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RESEARCH PROGRESS IN REVERSED CHERENKOV RADIATION IN DOUBLE-NEGATIVE METAMATERIALS

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Abstract—We review the research progress in reversed Cherenkov radiation in double-negative metamaterials (DNMs) starting from the first experimental verification of the DNMs reported in 2001, including theories, numerical computation and simulation and experiments. We also discuss the potential applications to particle detectors and high-power microwave or millimeter-wave devices, including the oscillators and amplifiers, and the formidable challenges needed to be resolved before the benefits of using such artificial materials can be harvested.

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1. INTRODUCTION

In 1967, Veselago introduced the concept of a left-handed (LH) medium with simultaneously negative permittivity and permeability and mentioned several unusual electromagnetic wave phenomena, such as the negative refractive index, the reversed Cherenkov radiation (CR), the reversed Doppler effect, and even reversal of radiation pressure to radiation tension [1]. Such material is called left-handed because the electric field $\overline{E}$, the magnetic field $\overline{H}$, and the wave vector $\overline{k}$ form a left-handed set, opposite to the normal types of material, which are right-handed (RH). LH material is not readily available in nature. Therefore, there was no progress on this topic for the next twenty nine years. In 1996, Pendry firstly proposed a thin wires structure with negative permittivity [2] and simply described how a thin wires structure leads to negative permittivity. After three years, he put forward a split-ring resonators (SRRs) structure with negative permeability [3] and simply described how a SRRs structure leads to negative permeability. In 2000, Smith theoretically studied a combined structure with these thin wires and SRRs to realize this artificial material having simultaneously negative permittivity and permeability [4]. After just one year, the first experimental verification of the double-negative metamaterials (DNMs) was reported [5]. Since then, a lot of effort world-wide have been spent studying the DNMs, its electromagnetic properties, and potential applications to sub-wavelength focusing, cloaking, antenna, microwave components such as phase compensation devices, sub-wavelength guided-wave structures, backward couples, filters, and diplexers, etc [6–18]. On the other hand, we have pioneered the researches on the reversed CR in the DNMs and the potential applications to particle physics and high-power electronics.

Cherenkov radiation (CR) is emitted when charged particles move in a material with a speed faster than the phase velocity of light in that material. The CR in normal RH media was experimentally observed by Cherenkov in 1934 [19], and later theoretically explained using macroscopic electromagnetic theory for the first time by Frank and Tamm in 1937 [20]. The phenomenological quantum theory of CR was developed by Ginzburg in 1940 [21].

This paper summarizes the main contributions and the authors' viewpoints on reversed CR in the metamaterials exhibiting negative effective parameters (an effective permittivity and an effective permeability which take negative values), including both theoretical research progress and experimental efforts to demonstrate the existence of backward radiation. And then we discuss the potential applications such as Cherenkov detectors and wave generation/amplification devices in details. Finally, we provide a perspective discussion of the significant challenges still lying ahead in this research field.

2. PROGRESS IN THEORETICAL AND NUMERICAL ANALYSIS

How about the CR in a novel artificial LH material, i.e., DNM, which is always considered as homogeneous? Veselago [1] stated that a DNM would induce reversal of CR without deriving the mathematical solution for CR in DNMs. Lu et al. in a group at MIT firstly addressed this issue both in dispersive and dissipative DNMs in detail and verified the Veselago's view [22]. The authors show the directions of power propagation are not exactly opposite to those of phase propagation when there is loss. In addition, Cherenkov angle and the radiation pattern of the CR at large angles close to 90° with respect to the particle motion are dependent on the loss. The use of DNMs might improve the Cherenkov detectors was also introduced. In [22], the unbounded DNMs were assumed. In another paper [23], Averkov and Makovenko theoretically investigated another case where the reversed CR was produced by an electron bunch moving in a vacuum above an isotropic DNM. In the case, the authors show that the wave vector magnitude in the plane of the interface of surface electromagnetic waves is larger than the corresponding wave vector magnitude of bulk electromagnetic waves, the energy flows in a left-handed material have been calculated, and the spectral density and the radiation pattern have been examined. However, how the material parameters affect the spectral density and the total radiated energy was not presented. Hence, it is not clear if the radiated power can be effectively detected.

For simplicity, the above papers assumed the DNMs as isotropic. In fact, a DNM is a composite medium with different materials such as the metallic strips for the SRRs and rods/dielectric materials for holding the strips. It is clear that the DNMs are essentially anisotropic rather than isotropic. Therefore, Based on Lu's work [24], Duan et al. developed the CR theory in the anisotropic DNMs. First, they investigated the CR in unbounded anisotropic DNMs [25]. They derived the analytical expressions for the CR condition, phase and group angles and explained the existence of the reversed CR. In addition, the spectral density and total radiated energy per unit path length were also obtained. Numerical results demonstrated the influences of material parameters of anisotropic DNMs on the spectral density and the total radiated energy. It turns out that the wave vector and time-averaged Poynting vector in anisotropic DNMs are nearly though not exactly anti-parallel, and the phase and group angles, the spectral density, and the total radiated energy in this case are quite different from those in isotropic DNMs. Second, they examined the CR in a waveguide filled with anisotropic DNMs [26], and compared this case with the unbounded case in details. A numerical example illustrates that the total radiated energy increases with increasing particle velocity, the radiated energy spectral density has different poles at the different frequencies for different anisotropic DNMs when the loss of the anisotropic DNM is smaller, and when the radius has the same order as the operating wavelength, the influence of the waveguide radius on the total radiated energy is smaller on the whole. Third, they further generalized reversed CR in anisotropic DNMs, and particularly discussed one of the most practical cases, i.e., CR in a waveguide partially filled with anisotropic DNMs in great detail [27]. Then they explored the effective ways of improving the total radiated energy: one is that the DNMs need scale to higher frequencies such as terahertz range, the other is that the intense electron beam should be adopted, addressed the issues of potential applications such as particle detectors and high-power sources, and at last, pointed out the serious challenges. We will further carry out these theoretical research works by using the self-consistent nonlinear theory to improve the radiated energy in order for us to detect the reversed CR in experiments.

As stated before, at this moment, a single electron produces very small energy in the GHz region when it moves through the DNMs [27]. An intense electron beam need to be used to detect its reverse CR. Currently, Bliokh et al. [28] have theoretically predicted beam instability when two electron beams pass through a slab of DNMs. This instability, which is inherent only for DNMs, originates from the backward CR and results in a self-modulation of the beams and radiation of electromagnetic waves. These waves leave the sample via the rear surface of the slab and form two shifted bright circles centered at the beams. The spectrum of the radiation has well-separated lines on top of a broad continuous spectrum. The radiation intensity and its spectrum can be tuned either by the beam current or by geometrical parameters. However, the authors did not show how much the radiated energy produced by two electron beams and whether the energy can be effectively detected, and the DNMs discussed there are assumed to be isotropic rather than anisotropic.

In addition, we should mention a related research work on the reverse CR [29]. This paper describes reverse CR produced by an electromagnetic shock wave propagating along a nonlinear transmission line. It can offer a helpful method to study the reversed CR in DNMs.

By the way, some research has also been devoted to the study of the backward CR in photonic crystals [30, 31], which are another artificial materials. But the mechanism here is different: the CR is actually due to the negative refraction in photonic crystals, and the effect is also combined with the transition radiation. The main difference between the photonic crystal and DNM is that in most cases, the effective permittivity, permeability, and refractive index can be defined in DNMs but not for a photonic crystal [24].

3. PROGRESS IN NUMERICAL STUDY OF THE DESIGNED DNMS FOR CR

As Wu et al. stated [32], most previously proposed LH media used the conducting rings with normals aligned in two orthogonal directions. They are of the type with one negative element in the permittivity tensor and two negative elements in the permeability tensor, which are useful only for TE (transverse electric) radiation. In 2007, the authors designed a new type of DNMs having two dimensions of negative permittivities and one dimension of negative permeability, which are demonstrated by using the CST MICROWAVE STUDIO 5.0. This new structure is suitable for the CR, because the CR is a typical TM (transverse magnetic) radiation. They further, used an antenna array to model a traveling current source, representing a single frequency component of a moving charged particle, and demonstrated the feasibility and foundation of observing backward CR experimentally with this kind of DNMs. Clearly, this method is based on the fact that a current source like electron beam can be modeled by an antenna array.

In addition, Antipov et al. [33, 34] discussed some theoretical aspects of particle interaction with an anisotropic and dispersive DNM placed in a waveguide. For this case, reverse CR is generated by charged particle propagation through the structure. Then the authors developed a simulation approach to determine the frequency and the level of excitation of waveguide modes from a charged particle beam. This method allows simulation of different waveguide cross sections, any transverse beam distribution, and any physical dispersion, of the medium. Shchegolkov et al. introduced a proposed measurement of the reversed CR effect in a metamaterial-loaded circular waveguide [35]. Currently, much experimental work on directly verifying the reversed CR in DNMs is being carrying out in their group.

4. PROGRESS IN EXPERIMENTAL RESEARCH

After summarizing the theoretical work and numerical work in the field of reverse CR, we review the corresponding experimental work. In 2002, Grbic and Eleftheriades reported that the coplanar waveguide (CPW)-based DNM was capable of supporting backward radiation-a characteristic analogous to reversed CR [36]. They experimentally demonstrated this characteristic by using current pulses in microwave radiating systems.

Currently, the experiment based on the new type of DNMs [32] is also carried out, where the metamaterial sample is realized with an array of split-ring resonator which, according to theoretical computation, is predicted to exhibit negative permeability in the φ direction and an array of two dimensional metallic rods which exhibits negative permittivity in both the z and ρ directions. The experiment is carried out in a microwave anechoic chamber. The metamaterial sample shows a negative refraction index in the frequency band between 7.9 GHz and 9.5 GHz. When a slot waveguide is put inside of the metamaterial to model the electronic beam, a backward radiation in the negative refraction band of the metamaterial is detected. As a comparison, a forward radiation in the frequency range from 11 GHz to 14 GHz, which corresponds to positive refraction band of the metamaterial, is detected [37].

Recently, Antipov et al. reported a design of a TM-mode based DNM-loaded waveguide and the first beam test of a DNM-loaded waveguide to indirectly verify the reversed CR by using an electron beam pulse [38]. This is the first experiment involving a beam of charged particles. Nevertheless, as the authors stated, they did not observe the reversed CR directly in the left-handed band and pointed out some issues of manufacturing tolerances and filtering of frequency content imposed by probes and cables, etc. in order to further verify the reversed CR directly. In recent communications with Dr. Antipov and Dr. Gai in Argonne National Laboratory [39], they emphasized that to demonstrate backward mode one had to design a directional coupler [40]. Design of a coupler is non-trivial. Particularly interface between metamaterial and air is tricky.

5. APPLICATIONS AND CHALLENGES

We next address the potential applications to Cherenkov detectors and high-power microwave devices. As we stated [27], the reversed CR has a distinct advantage that the detectors for the particle and the detectors for the reversed CR are naturally separated in the forward and the backward regions respectively so their physical interference is minimized, and the detection sensitivity can be improved. Optical DNMs would be perfect for Cherenkov detectors, which operate generally in the optical region [38]. Now, metamaterials have been realized in the terahertz range and up to optical frequencies [41-43]. Hence, we expect these research results to enable a new class of particle detectors. In addition, as we know, the traditional microwave devices, such as backward-wave oscillator (BWO) and traveling-wave tube (TWT) [44], are based on the CR effect. Similarly, we proposed an oscillator or amplifier based on the reversed CR which employs the DNMs rather than the traditional periodic guiding structure to guide the slow-waves. By tailoring the material parameters of DNMs, we can easily obtain a slower phase velocity, which means the oscillator or amplifier can operate at a low-voltage. Meanwhile, it is easier to synchronize the amplified electromagnetic waves with an intense electron beam, and the output power can be enhanced. We look forward to a novel radiation sources in microwave, millimeter wave, even in terahertz regime.

Certainly, there are still many challenging works needed to be done. We summarize them as follows. First, metamaterials, defined as effectively homogeneous media with properties not readily available in nature, in essence, are inhomogeneous. Currently, DNM fabricated at the optical regime are composed of only one layer at the propagation direction, and far away from being homogeneous [45]. So some research works on homogenization of the metamaterials should be further studied [46–49]. An effective approach to homogenization of the metamaterials is to minimize electrical size of the metamaterial elements [50]. Second, the DNMs are essentially anisotropic. It means they are sensitive to the polarization of the electromagnetic waves. Third, for the artificial DNMs composed of the resonance based structures and metallic wire structures, the loss, due to the finite conductivity of the conductors and the finite resistivity of the dielectrics, is inevitable. A significant part of this loss is represented by Ohmic losses in the metallic components of the particles. These Ohmic losses become larger with frequency which explains why metallic particle designs are not suitable for high frequency applications [51]. Thus the loss becomes a serious problem [52], which limits the potential applications of metamaterials such as perfect lens [53]. Currently, the most prominent next frontier of DNMs is to reduce their losses [54, 55]. Here, we summarize some methods such as a way to devise metamaterials, which avoids using metallic inclusions, by employing high dielectric constant materials, an alternative way to reduce losses in the left-handed metamaterial designs by increasing the inductance to the capacitance ratio L/C, and a simple way to choose the low-loss constituent metals. Fourth, because most of metamaterials are formed by resonant structure, there exist strong dispersion characteristics. As a result, the operating frequency band of the suggested high-power microwave devices by using the artificial media is narrower than that of the traditional high-power microwave devices. We propose an effective way of expanding the bandwidth by synthesizing the non-resonance structures. Fifth, the DNMs are also nonlinear [56–58]. Nevertheless, for simplicity, the current theoretical and numerical analyses of the reversed CR are under the assumption of linear DNMs. Sixth, how to accurately retrieve the constitute parameters of metamaterials (permittivity and permeability) in designing the actual DNMs should be further studied [59–61]. Seventh, there are still many open problems for fabrication and measurements of metamaterials, especially for operating frequencies up to millimeter, terahertz, infra-red, and visible range. This is because the required inclusion size p is much smaller than the wavelength in free space λ for these metamaterials (typical value: $p/\lambda < 0.1$). Furthermore, a typical DNM formed by the metals and dielectrics does not work well at a high-voltage.

Nevertheless, as Engheta and Ziolkowski stated [7], the ability to tailor material properties to achieve physical effects not thought to be possible only a few years ago is motivating a lot of activities in the very exciting research field. Also, we expect there will be impressive progresses in the near future for the research of reversed CR in the DNMs [62] and their applications to particle physics and high-power electronics.

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