STUDY OF LOSS EFFECT OF TRANSMISSION LINES AND VALIDITY OF A SPICE MODEL IN ELECTROMAGNETIC TOPOLOGY

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Abstract—This paper studies the loss effect of the line and the validity of a SPICE model for lossless transmission lines excited by an incident plane wave by using a simple structure — a single line over an infinite perfectly conducting ground excited by an incident electromagnetic field. The frequency domain current responses calculated by the Baum-Liu-Tesche (BLT) equation with the loss ignored are compared with those through the BLT equation with loss considered to study the loss effect of transmission lines. The loss effect study shows that the SPICE model for lossless transmission line can be used for nuclear electromagnetic pulse, ultra wideband, and the low-frequency range of high power microwave interaction with most systems. But for a sine or cosine wave of which the frequency equals the pole frequency of the line, the SPICE model may lead to significant error.

1. INTRODUCTION

Electromagnetic topology (EMT) is used to analyze electromagnetic interaction with complex electronic systems, and lots of studies have been done on it. The Baum-Liu-Tesche (BLT) equation is usually used in EMT to analyze transmission line networks. Alternatively, a SPICE model for multiconductor transmission lines can be directly used in EMT to analyze the transmission-line networks in the time domain.

Since the BLT equation was developed by Baum, Liu, and Tesche in 1978 [1, 2], lots of research has been done on it. The BLT equation

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was generalized to include nonuniform transmission lines in [3]. The effects of EM field propagation and coupling were included in the BLT equation in [4, 5], and apertures and cavities were included in [6]. But all the work above was done in the frequency domain, and relied on the inverse fast Fourier transform (IFFT) to get the transient responses. It was implicitly assumed that the system analyzed was invariant and linear. However, most electronic systems today are variant and nonlinear. In [7], Baum gave the time-domain form of the BLT equation, and recently Tesche illustrated the transient BLT equation by a simple example [8]. However, the transient BLT equation calls for costly temporal convolution. It is not convenient to use for nonlinear loads where a nonlinear matrix equation should be solved first to determine the reflection coefficients in the time domain [9].

Paul [10, 11] proposed a SPICE model for lossless multiconductor transmission lines excited by an incident electromagnetic field. The SPICE model can be used to predict the responses induced at the terminations of lossless transmission lines, and can be used for variant and nonlinear systems. It is usually used in circuit analysis. Since both the SPICE model and the BLT equation are based on the transmission-line equations, the SPICE model was proposed to be used in the EMT [12].

Because the SPICE model doesn't include the losses of the transmission lines and the loss effect can't be ignored in some cases, the loss effect and the cases when the SPICE model can be used in the EMT are studied in this paper. The cases when the loss can't be neglected are studied first with the BLT equation, and then the validity of the SPICE model is determined.

This paper is organized as follows. Section 2 gives simple reviews of the SPICE model and the BLT equation. Section 3 presents the problem and gives the computed results and analysis. Conclusions are made in Section 4.

2. THE SPICE MODEL AND BLT EQUATION

2.1. The SPICE Model for a Transmission Line Excited by an Incident Electromagnetic Field

The SPICE model for multiconductor transmission lines excited by an incident electromagnetic field proposed by Paul can be found in [10, 11]. Here we give a brief review of the SPICE model for a single lossless line above an infinite, perfectly conducting ground, as shown in Fig. 1.

The transmission-line equations for a single lossless line above an
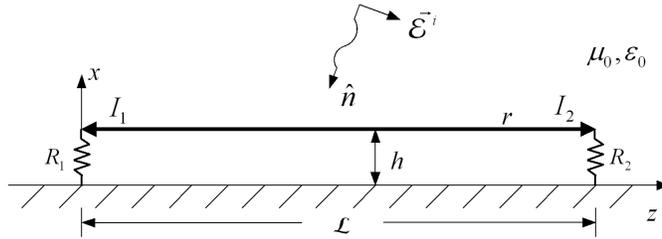
Figure 1. Single lossless line above an infinite, perfectly conducting ground.

infinite, perfectly conducting ground are

$$\frac{\partial}{\partial z}V\left(z,t\right)+L\frac{\partial}{\partial t}I\left(z,t\right)\,=\,V_F\left(z,t\right) \tag{1a}$$

$$\frac{\partial}{\partial z}I\left(z,t\right)+C\frac{\partial}{\partial t}V\left(z,t\right)\,=\,I_F\left(z,t\right), \tag{1b}$$

where L and C are the per-unit-length inductance and capacitance, respectively. $V(z,t)$ and $I(z,t)$ represent the voltage (with respect to the ground) and current. $V_F(z,t)$ and $I_F(z,t)$ are distributed voltage and current sources due to the incident field [13, 14].

The relations between the terminal voltages and currents of the line are given by [11],

$$V\left(0,t\right)-Z_CI\left(0,t\right)=V\left(\mathcal{L},t-T\right)-Z_CI\left(\mathcal{L},t-T\right)+V_0\left(t\right) \tag{2a}$$

$$V\left(\mathcal{L},t\right)+Z_CI\left(\mathcal{L},t\right)=V\left(0,t-T\right)+Z_CI\left(0,t-T\right)+V_{\mathcal{L}}\left(t\right), \tag{2b}$$

where Z_C is the characteristic impedance of the transmission line, T is the one-way line delay, and v is the velocity of propagation along the line.

When the incident electromagnetic field is a uniform plane wave and the cross section of the line is sufficiently small, $V_0(t)$ and $V_{\mathcal{L}}(t)$ have simple analytical expressions and are written as

$$V_0(t)=-2h\left[e_x-e_z\frac{\mathcal{L}}{v_x\left(T+T_z\right)}\right]\times\left[\mathcal{E}_0\left(t\right)-\mathcal{E}_0\left(t-T-T_z\right)\right] \tag{3a}$$

$$V_{\mathcal{L}}(t)=2h\left[e_x+e_z\frac{\mathcal{L}}{v_x\left(T-T_z\right)}\right]\times\left[\mathcal{E}_0\left(t-T\right)-\mathcal{E}_0\left(t-T_z\right)\right]. \tag{3b}$$

where e_x, e_y and e_z are the components of the incident electric field vector along the x, y, and z axes, respectively.

The definition of the angles for the incident field is shown in Fig. 2.
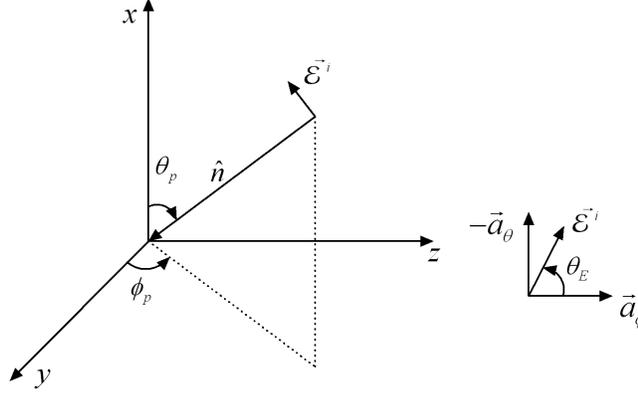
Figure 2. Definitions of the angles characterizing the incident plane-wave field.

2.2. The BLT Equation

The frequency domain BLT equation, based on transmission-line equations, can be used to compute the responses of the terminations of a transmission-line network and is the key equation of the EMT.

In the case of a single line over an infinite, perfectly conducting ground, the frequency domain BLT equation can be written in a simple form [15].

$$
\begin{bmatrix} \hat{V}(0) \\ \hat{V}(\mathcal{L}) \end{bmatrix} = \begin{bmatrix} 1 + \hat{\rho}_1 & 0 \\ 0 & 1 + \hat{\rho}_2 \end{bmatrix} \begin{bmatrix} -\hat{\rho}_1 & e^{\hat{\gamma}\mathcal{L}} \\ e^{\hat{\gamma}\mathcal{L}} & -\hat{\rho}_2 \end{bmatrix}^{-1} \begin{bmatrix} \hat{V}_1^s \\ \hat{V}_2^s \end{bmatrix} \tag{4a}
$$

$$
\begin{bmatrix} \hat{I}(0) \\ \hat{I}(\mathcal{L}) \end{bmatrix} = \frac{1}{\hat{Z}_C} \begin{bmatrix} 1 - \hat{\rho}_1 & 0 \\ 0 & 1 - \hat{\rho}_2 \end{bmatrix} \begin{bmatrix} -\hat{\rho}_1 & e^{\hat{\gamma}\mathcal{L}} \\ e^{\hat{\gamma}\mathcal{L}} & -\hat{\rho}_2 \end{bmatrix}^{-1} \begin{bmatrix} \hat{V}_1^s \\ \hat{V}_2^s \end{bmatrix}, \tag{4b}
$$

where $\hat{V}(0)$, $\hat{I}(0)$, $\hat{V}(\mathcal{L})$, and $\hat{I}(\mathcal{L})$ are the voltages and currents at the near end and far end, respectively. $\hat{\gamma}$ is the propagation constant of the transmission line, defined as

$$
\hat{\gamma} = \sqrt{\hat{Z}\hat{Y}} = \sqrt{\left(\hat{Z}_w + j\omega L\right)(G + j\omega C)} = \alpha + j\beta. \tag{5}
$$

$\hat{Z}_C$ is the characteristic impedance of the transmission line in the frequency domain, expressed as

$$
\hat{Z}_C = \sqrt{\hat{Z}\big/\hat{Y}} = \sqrt{\left(\hat{Z}_w + j\omega L\right)\big/(G + j\omega C)}. \tag{6}
$$

$\hat{\rho}_i (i = 1, 2)$ are the reflection coefficient of the loads, defined as

$$\hat{\rho}_i = \left(\hat{Z}_i - \hat{Z}_C \right) \Big/ \left(\hat{Z}_i + \hat{Z}_C \right), \qquad (7)$$

$\hat{V}_1^s$ and $\hat{V}_2^s$ are the source terms. In (5), $\hat{Z}$, $\hat{Y}$, L, C, and G are the per-unit-length impedance, admittance, external inductance, capacitance, and conductance of the transmission line, respectively. $\hat{Z}_w = R + L_i$ is the per-unit-length impedance of the conductor where R and L_i are the per-unit-length resistance and internal inductance of the conductor, respectively. $\hat{Z}_i (i = 1, 2)$ in (7) are the terminal impedances of the transmission line.

3. CALCULATED RESULTS AND ANALYSIS

3.1. Description of the Example

The simple structure as shown in Fig. 1 is used to study the loss effect of the conductor and the validity of the SPICE model. The height h and radius r of the wire are 10 mm and 1 mm, respectively. The terminators of the wire are R_1 and R_2. The material of the wire is copper of which the electric conductivity σ is 5.76×10^7 S/m. The per-unit-length resistance R of the wire can be approximated by [11]

$$R = \begin{cases} \dfrac{1}{\sigma \pi r^2} \Omega/\text{m} & f \leq f_0 \\[2mm] \dfrac{1}{2r} \sqrt{\dfrac{\mu f}{\pi \sigma}} \Omega/\text{m} & f \geq f_0 \end{cases} \qquad (8)$$

where R_{dc} is the dc per-unit-length resistance of the wire, defined as

$$R_{dc} = 1/(\sigma \pi r^2) \Omega/\text{m}, \qquad (9)$$

and f_0 is the frequency when the radius of the wire is twice the skin depths, expressed as

$$f_0 = 4/(\mu \sigma \pi r^2) \text{Hz}. \qquad (10)$$

Here μ is the permeability of the copper and can be approximated by μ_0.

Ignoring the per-unit-length conductance G and internal inductance L_i, which are very small, the propagation constant of (5) can be simplified as

$$\hat{\gamma} = \sqrt{(R + j\omega L)(j\omega C)} = \alpha + j\beta, \qquad (11)$$

where

$$\alpha = \sqrt{\frac{1}{2}\omega^2 LC \left[\sqrt{1 + \frac{R^2}{\omega^2 L^2}} - 1\right]} \qquad (12\text{a})$$

$$\beta = \sqrt{\frac{1}{2}\omega^2 LC \left[\sqrt{1 + \frac{R^2}{\omega^2 L^2}} + 1\right]} \qquad (12\text{b})$$

and the characteristic impedance of (6) can be written as

$$\hat{Z}_C = \sqrt{(R + j\omega L)/(j\omega C)} = Z_{c0}\sqrt{1 + R/(j\omega C)}, \qquad (13)$$

where Z_{C0} is the characteristic impedance of the lossless transmission line and equals $\sqrt{L/C}$.

If the loss of the transmission line is ignored, then the propagation constant of (11) becomes

$$\hat{\gamma} = j\omega\sqrt{LC} = j\omega/v \qquad (14)$$

and the characteristic impedance of (13) becomes

$$\hat{Z}_C = Z_{C0}. \qquad (15)$$

The direction of the incident electric field is shown in Fig. 2. The time waveform of the incident electric field here is a biexponential pulse, described by the expression $\mathcal{E}_0(t) = e^{-t/t_1} - e^{-t/t_2}$, where t_1 and t_2 equal 0.5 ns and 0.2 ns, respectively. The time waveform and the frequency spectrum of the incident electric field are shown in Fig. 3. The upper frequency is 5 GHz, above which the spectrum component is very small and the classical transmission line theory may be not correct for this structure.

3.2. Study of Loss Effect

The relations between the loss effect and the length of the transmission line $\mathcal{L}$, the frequency f, and the loads R_1 and R_2 are studied respectively to determine the cases when the loss should be considered.

To represent the differences between the lossy and lossless results, the relative difference is defined as

$$\text{relative difference } (\%) = \text{abs}\left(\left(\left|I_{\text{lossless}}\right| - \left|I_{\text{lossy}}\right|\right)\Big/\left|I_{\text{lossy}}\right|\right) \times 100\%. \qquad (16)$$

(a)

(b)
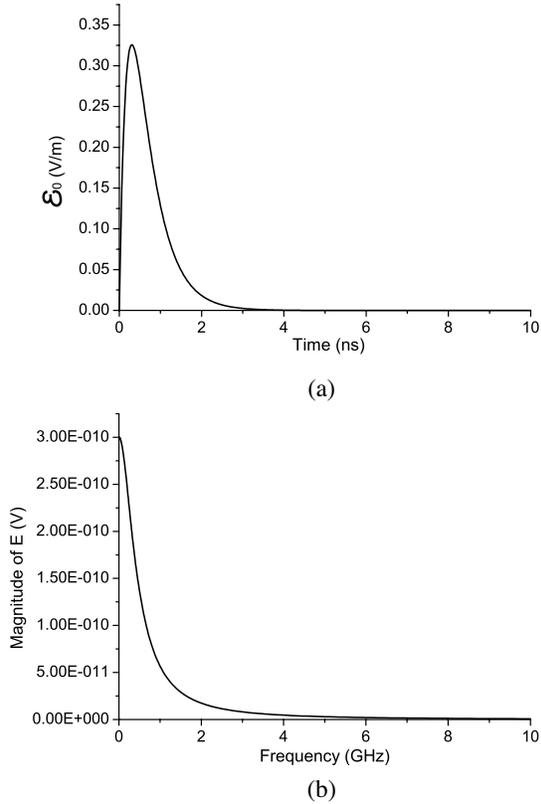
Figure 3. (a) The time waveform of the incident electric field; (b) The frequency spectrum of the incident electric field.

3.2.1. Relation between Loss Effect and Length

When $R_1 = 50\,\Omega$, $R_2 = 100\,\Omega$, and the angles $\theta_p = 0°$, $\phi_p = -90°$, $\theta_E = 90°$, the frequency-domain current responses I_1 of R_1 with the length $\mathcal{L} = 1\,\text{m}$, $\mathcal{L} = 10\,\text{m}$ and $\mathcal{L} = 100\,\text{m}$ are shown in Fig. 4. Fig. 4(b) and Fig. 4(c) only show the curves of a portion of the frequency range due to the very dense curves. The positive direction of I_1 is defined in Fig. 1. The relative differences with different lengths are shown in Fig. 5. The results show that the differences become large when the length $\mathcal{L}$ of the transmission line increases, and are larger at the poles than in other places. In the lossless case, $|I_1|$ has zeros. And the number of zeros increases with the increment of the length $\mathcal{L}$ which is because the oscillating period decreases with the length.
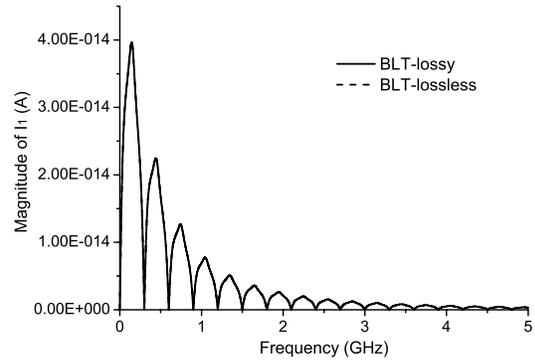
(a)
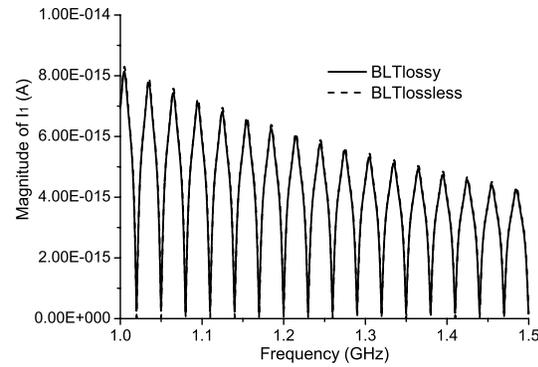
(b)
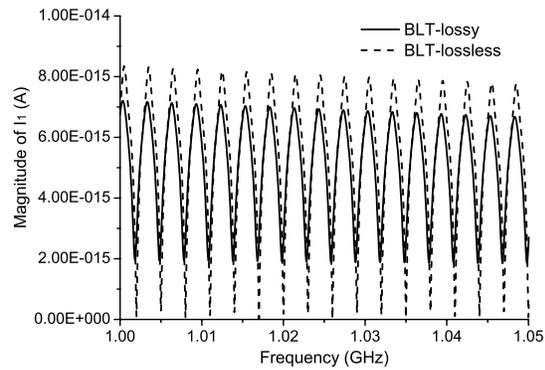
(c)

Figure 4. The frequency-domain current response of R_1 with different lengths of the transmission line. (a) $\mathcal{L} = 1\,\text{m}$; (b) $\mathcal{L} = 10\,\text{m}$; (c) $\mathcal{L} = 100\,\text{m}$.
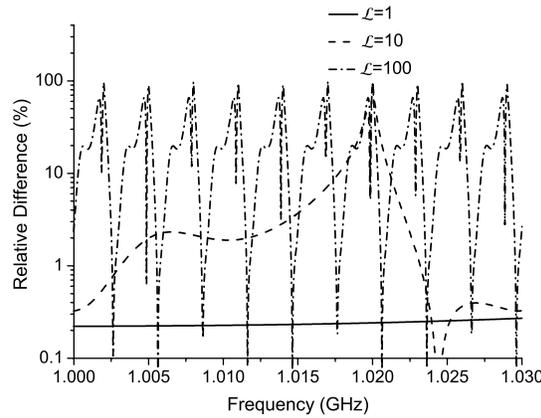
Figure 5. Relative difference with different lengths $\mathcal{L}$.

3.2.2. Relation between Loss Effect and Frequency

Figure 6 shows the relative difference of the current of R_1 changing with the frequency when R_1 equals $50\,\Omega$ and R_2 equals $100\,\Omega$. The length $\mathcal{L}$ of the transmission line is $1\,\mathrm{m}$, and the angles θ_p, ϕ_p, and θ_E are $0°$, $-90°$, and $90°$, respectively.

The loss effect generally increases with the frequency, due to the skin effect. When $\omega \ll R/L$, (12a) can be simplified as $\alpha \approx 1/2R(f)\sqrt{C/L}$, which increases with frequency.

When the length $\mathcal{L}$ is $10\,\mathrm{m}$ or $100\,\mathrm{m}$, similar results can be obtained. Fig. 7 shows the relative differences around $5\,\mathrm{GHz}$ with the length $10\,\mathrm{m}$ and $100\,\mathrm{m}$.

The relative difference is generally not more than 1% and 5% when the length $\mathcal{L}$ is $1\,\mathrm{m}$ and $10\,\mathrm{m}$, except for the poles, while the relative difference can be as high as 60% near $5\,\mathrm{GHz}$ when the length is $100\,\mathrm{m}$. But the relative difference for $100\,\mathrm{m}$ is generally below 10% when the frequency is below $100\,\mathrm{MHz}$.

3.2.3. Relation between Loss Effect and Loads

Broyd et al. pointed out that the loss of the transmission line can not be ignored when the terminal loads are small. Fig. 8 shows the relative differences with different terminal loads at low frequencies [16]. The length of the line and the incident angles are the same as those in Fig. 6. The relative differences decrease with the loads at low frequency, and the loss can not be ignored at low frequency when the terminal loads are small.
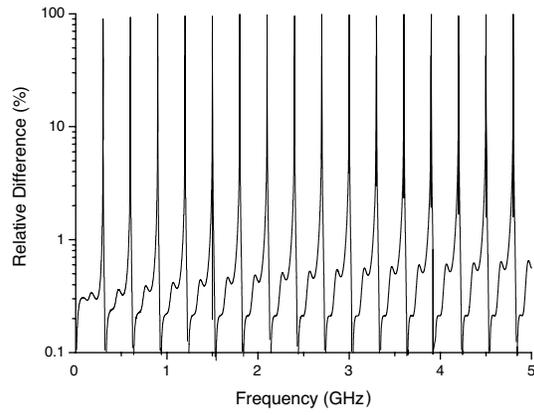
Figure 6. Relative difference with the frequency.

(a)

(b)

(c)
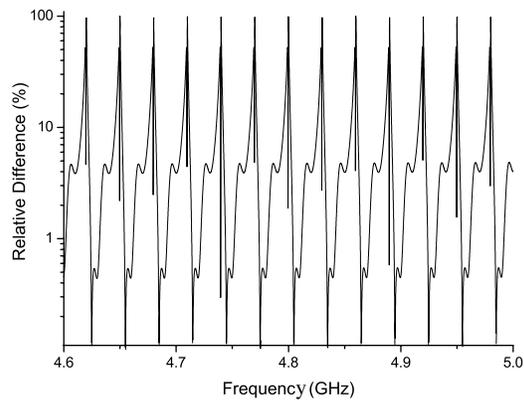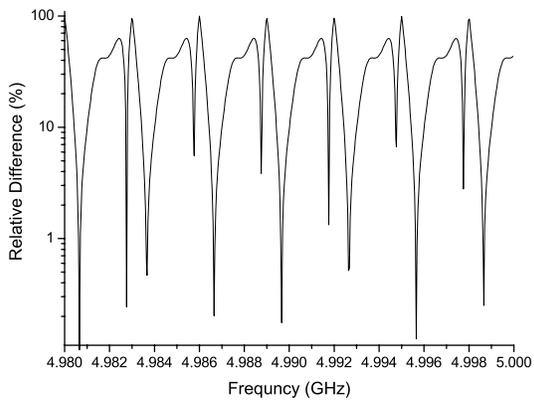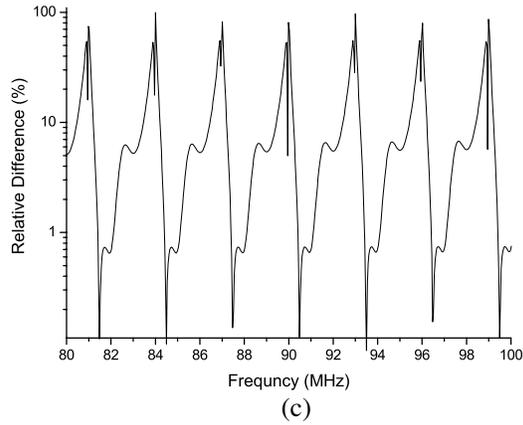
Figure 7. Relative difference with the frequency 2. (a) $\mathcal{L} = 10\,\text{m}$ near $5\,\text{GHz}$; (b) $\mathcal{L} = 100\,\text{m}$ near $5\,\text{GHz}$; (c) $\mathcal{L} = 100\,\text{m}$ near $100\,\text{MHz}$.
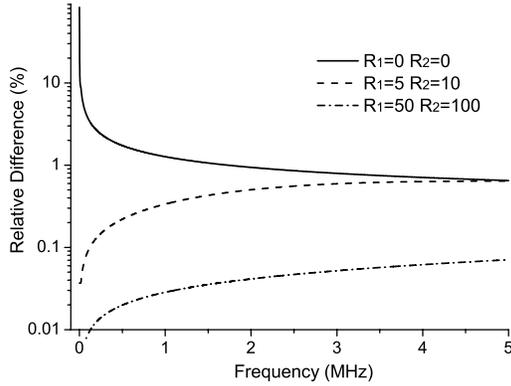

Figure 8. The relative difference with different terminal loads at low frequency.

When $\omega \ll R/L$, (12) can be simplified as

$$\alpha \approx \sqrt{\omega RC/2} \tag{17a}$$

$$\beta \approx \sqrt{\omega RC/2}. \tag{17b}$$

When ω satisfies $\beta\mathcal{L} \ll 2\pi$, the phase shift can be ignored and the transmission line can be treated as a circuit. There will be an error if the loss is ignored, because the resistance of the whole circuit is zero. The range of the low frequency when the loss can not be ignored decreases with the length $\mathcal{L}$.

In conclusion, the loss effect generally increases with the frequency and the length of the line, and may not be ignored at some poles and low frequency when the loads are small.

3.3. Validity of the SPICE Model

For electromagnetic pulse, such as nuclear electromagnetic pulse (NEMP), ultra wideband (UWB), and high power microwave (HPM), the number of poles is limited and the low frequency range when the loss effect may be not ignored are small compared with the whole frequency range, so the SPICE model for lossless transmission lines may be used for electromagnetic pulse interaction with electronic systems.

The transmission lines with lengths 1 m and 10 m meet the needs of small and large electronic systems, respectively, while those with the length 100 m almost meet the needs of all the electronic systems. The relative difference is generally not more than 1% for 1 m and 5% for 10 m when the frequency is below 5 GHz. While the length equals 100 m, the relative difference is generally below 10% for the frequency below 100 MHz. So the SPICE model for lossless transmission lines can be used for NEMP, UWB, and the low-frequency range of HPM interaction with most systems of which the sizes are less than one hundred meters.

The transient current of the case in Fig. 7(b) with $R_1 = 50\,\Omega$ and $R_2 = 100\,\Omega$ is shown in Fig. 9. The result of the SPICE model agrees well with that of the BLT equation, because the number of poles is limited and the component around 5 GHz is small compared with the whole frequency range.

Figure 10 shows the transient currents for the case with $R_1 = 0\,\Omega$
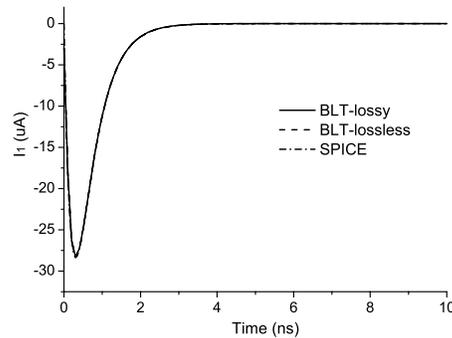
**Figure 9.** The transient current of the case in Fig. 7(b) with $R_1 = 50\,\Omega$ and $R_2 = 100\,\Omega$.
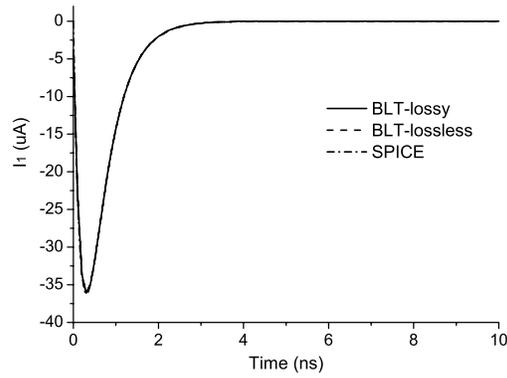
Figure 10. The transient current of the case in Fig. 6 with $R_1 = 0\,\Omega$ and $R_2 = 0\Omega$.
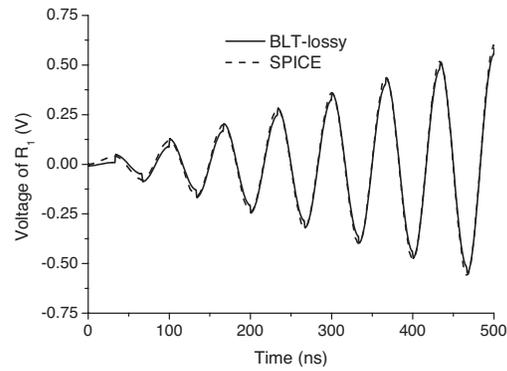

(a)
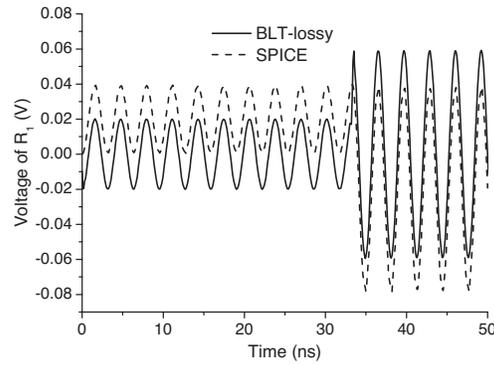
(b)

Figure 11. (a) The Voltage response of R_1 when f_0 equals $15\,\text{MHz}$; (b) The Voltage response of R_1 when f_0 equals $315\,\text{MHz}$.

and $R_2 = 0\,\Omega$ in Fig. 6. The results can be wrong at low frequencies when the terminations are very small, but the range of low frequencies is a small part of the whole frequency range, so the transient currents with the loss ignored agree well with those with the loss considered.

However, for a sine or cosine wave, whose frequency equals the pole frequency of the line, the SPICE model may lead to significant error. Figure 11 shows the voltage responses of R1 when the line of length $10\,\mathrm{m}$ is open at both ends and exited by a cosine wave, whose frequency f_0 equals $15\,\mathrm{MHz}$ and $315\,\mathrm{MHz}$. The incident angles θ_p, φ_p, and θ_E of the wave are $0°, -90°$, and $90°$, respectively. Both $15\,\mathrm{MHz}$ and $315\,\mathrm{MHz}$ are the pole frequencies of the line. The result of SPICE agrees well with that of the BLT equation when f_0 equals $15\,\mathrm{MHz}$, but has a significant error when f_0 equals $315\,\mathrm{MHz}$. This is because the loss effect increases with the frequency and is larger at the poles than in other places.

4. SUMMARY AND CONCLUSION

The loss effect of the line and the validity of the SPICE model for lossless transmission line have been studied by computing the current induced at the termination of a simple transmission line excited by an incident electromagnetic field. The loss effect of transmission lines generally increases with the length of the transmission line and frequency. At low frequencies where the terminations of the line are small, ignoring the loss can lead to error. At the poles of the frequency domain current response, the loss may not be ignored. Hence, the SPICE model can be used for nuclear electromagnetic pulse, ultra wideband, and the low-frequency range of high power microwave interaction with most systems, because the frequency range when the loss effect can't be ignored is small compared with the whole frequency range of the electromagnetic pulse But for a sine or cosine wave whose frequency equals the pole frequency of the line, the SPICE model may lead to significant error.

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