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SSOR PRECONDITIONED INNER-OUTER FLEXIBLE GMRES METHOD FOR MLFMM ANALYSIS OF SCATTERING OF OPEN OBJECTS

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Abstract—To efficiently solve large dense complex linear system arising from electric field integral equations (EFIE) formulation of electromagnetic scattering problems, the multilevel fast multipole method (MLFMM) is used to accelerate the matrix-vector product operations. The inner-outer flexible generalized minimum residual method (FGMRES) is combined with the symmetric successive over-relaxation (SSOR) preconditioner based on the near-part matrix of the EFIE in the inner iteration of FGMRES to speed up the convergence rate of iterative methods. Numerical experiments with a few electromagnetic scattering problems for open structures are given to demonstrate the efficiency of the proposed method.

1. INTRODUCTION

Electromagnetic integral equations are often discretized with the method of moments (MoM) [1–7], one of the most widespread and generally accepted techniques for electromagnetic problems. The formulation considered in this paper is the electric field integral equation (EFIE) as it is the most general and does not require any assumption about the geometry of the object. It is convenient to model objects with arbitrary shape using triangular patches; hence, RWG functions [8] are widely used to represent unknown current distributions. When iterative solvers are used to solve the MoM matrix equation, the fast multipole method (FMM) or multilevel fast

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multipole method (MLFMM) [9–15] can be used to accelerate the calculation of matrix–vector multiplications.

The system matrix resulted from EFIE is often an ill-conditioned matrix and results in the low convergence of the Krylov iterative method [16]. Iterative methods for solving linear systems are usually combined with a preconditioner that can be easily solved. For some practical problems, however, a natural and efficient choice of preconditioner may be one that cannot be easily solved by a direct method and thus may require an iterative method (called inner iteration) itself to solve the preconditioned equations. There also exist cases where the matrix operator contains inverses of some other matrices, an explicit form of which is not available. Then the matrix-vector multiplication can be obtained approximately through an inner iteration. The domain decomposition preconditioners [17], and some positive definite non-symmetric linear systems preconditioned by their symmetric parts are such examples. For these types of problems, the original iterative method will be called the outer iteration and the iterative method used for solving the preconditioner or forming the matrix-vector multiplication will be called the inner iteration.

In order to be able to enhance robustness of iterative solvers, we should be able to determine whether or not a given preconditioner is suitable for the problem at hand. If not, one can attempt another possible iterative method/preconditioner and switch periodically if necessary. It is desirable to be able to switch within the outer iteration instead of restarting. For the generalized minimum residual method (GMRES) algorithm, this can be easily accomplished with the help of a rather simple modification of the standard algorithm, referred to as the flexible GMRES (FGMRES) [18–21]. An important property of FGMRES is that it satisfies the residual norm minimization property over the preconditioned Krylov subspace just as in the standard GMRES algorithm. The FGMRES method with FFT technique is applied for the analysis of electromagnetic wave scattering from three-dimensional dielectric bodies with the large permittivity [19].

Moreover, the inner-outer flexible GMRES method can be combined with a preconditioner to further improve convergence as the conventional GMRES [18]. Simple preconditioners like the diagonal or diagonal blocks of the coefficient matrix can be effective only when the matrix has some degree of diagonal dominance [22]. Incomplete LU (ILU) factorizations have been successfully used for nonsysmmetric dense systems [23]. But the factorization is often very ill-conditioned that makes the triangular solvers highly unstable and the use of the ILU preconditioner may be totally ineffective [24]. Sparse approximate inverse (SAI) preconditioning techniques have been successfully used with the MLFMM [25, 26], however, the construction cost is usually very high. The symmetric successive over-relaxation (SSOR) [18, 27] preconditioner has the advantage of very small construction cost, which makes it preferable over general-purpose preconditioners such as ILU and SAI preconditioner techniques. Furthermore, SSOR preconditioning technique contains more information of the coefficient matrix when compared with a diagonal/block diagonal matrix, which is perhaps efficient only for very long and narrow structures. In this investigation, the SSOR preconditioning scheme is used to accelerate the inner-outer FGMRES method, which is proposed to speed up the convergence of GMRES algorithm [18]. The symmetric successive over-relaxation (SSOR) preconditioner is constructed based on the near-part matrix of the EFIE impedance matrix in the inner iteration of FGMRES. Numerical examples are given to demonstrate the accuracy and efficiency of the SSOR preconditioned inner-outer FGMRES algorithm in radar cross section (RCS) calculations

This paper is outlined as follows. Section 2 gives an introduction of EFIE formulation combined with MLFMM. Section 3 describes the details to construct the SSOR preconditioned inner-outer FGMRES algorithm based on the near-field matrix in MLFMM implementation. Numerical experiments with a few electromagnetic wave scattering problems are presented to demonstrate the efficiency of the SSOR preconditioned FGMRES algorithm in Section 4. Section 5 gives some conclusions and comments.

2. EFIE FORMULATION AND MLFMM

The EFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [1]. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows:

$$\sum_{n=1}^{N} A_{mn} I_n = b_m, \qquad m = 1, 2, \ldots, N \tag{1}$$

where

$$Z_{mn} = jk \int_s \mathbf{f}_m(\mathbf{r}) \cdot \int_{s'} \left(\bar{\mathbf{I}} + \frac{1}{k^2} \nabla\nabla\cdot \right) [G(\mathbf{r}, \mathbf{r}')\mathbf{f}_n(\mathbf{r}')] ds ds' \tag{2}$$

and

$$b_m = \frac{1}{\eta} \int_s \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{E}^i(\mathbf{r}) ds, \qquad G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{3}$$

where $G(\mathbf{r}, \mathbf{r}')$ refers to the Green's function in free space and $\{I_n\}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions $\mathbf{f}_m$. Also, as usual, $\mathbf{r}$ and $\mathbf{r}'$ denote the observation and source point locations. $\mathbf{E}^i(\mathbf{r})$ is the incident excitation plane wave, and η and k denote the free space impendence and wave number, respectively. Once the matrix equation (1) is solved by numerical matrix equation solvers, the expansion coefficients $\{I_n\}$ can be used to calculate the scattered field and RCS. In the following, we use A to denote the coefficient matrix in equation (1), $x = \{I_n\}$, and $b = \{b_m\}$ for simplicity. Then, the EFIE matrix equation (1) can be symbolically rewritten as:

$$Ax = b \tag{4}$$

The basic idea of the fast multipole method (FMM) is to convert the interaction of element-to-element to the interaction of group-to-group. Here a group includes the elements residing in a spatial box. The mathematical foundation of the FMM is the addition theorem for the scalar Green's function in free space. Using the FMM, the matrix-vector product Ax can be written as:

$$Ax = A_N x + A_F x \tag{5}$$

where A_N is the near part of A and A_F is the far part of A.

In the FMM, the calculation of matrix elements in A_N remains the same as in the method of moments (MoM) procedure. However, those elements in A_F are not explicitly computed and stored. Hence they are not numerically available in the FMM. It has been shown that the operation complexity of FMM to perform Ax is $O(N^{1.5})$. If the FMM is implemented in multilevel, the total cost can be reduced further to $O(N\log N)$ [9].

3. SSOR PRECONDITIONED INNER-OUTER FGMRES

Consider the iterative solution of large linear systems of equations as shown in (4). Let $x_0 \in C^n$ be an initial guess for this linear system, $r_0 = b - Ax_0$ be its corresponding residual and M^{-1} the right preconditioner. The Krylov subspace algorithm with right preconditioning solves the modified system:

$$AM^{-1}(Mx) = b \tag{6}$$

Clearly the matrix AM^{-1} need not be formed explicitly: we only need to solve $Mz = v$ whenever such an operation is required. In some cases, solving a linear system with the matrix M consists of forming an approximate solution by performing one or a few steps of a relaxation type method, or a few Chebyshev iterations. It is natural that the preconditioners is used not only a single step during an outer iteration, but as many as are needed to solve a linear system within a given tolerance. The preconditioner may be no longer constant but is allowed to vary from one step to another in the outer iteration. A similar situation in which the preconditioner is not constant is when another Krylov subspace method is used as a preconditioner. These lead us to raise the question of whether or not it is possible to accommodate such variations in the preconditioners and still obtain an algorithm that satisfies an optimality property similar to the one satisfied by the original iterative method. This question has been avoided in the past because in the Hermitian case there does not seem to have a version of the usual preconditioned conjugate gradient algorithm that satisfies a short vector recurrence and that allows the preconditioner to vary at each step. In the non-Hermitian case and for methods that do not rely on short vector recurrences, such as GMRES, variations in the preconditioner can be handled without difficulty. The preconditioner M is constant in conventional preconditioned GMRES algorithm [18]. However, in Flexible GMRES (FGMRES) [18], the preconditioner is no longer constant but is allowed to vary from one step to another in the outer iteration.

The FGMRES algorithm is classically described by

(1) Start: Choose x_0 and a dimention m of the Krylov subspaces. Define an $(m+1)\times m$ matrix $\overline{H}_m$ and initialize all its entries $h_{i,j}$ to zero.

(2) Arnoldi process:

(a) compute $r_0 = b - Ax_0$, $\beta = \|r_0\|_2$ and $v_1 = r_0/\beta$.

(b) for $j = 1, \ldots, m$ do

- compute $z_j := M_j^{-1} v_j$;
- compute $w := Az_j$;
- For $i = 1, \ldots, j$, do
  $h_{i,j} := (w, v_j)$
  $w := w - h_{i,j} v_i$
  Enddo
- compute $h_{j+1,j} = \|w\|_2$ and $v_{j+1} = w/h_{j+1,j}$

Enddo

(c) Define $Z_m := [z_1, \ldots, z_m]$.

(3) Form the approximate solution: Compute $x_m = x_0 + Z_m y_m$ where $y_m = \arg\min_y \left\|\beta e_1 - \overline{H_m} y\right\|_2$ and $e_1 = [1, 0, \ldots, 0]^T$.

(4) Restart: If satisfied stop, else set $x_m \rightarrow x_0$ and go to 2.

As can be observed, the only difference from the standard GMRES version is that we now save the preconditioned vector z_j and update the solution using these vectors. Note that we can define z_j in step (2) without reference to any preconditioner, i.e., we can simply pick a given new vector z_j. In FGMRES algorithm, the preconditioning operation $z_j := M_j^{-1} v_j$ can be thought of as a means of approximately solving the matrix system

$$M_j z_j = v_j \tag{7}$$

where M_j^{-1} is the preconditioner. This is referred to as an inner iteration. In this investigation, the preconditioner M is taken to be the near part impedance matrix A_N.

For the inner iteration any method can be used, though for practical purposes it should be a fast approximate solve. In conventional FGMRES algorithm, the GMRES algorithm is used for the preconditioning, or inner iterations. In this paper, we have SSOR preconditioned GMRES for the inner iterations, and FGMRES as the (outer) flexible method, denoted as SSOR-FGMRES. In the SSOR preconditioning scheme [27], the SSOR preconditioned inner iteration matrix system is as the following form

$$M_{\text{SSOR}} A_N z_j = v_j \tag{8}$$

The SSOR preconditioner M_{SSOR} is constructed based on the near part impedance matrix A_N as the following form

$$M_{\text{SSOR}} = \left(\widetilde{D} + L\right) \left(\widetilde{D}\right)^{-1} \left(\widetilde{D} + U\right) \tag{9}$$

where $A_N = L + D + U$ in (9), L is the lower triangular matrix A_N, D is the diagonal matrix of A_N, U is the upper triangular matrix A_N, and $\tilde{D} = (1/\omega)D$, $0 < \omega < 2$ (ω is the relaxation parameter).

The additional cost needed by the SSOR-FGMRES algorithm over the conventional FGMRES algorithm is only in the extra memory required to save the SSOR preconditioner M_{SSOR} in the inner iteration. On the other hand, the added advantage of faster convergence may certainly be worth this extra cost. There are a few applications in which this flexibility can be quite helpful, especially in the context of developing robust iterative methods or for developing preconditiomers for massively parallel computers.

In the next section, the convergence of the SSOR-FGMRES algorithm combined with MLFMM technique is discussed in detail for electromagnetic scattering by open conducting conductors.

4. NUMERICAL EXPERIMENTS

In this section, we show some numerical results for open conducting structures that illustrate the effectiveness of the proposed SSOR preconditioned inner-outer FGMRES (SSOR-FGMRES) for the solution of the EFIE linear systems in electromagnetic scattering problems. The EFIE linear systems based on the RWG basis functions are solved with MLFMM accelerated Krylov iterative methods. All numerical experiments are performed on a Pentium 4 with 2.9 GHz CPU and 2 GB RAM in single precision. The restarted version of GMRES(m) algorithm is used as iterative method, where m is the dimension size of Krylov subspace for GMRES. Additional details and comments on the implementation are given as follows:

- Zero vector is taken as initial approximate solution for all examples and all systems in each example
- The iteration process is terminated when the normalized backward error is reduced by 10^{-3} for all the examples.
- $m = 30$ is used as the dimension of the Krylov subspace for the restarted GMRES algorithms.
- The inner stop precision is taken to be 10^{-1} in both the FGMRES and SSOR-FGMRES algorithms.

We investigated the performance of the presented SSOR-FGMRES algorithm on three open conducting structures and one closed structures. The first example is a metallic open-cone, which

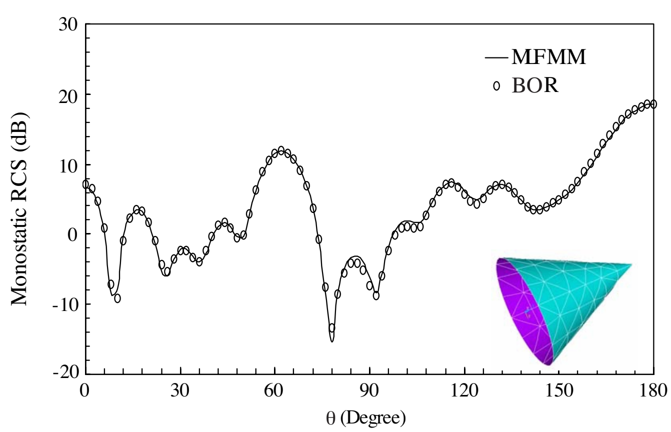

Figure 1. The monostatic RCS for vertical polarization at 3.0 GHz for an open cone.

has a height of 20 cm and a base whose diameter is also 20 cm. The open cone is discretized with 1922 planar triangular patches for RWG basis functions with 2851 unknowns at 3 GHz. The second example is a closed missile structure, which is discretized with 5212 planar triangular patches for RWG basis functions with 7818 unknowns with 7818 unknowns at 200 MHz. The three example is a $2.5\lambda \times 2.5\lambda \times 3.75\lambda$ open-cavity with 10568 unknowns for RWG basis functions at 300 MHz (λ is wavelength in free space). The last example is a box-plate perfectly electrically conducting (PEC) scatterer consisting of a plate of size ($1\,\text{m} \times 1\,\text{m}$) placed on an $2\,\text{m} \times 1\,\text{m}$ large plate having a thickness of 0.3 m. The box-plate is discretized with 232032 planar triangular patches for RWG basis functions with 347804 unknowns at 5 GHz. The MLFMM was employed for the computation of the matrix/vector products involving non-near zone interaction elements in the computation. As shown in Figure 1, the monostatic vertical polarized RCS curve of the cone at 3.0 GHz is compared with that of a BOR (Body of Revolution) analysis. The size dimension of the group on the finest level is 0.2 wavelength (0.2λ) and the MLFMM with two levels was employed for the cone. It can be found that the results using the MLFMM are in good agreement with that of the BOR analysis [28]. The monostatic vertical polarized RCS curves of the missile example at 200 MHz and the cavity example at 300 MHz are given in Figure 2 and Figure 3, respectively. The size dimension of the group on the finest level is quarter of the wavelength (0.25λ) and the MLFMM with three levels was employed for both the missile and

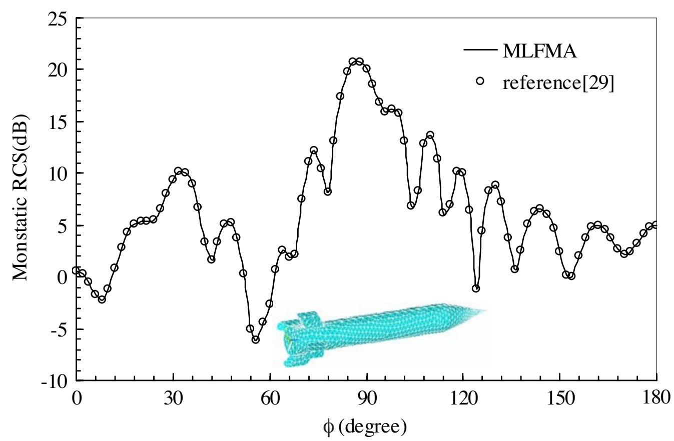

Figure 2. The monostatic RCS for vertical polarization at 200 MHz for a missile.

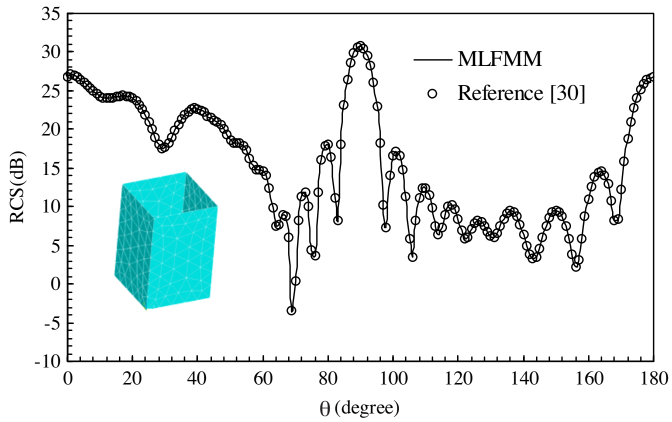

Figure 3. The monostatic RCS for vertical polarization at 300 MHz for an open cavity.

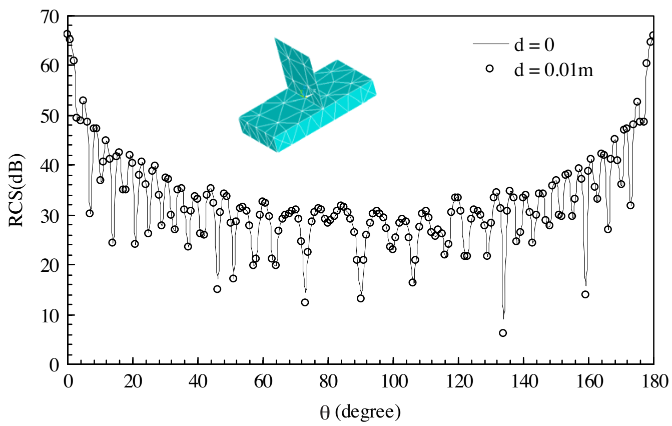

Figure 4. The bistatic RCS at 5 GHz for a box-plate example.

the cavity examples. Figure 4 gives the numerical results of Bistatic RCS for box-plate example. For this example, the size dimension of the group on the finest level is 0.5 wavelength (0.5λ) and the MLFMM with 5 levels was employed. To verify the accuracy of the method, a closed scatterer consisting of a plate of size ($1\,\text{m} \times 1\,\text{m}$) having a thickness of 0.01 m placed on the above plate is considered in Figure 4. It can be obseverd that our results agree well with those in [29, 30].

As shown in Figures 5–7, the convergence histories of proposed SSOR-FGMRES algorithm for system 1 are given for the first three

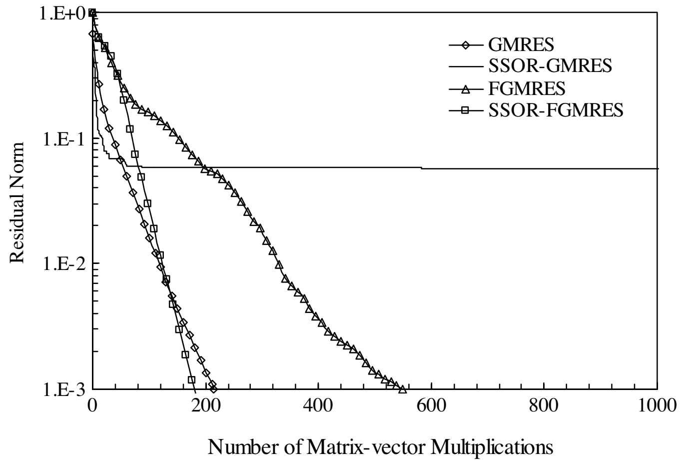

Figure 5. Convergence history of GMRES algorithms for system 1 on the open-cone example.

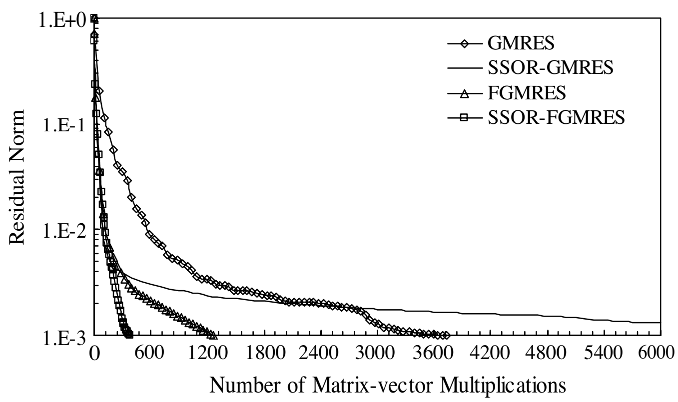

Figure 6. Convergence history of GMRES algorithms for system 1 on the missile example.

examples in monostatic RCS computation. The convergence curves of the GMRES, SSOR preconditioned GMRES (SSOR-GMRES), inner-outer FGMRES algorithms are included for comparison. For the box-plate examples in bistatic RCS computation with single right-hand side, the convergence histories are given in Figure 8. In these computations, the MLFMM was employed for accelerating the computation of the matrix/vector products. The value of relaxation parameter in SSOR-GMRES and SSOR-FGMRES algorithms is 0.6,

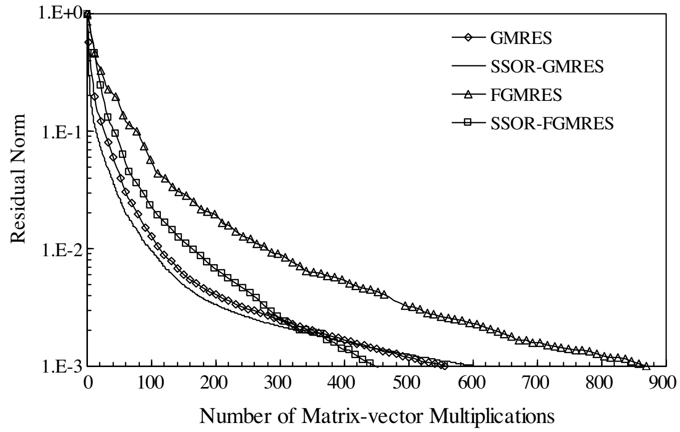

Figure 7. Convergence history of GMRES algorithms for system 1 on the open-cavity example.

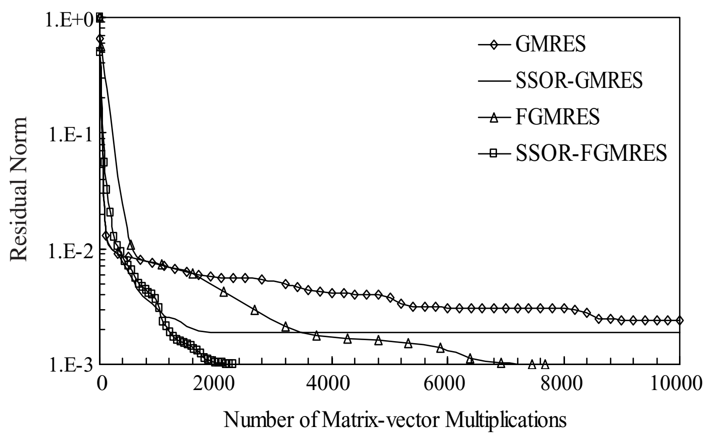

Figure 8. Convergence history of GMRES algorithms on the box-plate example.

inner iteration number in FGMRES and SSOR-FGMRES algorithms is taken to be 10. It can be found that the proposed SSOR-FGMRES algorithm converges fastest than the other algorithms for all the four examples.

As shown in Table 1, the number of matrix-vector multiplication and solution time with the above preconditioned algorithms for are given for system 1 in monostatic RCS computation on open-cavity, missile and open-cone examples, where * refers to no convergence

Table 1. Number of matrix-vector multiplications and solution time (in seconds) for open structures.

	Number of matrix-vector multiplications				Solution time			
Example	**GMRES**	**SSOR-GMRES**	**FGMRES**	**SSOR-FGMRES**	**GMRES**	**SSOR-GMRES**	**FGMRES**	**SSOR-FGMRES**
Open-cone	215	$>10^4$	550	187	104s	*	30s	10s
Missile	3738	>6000	1268	378	1358s	*	484s	128s
Open-cavity	550	603	869	451	410s	617s	342s	224s
Box-plate	$>10^4$	$>10^4$	7678	2298	*	*	183781s	46392s

after maximum iterations. The number of matrix-vector multiplication and solution time for the box-plate in bistatic RCS computation are also given in Table 1. It is noted that the number of matrix-vector multiplications (MVM) includes both the number of MVM in the inner iteration and the one in the outer iteration in the FGMRES and SSOR-FGMRES algorithms. It can be found that the proposed SSOR-FGMRES algorithm can save much time than other algorithms. For both open-cavity example and open-cone one, the solution time of the FMGRES algorithm is less than the GMRES. However, the number of matrix-vector multiplications for FGMRES is more than that of GMRES. This is mainly due to the use of the near-part impedance matrix on the inner iteration of the FGMRES algorithm. Furthermore, small size of the near-part impedance matrix A_N results in the low cost for implementation of matrix-vector multiplications in the inner iteration. When compared with FGMRES, the improved SSOR-FGMRES save the solution time by a factor of 3.0 for the open cone example, 3.8 for the missile, 1.5 for the open cavity and 4.0 for the box-plate. This demonstrates the efficiency of the newly proposed SSOR preconditioned FGMRES algorithm for the open structures.

A critical question in the use of the SSOR preconditioned inner-outer FGMRES algorithm is to what precision the preconditioner should be solved, i.e., what stopping threshold should be used in the inner iteration. In other words, we wish to balance the inner and outer iterations so that the total number of operations is minimized. In order to answer this question, we vary the inner iteration number from 5 to 75 and the total number of matrix-vector multiplications and CPU time of SSOR-FGMRES are given in Figure 9 for the open cone example. In these computations, the value of relaxation parameter in the SSOR-FGMRES algorithms is taken to be 0.6. It can be observed

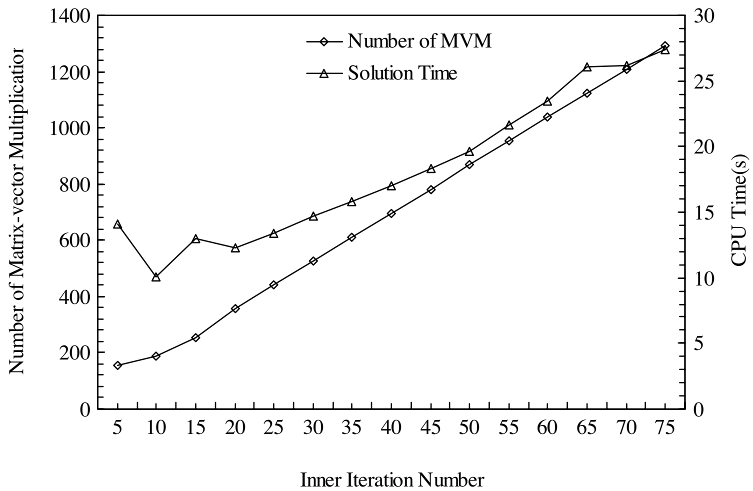

Figure 9. Number of matrix-vector multiplications and solution time with SSOR-FGMRES algorithm for the different inner iteration numbers on the open-cone example.

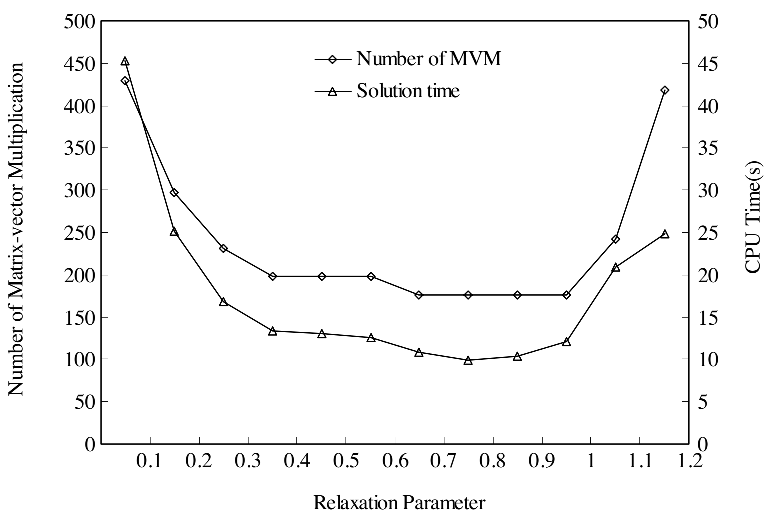

Figure 10. Number of matrix-vector multiplications and solution time with SSOR-FGMRES algorithm for the different relaxation parameters on the open cone example.

that the number of matrix-vector multiplications and solution time is almost increased with the increasing of the inner iteration. When the inner iteration number is taken to be 10, the minimal solution time is 10.1 seconds and the number of matrix-vector multiplications is 187.

Another important factor of the convergence rate for the proposed SSOR-FGMRES algorithm is the relaxation parameter. We vary the relaxation parameter from 0.1 to 1.2 and the total number of matrix-vector multiplications (MVM) and solution time of SSOR-FGMRES are given in Figure 10. In these computations, the inner iteration number in the SSOR-FGMRES algorithms is taken to be 10. It can be observed that the number of MVM and solution time of SSOR-FGMRES decrease rapidly when the value of the relaxation parameter is increased from 0.1 to 0.3. The total MVM number and solution time is nearly invariant with the value of the relaxation parameter varying from 0.4–1.0. When the value of the relaxation parameter is increased from 1.1 to 1.2, the total MVM number and solution time increases rapidly. When the relaxation parameter is taken to be 0.8, the minimal number of MVM and the minimal solution time is 176 and 9.9 seconds, respectively.

In order to further investigate the performance of the proposed SSOR-FGMRES algorithm, the number of matrix-vector multiplications (MVM) with FGMRES and SSOR-FGMRES algorithms is given for the different incident angles in monostatic RCS computation for

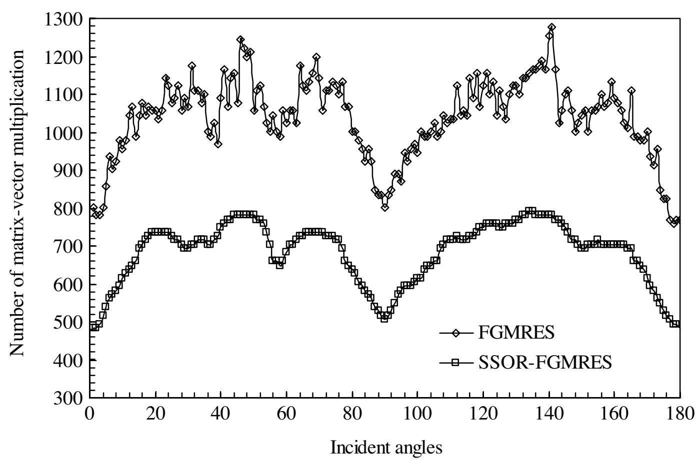

Figure 11. Number of matrix-vector multiplications with FGMRES and SSOR-FGMRES algorithms for the different incident angles (or systems) on the open cavity.

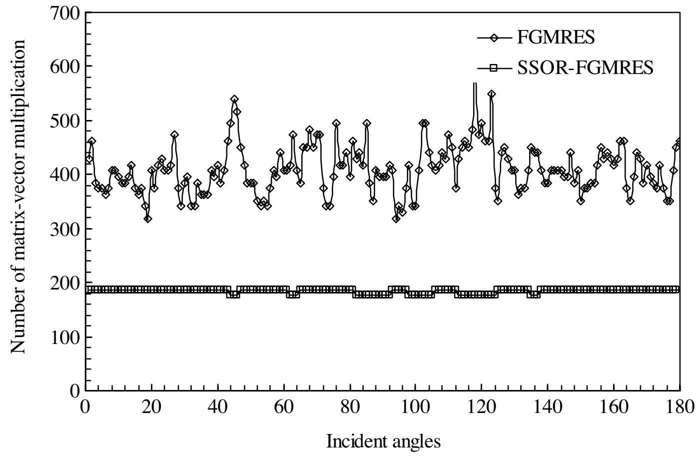

Figure 12. Number of matrix-vector multiplications with FGMRES and SSOR-FGMRES algorithms for the different incident angles (or systems) on the open cone.

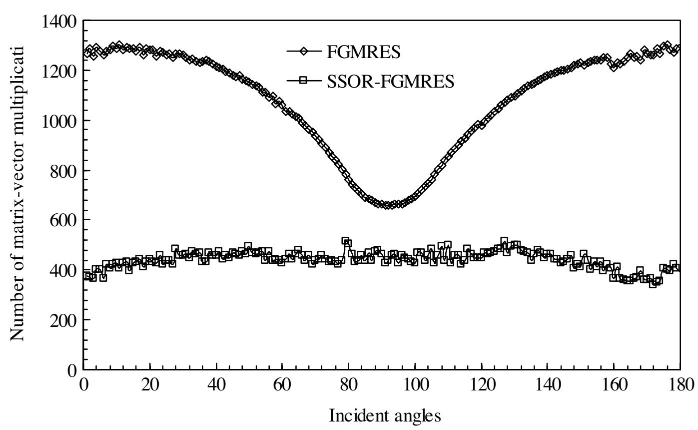

Figure 13. Number of matrix-vector multiplications with FGMRES and SSOR-FGMRES algorithms for the different incident angles (or systems) on the missile.

open-cone, open-cavity and missile examples in Figures 11–13. The sets of angles of interest for the monstatic RCS of the missile example vary from 0 to 180 degree in φ direction when θ is fixed at 90 degree. For both open cavity and open cone examples, the sets of angles of interest for the monstatic RCS vary from 0 to 180 degree in θ direction when φ is fixed at 0 degree. The increasing step is one degree and the total number of right-hand sides to be solved for the complete monostatic RCS calculation is 181 for each example. In these computations, the inner iteration number in both FGMRES and SSOR-FGMRES algorithms is taken to be 10. The value of relaxation parameter in SSOR-FGMRES is 0.6. It can be observed that the SSOR-FGMRES algorithm can greatly improve the convergence. It can also be found that the number of MVM for FGMRES varies largely from one system to another for each example. The number of MVM for SSOR-FGMRES varies slowly on open-cavity example and is almost constant on open-cone example. As shown in Table 2, the cumulated number of matrix-vector multiplications and the total elapsed solution time for a complete monostatic RCS calculation are summarized for each example using FGMRES and SSOR-FGMRES algorithms. Depending on the geometry, the overall gain ranges from a factor of 1.6 to 2.7 for both computational time and total number of matrix-vector multiplications.

Table 2. Total number of matrix-vector multiplications and solution time (in seconds) for a complete monostatic RCS calculation.

	Number of matrix-vector multiplications		Solution time	
Geometry	FGMRES	SSOR-FGMRES	FGMRES	SSOR-FGMRES
Open cavity	186626	122958	84785 s	54860 s
Open -cone	73788	33220	8968 s	3404 s
Missile	79410	195539	72813 s	30098 s

5. CONCLUSIONS AND COMMENTS

In this paper, the SSOR-preconditioned inner-outer FGMRES algorithm is presented for solving EFIE with single and multiple right-hand sides in both bistatic and monostatic RCS calculation for open structures. The symmetric successive over-relaxation (SSOR) preconditioner is constructed based on the near-part matrix of the EFIE in the inner iteration of FGMRES. The combined effect of the prescribed SSOR preconditioner coupled with inner-outer FGMRES algorithm is very beneficial for the convergence of the iterative methods. It can be found that the proposed method can reduce both the number of matrix-vector multiplications and the computational time significantly with low cost for construction and implementation of preconditioner.

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