A SIMPLE APPROACH FOR EVALUATING THE RECIPROCITY OF MATERIALS WITHOUT USING ANY CALIBRATION STANDARD

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Abstract—A simple approach for evaluation of the reciprocity of materials using raw scattering parameter measurements is proposed. This approach not only reduces the overall measurement time but also eliminates the need for calibrating the measurement system since it uses calibration-independent measurements. We have derived a metric function for reflecting and nonreflecting cells, which are used to house the sample under test. This function does not depend on electrical properties of materials and their lengths, and whether the cell is reflecting. We have also investigated the effects of the sample length and air pockets between sample external surfaces and cell inner walls on the performance of the evaluation of sample reciprocity.

1. INTRODUCTION

Microwave engineering requires precise knowledge on electromagnetic properties of materials at microwave frequencies since microwave communications are playing more and more important roles in military, industrial, and civilian life [1–3]. For these reasons, various microwave
techniques, each with its unique advantages and constraints [1], are introduced to characterize the electrical properties of materials.

Microwave nonresonant methods are widely used for broadband materials characterization [1]. However, these methods require some sort of calibration before measurements [2]. This is because these methods are generally adapted to a specific application and sensitive to more than one variable. It has recently been shown that calibration-independent microwave nonresonant methods can effectively be utilized to eliminate this need [4–9]. These methods are very attractive because the accuracy of calibration-dependent techniques is limited by the requirement of a full two- or one-port calibration using a set of standards which inevitably cause errors due to their imperfections [6]. In addition to the advantage that they remove the need for calibration, they as well have wide frequency coverage so that they are ready candidates for broadband applications and reduce the overall measurement time.

Calibration-independent techniques utilize the measured uncalibrated (raw) scattering (S-) parameters to extract the electrical properties of materials. Therefore, a technique which auto-monitors the performance of S-parameter measurements gains importance since such a technique may give an insight on the accuracy of measurements. We have recently proposed two techniques to monitor the accuracy of S-parameter measurements before determining the electrical properties of materials [10, 11]. There are three main advantages of these techniques as: a) they are applicable to different calibration-independent methods available in the literature; b) they can be employed for non-magnetic and magnetic materials; and c) they are not functions of sample thickness. The connection between the reciprocity of the sample and the raw S-parameter measurements has not yet been investigated. The motivation of this work is to analyze and propose a feasible and simple method for monitoring this connection. This investigation is very important since it allows us to monitor the reciprocity of materials before measuring their electrical properties. Another importance of the investigation is that it can show the level of reciprocity of the materials.

The organization of the paper is as follows. First, the model for the problem is given in Section 2. Then, in Section 3, we derive a metric function for evaluation of the reciprocity of magnetic or nonmagnetic materials. Next, raw S-parameter measurements of five polystyrene, polyvinyl-chloride (PVC) and Plexiglas samples with various lengths are shown for validation of the proposed method in Section 4.
2. MODEL FOR THE PROBLEM

We consider the measurement configurations shown in Fig. 1. While Fig. 1(a) illustrates the empty cell connection, Fig. 1(b) shows the measurement configuration where the sample with length $L$ is arbitrarily located into a measurement cell (a waveguide section).

![Measurement configurations for evaluation of the reciprocity of materials using uncalibrated scattering parameter measurements from a vector network analyzer (VNA).](image)

**Figure 1.** Measurement configurations for evaluation of the reciprocity of materials using uncalibrated scattering parameter measurements from a vector network analyzer (VNA).

The two ports referred to as $X$ and $Y$ in Fig. 1 are used as transitions between a vector network analyzer (VNA) and the configurations in Figs. 1(a) and 1(b). These ports include source and load match errors, tracking errors, hardware imperfection of VNA [8–11]. It is assumed that $X$ and $Y$ are unequal and are unchanged for each configuration in Fig. 1. In implementing the method, firstly raw $S$-parameters of the configuration in Fig. 1(a) are measured. Then, we locate the sample into the cell. Next, we measure the raw $S$-parameters of this new measurement configuration (Fig. 1(b)). Finally, the relative complex permittivity ($\varepsilon$) is extracted from these measurements by the method.
A microwave network with an arbitrary number of ports can be characterized by using S-parameter presentation. However, in practice, many microwave networks consist of a cascade connection of two or more two-port networks (e.g., X and Y in Fig. 1). In these circumstances, it is convenient to use ABCD matrix [7, 12, 13] or wave cascading matrix (WCM) presentations [14] of such microwave networks. For the mathematical analysis in this paper, we utilize the wave cascading matrix (WCM) since it is useful in calibration/error correction problems [6, 15]. We denote the two-port WCM matrices, $T_X$, $T_Y$, $T_I$, $T_{II}$, $T_{III}$, and $T_{IV}$, respectively, for modeling the transitions X and Y, air regions with lengths $L_{01}$, $L_{02}$, and $L$, and the sample (Fig. 1). For each of these ports, we can write their theoretical expressions for the derivation of $\varepsilon$. Since our method eliminates the need for knowledge on $T_X$ and $T_Y$, we will only deal with $T_I$, $T_{II}$, $T_{III}$, and $T_{IV}$ for $\varepsilon$ determination. The expressions for these ports can be obtained by finding electric and magnetic fields in each port, which can be derived from their vector potentials (or Hertzian vectors), $\vec{A}$ and $\vec{F}$ [16] as

$$\vec{E}^{(n)} = -j\omega \vec{A}^{(n)} - \frac{1}{\omega \mu^{(n)} \varepsilon^{(n)}} \nabla \left( \nabla \cdot \vec{A}^{(n)} \right) - \frac{1}{\varepsilon^{(n)}} \nabla \times \vec{F}^{(n)}, \quad (1)$$

$$\vec{H}^{(n)} = \frac{1}{\mu^{(n)}} \nabla \times \vec{A}^{(n)} - j\omega \vec{F}^{(n)} - \frac{1}{\omega \mu^{(n)} \varepsilon^{(n)}} \nabla \left( \nabla \cdot \vec{F}^{(n)} \right), \quad (2)$$

where $n = I, II, III$ and IV. Assuming that the rectangular waveguide operates in the dominant mode (TE$_{10}$) and assuming that the sample has a flat surface and there is no air gap between the sample external surfaces and inner waveguide walls, we have $\vec{A}^{(n)} = 0$ and $\partial F_z^{(n)}/\partial y = 0$ [16]. Then, the electric vector potential can be written for the two port networks I, II, III and IV as

$$F_z^{(n)}(x, z) = \cos \left( \frac{2\pi}{\lambda_c} x \right) \left[ C_{1n} e^{-\gamma_n z} + C_{2n} e^{\gamma_n z} \right], \quad (3)$$

where

$$\gamma_n = j\frac{2\pi}{\lambda_0} \sqrt{\varepsilon^{(n)} \mu^{(n)}} - \frac{\lambda_0^2}{\lambda_c^2}. \quad (4)$$

Here, $C_{1n}$ and $C_{2n}$ are constants (real or complex); $\lambda_0 = c/f$ and $\lambda_c = c/f_c$ correspond to the free-space and cut-off wavelengths; $f$, $f_c$, and $c$ are, respectively, the operating and cut-off frequencies and the speed of light; and $\varepsilon^{(n)} = \varepsilon'_n - j\varepsilon''_n$ and $\mu^{(n)} = \mu'_n - j\mu''_n$ are the relative complex permittivity and relative complex permeability of regions I, II, III and IV.
Using the electric vector potentials in (3), electric and magnetic fields can be determined from (1) and (2) for each region (I, II, III and IV) [16]. Here, we assume that the measurement cell in Fig. 1 is homogenous, isotropic and non-reflecting and that the sample is homogenous and isotropic. Applying boundary conditions (continual of electric and magnetic fields at each region interface), $S$-parameters for each region can be derived as

$$S_{11}^{(n)} = S_{22}^{(n)} = 0, \quad S_{21}^{(n)} = S_{12}^{(n)} = \alpha_n, \quad n = I, II, III$$

(5)

$$S_{11}^{(IV)} = S_{22}^{(IV)} = \Gamma \frac{(1 - T^2)}{1 - T^2}, \quad S_{21}^{(IV)} = S_{12}^{(IV)} = T \frac{(1 - \Gamma^2)}{1 - T^2}$$

(6)

where

$$\alpha_I = e^{-\gamma L_{01}}, \quad \alpha_{II} = e^{-\gamma L_{02}}, \quad \alpha_{III} = e^{-\gamma L}.$$  

(7)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad T = \exp (-\gamma L),$$

(8)

$$Z_L = j\omega \mu_0 \mu_r / \gamma, \quad Z_0 = j\omega / \gamma,$$

(9)

$$\gamma_0 = j2\pi / \lambda_0 \sqrt{1 - \lambda_0^2 / \lambda_c^2}, \quad \gamma = j2\pi / \lambda_0 \sqrt{\varepsilon_r \mu_r - \lambda_0^2 / \lambda_c^2}.$$  

(10)

where $\gamma_0$, $Z_0$, $\gamma$, $Z_L$ represent, respectively, the propagation constants and impedances of the air-filled region and sample-filled region in the cell; $L_{01}$, $L_{02}$ and $L$ are, respectively, the lengths of air regions inside the cell; $\Gamma$ and $T$ are the first reflection and transmission coefficients of the sample; and $\varepsilon_r = \varepsilon'_r - j\varepsilon''_r$ and $\mu_r = \mu'_r - j\mu''_r$ are the relative complex permittivity and relative complex permeability of the sample.

Using $S$-parameters, we can write the WCM matrices in each region as

$$T_n = \begin{bmatrix} \alpha_n & 0 \\ 0 & 1/\alpha_n \end{bmatrix}, \quad n = I, II, III$$

(11)

$$T_{IV} = \frac{1}{S_{21}^{(IV)}} \begin{bmatrix} S_{21}^{(IV)} & S_{12}^{(IV)} & S_{11}^{(IV)} \\ S_{12}^{(IV)} & S_{22}^{(IV)} & -S_{22}^{(IV)} \\ -S_{22}^{(IV)} & 1 \end{bmatrix}.$$  

(12)

In the same manner, whole WCM matrices of each configuration in Fig. 1 as

$$M_a = T_X T_I T_{II} T_{III} T_Y, \quad M_b = T_X T_I T_{IV} T_{III} T_Y,$$

(13)

where

$$M_i = \frac{1}{S_{21i}} \begin{bmatrix} (S_{12i}, S_{21i} - S_{11i}, S_{22i}) & S_{11i} \end{bmatrix}, \quad i = a, b,$$

(14)
3. A METRIC FUNCTION FOR EVALUATION OF THE RECIPROCITY

In recent studies, two simple approaches have been proposed to decrease any errors arising from non-repeatable raw $S$-parameter measurements in different measurement environments [17, 18]. In the former method, an error cost function (Eq. (14) in [17]) over the frequency band was defined to minimize the repeatability errors as follows. Firstly, independent measurements of each cell connection over the entire frequency band are carried out. Next, the error cost function for each measurement at a given frequency is calculated. Then, the minimum of calculated error costs over the entire frequency band is taken as a measure for the effective $\varepsilon_r$ determination. This approach is not suitable because it takes time and to obtain a very highly accurate $\varepsilon_r$ determination more than 10 independent measurements must be done. On the other hand, the latter approach employs different multiplications of measured $S$-parameters and their inverses producing the same trace [18]. Because this approach is very effective and less operator dependent, we will employ it in our analysis. In this research paper, we will adopt the main concept of the latter approach [18] and derive a metric function for evaluation of the reciprocity of materials using measured raw $S$-parameters. To elaborate on this, we firstly obtain

$$M_b M_a^{-1} = T_X T_{01} T_S T_{0S}^{-1} (T_X T_{01})^{-1}$$
$$M_a M_b^{-1} = T_X T_{01} T_{0S} T_S^{-1} (T_X T_{01})^{-1},$$
$$M_a^{-1} M_b = (T_{02} T_Y)^{-1} T_{0S}^{-1} T_S T_{02} T_Y,$$
$$M_b^{-1} M_a = (T_{02} T_Y)^{-1} T_S^{-1} T_{0S} T_{02} T_Y.$$ (15)

where $'^{-1}$ means the inverse of a square matrix $'$. Then, applying a property (trace) of similar matrices to the expressions in (15) and (16) [19], we derive

$$F_m = \frac{1}{2} \left[ \frac{Tr (M_b M_a^{-1})}{Tr (M_a M_b^{-1})} + \frac{Tr (M_a^{-1} M_b)}{Tr (M_b^{-1} M_a)} \right] = \frac{S_{12}^{(IV)}}{S_{21}^{(IV)}}.$$ (17)

where $Tr(\cdot)$ denotes the trace of a square matrix ‘i’. It is obvious that the metric function in (17) solely depends upon measured raw
S-parameter measurements of the configurations in Fig. 1. If the cell was a reflecting, non-standard or non-uniform cell [20, 21], we would attain the same result in (17) since the difference between the two configurations in Fig. 1 is the presence of the sample [11]. That is, the method is based upon a relative calibration procedure which eliminates the effects of two port WCM matrices $T_X$, $T_Y$, $T_I$, $T_{II}$ and $T_{III}$ on the reciprocity evaluation of materials and uses the same measurement cell for measurement configurations in Figs. 1(a) and 1(b). If the sample is reciprocal (transmission symmetrical), the metric function, $F_m$, will be equal to one. In addition, $F_m$ can also output the level of transmission asymmetricity of materials if $F_m$ is different than one.

It is seen from (17) that the proposed method solely uses calibration-independent $S$-parameter measurements for evaluation of the reciprocity of materials. In this respect, it does not make any use of measurements from any absolute calibration standard which has definite electrical properties over long frequency bands. However, it should be pointed out that the proposed method relies on a relative calibration method.

4. EXPERIMENTAL RESULTS

A general purpose waveguide measurement set–up operating at X-band was used for validation of the proposed method [22, 23] as shown in Fig. 1. A HP8720C VNA is connected as a source and measurement equipment. It has a 1Hz frequency resolution (with option 001) and 8 ppm (parts per million) frequency accuracy. The waveguide used in measurements has a width of 22.86 mm ($f_c \approx 6.555$ GHz).

We assumed a single-mode transmission (TE$_{10}$) through the sample in Section 2. This condition for empty and specimen-filled sections of the waveguide will not be consistent for a dielectric sample with $\varepsilon_r > 4$ [24]. In this case, higher-order modes will appear. Using samples with a thickness less than one-half guided wavelength of the fundamental mode in the sample will suppress these modes [25]. Another option could be using two extra waveguide sections with lengths greater than 70 mm (greater than two free-space wavelengths) between the sample and coaxial-to-waveguide adapters for thick samples [25, 26]. This is because higher-order modes will die out drastically in a short distance away from the sample and real measurements are performed near the adapters [26].

We prepared four polystyrene and Plexiglas samples (10 mm long as first test samples and 44.38 mm long as second test samples) and a 44.38 mm long polyvinyl-chloride (PVC) sample to validate the proposed method. Before validation of the proposed method, we
employed the self-checking technique [10, 11] in order to ensure that the accuracy of measurements is sufficient. The technique auto-monitors the performance of measurements before measuring the lengths of extra cells and extracting the electrical properties of materials from raw $S$-parameters. For example, Fig. 2 demonstrates the dependency of this metric function, $F_{c2}$, (Eq. 29 in [11]) over X-band for 10 mm long Plexiglas and polystyrene samples.

Figure 2. The dependency of $F_{c2}$ over X-band for ensuring the performance of measurements for validation of the proposed method.

It is expected that the value of the metric function $F_{c2}$ should be equal to zero [11]. It is seen from Fig. 2 that the maximum and minimum levels of this metric function for two samples throughout X-band are around 0.015. This shows a very good agreement with the theory and the measurements. Assuming that the sample perfectly fits into the waveguide, the level of measured $F_{c2}$ in Fig. 2 demonstrates how flat the surfaces of the samples are prepared. This is because the measured WCM matrices for the configurations in Fig. 1 contain both reflection and transmission $S$-parameters and reflection measurements are mainly affected by surface roughness more than transmission measurements [27, 28]. In addition, the relatively smooth dependency of the measured $F_{c2}$ in Fig. 2 exhibits that the sample is fairly homogeneous.

After validation of the performance of measurements, we carried out measurements for validation of the metric function in (17). Because this function does not depend on whether the line is reflecting, we validated the metric function for two cases: a) when a nonreflecting cell is utilized and b) when a reflecting cell is employed. For the first case, we firstly measured raw $S$-parameters of a 44.38 mm long empty waveguide section, and then arbitrarily placed each first test sample into the cell. Next, raw $S$-parameters of this new configuration
are measured. Finally, the dependency of $F_m$ in (17) over X-band was drawn. For the second case, we initially loaded the cell fully with the PVC sample and then measured raw $S$-parameters of this configuration. This cell will behave like an empty reflecting cell [11]. Next, we located each second test sample into the 44.38 mm long cell and measured their raw $S$-parameters. Finally, we drew the dependency of $F_m$ in (17) over X-band. For example, Figs. 3 and 4 demonstrate such dependencies for two different test samples.

![Figure 3](image1.png)

**Figure 3.** The dependency of $F_m$ over X-band for validation of the metric function for nonreflecting cells using two 10 mm long polystyrene and Plexiglas samples.

![Figure 4](image2.png)

**Figure 4.** The dependency of $F_m$ over X-band for validation of the metric function for reflecting cells using two 44.38 mm long polystyrene and Plexiglas samples.

It is seen from Figs. 3 and 4 that all curves have smooth variations over the frequency band (X-band). In addition, they are almost equal
to one, which clearly verifies that all test samples are reciprocal. The measurement results in Figs. 3 and 4 are in good agreement with those of the traditional method which measures the forward and reverse transmission S-parameters after the calibration of the measurement cell in Fig. 1. However, the curves in Fig. 4 are more sensitive to frequency and have more ripples. There are two main reasons for this if we assume that first and second test samples perfectly fit into the waveguide and they are homogenous. First, the PVC sample which is used for simulating a reflecting cell may have some inhomogeneities. Second, there might be air pockets between external surfaces of the PVC sample and inner waveguide walls. In order to assess these reasons, first, we used different 44.38 mm long PVC samples. We observed that using different PVC samples did not significantly change the curves in Fig. 4. This assures that the prepared PVC samples in our lab can be assumed homogeneous. Second, we applied some silver paste on edges of test samples, which are contact with the waveguide walls. Then, we re-measured raw S-parameters and drew the dependency of $F_m$ over X-band. We observed that employing the silver paste greatly reduced the ripples in the dependency of $F_m$ in Fig. 4. As a result, we conclude that air gaps between the sample outer surfaces and inner waveguide walls may considerably alter the dependency of $F_m$.

We also used different lengths of first test samples in order to analyze the effect of sample thickness on the dependency of $F_m$. We note that increasing the sample thickness makes the dependency have a flat curve over the frequency band. The main reason for this is that wider samples are more homogenous than thinner ones.

In Figs. 3 and 4, we showed that the derived metric function, $F_m$, works well for reciprocal materials. In order to fully assess the validity and performance of the derived metric function for all materials, the dependency of $F_m$ over the frequency band for nonreciprocal samples should have been obtained. However, in our measurement laboratory, we do not have enough facility to measure the uncalibrated $S$-parameters of a ferrite material under a DC or steady magnetic field in one direction. Such a measurement can be performed by using a circulator. As it is well known, this microwave component allows wave propagation in one direction while it does not in another direction [29]. We predict two key results from measurements of non-reciprocal materials. First, we expect that the proposed method will work for non-reciprocal materials. This is because, as shown in the theoretical analysis in Section 3, the proposed method removes the effect of error matrices $(X \text{ and } Y)$ before and after the measurement cell with/out material under test. Non-transmission or weakly-transmission in one way compared to that in another way will not affect
the general discussions and comments made from the measurements of reciprocal materials (Figs. 3 and 4). Second, the proposed method will evaluate the reciprocity of non-reciprocal materials with no thickness dependency. This situation is analogous to measuring the resistivity of an arbitrary length of copper bar stock. From the electrical measurements alone, the resistance, which is a property for the actual bulk material, cannot be determined. None dependency of $F_m$ on sample thickness over the frequency band is demonstrated from the reciprocity measurements of two reciprocal materials (Figs. 3 and 4).

It should be pointed out that the accuracy of measured $S_{21}$ or $S_{12}$ will drastically decrease for non-reciprocal materials or reciprocal materials which possess an attenuation more than approximately 40 dB depending on the model of the VNA and frequency region the measurements are carried out [25]. The effect of this measurement uncertainty on the reciprocity evaluation of materials by the proposed method can be monitored from (14). However, we note that this uncertainty will also significantly affect the reciprocity measurements from corrected or calibrated $S$-parameter measurements. However, the methodology presented in this study at its present form is not valid for anisotropic materials [30].

5. CONCLUSION

A simple, yet promising, method is proposed to assess the reciprocity of materials using uncalibrated $S$-parameter measurements. A metric function, which is applicable to reflecting and nonreflecting cells, is derived for this purpose. This function does not rely on electrical properties of materials and their lengths. We validated the method using uncalibrated scattering parameter measurements of some low-loss materials.

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