Progress In Electromagnetics Research, PIER 90, 151–169, 2009

ROBUST ADAPTIVE BEAMFORMING BASED ON PARTICLE FILTER WITH NOISE UNKNOWN

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Abstract—Adaptive beamforming, which uses a weight vector to maximize the signal-to-interference-plus-noise ratio (SINR), is often sensitive to estimation error and uncertainty in the parameters, such as direction of arrival (DOA), steering vector and covariance matrix. Robust beamforming attempts to mitigate this sensitivity and diagonal loading in sample covariance matrix can improve the robustness. In this paper, beamformer based on particle filter (PF) is proposed to improve the robustness by optimizing the diagonal loading factor in sample covariance matrix. In the proposed approach, the level of diagonal loading is regarded as a group of particles and optimized using PF. In order to compute the post probability of particles beyond the knowledge of noise, a simplified cost function is derived first. Then, a statistical approach is developed to decide the level of diagonal loading. Finally, simulations with several frequently encountered types of estimation error are conducted. Results show a better performance of the proposed beamformer than other typical beamformers using diagonal loading. In particular, the prominent advantage of the proposed approach is that it can perform well even noise and error in the steering vector are unknown.

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1. INTRODUCTION

Compared with the traditional data-independent beamformers, the adaptive beamformers can have much better interference rejection capability [1–3] and some can improve the resolution of the direction of arrival (DOA) [6]. In the past decades, adaptive beamforming is always regarded as a hot topic by researchers. It has been widely used in radar, sonar, radio astronomy, wireless communications, microphone array speech processing, medical imaging and other areas [4, 5, 23]. It is well known that adaptive beamformers can suffer significant performance degradation when the environment, sources, or sensor array is violated and this may cause a mismatch between the assumed and actual signal steering vectors. A lot of algorithms have been proposed to improve the robustness of adaptive beamforming [6–9]. However, these methods mostly focus on optimizing the direction of arrival (DOA) or signal steering vector only.

In practice, the sample covariance is estimated with error, and so it is uncertain. Especially, the deadly disadvantage of sample matrix inversion (SMI) is that it is generally ill-conditioned and leads to significant degradation of performance. Therefore, some methods are developed to further improve the performance of the SMI technique. The use of diagonal loading has usually been regarded as the natural complement to the SMI technique. In this paper, we focus on improving the robustness of adaptive beamforming by optimizing the level of diagonal loading.

In the past decades, many works have been done on diagonal loading optimization. The first presented use of diagonal loading can date back to the work of Capon [10]. Two years later, Riley gave in [11] the first indiction that diagonal loading was useful in order to improve the conditioning of the augmented matrix, thereby facilitating its numerical inversion. Later, in the context of the regularization of ill-posed problems, diagonal techniques are more extensively analyzed [12]. Obviously, diagonal loading has been the most popular method to alleviate the losses of using finite sample size estimate of the true covariance matrix [13]. However, the decision of the loading level is still a difficult problem. To solve this problem, some ideas can be seen in literatures [13–18]. In [14], the author pointed out the loading level should be chosen to be higher than the noise level but much lower than the smallest interference eigenvalue. However, the problem is that the level of noise and interference is usually unknown. Calson [13] suggests to fix the value at the level 10 dB above the white noise power. It is clear that getting the accurate white noise level is still not easy. Another normal method is to fix the value at 10 times of the minimum eigenvalue of the sample matrix. These methods are indeed effective in many practical cases. However, while the desired signal is present in training snapshots, the performance will decrease severely. The method by computing the eigenvalues of the sample matrix to estimate the loading factor is given in [15]. This method can combat the finite sample size effect, but the main drawback is that it must estimate the dimension of the interference subspace at first. Recently, some methods based on iteration [16, 17] are developed to loading factor optimization. These two methods use the linearization of weights by Taylor series at the vicinity where the loading factor value is zero. They are effective in some situations and better than fixed loading methods. However, the linearization prevents the convergence of these methods and it is very important for iterative methods. Specially, the variance of noise must be used at the step of initialization in [16]. It means that the noise need be estimated at first. In [14], the relationship between beamformer based on worst-case optimization and loading factor based SMI (LSMI) beamformer is discussed. The result illustrates that the worst case optimization based beamformer belongs to the class of diagonal loading techniques as LSMI beamformer. However, the shortcoming of this beamformer is that the optimized performance is based on the norm of the steering vector distortion which must be bounded at first.

In this paper, we focus on using the statistical theory to loading factor optimization and propose an approach based on particle filter (PF). To the best of our knowledge, using PF to decide the level of diagonal loading of sample covariance matrix is new. In the proposed approach, the value of diagonal loading is regarded as a discrete random variables with a prior probability. The aim of this approach is the maximization of empirical Rayleigh quotient. And we have derived a simplified cost function to compute the posterior probability of these particles. Simulations also manifest that it can perform better than typical diagonal loading methods. In addition, the meaningful advantage of the proposed approach is that the level and type of noise can not be concerned.

The paper is organized as follows. Section 2 contains background material. The SMI beamformer based on particle filter is developed in Section 3. Performance examples are presented in Section 4. Conclusion is given in Section 5.

2. BACKGROUND

We use the standard narrowband beamforming model in which a set of P narrowband plane wave signals with known center frequency, impinge on an array of N sensors, where $P < N$. Assume one is the desired signal from the direction θ_s and the remaining are interference from the directions θ_p $(p = 1, 2, \ldots, P-1)$, the $N \times 1$ vector of received signals is given by

$$\boldsymbol{x}(k) = \boldsymbol{s}(k) + \boldsymbol{i}(k) + \boldsymbol{n}(k) = s(k)\boldsymbol{a}(\theta_s) + \sum_{p=1}^{P-1} \boldsymbol{a}(\theta_p) i_p(k) + \boldsymbol{n}(k) \quad (1)$$

where k is the time index, $\boldsymbol{s}(k), \boldsymbol{i}(k)$, and $\boldsymbol{n}(k)$ are the desired signal, interference and noise, respectively. $s(k)$ is the signal waveform and $i_p(k)$ is the waveform of the pth interference. Here, $\boldsymbol{a}$ is the steering vector of wave. The source and noise waveforms are assumed to be sample functions of zero-mean random processes, and successive snapshots of both the source and noise are assumed to be statistically independent.

When $s(k)$ is uncorrelated with the noise and interference, the received covariance matrix can be written as

$$\boldsymbol{R}_x = E\left\{\boldsymbol{x}(k)\boldsymbol{x}(k)^H\right\} = \sigma_s^2 \boldsymbol{a}(\theta_s)\boldsymbol{a}(\theta_s)^H + \boldsymbol{R}_{i+n} \quad (2)$$

where σ_s^2 is the desired signal power

$$\sigma_s^2 = E\left\{|s(k)|^2\right\} \quad (3)$$

and $\boldsymbol{R}_{i+n}$ is the interference plus noise covariance matrix, and $(\cdot)^H$ stands for the Hermitian transpose.

The narrowband beamformer is a linear filter consisting of N complex weights. The output of the beamformer $y(k)$ is an estimate of the desired signal and has the form as

$$y(k) = \hat{s}(k) = \boldsymbol{w}^H \boldsymbol{x}(k) \quad (4)$$

The weight vector can be found from the maximum of the signal-to-interference-plus-noise ratio (SINR) and it is equivalent to [18]

$$\min_{\boldsymbol{w}} \ \boldsymbol{w}^H \boldsymbol{R}_{i+n} \boldsymbol{w} \ \ s.t. \ \ (\boldsymbol{a}(\theta_s))^H \boldsymbol{w} = 1 \quad (5)$$

The optimal weight vector of (5) has the form as [22]

$$w = \xi \boldsymbol{R}_{i+n}^{-1} \boldsymbol{a}(\theta_s) \quad (6)$$

where ξ is a scale factor. When $\xi = \sigma_s^2$, the weights correspond to spatial Winer filter, and when $\xi = (\boldsymbol{a}(\theta_s)^H)\boldsymbol{R}_{i+n}^{-1}\boldsymbol{a}(\theta_s)$, the minimum variance distortionless response (MVDR) weights are obtained. When

$\boldsymbol{R}_{i+n}$, σ_s^2 and $\boldsymbol{a}(\theta_s)$ are known exactly, the optimal weights can be obtained. The weight vector here also maximizes the output SINR.

Yet, the second-order statistics $\boldsymbol{R}_{i+n}$ and σ_s^2 are usually unknown and fluctuating, and the beamformer weights are computed adaptively using estimation based on data collected at the sensors. There are many methods to improve the numerical stability, the rate and performance of convergence [19, 21]. Block-adaptive method such as SMI is a typical method which collects a block data, estimates and inverts the sample covariance matrix, then updates the beamformer weights each time a new block of data is received. The weights are updated every K samples using the K sample covariance matrix as

$$\hat{\boldsymbol{R}}_K = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{x}(k)\boldsymbol{x}(k)^H \tag{7}$$

where $\hat{\boldsymbol{R}}_K$ is the maximum likelihood estimate of $\boldsymbol{R}_x$. Accordingly, the weight vector of SMI beamformer becomes

$$\hat{\boldsymbol{w}}_{SMI} = \frac{\hat{\boldsymbol{R}}_K^{-1}\boldsymbol{a}(\theta_s)}{\boldsymbol{a}(\theta_s)^H\hat{\boldsymbol{R}}_K^{-1}\boldsymbol{a}(\theta_s)} \tag{8}$$

One of the most popular approaches is called LSMI. The beamformer will perform better while the value of diagonal loading factor is incorporated in sample covariance matrix estimate [16]. This kind of sample matrix can be denoted as

$$\hat{\boldsymbol{R}}_{DL} = \hat{\boldsymbol{R}}_K + \delta\boldsymbol{I} \tag{9}$$

where δ is the value of diagonal loading factor and $\boldsymbol{I}$ is the identity matrix.

The LSMI weight vector can be reformed as [16]

$$\hat{\boldsymbol{w}}_{DL} = \left(\hat{\boldsymbol{R}}_K + \delta\boldsymbol{I}\right)^{-1}\boldsymbol{a}(\theta_s) \tag{10}$$

Another popular approach to robust adaptive beamforming is the worst case optimization based beamformer [18]. This approach assumes that the norm of the steering vector distortion $\boldsymbol{\Delta}$ can be bounded by known constant $\varepsilon > 0$. That is

$$\|\boldsymbol{\Delta}\| < \varepsilon \tag{11}$$

Then, the actual signal steering vector belongs to the set

$$\Gamma(\varepsilon) \triangleq \{\boldsymbol{c}|\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{e}, \|\boldsymbol{e}\| \leq \varepsilon\} \tag{12}$$

Finally, the worst case optimization based SINR analysis problem of finding a optimum weight vector can be written as

$$\min_{w} \ \boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w}, \quad s.t. \quad \left|\boldsymbol{w}^H \boldsymbol{a} - 1\right|^2 = \varepsilon^2 \boldsymbol{w}^H \boldsymbol{w} \tag{13}$$

The solution of (13) can be found by minimizing the function

$$J(\boldsymbol{w}, \lambda_1) = \boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w} + \lambda_1 \left(\varepsilon^2 \boldsymbol{w}^H \boldsymbol{w} - \left|\boldsymbol{w}^H \boldsymbol{a} - 1\right|^2\right) \tag{14}$$

where λ_1 is a Lagrange multiplier. Taking the gradient of (14), the weight vector of worst case optimization based beamformer can be denoted as [18]

$$\boldsymbol{w} = \frac{\lambda_1}{\lambda_1 \boldsymbol{a}^H \left(\hat{\boldsymbol{R}}_K + \lambda_1 \varepsilon^2 \boldsymbol{I}\right)^{-1} \boldsymbol{a} - 1} \left(\hat{\boldsymbol{R}}_K + \lambda_1 \varepsilon^2 \boldsymbol{I}\right)^{-1} \boldsymbol{a} \tag{15}$$

which shows this robust beamformer belongs to the class of diagonal loading techniques. For simplicity, we call it worst case beamformer in brief. At the same time, from (15), we can know that the weight vector is sensitive to the value ε. It means that the performance of this beamformer will degrade if ε is not known exactly. However, ε is not only difficult to be known exactly, but also variable in the whole process. It means that it will affect the performance of this beamformer if there is an error in ε.

Given the structure of SMI with diagonal loading, the optimum value of loading factor can be stated as

$$\delta_{opt} = \arg\ \max(\gamma_{out}) \tag{16}$$

where γ_{out} denotes the output SINR.

3. SMI BEAMFORMER BASED ON PARTICLE FILTER

3.1. Simplified Cost Function

To solve the problem of (16), we can write the output SINR ($\hat{\gamma}_{out}$) as

$$\hat{\gamma}_{out} = \frac{\left|\boldsymbol{w}^H \boldsymbol{a}(\theta_s)\right|^2}{\boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w}} \tag{17}$$

Thus, the problem of finding a weight vector that maximizes output SINR can be written as

$$\min_{w} \ \boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w}, \quad s.t. \quad \left|\boldsymbol{w}^H \boldsymbol{a}(\theta_s)\right|^2 = 1 \tag{18}$$

Because $(\boldsymbol{w}^H \boldsymbol{a}(\theta_s))^2$ is not less than one, optimization problem (18) can be reformed as

$$\min_{w} \ \boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w}, \quad s.t. \ \ln\left(\boldsymbol{w}^H \boldsymbol{a}(\theta_s)\right)^2 = 0 \tag{19}$$

Then, the cost function $J(\boldsymbol{w})$ of this problem can be written as

$$J(w) = \boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w} + \lambda \ln\left(\boldsymbol{w}^H \boldsymbol{a}(\theta_s)\right)^2 \tag{20}$$

where λ is the Lagrange multiplier. To obtain the minimum value of the cost function, take the gradient of $J(\boldsymbol{w})$ equal to 0. That is to say

$$\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}} = 0 \tag{21}$$

The expression for λ becomes

$$\lambda = \frac{\boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w}}{2\ln\left(\boldsymbol{w}^H \boldsymbol{a}(\theta_s)\right)} \tag{22}$$

Substituting (22) into (20), the minimized cost function can be denoted as

$$J(\boldsymbol{w}) = \boldsymbol{w}^H \hat{\boldsymbol{R}}_K \boldsymbol{w}\left[1 + \frac{1}{2}\ln\left(\boldsymbol{w}^H \boldsymbol{a}(\theta_s)\right)\right] \tag{23}$$

According to Equation (10), $\boldsymbol{w}$ is the function of δ. Therefore, the cost function can be written as the function of δ, which is

$$\hat{J}(\delta) = \boldsymbol{w}^H(\delta) \hat{\boldsymbol{R}}_K \boldsymbol{w}(\delta)\left[1 + \frac{1}{2}\ln\left(\boldsymbol{w}^H(\delta) \boldsymbol{a}(\theta_s)\right)\right] \tag{24}$$

where $\hat{J}(\delta)$ denotes the estimation of the cost function while loading level is δ.

In our approach, this simplified cost function will be used to reconstitute the measurement equation.

3.2. Proposed Beamformer

In the proposed approach, we regard the value of diagonal loading factor as a discrete random variable which have L particles. Then, the approximate post probability of these particles can be obtained by the measurement equation that will be discussed next.

PF is the state-of-art solution to nonlinear and non-Gaussion problems [20]. In our approach, the state vector $\boldsymbol{\delta}_k$ is estimated on the sequence of all available measurements, which are $\boldsymbol{J}_k = \{\boldsymbol{J}_i, i =$

$1, 2, \ldots, k\}$ up to the time k. It has been shown that the performance of PF is much better than that of the extended Kalman Filter (EKF) [21]. To use PF based adaptive beamforming method, the state equation should be formulated first.

$$\boldsymbol{\delta}_{k+1} = \boldsymbol{\delta}_k + \boldsymbol{\nu}_k \tag{25}$$

where $\boldsymbol{\delta}_k$ denotes the value of loading factor and $\boldsymbol{\nu}_k$ denotes the system noise at time k. The initial density of the state vector can be denoted as $p(\boldsymbol{\delta}_0) = p(\boldsymbol{\delta}_{\mathbf{0}}|\hat{\boldsymbol{J}}_0)$. Thus, the received data can be denoted as

$$\boldsymbol{x}_{k+1} = \boldsymbol{w}^H(\boldsymbol{\delta}_k)\boldsymbol{a}(\theta_s) + \boldsymbol{n}_k \tag{26}$$

where $\boldsymbol{n}_k$ is the measurement noise at time k. According to (10), $\boldsymbol{w}(\boldsymbol{\delta}_k)$ can be written as

$$\boldsymbol{w}(\boldsymbol{\delta}_k) = \left(\hat{\boldsymbol{R}}_K + \boldsymbol{\delta}_k \boldsymbol{I}\right)^{-1} \boldsymbol{a}(\theta_s) \tag{27}$$

and

$$\boldsymbol{y}(k) = \boldsymbol{w}^H(\boldsymbol{\delta}_k)\boldsymbol{a}(\theta_s) \tag{28}$$

Obviously, equations including (26), (27) and (28) can be regarded as a group of measurement equation. Because the level and type of noise are usually unknown, computing the post probability of $\boldsymbol{\delta}_k$ is very difficult. In the proposed approach, a new measurement equation is reconstituted using the simplified cost function $\hat{J}(\delta)$. That is

$$\hat{\boldsymbol{J}}_k = \boldsymbol{J}_k + \boldsymbol{\varepsilon}_k \tag{29}$$

Here, $\hat{\boldsymbol{J}}_k$ can be estimated from (24); $\boldsymbol{J}_k$ is the true value of cost function according to $\boldsymbol{\delta}_k$; and $\boldsymbol{\varepsilon}_k$ is the measurement noise.

It is reasonable to assume the post probability of particles approximate normal distribution ($N(\mu, \sigma^2)$) if the number of particles is large enough. Thus, the probability distribution of particles can be estimated from the simplified cost function.

There are four steps to finish a PF process after the state equation (25) and the measurement equation (29) have been built. They are initialization, prediction, update and resampling.

Firstly, in the initialization procedure, we set the particles $\delta_0^i = 0$, $(i = 0, 1, \ldots, L)$. System noise $\boldsymbol{\nu}_0$ meets a Gaussian distribution with mean = 1 and deviation = 1. To improve the efficiency and particle diversities, we set $\boldsymbol{\nu}_k$ to a Gaussian distribution $N\left(\mu, \sigma_\nu^2\right)$. In our simulation, we set $\mu = 0$ and $\sigma_\nu^2 = 0.1\hat{\delta}_{k-1}^2$. In order to simplify the computation, the initial density of the state vector is all set to $p(\delta_0^i) = \frac{1}{L}$.

Then, the predition step is conducted after initialization. At time $k+1$, new particles are generated by passing the resampled particles at time k through the system model as (25).

Next, the steps to update weights once in the proposed beamformer at time k can be concluded as follows.

For $i = 1, \ldots, L$

$$(1)\quad \hat{\boldsymbol{R}}_{DL}^{-1}(i) = \left((1/K)\sum\nolimits_{j=k-K}^{k} \boldsymbol{x}_j \boldsymbol{x}_j^H + \delta_k^i I\right)^{-1}$$

$$(2)\quad \boldsymbol{w}_k^i = \left(\hat{\boldsymbol{R}}_{DL}(i)\right)^{-1} \boldsymbol{a}(\theta_s)$$

$$(3)\quad \hat{J}_k^i = \left(\boldsymbol{w}_k^i\right)^H \hat{\boldsymbol{R}}_K \boldsymbol{w}_k^i \left[1 + \frac{1}{2}\ln\left(\boldsymbol{w}_k^i\right)^H \boldsymbol{a}(\theta_s)\right]$$

$$(4)\quad \mu_k = \min\left(\hat{J}_k^i\right), \quad \sigma_k^2 = (1/L)\sum\nolimits_{i=1}^{L}\left(\hat{J}_k^i - \mu_k\right)^2$$

$$(5)\ p'\left(\delta_k^i|\hat{\boldsymbol{J}}_k\right) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(\hat{J}_k^i - \mu_k\right)^2}{2\sigma_k^2}\right)$$

$$(6)\quad c = \sum\nolimits_{i=1}^{L} p'\left(\delta_k^i|\hat{\boldsymbol{J}}_k\right)$$

$$(7)\quad p\left(\delta_k^i|\hat{\boldsymbol{J}}_k\right) = \frac{1}{c}\left(p'\left(\delta_k^i|\hat{\boldsymbol{J}}_k\right)\right)$$

Resampling is another critical step in the proposed beamformer. Via resampling, which multiplies the particles with high weights and discards the particles with low weights, more particles are distributed in domains of higher posterior probability. Therefore, the estimation will be improved. The resampling method we adopted is the same as Generic Particle Filter in [20].

With the knowledge of the weights that characterize the posterior density of the particles, we use the maximum a posteriori (MAP) to compute an optimal state value $\hat{\delta}$. The estimated value is

$$\hat{\delta} = \arg \max_{\delta_k^i}(p_k^i) \tag{30}$$

Consequently, the weight vector of the proposed beamformer $\hat{\boldsymbol{w}}_{PF}$ can be written as

$$\hat{\boldsymbol{w}}_{PF} = \left(\boldsymbol{R}_x + \hat{\delta}\boldsymbol{I}\right)^{-1} \boldsymbol{a}(\theta_s) \tag{31}$$

4. NUMERICAL EXAMPLES

In this section, five beamformers are compared in terms of mean output SINR, diagonal loading level or beampattern. They include the optimum beamformer with known covariance matrix R_x (OPTIMAL), the proposed PF-based beamformer (31), worst case optimization based beamformer proposed in [18], typical SMI beamformer with no diagonal loading (8) and the extensively used beamformer with the loading level $\delta = 10\sigma_n^2$, ten times to the power of noise (10). Here, consider a uniform linear array with $N = 10$ omnidirectional sensors, in which the spacing between the elements is half of the wavelength of the incident wave. In all examples, assume the direction of desired signal is always 3°, the incident angles of two interfering sources are 30° and 50°, respectively. And the ratio of interference and noise (INR) in a single sensor is equal to 25 dB. To the proposed beamformer, the number of particles is $L = 300$ from Fig. 1 to Fig. 6. The initial prior probability $p(\delta_0^i)$, $(i = 1, 2, \ldots, L)$ is all $1/L$. All simulations are conducted while the signal is always present in the training data. The results in all simulations are averaged with 200 Monte-Carlo trials.

4.1. Known Signal Steering Vector

In this subsection, the performance of beamformers vs. the signal-to-noise ratio (SNR) when there is no error in signal steering vector is discussed. The results are drawn on Fig. 1 and Fig. 2. The bound of norm ε for worst case optimization based beamformer is set to 0.

Figure 1 compares the performance of beamformers in terms of the

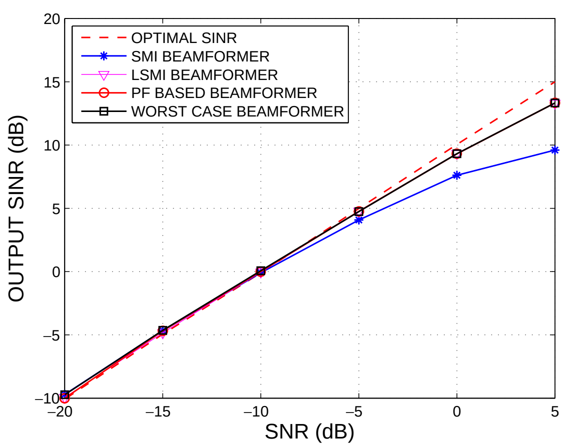

Figure 1. Output SINR vs. SNR with known signal steering vector.

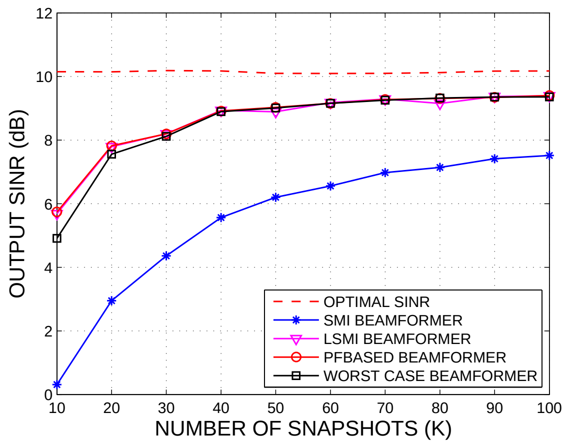

Figure 2. Output SINR vs. K with known signal steering vector.

mean output SINR versus SNR at the number of snapshots $K = 50$. This figure tells us that the proposed PF-based beamformer is almost no different in performance with LSMI beamformer and worst case optimization based beamformer when signal steering vector is known perfectly. The performance of SMI beamformer is the worst because there is no diagonal loading. It illustrates that diagonal loading is important for adaptive beamforming.

Figure 2 shows the output SINR versus the number of snapshots (K) at SNR $= 0$. It can be observed that the proposed beamformer performs better than the worst case optimization based beamformer at low K. It reflects that the proposed beamformer is more robust. Additionally, the improvement space of beamformers except SMI beamformer is little while K is more than 40. Therefore, $K = 50$ is selected in the following simulations.

4.2. Error in Direction of Desired Signal

In this subsection, a case with error $\Delta\theta_s = 2^\circ$ in the direction of desired signal is considered. Here, we assume that the presumed DOA is 5° and the actual DOA is 3°. The norm bound of signal steering vector ε in worst case beamformer is set to 3. Simulation results are shown from Fig. 3 to Fig. 5.

Figure 3 shows the performance comparison versus SNR. This figure clearly demonstrates that the proposed beamformer enjoys the best performance among four approaches. And that of SMI beamformer is the worst. The output SINR of beamformers except SMI beamformer is improved while SNR increases. Especially, using

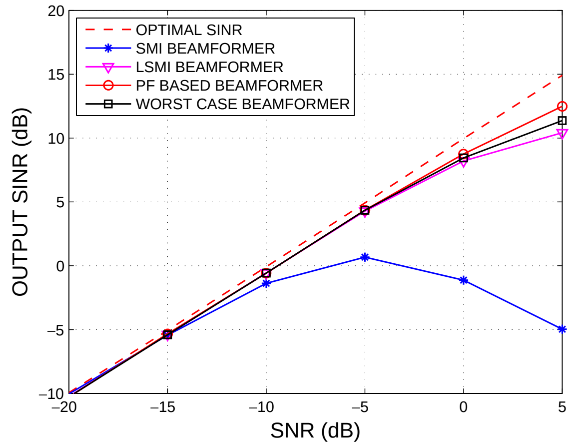

Figure 3. Output SINR vs. SNR with error in the direction of desired signal.

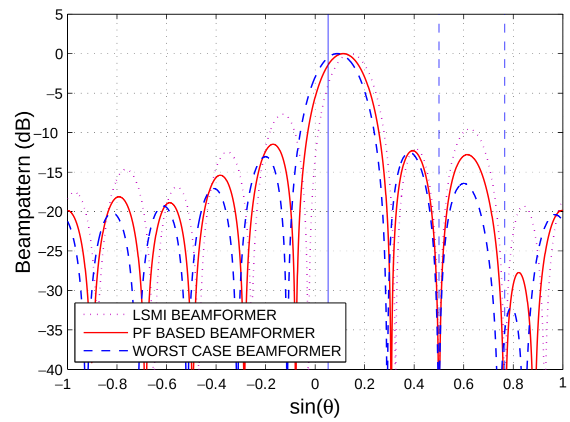

Figure 4. Beampatterns of beamformers at SNR = 5 dB.

the proposed beamformer leads to an improvement by more than 2.1 dB over LSMI beamformer and by 0.8 dB over the worst case beamformer at the point SNR = 5 dB. It can explain that not only diagonal loading benefits to the performance of beamformers, but also the performance is varying with the value of diagonal loading. In addition, the better performance of the proposed beamformer is due to a better loading level derived by PF compared with other beamformers.

Figure 4 shows the performance comparison among three beamformers from the viewpoint of beampattern. This simulation is condcuted at SNR = 5 dB. The variable $\sin(\theta)$ on horizontal coordinates is the sine value of direction θ. Dash dotted line denotes the direction of interference and solid line denotes the DOA. From Fig. 3, we have known the output SINR of the proposed beamformer is better than the worst case based beamformer at SNR $= 5$ dB. However, the gain of proposed beamformer at the desired direction is slightly lower than that of worst case beamformer. It is not beneficial to the performance. We can make a detail observation near the $50°$ direction; the zero subsidence of PF-beamformer at that direction is deeper than that of worst case based beamformer. It can explain its relatively better performance over worst case beamformer drawn on Fig. 3 at this SNR.

In Fig. 5, the loading values of beamformers vs. SNR are listed. Here, the value in longitudinal axis is normalized by the noise variance σ_n^2. Contrast to Fig. 1, we can also conclude that optimizing the diagonal loading level can improve the robustness of adaptive beamforming. In addition, the optimized value of diagonal loading is related to the noise level. At low SNR, the loading level of PF based beamformer is small and it is large at high level. It changes with the varying of noise level. Hence, the PF based have better performance than others.

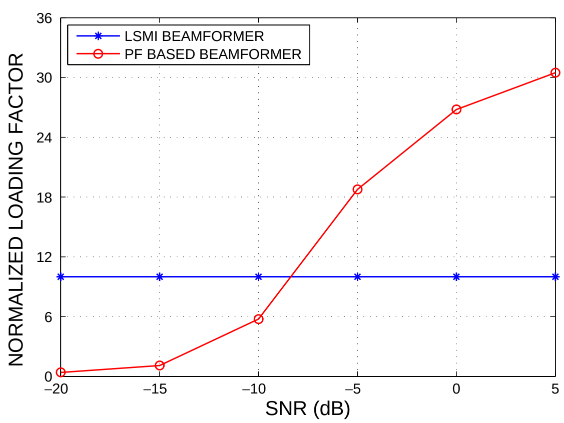

Figure 5. Normalized loading factor vs. SNR.

4.3. Error in Signal Steering Vector Due to Local Scattering

In this subsection, we assume that the desired signal arrives with four paths because of local scattering. One is the direct path with the signal $s_0(k)$ and the steering vector $\boldsymbol{a}$. Thus, the model of signal steering vector in this subsection can be formulated as

$$\hat{\boldsymbol{a}}(k) = s_0(k)\boldsymbol{a} + \sum_{m=1}^{3} s_m(k)\boldsymbol{b}(\theta_m) \tag{32}$$

where $s_m(k)$, $(m = 1, 2, 3)$ are i.i.d zero-mean complex Guassian random variables, $\boldsymbol{b}(\theta_m)$ are the signal steering vector of path with DOA θ_m. DOAs θ_m are drawn from a random generator with mean $= 3^\circ$ and deviation $= 2^\circ$. The bound of norm for worst case based beamformer is set to $\varepsilon = 3$. It is a relatively bad environment because the signal steering vector will change at every sample time. Simulation results are shown in Fig. 6.

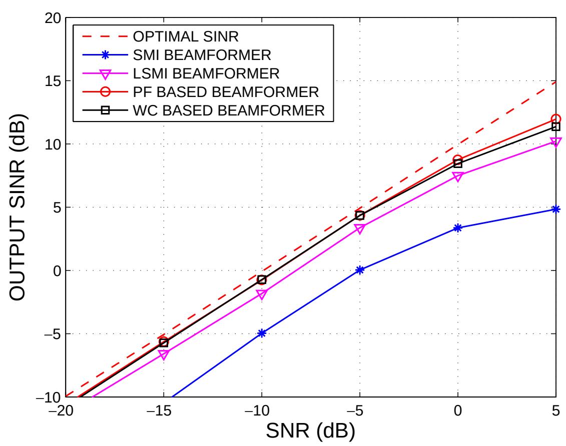

Figure 6. Output SINR vs. SNR with local scattering.

Compared to Fig. 3, we can find that the output SINR of worst case based beamformer has small changes. It means that this beamformer is very stable and not related to the kind of error only if the norm of error does not exceed its bound. Its shortcoming is performance loss because this beamformer always optimizes its performance with the worst case defined by error bound, even if it is not so bad. Therefore, the loading level of worst case is decided based on the worst case denoted by error bound. On the contrary, the loading level of proposed beamformer is decided by the actual situation. It can explain why the proposed beamformer performs better than the worst case based beamformer. In addition, the LSMI beamformer performs worse than worst case based beamformer and PF-based beamformer in Fig. 3. It illustrates that LSMI beamformer is not very robust in this case. Additionally, the proposed beamformer performs best from both of the two figures. It is clear that using PF in adaptive beamforming is effective to improve the robustness.

4.4. The Relationship between Performance and Particle Number

To PF-based beamformer, the particle number is related to the performance of beamformer. Generally, the estimation will be more accurate if higher particle number is used. To study the performance under the condition of different particle numbers, we conducted simulations with particle numbers $L = 50$, $L = 250$ and $L = 1250$. The results are shown in Fig. 7. The conclusion can be made that the performance is improved while the particle number is increased. However, the improvement is slight after the particle number exceeds 250. In addition, higher particle number is needed at high SNR because there is low noise variance.

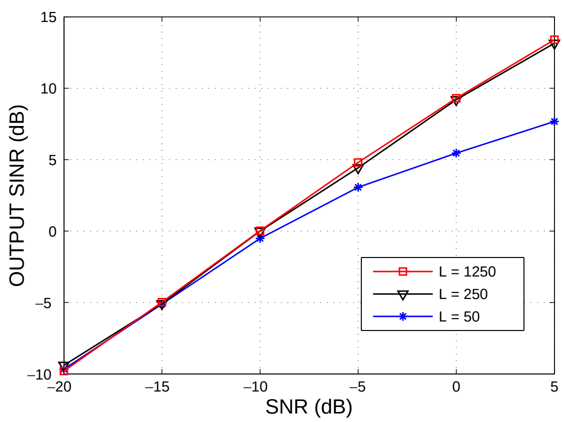

Figure 7. Output SINR of PF-based beamformer vs. SNR with different number of particles.

4.5. Approximated Cramer-Rao Bound and Complexity Analysis

In this subsection, we consider the Cramer-Rao Bound (CRB) of loading level in terms of fundamental noise properties. Using standard notations we consider independent noise sources, $\boldsymbol{Q}_k = E\left\{\boldsymbol{\nu}_k\boldsymbol{\nu}_k^T\right\}$ and $\boldsymbol{R}_k = E\left\{\boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k^T\right\}$. Here, $\boldsymbol{\nu}_k$ and $\boldsymbol{\varepsilon}_k$ are the system and measurement noises according to state equation (25) and measurement equation (29).

Because the optimal loading level is unknown, the optimization of diagonal loading level can not be directly equivalent to the tracking problem. We must have a base in order to compute the root mean square error (RMSE) of our approach. In our simulation, we regard the loading level $\boldsymbol{\delta}$ computed by PF-based beamformer as the base when the received matrix $\boldsymbol{R}$ and the noise variance are known. Hence, we call this CRB approximated CRB (ACRB). Thus, the ACRB can be formulated as [24]

$$\boldsymbol{P}_{k+1} = \left(\boldsymbol{P}_k^{-1} + E\left\{\boldsymbol{\varphi}\left(\boldsymbol{\delta}_k\right)\boldsymbol{R}_k^{-1}\boldsymbol{\varphi}^T\left(\boldsymbol{\delta}_k\right)\right\}\right)^{-1} + \boldsymbol{Q}_k \qquad (33)$$

where $(\cdot)^T$ stands for the transpose operation and $\boldsymbol{\varphi}\left(\boldsymbol{\delta}_k\right) = \frac{\hat{\boldsymbol{J}}_k - \hat{\boldsymbol{J}}_{k-1}}{\hat{\boldsymbol{\delta}}_k - \hat{\boldsymbol{\delta}}_{k-1}}$.

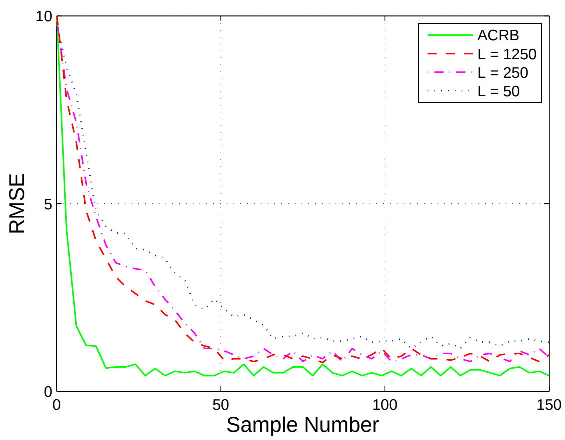

Figure 8. Performance of RMSE vs. sample number at SNR = 5.

Identical Monte Carlo simulations are performed in Fig. 8 using the number of particles 50, 250, 1250. Fig. 8 shows the resulting Monte Carlo RMSE. The figure also shows the ACRB for the proposed approach. The RMSE naturally decreases with increasing particle number, although not monotonically. For the particle number is which is 250 or higher, the average error after convergence almost equal. A large number of particles converge fast because a large number of particles give a more correct description of filter density.

The main shortcoming of the proposed approach is relatively more complex than other approaches mentioned in paper. A comparison of the computation times is shown in Table 1. The simulations are implemented on a 550-MHz AMD Athlon processor using MATLAB 7.1. Same as expected, the computation time for the proposed beamformer is much higher than worst case based beamformer and LSMI beamformer. However, these times are obtained on a serial computer and much reduction in computation times can be expected for the proposed beamformer when implemented in parallel. In addition, the computation time will be reduced greatly if we adopt DSP or FPGA to implement the approach [25]. It means that it is possible to use our approach in the real time system.

Table 1. Comparison of the computation times between beamformers.

Beamformer	PF-Based Beamformer (L = 50)	PF-Based Beamformer (L = 250)	PF-Based Beamformer (L = 1250)	LSMI Beamformer	Worst Case Beamformer
Computation Time(second)	5.3	8.2	27	< 1	< 1

5. CONCLUSIONS

Beamformer based on PF is developed to improve the robustness in adaptive beamforming by optimizing the level of diagonal loading factor. In order to perform PF efficiently, a simplified cost function is derived. Simulation results indicate that the proposed beamformer outperforms the typical worst case optimization based and LSMI beamformers. In particular, the main advantage of the proposed beamformer over typical methods is that it can perform well with unknown noise or error in signal steering vector. The disadvantage of this approach is relatively complex computation. However, it can be solved by adopting the higher speed microchip in implementation and it is valuable in many cases.

ACKNOWLEDGMENT

This work was supported by Natural Science Foundation of China (No. 60531020) and Natural Science Foundation of Zhejiang Province (No. Y106384).

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