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ENHANCED REFRACTOMETRIC OPTICAL SENSING BY USING ONE-DIMENSIONAL TERNARY PHOTONIC CRYSTALS

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Abstract—One-dimensional ternary photonic crystals are suggested as refractometric sensing elements, for sensing very small refractive index changes of a medium. These one-dimensional ternary photonic crystals based refractometric sensing elements are not only remarkably smaller, but are also more sensitive than one-dimensional binary photonic crystal based sensing element recently suggested by researchers.

1. INTRODUCTION

In past years, photonic band gap structures [1, 2] attracted lot of attention of researchers [3–31], due to their enormous applications in optical communications, optoelectronics and optical instrumentation. These photonic band gap structures are multilayer structures formed by using two or more materials. These multilayered structures lead to formation of photonic band gaps or stop bands, in which propagation of electromagnetic waves of certain wavelengths are prohibited. However, these bands or ranges depend upon a number of parameters such as refractive indices of materials, filling fraction and angle of incidence. If all other parameters are kept constant, then any change in refractive index of a material will change the photonic band gaps and ranges of transmission. By monitoring transmission or reflection band change or shift, a slight change in refractive index of a structural material can be detected. Based on this principle, recently Hopman et al. [32] suggested a quasi-one dimensional photonic crystal based compact

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building block for refractometric optical sensing. The suggested one-dimensional binary photonic band gap structure [32] was 76 μm thick and it produced a wavelength band shift of 0.8 nm for a refractive index change of 0.05.

Recently, one-dimensional periodic ternary photonic crystals attracted attention of researchers due to their superior performance over one-dimensional periodic binary crystals in omni directional reflection [33] and tunable optical filtering [34]. These one-dimensional ternary photonic band gap crystals are formed by periodic repetition of three different material layers instead of two different materials as in case of conventional one-dimensional binary photonic band gap structures. Actually photonic crystals can be classified into categories like binary, ternary, quaternary or so on, according to the number of layers in one period. Binary photonic crystal contains two layers in a period, ternary photonic crystal contains three layers in a period, quaternary photonic crystal contains four layers in a period and so on. Finally, these periods are repeated several times to form photonic crystals.

These one-dimensional ternary photonic crystals are not only superior omni directional reflectors [33] and tunable optical filters [34] but in this paper, it is shown that they are superior refractometric optical sensing elements also, when compared to one-dimensional binary photonic crystals. It is shown in this paper that sixty three times smaller and twenty two times more sensitive refractometric sensing elements can be designed with one-dimensional ternary photonic crystals as compared to one-dimensional binary photonic crystal [32]. With proper arrangements, this kind of refractometric sensing elements can be used in material adulteration sensors and refractometer systems, for enhanced detection of adulteration and determination refractive index of gases or liquids or suitable materials.

2. THEORY

The periodic structure consisting of alternate layers of refractive indices n_1 (first material layer), n_2 (second material layer) and $n_3 + \Delta n$ (third material layer) with thicknesses d_1, d_2 and d_3 respectively is depicted in Figure 1. $d = d_1 + d_2 + d_3$ is the period of the lattice. Here, the third material layer in this structure could be of materials of which refractive index could be varied.

It is assumed that the incident media is air ($n_0 = 1.0$). Light is incident on the multilayer at an angle θ_0. For the s wave, the

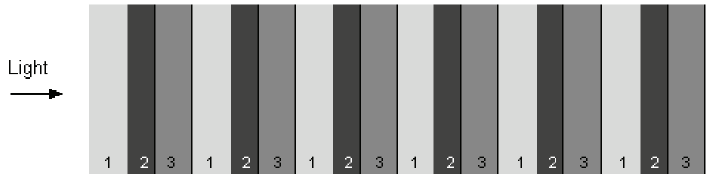

Figure 1. Structure of one-dimensional ternary photonic crystal sensing element.

characteristic matrix [35] $M[d]$ of one period is given by

$$M\left[d\right]=\prod_{i=1}^{l}\begin{bmatrix}\cos\beta_i & \frac{-i\sin\beta_i}{p_i}\\ -ip_i\sin\beta_i & \cos\beta_i\end{bmatrix}=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix} \tag{1}$$

where, $l = 3$ (1, 2 and 3 signify the layers of refractive indices n_1, n_2 and n_3 respectively) $\beta_1 = \frac{2\pi n_1 d_1\cos\theta_1}{\lambda_0}$, $\beta_2 = \frac{2\pi n_2 d_2\cos\theta_2}{\lambda_0}$, $\beta_3 = \frac{2\pi(n_3+\Delta n)d_3\cos\theta_3}{\lambda_0}$, λ_0 is the free space wavelength, $p_1 = n_1\cos\theta_1$, $p_2 = n_2\cos\theta_2$ and $p_3 = (n_3+\Delta n)\cos\theta_3$, where, Δn is the change in refractive index of the third material layer due to introduction or adsorption of any material. θ_1, θ_2 and θ_3 are the ray angles inside the layers 1, 2 and 3 respectively and are related to the angle of incidence θ_0 by

$$\cos\theta_1=\left[1-\frac{n_0^2\sin^2\theta_0}{n_1^2}\right]^{\frac{1}{2}},$$

$$\cos\theta_2=\left[1-\frac{n_0^2\sin^2\theta_0}{n_2^2}\right]^{\frac{1}{2}} \text{ and } \cos\theta_3=\left[1-\frac{n_0^2\sin^2\theta_0}{(n_3+\Delta n)^2}\right]^{\frac{1}{2}} \tag{2}$$

The matrix $M\left[d\right]$ in Equation (1) is unimodular as $|M[d]| = 1$.

For an N period structure, the characteristic matrix of the medium is given by

$$\left[M\left(d\right)\right]^N=\begin{bmatrix}M_{11}U_{N-1}\left(a\right)-U_{N-2}\left(a\right) & M_{12}U_{N-1}\left(a\right)\\ M_{21}U_{N-1}\left(a\right) & M_{22}U_{N-1}\left(a\right)-U_{N-2}\left(a\right)\end{bmatrix}$$
$$\equiv\begin{bmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{bmatrix}$$

where,

$$
\begin{aligned}
M_{11} &= \left(\cos\beta_1\cos\beta_2\cos\beta_3 - \frac{p_2\sin\beta_1\sin\beta_2\cos\beta_3}{p_1}\right. \\
&\quad \left. -\frac{p_3\cos\beta_1\sin\beta_2\sin\beta_3}{p_2} - \frac{p_3\sin\beta_1\cos\beta_2\sin\beta_3}{p_1}\right) \\
M_{12} &= -i\left(\frac{\sin\beta_1\cos\beta_2\cos\beta_3}{p_1} + \frac{\cos\beta_1\sin\beta_2\cos\beta_3}{p_2}\right. \\
&\quad \left. +\frac{\cos\beta_1\cos\beta_2\sin\beta_3}{p_3} - \frac{p_2\sin\beta_1\sin\beta_2\sin\beta_3}{p_1p_3}\right) \\
M_{21} &= -i\left(p_1\sin\beta_1\cos\beta_2\cos\beta_3 + p_2\cos\beta_1\sin\beta_2\cos\beta_3\right. \\
&\quad \left. +p_3\cos\beta_1\cos\beta_2\sin\beta_3 - \frac{p_1p_3\sin\beta_1\sin\beta_2\sin\beta_3}{p_2}\right) \\
M_{22} &= \left(\cos\beta_1\cos\beta_2\cos\beta_3 - \frac{p_1\sin\beta_1\sin\beta_2\cos\beta_3}{p_2}\right. \\
&\quad \left. -\frac{p_2\cos\beta_1\sin\beta_2\sin\beta_3}{p_3} - \frac{p_1\sin\beta_1\cos\beta_2\sin\beta_3}{p_3}\right)
\end{aligned}
$$

U_N are the Chebyshev polynomials of the second kind

$$U_N\left(a\right) = \frac{\sin[(N+1)\cos^{-1}a]}{[1-a^2]^{\frac{1}{2}}} \tag{3}$$

where,

$$a = \frac{1}{2}\left[M_{11} + M_{22}\right] \tag{4}$$

The transmission coefficient of the multilayer is given by

$$t = \frac{2p_0}{\left(m_{11} + m_{12}p_0\right)p_0 + \left(m_{21} + m_{22}p_0\right)} \tag{5}$$

and the transmissivity for this structure can be written in terms of transmission coefficient as

$$T = |t|^2 \tag{6}$$

where,

$$p_0 = n_0\cos\theta_0 = \cos\theta_0. \tag{7}$$

The transmissivity of the structure, for p wave can be obtained by using expressions (1)–(6) with the following values of p

$$p_1 = \frac{\cos\theta_1}{n_1},\ p_2 = \frac{\cos\theta_2}{n_2},\ p_3 = \frac{\cos\theta_3}{(n_3+\Delta n)}\ \text{ and }\ p_0 = \frac{\cos\theta_0}{n_0} \tag{8}$$

3. RESULTS AND DISCUSSION

To study the sensing performance of this one-dimensional ternary photonic crystal, two one-dimensional ternary photonic crystals with different structural parameters were considered. In the first one-dimensional ternary photonic crystal, values of various parameters were n_1=1.37, $n_2 = 4.35$, $d_1 = 90$ nm, $d_2 = 20$ nm and $d_3 = 90$ nm, $N = 6$ with starting value of $n_3 = 3.6$. For normal incidence of light, the transmission spectrum for this one-dimensional ternary photonic crystal was plotted for different values of Δn and wavelength shifts were calculated. Table 1 shows the wavelength shift ($\Delta\lambda$) with different values of Δn, for this first one-dimensional ternary photonic crystal. It was found that this one-dimensional ternary photonic crystal exhibits narrow transmission peak near photonic band edge, which shifts by 0.35 nm for each refractive index change of 0.001. To show this transmission peak shift distinctly, values of Δn were taken proportionally larger for plotting Figure 2. Figure 2 shows the transmission peaks at refractive index difference of 0.01. The black coloured transmission peak (centered at 1510.5 nm) is for $\Delta n =$ 0.00. The red coloured transmission peak (centered at 1514.0 nm) is for $\Delta n = 0.01$. The blue coloured transmission peak (centered at 1517.5 nm) is for Δn =0.02. The pink coloured transmission peak (centered at 1521.0 nm) is for $\Delta n = 0.03$ and the green coloured transmission peak (centered at 1524.5 nm) is for $\Delta n = 0.04$. It was found that, the transmission peak shifts by 3.5 nm for each refractive index change of 0.01, which is proportional to transmission peak shift of 0.35 nm for each refractive index change of 0.001.

Next, another one-dimensional ternary photonic crystal with values of lattice parameters as $n_1 = 1.52$, $n_2 = 4.35$, $d_1 = 90$ nm, $d_2 = 20$ nm and $d_3 = 90$ nm, $N = 6$ with starting value of $n_3 = 3.6$ was considered. For normal incidence of light on this one-dimensional ternary photonic crystal, the transmission spectrum was plotted for different values of Δn and wavelength shifts were calculated. Table 2 shows the wavelength shift ($\Delta\lambda$) with different values of Δn, for this second one-dimensional ternary photonic crystal. It was found that this one-dimensional ternary photonic crystal exhibits narrow transmission peak near the forbidden wavelength band edge, which shifts by 0.35 nm for each refractive index change of 0.001. Again, to show this transmission peak shift distinctly, values of Δn were taken proportionally larger for plotting Figure 3. Figure 3 shows the transmission peaks at refractive index difference of 0.01. The black coloured transmission peak (centered at 1518.0 nm) is for $\Delta n =$ 0.00. The red coloured transmission peak (centered at 1521.5 nm) is for Δn = 0.01. The blue coloured transmission peak (centered at 1525.0 nm) is for Δn =0.02. The pink coloured transmission peak (centered at 1528.5 nm) is for $\Delta n = 0.03$ and the green coloured transmission peak (centered at 1532.0 nm) is for $\Delta n = 0.04$. It was observed that, the transmission peak shifts by 3.5 nm for each refractive index change of 0.01, which is proportional to transmission peak shift of 0.35 nm for each refractive index change of 0.001.

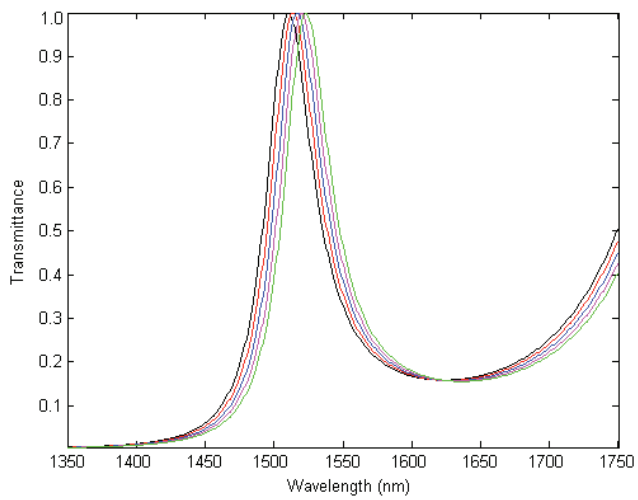

Figure 2. Transmission peaks for first one-dimensional ternary photonic crystal with different values of Δn.

It is clearly evident from Tables 1–2 and Figures 2–3 that the transmission peaks exhibited near forbidden wavelength band edge by one-dimensional ternary photonic crystals shift toward higher wavelengths with increase in Δn. These transmission peak shifts are very easily detectable with existing optoelectronic devices.

From above observations, a relation between wavelength shift $\Delta\lambda$ and refractive index change Δn can be found for one-dimensional ternary photonic crystal as $\Delta n = 2.857143 \times 10^6 \Delta\lambda$. Using these relations, the change in refractive index with respect to initial reference material can be directly calculated for any arbitrary wavelength shift, for the given two one-dimensional ternary photonic crystals. Similarly, relation between Δn and $\Delta\lambda$ for any material combination can be derived, which makes it possible to calculate refractive index change and hence exact refractive index by measuring transmission peak shift.

Table 1. Wavelength shifts in first one-dimensional ternary photonic crystal with different values of Δn.

Δn	$n_3 + \Delta n$	Transmission peak center	$\Delta\lambda$
0.000	3.600	1510.50 nm	0.00 nm
0.001	3.601	1510.85 nm	0.35 nm
0.002	3.602	1511.20 nm	0.35 nm
0.003	3.603	1511.55 nm	0.35 nm
0.004	3.604	1511.90 nm	0.35 nm
0.005	3.605	1512.25 nm	0.35 nm
0.006	3.606	1512.60 nm	0.35 nm
0.007	3.607	1512.95 nm	0.35 nm
0.008	3.608	1513.30 nm	0.35 nm
0.009	3.609	1513.65 nm	0.35 nm
0.010	3.610	1514.00 nm	0.35 nm

Table 2. Wavelength shifts in second one-dimensional ternary photonic crystal with different values of Δn.

Δn	$n_3 + \Delta n$	Transmission peak center	$\Delta\lambda$
0.000	3.600	1518.00 nm	0.00 nm
0.001	3.601	1518.35 nm	0.35 nm
0.002	3.602	1518.70 nm	0.35 nm
0.003	3.603	1519.05 nm	0.35 nm
0.004	3.604	1519.40 nm	0.35 nm
0.005	3.605	1519.75 nm	0.35 nm
0.006	3.606	1520.10 nm	0.35 nm
0.007	3.607	1520.45 nm	0.35 nm
0.008	3.608	1520.80 nm	0.35 nm
0.009	3.609	1521.15 nm	0.35 nm
0.010	3.610	1521.50 nm	0.35 nm

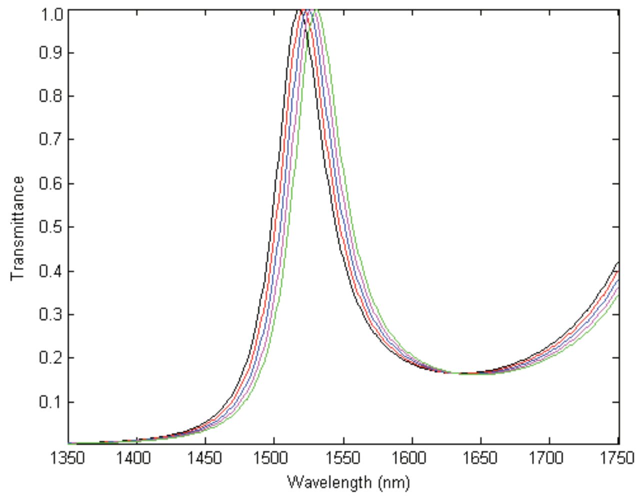

Figure 3. Transmission peaks for second one-dimensional ternary photonic crystal with different values of Δn.

4. CONCLUSIONS

One-dimensional ternary photonic crystals are suggested as refractometric sensing elements. The superiority of one-dimensional ternary photonic crystal over one-dimensional binary photonic crystal was shown for refractometric sensing application. It was observed that slight refractive index change in material layers of the ternary structure causes a sufficiently large transmission peak shift, which can be very easily detected by monitoring transmission spectrum, with the help of existing optoelectronic devices. An expression of relation between peak shift and refractive index change was also found to find the changed refractive index by measuring transmission peak shift. The sizes of the proposed one-dimensional ternary photonic crystal based sensing elements considered are much less than the refractometric sensing block suggested by using one-dimensional binary photonic crystal by researchers [32] recently. The thickness of one-dimensional ternary photonic crystal is 1200 nm which is only remarkably 1.5789% of the thickness of the previously suggested one-dimensional binary photonic crystal sensing block [32]. Further, the transmission peak shifts observed in these one-dimensional ternary photonic crystal first and second structures are 3.5 nm for refractive index change of 0.01, while in case of reference [32] it is 0.8 nm for a refractive index change of 0.05, which means that this one-dimensional ternary photonic crystal is 21.875 times more sensitive to refractive changes than structure reported in reference [32]. Therefore, it is suggested that one dimensional ternary photonic crystal can be substituted in place of one-dimensional binary photonic crystal for enhancing sensing performance with an additional advantage of reduced size.

These one-dimensional ternary photonic crystal sensing elements find potential applications in fluid (petrol, diesel etc.) or material adulteration sensors or refractometer systems. The use of one-dimensional ternary photonic crystals is not limited to enhanced refractive index sensing, but it can be further extended to enhanced temperature sensing by infiltrating the third material layer with liquid crystals, in which, refractive index changes can be thermally induced.

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REFERENCES

1. Yablonovitch, E., "Inhibited spontaneous emission in solid state physics and electronics," *Phys. Rev. Lett.*, Vol. 58, 2059–2062, 1987.
2. Yablonovitch, E., "Photonic band gap structures," *Journal of Optical Society of America B*, Vol. 10, 283–295, 1993.
3. Li, Y. Y., P. F. Gu, M. Y. Li, H. Yan, and X. Liu, "Research on the wide-angle and broadband 2D photonic crystal polarization splitter," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 2, 265–273, 2006.
4. Fu, Y. Q., Q. R. Zheng, Q. Gao, and G. H. Zhang, "Mutual coupling reduction between large antenna arrays using electromagnetic bandgap (EBG) structures," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 6, 819–825, 2006.
5. Li, B., B. Wu, and C. H. Liang, "High gain circular waveguide array antenna using electromagnetic band-gap structure," *Journal*

of Electromagnetic Waves and Applications, Vol. 20, No. 7, 955–966, 2006.

6. Aissaoui, M., J. Zaghdoudi, M. Kanzari, and B. Rezig, "Optical properties of the quasi-periodic one-dimensional generalized multilayer fibonacci structures," *Progress In Electromagnetics Research*, PIER 59, 69–83, 2006.
7. Rojas, J. A. M., J. Alpuente, J. Piñeiro, and R. Sánchez, "Rigorous full vectorial analysis of electromagnetic wave propagation in 1D," *Progress In Electromagnetics Research*, PIER 63, 89–105, 2006.
8. Banerjee, A., S. K. Awasthi, U. Malaviya, and S. P. Ojha, "Design of a nano-layered tunable optical filter," *J. Mod. Opt.*, Vol. 53, No. 12, 1739–1752, 2006.
9. Li, L., C.-H. Liang, and C.-H. Chan, "Waveguide end-slot phased array antenna integrated with electromagnetic bandgap structures," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 2, 161–174, 2007.
10. Zheng, Q. R., B. Q. Lin, Y. Q. Fu, and N. C. Yuan, "Characteristics and applications of a novel compact spiral electromagnetic band-gap (EBG) structure," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 2, 199–213, 2007.
11. Villa-Villa, F., J. A. Gaspar-Armenta, and A. Mendoza-Suárez, "Surface modes in one dimensional photonic crystals that include left handed materials," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 4, 485–499, 2007.
12. Wu, C.-J. and S. Gwo, "Calculations of optical properties of an annular dielectric mirror," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 6, 821–827, 2007.
13. Rojas, J. A. M., J. Alpuente, P. López-Espí, and P. García, "Accurate model of electromagnetic wave propagation in unidimensional photonic crystals with defects," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1037–1051, 2007.
14. Castaldi, G., V. Galdi, V. Pierro, and I. M. Pinto, "Radiation from fibonacci-type quasiperiodic arrays on dielectric substrates," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 9, 1231–1245, 2007.
15. Lu, W. B. and T. J. Cui, "Efficient method for full-wave analysis of large-scale finite-sized periodic structures," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 14, 2157–2168, 2007.

16. Edalati, A., H. Boutayeb, and T. A. Denidni, "Band structure analysis of reconfigurable metallic crystals: Effect of active elements," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 15, 2421–2430, 2007.
17. Xu, H.-J., Y.-H. Zhang, and Y. Fan, "Analysis of the connection between k connector and microstrip with electromagnetic bandgap (EBG) structures," *Progress In Electromagnetics Research*, PIER 73, 239–247, 2007.
18. Watanabe, K. and K. Yasumoto, "Two-dimensional electromagnetic scattering of non-plane incident waves by periodic structures," *Progress In Electromagnetics Research*, PIER 74, 241–271, 2007.
19. Zandi, O., Z. Atlasbaf, and K. Forooraghi, "Flat multilayer dielectric reflector antennas," *Progress In Electromagnetics Research*, PIER 72, 1–19, 2007.
20. Golmohammadi, S., M. K. Moravvej-Farshi, A. Rostami, and A. Zarifkar, "Spectral analysis of the fibonacci-class one-dimensional quasi-periodic structures," *Progress In Electromagnetics Research*, PIER 75, 69–84, 2007.
21. Srivastava, S. K. and S. P. Ojha, "Enhancement of omnidirectional reflection bands in one-dimensional photonic crystals with left-handed materials," *Progress In Electromagnetics Research*, PIER 68, 91–111, 2007.
22. Chandran, A. R., M. Gopikrishna, C. K. Aanandan, P. Mohanan, and K. Vasudevan, "Scattering behaviour of fractal based metallo-dielectric structures," *Progress In Electromagnetics Research*, PIER 69, 323–339, 2007.
23. Singh, S. K., J. P. Pandey, K. B. Thapa, and S. P. Ojha, "Structural parameters in the formation of omnidirectional high reflectors," *Progress In Electromagnetics Research*, PIER 70, 53–78, 2007.
24. Ibrahim, A.-B. M. A. and P. K. Choudhury, "Relative power distributions in omniguiding photonic band-gap fibers," *Progress In Electromagnetics Research*, PIER 72, 269–278, 2007.
25. Srivastava, S. K. and S. P. Ojha, "Omnidirectional reflection bands in one-dimensional photonic crystal structure using fullerene films," *Progress In Electromagnetics Research*, PIER 74, 181–194, 2007.
26. El-Dahshory, M. A., A. M. Attiya, and E. A. Hashish, "Design equations of two-dimensional dielectric photonic band gap structures," *Progress In Electromagnetics Research*, PIER 74, 319–340, 2007.

27. Srivastava, R., S. Pati, and S. P. Ojha, "Enhancement of omnidirectional reflection in photonic crystal heterostructures," *Progress In Electromagnetics Research B*, Vol. 1, 197–208, 2008.
28. Srivastava, R., S. Srivastava, and S. P. Ojha, "Negative refraction by photonic crystal," *Progress In Electromagnetics Research B*, Vol. 2, 15–26, 2008.
29. Prokopovich, V., A. V. Popov, and A. V. Vinogradov, "Analytical and numerical aspects of bragg fiber design," *Progress In Electromagnetics Research B*, Vol. 6, 361–379, 2008.
30. Srivastava, R., K. B. Thapa, S. Pati, and S. P. Ojha, "Omni-direction reflection in one dimensional photonic crystal," *Progress In Electromagnetics Research B*, Vol. 7, 133–143, 2008.
31. Minin, I. V., O. V. Minin, Y. R. Triandaphilov, and V. V. Kotlyar, "Subwavelength diffractive photonic crystal lens," *Progress In Electromagnetics Research B*, Vol. 7, 257–264, 2008.
32. Hopman, W. C. L., P. Pottier, D. Yudistira, J. V. Lith, P. V. Lambeck, R. M. De, L. Rue, A. Driessen, H. J. W. M. Hoekstra, and R. M. de Ridder, "Quasi-one-dimensional photonic crystal as a compact building-block for refractometric optical sensors," *IEEE J. Selected Topics in Quantum Electronics*, Vol. 11, No. 1, 11–16, 2005.
33. Awasthi, S. K., U. Malaviya, and S. P. Ojha, "Enhancement of omnidirectional total-reflection wavelength range by using one-dimensional ternary photonic bandgap material," *J. Opt. Soc. Am. B*, Vol. 23, 2566–2571, 2006.
34. Awasthi, S. K. and S. P. Ojha, "Design of a tunable optical filter by using one-dimensional ternary photonic band gap material," *Progress In Electromagnetic Research M*, Vol. 4, 117–132, 2008.
35. Born, M. and E. Wolf, "Basic properties of electromagnetic fields," *Principles of Optics*, 58–69, Cambridge University Press, U.K., 1980.