$z < 0$]:

For the same mode propagation for any incident field as in structure1 as shown in Figure 2 for the normalized dominant mode the value of H_y is given by

$$\begin{aligned} H_y &= \cos(\pi x/2a_2) \cdot e^{-\gamma_{10} z} \quad x < a_2 \\ &= 0 \quad\quad\quad\quad\quad\quad\quad\quad\quad x > a_2 \end{aligned} \tag{20a}$$

at the junction the value of H_y is given by

$$\begin{aligned} H_{y|z=0} &= \cos(\pi x/2a_2). \quad x < a_2 \\ &= 0 \quad\quad\quad\quad\quad\quad x > a_2 \end{aligned} \tag{20b}$$

using Equations (4) and (6) the value of H_{10} can be obtained as

$$H_{10} = (2/(a_1) \cdot b_1) \cdot \int_0^{a_2} \int_0^{b_1} \cos(\pi x/2a_1) \cdot \cos(\pi x/2a_1)\, dxdy \tag{21a}$$

This reduces to

$$\begin{aligned} H_{10} \;=\; & (2 \cdot a_1/\pi) \cdot [\{\sin\{(\pi \cdot a_1)(1/2a_1 + 1/2a_2)\}/(a_2 + a_1)\} \\ & \{\sin\{(\pi \cdot a_2)(1/2a_2 - 1/2a_1)\}/(a_1 - a_2)\}] \end{aligned} \tag{21b}$$

(ii) Propagation in the Bigger Guide:

Imposing the boundary conditions on H_y we have for any general TM_{mn} mode

$$H_{mn} = (2\varepsilon_n/a_1 \cdot b_1) \cdot \int_0^{a_2} \int_0^{b} H_{y|z=0} \cos(m\pi x/2a_1) \cdot \cos(n\pi y/2b)\, dxdy \tag{22}$$

which gives: for $n \neq 0$

$$\begin{aligned} H_{mn} \;=\; & \left(2 \cdot \varepsilon_n/a_1 \cdot n \cdot \pi^2\right) \\ & \cdot \sin(n\pi/2) \cdot [\{\sin(\pi/2 + m\pi a_2/2a_1)/(1/2a_2 + m/2a_1)\} \\ & + [\{\sin(\pi/2 - m\pi a_2/2a_1)/(1/2a_2 - m/2a_1)\} \end{aligned} \tag{23}$$

for $n = 0$

$$\begin{aligned} H_{m0} \;=\; & (2 \cdot \varepsilon_n/a_1 \cdot \pi)\, [\{\sin(\pi/2 + m\pi a_2/2a_1)/(1/2a_2 + m/2a_1)\} \\ & + [\{\sin(\pi/2 - m\pi a_2/2a_1)/(1/2a_2 - m/2a_1)\} \end{aligned} \tag{24}$$

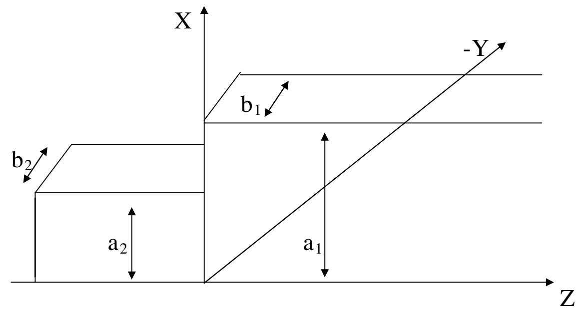

Figure 3. 3D step.

2.2.2. Derivation of Power

As for the previous structure the total power can be represented as

$$P'_{total} = P'_{dominant} + P'_{mn}$$

Voltage across the centre of the junction ($z = 0$), is $V = c$ for the structure in Figure 2. The junction admittance will then be given by

$$Y_j = P^*/|V|^2 = P^*/{a_2}^2 \tag{25}$$

the imaginary part of Y_j gives the junction susceptance B_j for the x-directed step.

2.3. Three Dimensional Step

For a Waveguide junction where the discontinuity is both x and y directed given in Figure 3, the analysis for the three dimensional step is done as for the above two structures. the value of H_y at the junction would be given by

$$\begin{aligned} H_{y|z=0} &= \cos(\pi x/2a_2). \quad & x < a_2 \\ &= 0 & x > a_2 \end{aligned} \tag{26}$$

The non zero modal amplitudes for the general TM_{mn} modes propagating in the bigger guide are thus given by

$$H_{mn} = (2\varepsilon_n/a_1 \cdot b_1) \cdot \int_0^{a_2}\int_0^{b_2} H_{y|z=0} \cdot \cos(m\pi x/2a_1) \cdot \cos(n\pi y/2b_1)\, dxdy \tag{27}$$

The junction admittance is obtained by using Equations (16) and (27) where P is the z-directed power in the bigger guide. The junction susceptance B_j is the imaginary part of this junction admittance.

Table 1. Susceptance values for different permittivities for step size ($a_2/a_1 = b_2/b_1 = .5$).

ε_d	4	6	8	10	12	14	16
$-B_j$	8.507	10.314	12.606	16.61	29.562	−4.02E-08	−1.53iE-8

3. RESULTS AND DISCUSSION

The junction susceptance for the three proposed structures has been calculated at 21 GHz for the dielectric of permittivity 12. The plots of the susceptance B_j at the junction verses the step ratios for the structures 1 and 2 are as shown in the graphs Figures 4, 5 and 6. Their behavior is in agreement with the results for the similar structures in metallic waveguide [9]. The susceptance for the first case of discontinuity (y-directed step) is capacitive (positive) and hence the junction can be termed as capacitive waveguide junction, the x-directed step having a negative susceptance is an inductive junction.

The behavior of the graphs can be understood as on increasing

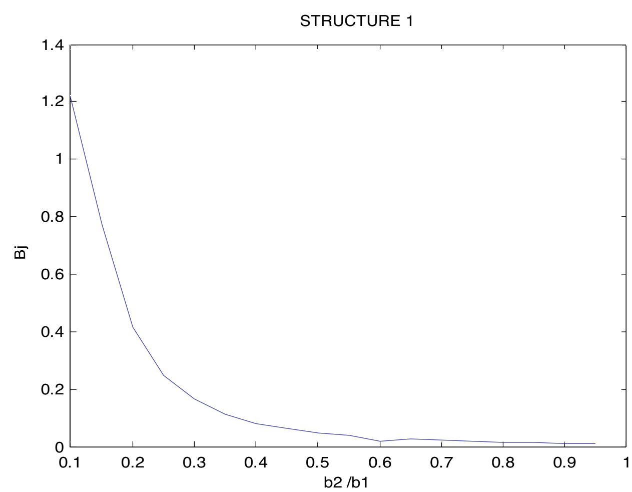

Figure 4. Susceptance verses step ratio for the y-directed step.

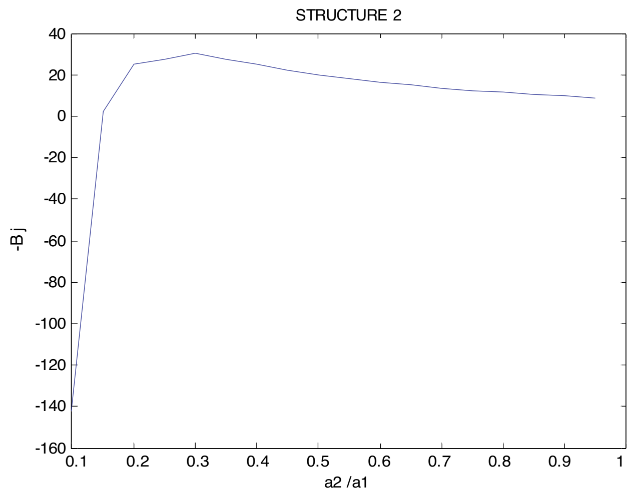

Figure 5. Susceptance verses step ratio for the x-directed step.

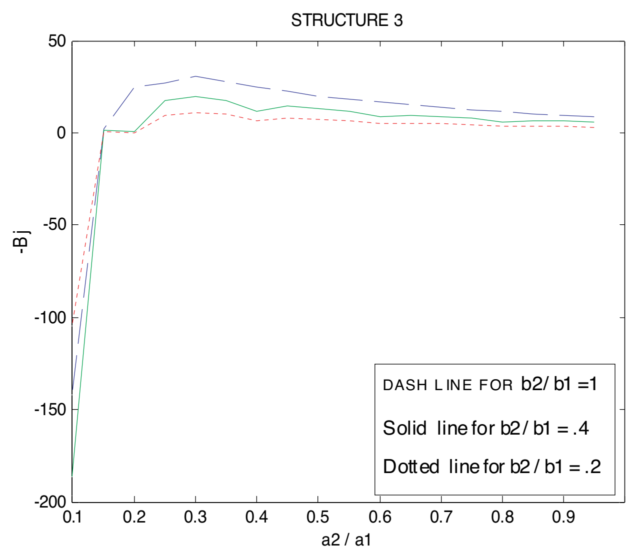

Figure 6. Susceptance verses step ratio for the 3D step.

the step size the value of susceptance increases. When both the guides are of same dimensions susceptances converge to zero value indicating that the whole power gets transmitted through the junction and there is no localization of energy at the step. The susceptance values with

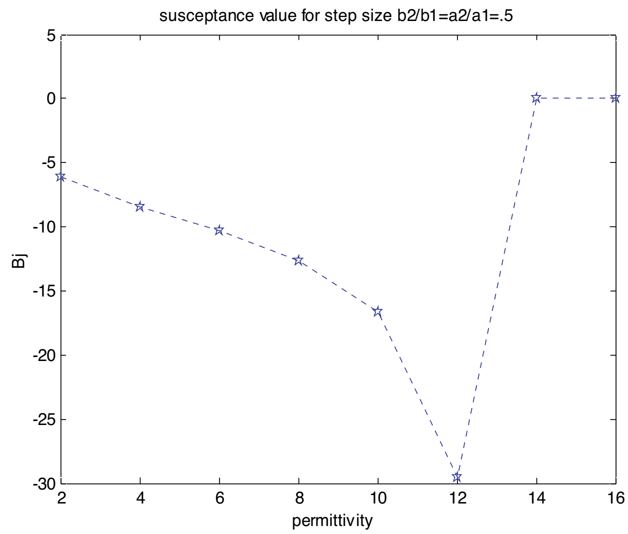

Figure 7. Susceptance verses permittivity for step size ($a_2/a_1 = b_2/b_1 = .5$).

fixed step ratios of $a_2/a_1 = b_2/b_1 = 0.5$ and different permittivity (ε_d) for the structure of Figure 3 are as given in Table 1 and shown in the graph. All numerical results have been computed using MATH-CAD software.

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