ENHANCED REFRACTOMETRIC OPTICAL SENSING BY USING ONE-DIMENSIONAL TERNARY PHOTONIC CRYSTALS

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Abstract—One-dimensional ternary photonic crystals are suggested as refractometric sensing elements, for sensing very small refractive index changes of a medium. These one-dimensional ternary photonic crystals based refractometric sensing elements are not only remarkably smaller, but are also more sensitive than one-dimensional binary photonic crystal based sensing element recently suggested by researchers.

1. INTRODUCTION

In past years, photonic band gap structures [1,2] attracted lot of attention of researchers [3–31], due to their enormous applications in optical communications, optoelectronics and optical instrumentation. These photonic band gap structures are multilayer structures formed by using two or more materials. These multilayered structures lead to formation of photonic band gaps or stop bands, in which propagation of electromagnetic waves of certain wavelengths are prohibited. However, these bands or ranges depend upon a number of parameters such as refractive indices of materials, filling fraction and angle of incidence. If all other parameters are kept constant, then any change in refractive index of a material will change the photonic band gaps and ranges of transmission. By monitoring transmission or reflection band change or shift, a slight change in refractive index of a structural material can be detected. Based on this principle, recently Hopman et al. [32] suggested a quasi-one dimensional photonic crystal based compact

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building block for refractometric optical sensing. The suggested one-
dimensional binary photonic band gap structure [32] was 76 µm thick
and it produced a wavelength band shift of 0.8 nm for a refractive index
change of 0.05.

Recently, one-dimensional periodic ternary photonic crystals
attracted attention of researchers due to their superior performance
over one-dimensional periodic binary crystals in omni directional
reflection [33] and tunable optical filtering [34]. These one-dimensional
ternary photonic band gap crystals are formed by periodic repetition
of three different material layers instead of two different materials
as in case of conventional one-dimensional binary photonic band gap
structures. Actually photonic crystals can be classified into categories
like binary, ternary, quaternary or so on, according to the number of
layers in one period. Binary photonic crystal contains two layers in
a period, ternary photonic crystal contains three layers in a period,
quaternary photonic crystal contains four layers in a period and so
on. Finally, these periods are repeated several times to form photonic
crystals.

These one-dimensional ternary photonic crystals are not only
superior omni directional reflectors [33] and tunable optical filters [34]
but in this paper, it is shown that they are superior refractometric
optical sensing elements also, when compared to one-dimensional
binary photonic crystals. It is shown in this paper that sixty three
times smaller and twenty two times more sensitive refractometric
sensing elements can be designed with one-dimensional ternary
photonic crystals as compared to one-dimensional binary photonic
crystal [32]. With proper arrangements, this kind of refractometric
sensing elements can be used in material adulteration sensors and
refractometer systems, for enhanced detection of adulteration and
determination refractive index of gases or liquids or suitable materials.

2. THEORY

The periodic structure consisting of alternate layers of refractive indices
\( n_1 \) (first material layer), \( n_2 \) (second material layer) and \( n_3 + \Delta n \) (third
material layer) with thicknesses \( d_1, d_2 \) and \( d_3 \) respectively is depicted
in Figure 1. \( d = d_1 + d_2 + d_3 \) is the period of the lattice. Here, the
third material layer in this structure could be of materials of which
refractive index could be varied.

It is assumed that the incident media is air \( (n_0 = 1.0) \). Light
is incident on the multilayer at an angle \( \theta_0 \). For the s wave, the
The characteristic matrix \[ M[d] \] of one period is given by

\[
M[d] = \prod_{i=1}^{l} \begin{bmatrix}
\cos \beta_i & -i \frac{\sin \beta_i}{p_i} \\
-ip_i \sin \beta_i & \cos \beta_i
\end{bmatrix} = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\] (1)

where, \( l = 3 \) (1, 2 and 3 signify the layers of refractive indices \( n_1, n_2 \) and \( n_3 \) respectively) \( \beta_1 = \frac{2\pi n_1 d_1 \cos \theta_1}{\lambda_0}, \beta_2 = \frac{2\pi n_2 d_2 \cos \theta_2}{\lambda_0}, \beta_3 = \frac{2\pi(n_3 + \Delta n)d_3 \cos \theta_3}{\lambda_0} \), \( \lambda_0 \) is the free space wavelength, \( p_1 = n_1 \cos \theta_1, p_2 = n_2 \cos \theta_2 \) \) and \( p_3 = (n_3 + \Delta n) \cos \theta_3 \), where, \( \Delta n \) is the change in refractive index of the third material layer due to introduction or adsorption of any material. \( \theta_1, \theta_2 \) and \( \theta_3 \) are the ray angles inside the layers 1, 2 and 3 respectively and are related to the angle of incidence \( \theta_0 \) by

\[
\cos \theta_1 = \left[ 1 - \frac{n_0^2 \sin^2 \theta_0}{n_1^2} \right]^{\frac{1}{2}},
\]
\[
\cos \theta_2 = \left[ 1 - \frac{n_0^2 \sin^2 \theta_0}{n_2^2} \right]^{\frac{1}{2}} \] and \( \cos \theta_3 = \left[ 1 - \frac{n_0^2 \sin^2 \theta_0}{(n_3 + \Delta n)^2} \right]^{\frac{1}{2}} \) (2)

The matrix \( M[d] \) in Equation (1) is unimodular as \( |M[d]| = 1 \).

For an \( N \) period structure, the characteristic matrix of the medium is given by

\[
[M(d)]^N = \begin{bmatrix}
M_{11} U_{N-1}(a) - U_{N-2}(a) & M_{12} U_{N-1}(a) \\
M_{21} U_{N-1}(a) & M_{22} U_{N-1}(a) - U_{N-2}(a)
\end{bmatrix}
\]
\[
\equiv \begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\]
where,

\[ M_{11} = \left( \cos \beta_1 \cos \beta_2 \cos \beta_3 - \frac{p_2 \sin \beta_1 \sin \beta_2 \cos \beta_3}{p_1} \right. \]
\[ \left. - \frac{p_3 \cos \beta_1 \sin \beta_2 \sin \beta_3}{p_2} - \frac{p_3 \sin \beta_1 \cos \beta_2 \sin \beta_3}{p_1} \right) \]
\[ M_{12} = -i \left( \sin \beta_1 \cos \beta_2 \cos \beta_3 + \frac{p_1 \sin \beta_1 \sin \beta_2 \cos \beta_3}{p_2} \right. \]
\[ \left. + \frac{p_2 \cos \beta_1 \cos \beta_2 \sin \beta_3}{p_3} - \frac{p_2 \sin \beta_1 \sin \beta_2 \sin \beta_3}{p_1 p_3} \right) \]
\[ M_{21} = -i \left( p_1 \sin \beta_1 \cos \beta_2 \cos \beta_3 + p_2 \cos \beta_1 \sin \beta_2 \cos \beta_3 \right. \]
\[ \left. + p_3 \cos \beta_1 \cos \beta_2 \sin \beta_3 - \frac{p_1 p_3 \sin \beta_1 \sin \beta_2 \sin \beta_3}{p_2} \right) \]
\[ M_{22} = \left( \cos \beta_1 \cos \beta_2 \cos \beta_3 - \frac{p_1 \sin \beta_1 \sin \beta_2 \cos \beta_3}{p_2} \right. \]
\[ \left. - \frac{p_2 \cos \beta_1 \sin \beta_2 \sin \beta_3}{p_3} - \frac{p_1 \sin \beta_1 \cos \beta_2 \sin \beta_3}{p_3} \right) \]

\( U_N \) are the Chebyshev polynomials of the second kind

\[ U_N (a) = \frac{\sin[(N + 1) \cos^{-1} a]}{\sqrt{1 - a^2}} \]  

(3)

where,

\[ a = \frac{1}{2} \left[ M_{11} + M_{22} \right] \]  

(4)

The transmission coefficient of the multilayer is given by

\[ t = \frac{2p_0}{(m_{11} + m_{12}p_0) p_0 + (m_{21} + m_{22}p_0)} \]  

(5)

and the transmissivity for this structure can be written in terms of transmission coefficient as

\[ T = |t|^2 \]  

(6)

where,

\[ p_0 = n_0 \cos \theta_0 = \cos \theta_0. \]

(7)

The transmissivity of the structure, for \( p \) wave can be obtained by using expressions (1)–(6) with the following values of \( p \)

\[ p_1 = \frac{\cos \theta_1}{n_1}, \quad p_2 = \frac{\cos \theta_2}{n_2}, \quad p_3 = \frac{\cos \theta_3}{(n_3 + \Delta n)} \quad \text{and} \quad p_0 = \frac{\cos \theta_0}{n_0} \]  

(8)
3. RESULTS AND DISCUSSION

To study the sensing performance of this one-dimensional ternary photonic crystal, two one-dimensional ternary photonic crystals with different structural parameters were considered. In the first one-dimensional ternary photonic crystal, values of various parameters were $n_1=1.37$, $n_2 = 4.35$, $d_1 = 90\text{ nm}$, $d_2 = 20\text{ nm}$ and $d_3 = 90\text{ nm}$, $N = 6$ with starting value of $n_3 = 3.6$. For normal incidence of light, the transmission spectrum for this one-dimensional ternary photonic crystal was plotted for different values of $\Delta n$ and wavelength shifts were calculated. Table 1 shows the wavelength shift ($\Delta \lambda$) with different values of $\Delta n$, for this first one-dimensional ternary photonic crystal. It was found that this one-dimensional ternary photonic crystal exhibits narrow transmission peak near photonic band edge, which shifts by 0.35 nm for each refractive index change of 0.001. To show this transmission peak shift distinctly, values of $\Delta n$ were taken proportionally larger for plotting Figure 2. Figure 2 shows the transmission peaks at refractive index difference of 0.01. The black coloured transmission peak (centered at 1510.5 nm) is for $\Delta n = 0.00$. The red coloured transmission peak (centered at 1514.0 nm) is for $\Delta n = 0.01$. The blue coloured transmission peak (centered at 1517.5 nm) is for $\Delta n = 0.02$. The pink coloured transmission peak (centered at 1521.0 nm) is for $\Delta n = 0.03$ and the green coloured transmission peak (centered at 1524.5 nm) is for $\Delta n = 0.04$. It was found that, the transmission peak shifts by 3.5 nm for each refractive index change of 0.01, which is proportional to transmission peak shift of 0.35 nm for each refractive index change of 0.001.

Next, another one-dimensional ternary photonic crystal with values of lattice parameters as $n_1 = 1.52$, $n_2 = 4.35$, $d_1 = 90\text{ nm}$, $d_2 = 20\text{ nm}$ and $d_3 = 90\text{ nm}$, $N = 6$ with starting value of $n_3 = 3.6$ was considered. For normal incidence of light on this one-dimensional ternary photonic crystal, the transmission spectrum was plotted for different values of $\Delta n$ and wavelength shifts were calculated. Table 2 shows the wavelength shift ($\Delta \lambda$) with different values of $\Delta n$, for this second one-dimensional ternary photonic crystal. It was found that this one-dimensional ternary photonic crystal exhibits narrow transmission peak near the forbidden wavelength band edge, which shifts by 0.35 nm for each refractive index change of 0.001. Again, to show this transmission peak shift distinctly, values of $\Delta n$ were taken proportionally larger for plotting Figure 3. Figure 3 shows the transmission peaks at refractive index difference of 0.01. The black coloured transmission peak (centered at 1518.0 nm) is for $\Delta n = 0.00$. The red coloured transmission peak (centered at 1521.5 nm) is
for $\Delta n = 0.01$. The blue coloured transmission peak (centered at 1525.0 nm) is for $\Delta n = 0.02$. The pink coloured transmission peak (centered at 1528.5 nm) is for $\Delta n = 0.03$ and the green coloured transmission peak (centered at 1532.0 nm) is for $\Delta n = 0.04$. It was observed that, the transmission peak shifts by 3.5 nm for each refractive index change of 0.01, which is proportional to transmission peak shift of 0.35 nm for each refractive index change of 0.001.

\[ \text{Figure 2. Transmission peaks for first one-dimensional ternary photonic crystal with different values of } \Delta n. \]

It is clearly evident from Tables 1–2 and Figures 2–3 that the transmission peaks exhibited near forbidden wavelength band edge by one-dimensional ternary photonic crystals shift toward higher wavelengths with increase in $\Delta n$. These transmission peak shifts are very easily detectable with existing optoelectronic devices.

From above observations, a relation between wavelength shift $\Delta \lambda$ and refractive index change $\Delta n$ can be found for one-dimensional ternary photonic crystal as

$$\Delta n = 2.857143 \times 10^6 \Delta \lambda.$$

Using these relations, the change in refractive index with respect to initial reference material can be directly calculated for any arbitrary wavelength shift, for the given two one-dimensional ternary photonic crystals. Similarly, relation between $\Delta n$ and $\Delta \lambda$ for any material combination can be derived, which makes it possible to calculate refractive index change and hence exact refractive index by measuring transmission peak shift.
Table 1. Wavelength shifts in first one-dimensional ternary photonic crystal with different values of $\Delta n$.

<table>
<thead>
<tr>
<th>$\Delta n$</th>
<th>$n_3 + \Delta n$</th>
<th>Transmission peak center</th>
<th>$\Delta \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>3.600</td>
<td>1510.50 nm</td>
<td>0.00 nm</td>
</tr>
<tr>
<td>0.001</td>
<td>3.601</td>
<td>1510.85 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.002</td>
<td>3.602</td>
<td>1511.20 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.003</td>
<td>3.603</td>
<td>1511.55 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.004</td>
<td>3.604</td>
<td>1511.90 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.005</td>
<td>3.605</td>
<td>1512.25 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.006</td>
<td>3.606</td>
<td>1512.60 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.007</td>
<td>3.607</td>
<td>1512.95 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.008</td>
<td>3.608</td>
<td>1513.30 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.009</td>
<td>3.609</td>
<td>1513.65 nm</td>
<td>0.35 nm</td>
</tr>
<tr>
<td>0.010</td>
<td>3.610</td>
<td>1514.00 nm</td>
<td>0.35 nm</td>
</tr>
</tbody>
</table>

Table 2. Wavelength shifts in second one-dimensional ternary photonic crystal with different values of $\Delta n$.

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<td>0.35 nm</td>
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<td>0.003</td>
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<td>3.604</td>
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</tr>
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</table>
4. CONCLUSIONS

One-dimensional ternary photonic crystals are suggested as refractometric sensing elements. The superiority of one-dimensional ternary photonic crystal over one-dimensional binary photonic crystal was shown for refractometric sensing application. It was observed that slight refractive index change in material layers of the ternary structure causes a sufficiently large transmission peak shift, which can be very easily detected by monitoring transmission spectrum, with the help of existing optoelectronic devices. An expression of relation between peak shift and refractive index change was also found to find the changed refractive index by measuring transmission peak shift. The sizes of the proposed one-dimensional ternary photonic crystal based sensing elements considered are much less than the refractometric sensing block suggested by using one-dimensional binary photonic crystal by researchers [32] recently. The thickness of one-dimensional ternary photonic crystal is 1200 nm which is only remarkably 1.5789% of the thickness of the previously suggested one-dimensional binary photonic crystal sensing block [32]. Further, the transmission peak shifts observed in these one-dimensional ternary photonic crystal first and second structures are 3.5 nm for refractive index change of 0.01, while in case of reference [32] it is 0.8 nm for a refractive index change of 0.05, which means that this one-dimensional ternary photonic crystal

![Figure 3. Transmission peaks for second one-dimensional ternary photonic crystal with different values of $\Delta n$.](image)
is 21.875 times more sensitive to refractive changes than structure reported in reference [32]. Therefore, it is suggested that one dimensional ternary photonic crystal can be substituted in place of one-dimensional binary photonic crystal for enhancing sensing performance with an additional advantage of reduced size.

These one-dimensional ternary photonic crystal sensing elements find potential applications in fluid (petrol, diesel etc.) or material adulteration sensors or refractometer systems. The use of one-dimensional ternary photonic crystals is not limited to enhanced refractive index sensing, but it can be further extended to enhanced temperature sensing by infiltrating the third material layer with liquid crystals, in which, refractive index changes can be thermally induced.

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