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MUTUAL INDUCTANCE CALCULATION FOR NON-COAXIAL CIRCULAR AIR COILS WITH PARALLEL AXES

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Abstract—We present a practical and simple method for calculating the mutual inductance between two non-coaxial circular coils with parallel axes. All possible circular coils such as coils of rectangular cross section, thin wall solenoids, thin disk coils (pancakes) and circular filamentary coils are taken into consideration. We use Grover's formula for the mutual inductance between two filamentary circular coils with parallel axes. The filament method is applied for all coil combinations, for coils of the rectangular cross section and for thin coils. We consider that the proposed method is very simple, accurate and practical for engineering applications. Computed mutual inductance values obtained by the proposed method have been verified by previously published data and the software Fast-Henry. All results are in a very good agreement. This method can be used in various electromagnetic applications such as coil guns, tubular linear motors, transducers, actuators and biomedical implanted sensors.

1. INTRODUCTION

In various electromagnetic applications, an inductive link consists of two coils, forming a loosely coupled transformer. The primary coil generates a magnetic field that is partly picked up by the secondary coil. In this way power can be transferred wirelessly. This power system should be optimized towards maximal transfer efficiency and misalignment tolerance: a minimal amount of power transfer is guaranteed within certain limits of coil separation and lateral and angular misalignment [1]. A decrease in power transfer efficiency of the

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inductive power system can be caused by the lower mutual inductance due to the coil misalignment. This means that in the formula of the mutual inductance between two coils, misalignments (lateral and angular) have to be taken into consideration. In [2], we presented a relatively easy approach to calculate the mutual inductance between circular coils with inclined axes in air. In this paper, we study the case of lateral misalignment (parallel axes), where we calculate the mutual inductance of noncoaxial circular coils with negligible section and the rectangular cross section. The problem of the accurate and fast calculation of the mutual inductance of circular coils in air has a long history in electrical engineering. Many contributions to the problem of mutual inductance calculation for circular coils have been made in the literature [3–6]. Many contributions have been based on the application of Maxwell's formula, Neumann's formula and the Biot-Savart law [7–20]. The mutual inductance of circular coils can be obtained in analytical or semi-analytical forms expressed over elliptic integrals of the first, second and third kind, Heuman's Lambda function, Bessel functions, and Legendre functions. In addition, the problem can be solved using numerical methods, such as the finite element method (FEM) and the boundary element method (BEM). In this paper, we propose a relatively easy approach based on the filament method, where coils of rectangular cross section are replaced by a set of filamentary circular coils [2]. The mutual inductance of such coils has been given by Grover [3], whose formula takes into consideration two filamentary circular coils with parallel axes (lateral misalignment) and yields Maxwell's formula in the case of coaxial coils. It can be useful for the calculation of the mutual inductance between all noncoaxial circular coil configurations with parallel axes in the filament method treatment. We also provide the modified Grover's formula in the singular cases. According to our knowledge up until now there have been few papers or books that deal with the mutual inductance calculation between circular coils with parallel axes [3–6]. In these books the mutual inductance has been calculated using series that converge slowly. Recently a very interesting and useful semi analytical approach was presented where the mutual inductance between some circular coils with parallel axes is calculated using Bessel functions [16, 17]. The method presented in this paper can be a good alternative to numerical methods, such as FEM and BEM. We present many examples, that confirm our statement.

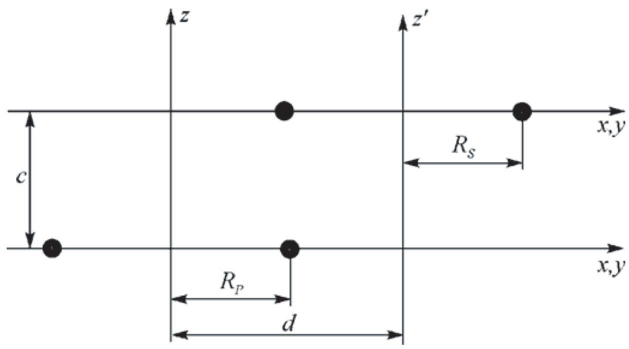

Figure 1. Filamentary circular coils with lateral misalignment (parallel axes).

2. BASIC EXPRESSIONS

The mutual inductance between two noncoaxial filamentary circular coils with parallel axes (See Fig. 1), one with radius R_P, and the other with radius R_S, with a distance d between their axes (lateral misalignment), can be calculated as in [3],

R_P — radius of the primary coil
R_S — radius of the secondary coil
c — distance between plans of coils
d — distance between axes

$$M = \frac{\mu_0}{\pi}\sqrt{R_P R_S}\int_0^{\pi} \frac{\left(1 - \frac{d}{R_S}\cos\phi\right)\Phi(k)}{\sqrt{V^3}}\, d\phi \tag{1}$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad k^2 = \frac{4\alpha V}{(1+\alpha V)^2 + \beta^2},$$
$$V = \sqrt{1 + \frac{d^2}{R_S^2} - 2\frac{d}{R_S}\cos\phi}, \quad \Phi(k) = \left(\frac{2}{k} - k\right)K(k) - \frac{2}{k}E(k)$$

ϕ — angle of the integration at any point of the secondary coil of the radius R_S [3]
$K(k)$ — complete elliptic integral of the first kind, [22]

$E(k)$ — complete elliptic integral of the second kind, [22]
$\mu_0 = 4\pi \times 10^{-7}$ H/m — the magnetic permeability of vacuum

In all expressions of the mutual inductance the radius of the primary coil R_P is larger than the radius of the secondary coil R_S. The kernel function of (1) is singular in the case $c \neq 0$ and $R_S = d$. Following equation is a modified version for this case

$$M = \frac{\mu_0}{2\pi} R_P \int_0^{\pi} \sqrt{(1+\alpha V)^2 + \beta^2}\,\Psi(k)d\phi \tag{2}$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad k^2 = \frac{4\alpha V}{(1+\alpha V)^2 + \beta^2},$$
$$V = 2\sin\left(\frac{\phi}{2}\right), \quad \Psi(k) = \left[\left(1 - \frac{k^2}{2}\right)K(k) - E(k)\right]$$

Also the kernel function of (1) begins singular in the cases $c = 0$, $R_P = R_S = d$ and $c = 0$, $d = 2R_P = 2R_S$. These singular cases can be solved using Bessel functions, [16]. If the distance between axes is $d = 0$, Equation (1) becomes the well-known Maxwell's formula for the mutual inductance of two coaxial circular coils,

$$M = \frac{2\mu_0\sqrt{R_P R_S}}{k}\left[\left(1 - \frac{k^2}{2}\right)K(k) - E(k)\right] = \mu_0\sqrt{R_P R_S}\,\Phi(k) \tag{3}$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad k^2 = \frac{4\alpha}{(1+\alpha)^2 + \beta^2},$$
$$\Phi(k) = \left(\frac{2}{k} - k\right)K(k) - \frac{2}{k}E(k)$$

Formula (1) is the basic formula for the calculation of the mutual inductance of all non-coaxial circular coils with parallel axes using the filament method. In this paper, we use Romberg numerical integration for solving the simple integral in the treatment of the mutual inductance. In the singular cases, Gaussian numerical integration is recommended.

3. CALCULATION METHOD

Two non-coaxial circular coils of rectangular cross section with parallel axes (general case)

Let us take into consideration the system of two non-coaxial circular coils of rectangular cross section with parallel axes, as shown in Fig. 2(a), with N_1 and N_2 being the number of turns of the windings. It is assumed that the coils are compactly wound and the insulation on the wires is thin, so that the electrical current can be considered uniformly distributed over the whole cross sections of the winding. The corresponding dimensions of these coils are shown in Fig. 2(a). The cross sectional area of the first coil I is divided into $(2K+1)$ by $(2N+1)$ cells and the second coil II into $(2m+1)$ by $(2n+1)$ cells (see Fig. 2(b)). Each cell in the first coil I contains one filament, and the current density in the coil cross section is assumed to be uniform, so that the filament currents are equal. The same assumption applies to the second coil II, [2]. This means that it is possible to apply (1) to filament pairs in two coils.

Using the filament method and the approach given by [2] the mutual inductance between two circular coils of rectangular cross section with parallel axes is given by,

$$M = \frac{N_1 N_2 \sum_{g=-K}^{g=K} \sum_{h=-N}^{h=N} \sum_{p=-m}^{p=m} \sum_{l=-n}^{l=n} M(h,l,g,p)}{(2K+1)(2N+1)(2m+1)(2n+1)} \tag{4}$$

where

$$M(h,l,g,p) = \frac{\mu_0}{\pi}\sqrt{R_P(h) R_S(l)} \int_0^{\pi} \frac{\left(1 - \frac{d}{R_S(l)}\cos\phi\right)\Phi(k)}{\sqrt{V^3}} d\phi$$

$$\alpha(h,l) = \frac{R_S(l)}{R_P(h)}, \quad \beta(h,g,p) = \frac{z(g,p)}{R_P(h)},$$

$$k^2(h,l,g,p) = \frac{4\alpha(h,l)V(l)}{(1+\alpha(h,l)V(l))^2 + \beta^2(h,g,p)}$$

$$V(l) = \sqrt{1 + \frac{d^2}{R_S^2(l)} - 2\frac{d}{R_S(l)}\cos\phi}$$

$$R_P(h) = R_P + \frac{h_P}{(2N+1)}h; \quad h = -N, \ldots, 0, \ldots, N,$$

$$R_S(l) = R_S + \frac{h_S}{(2n+1)}l; \quad l = -n, \ldots, 0, \ldots, n$$

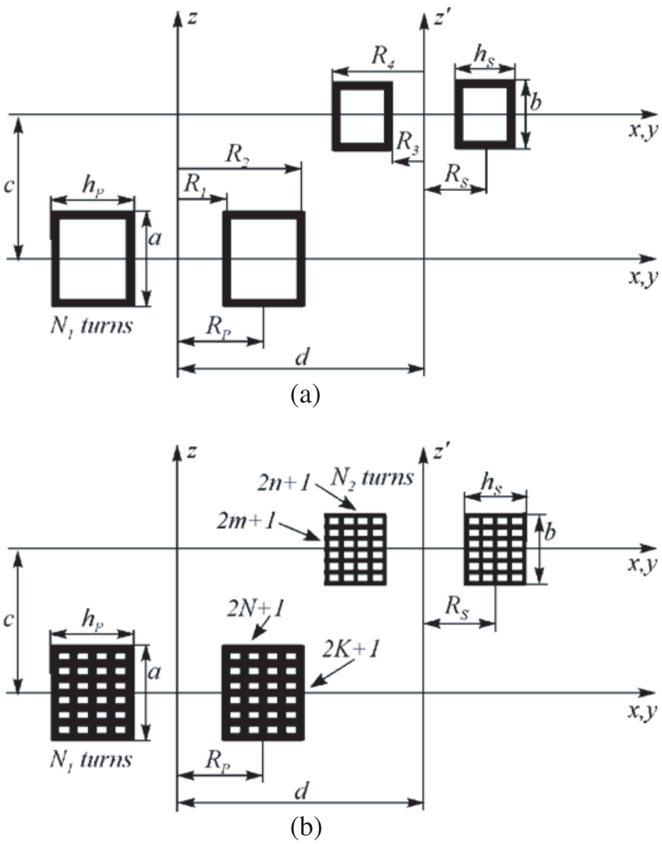

Figure 2. (a) Two circular coils of rectangular cross section with lateral misalignment (parallel axes). (b) Configuration of mesh coils: two circular coils of rectangular cross section with lateral misalignment (parallel axes).

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\begin{aligned}
&R_P = \frac{R_1 + R_2}{2}, \quad R_S = \frac{R_3 + R_4}{2}, \\
&h_P = R_2 - R_1, \quad h_S = R_4 - R_3 \\
&z(g, p) = c - \frac{a}{(2K+1)} g + \frac{b}{(2m+1)} p \\
&g = -K, \ldots, 0, \ldots, K; \quad p = -m, \ldots, 0, \ldots, m \\
&\Phi(k) = \left(\frac{2}{k} - k\right) K(k) - \frac{2}{k} E(k)
\end{aligned}
$$

Thus, we obtain the expression for the mutual inductance of the proposed coil configurations by using the filament method. In the case of coaxial coils ($d = 0$) one may use Equation (3). In the singular cases ($d = R_S$, $c \neq 0$ or $c = 0$, $d = R_S = R_S$), Equations (2) and (1) must be used to calculate the mutual inductance. Equation (4) can be used as the general formula to calculate the mutual inductance of all non-coaxial circular configurations with parallel axes. In the case where the radius of the primary coil R_P is not larger than the radius of the secondary coil R_S it is possible to apply previous expressions. We simply choose the coil of the larger radius to be the primary coil and apply either equation (1) or equation (2) in general equation (4).

4. EXAMPLES

4.1. Example 1

(a) Two circular coils of rectangular cross section with parallel axes

Two reactance coils of rectangular cross section with parallel axes have dimensions, (Dwight) [4]: $R_P = 7.8232\,\text{cm}$, $R_S = 11.7729\,\text{cm}$, $a = 14.2748\,\text{cm}$, $b = 2.413\,\text{cm}$, $h_P = 1.397\,\text{cm}$, $h_S = 4.1529\,\text{cm}$, $c = 7.366\,\text{cm}$, $d = 30.988\,\text{cm}$. The numbers of turns are $N_1 = 1142$ and $N_2 = 516$. Calculate the mutual inductance between reactance coils.

Two corresponding values of the mutual inductance are given, calculated and measured [4] (Dwight),

$$M_{Calculated} = 1.43\,\text{mH}$$
$$M_{Measured} = 1.47\,\text{mH}$$

This work (4) gives the mutual inductance,

$$M_{This\ Work} = 1.42262284\,\text{mH}$$

The number of subdivisions was $N = K = n = m = 12$.

4.2. Example 2

(b) A circular coil of rectangular cross section and a solenoid (thin wall) with parallel axes

This coil configuration can be obtained from the general case (4) by replacing $h_S = 0$ and omitting the sum for the variable l.

Let take a system of two coils: a solenoid and a coil of rectangular cross section with parallel axes, (Dwight) [4]. The dimensions of the coils are: $R_P = R_S = 10\,\text{cm}$, $a = 22\,\text{cm}$, $b = 12\,\text{cm}$, $h_P = 2\,\text{cm}$,

$c = 20\,\text{cm}$, and $d = 20\,\text{cm}$. The mutual inductance is given as a function of coil turns (Dwight),

$$M = 0.03696 N_1 n_2\,\mu\text{H}$$

where $n_2 = N_2/b$.

$$M = 0.00308 N_1 N_2\,\mu\text{H}$$

If $N_1 = N_2 = 100$ the mutual inductance is,

$$M = 30.8\,\mu\text{H}$$

Applying modified Expression (4) of this work ($d = 2R_P = 2R_S$), we obtain,

$$M_{\textit{This Work}} = 30.8326\,\mu\text{H}$$

The number of subdivision was $K = n = m = 30$.

4.3. Example 3

(c) A circular coil of rectangular cross section and a thin disk coil (pancake) with parallel axes

This coil configuration can be obtained from the general case (4) by replacing $b = 0$ and omitting the sum for the variable p.

In this example the mutual inductance between a thin disk coil and a coil of rectangular cross section with parallel axes is calculated. The dimensions of coils are, $R_P = R_S = 20\,\text{cm}$, $a = 5\,\text{cm}$, $h_P = h_S = 5\,\text{cm}$, $c = 30\,\text{cm}$, $d = 30\,\text{cm}$ and the number of turns $N_1 = 100$ and $N_2 = 150$.

Using (4) the mutual inductance is,

$$M_{\textit{This Work}} = 252.5128\,\mu\text{H}$$

The number of subdivision was $N = n = m = 15$.

Applying the software *FastHenry*, [21] the mutual inductance is,

$$M_{\textit{Fast-Henry}} = 252.4575\,\mu\text{H}$$

4.4. Example 4

(d) A circular coil of rectangular cross section and a filamentary circular coil with parallel axes

It is clear that this coil configuration can be obtained from the general case (4) by replacing $b = h_S = 0$, $N_2 = 1$ and omitting the two sums for the variables p and l.

The system of consideration is the combination of a thin filamentary coil and a coil of rectangular cross section. Their axes are parallel. The dimensions of coils are $R_P = R_S = 20\,\text{cm}$, $a = 10\,\text{cm}$, $h_P = 5\,\text{cm}$, $c = 20\,\text{cm}$, $d = 20\,\text{cm}$ and the number of turns $N_1 = 150$ and $N_2 = 1$.

The modified Expression (4) gives the mutual inductance,

$$M_{This\ Work} = 7.8531\,\mu\text{H}$$

It is necessary to use the modified Formula (2) (singular case) in the modified Expression (5).

Applying the software *FastHenry*, [21] the mutual inductance is,

$$M_{FastHenry} = 7.8684\,\mu\text{H}$$

The number of subdivision was $n = m = 30$.

4.5. Example 5

(e) Two disk coils (pancakes) with parallel axes

This coil configuration can be obtained from the general case (4) by replacing $a = b = 0$ and omitting the two sums for the variables g and p.

The mutual inductance between two non-coaxial pancake coils placed in the different planes has been calculated for different coil positions (Conway), [16]. Given: $R_P = 1.25\,\text{m}$, $R_S = 1.6\,\text{m}$, $h_P = 0.5\,\text{m}$, $h_S = 0.8\,\text{m}$. ($R_1 = 1\,\text{m}$).

In Table 1, we show the values of the mutual inductance obtained by this approach and that of [16]. The number of subdivisions was $N = n = 100$.

4.6. Example 6

(f) Two solenoids (thin walls) with parallel axes

Obviously this coil configuration can be obtained from the general Case (4) replacing $h_P = h_S = 0$ and omitting two sums regarding variables h and l.

We find the mutual inductance between two loosely coupled coils (two wall solenoids) for which the given constants are: $R_P = R_S = 2.5\,\text{cm}$ and lengths $a = b = 5\,\text{cm}$, $d = 25\,\text{cm}$, $c = 0\,\text{cm}$, $N_1 = N_2 = 125$, (Grover), [3].

Following Grover [3] the mutual inductance is

$$M = -0.3826\,\mu\text{H}$$

Table 1. Mutual inductance calculation (two pancakes) — $M/(N_1 N_2 R_1)$.

d (m)	c (m)	M(µH/m) This work	M(µH/m) [16] (Conway)
0.0	0.25	2.3115	2.3115
0.2	0.25	2.2720	2.2720
0.5	0.25	2.0722	2.0722
1.0	0.25	1.4575	1.4575
1.5	0.25	0.7966	0.7966
2.0	0.25	0.2379	0.2379
2.5	0.25	−0.1638	−0.1638
0.0	0.5	1.7176	1.7176
0.2	0.5	1.6907	1.6907
0.5	0.5	1.5543	1.5543
1.0	0.5	1.1406	1.1406
2.0	0.5	0.2381	0.2381
2.5	0.5	−0.0566	−0.0566
0.0	1.0	0.9906	0.9906
0.2	1.0	0.9778	0.9778

Using the presented work the mutual inductance is,

$$M = -0.38257616\,\mu\text{H}$$

The number of subdivisions was $N = n = 50$. The negative values of the mutual inductance are caused by the coil arrangement.

4.7. Example 7

(g) A solenoid (thin wall) and a thin disk coil (pancake) with parallel axes

This coil configuration can be obtained from the general case (4) by replacing $h_P = b = 0$ and omitting the two sums for variables h and p.

We calculate the mutual inductance between a thin disk coil and thin wall solenoid with parallel axes. The coil dimensions and the number of turns are: $R_P = R_S = 10\,\text{mm}$, $a = 10\,\text{mm}$, $h_S = 10\,\text{mm}$, $c = 20\,\text{mm}$, $d = 20\,\text{mm}$ and the number of turns $N_1 = N_2 = 100$.

Applying the method presented in this work the mutual inductance is,

$$M = 3.5079\,\mu\text{H}$$

The number of subdivision was $K = n = 50$. This case can be used as a benchmark problem for testing other methods.

Using the software *FastHenry* [21] the mutual inductance is,

$$M_{FastHenry} = 3.1894\,\mu\text{H}$$

Results obtained by these two methods differ by about 9.1%. The software *FastHenry* takes into consideration coil turns of real cross section, and they are not round. Thus, discrepancies between the results are expected because our approach uses thin filament coils to replace real configurations [2]. However, the values of the mutual inductance obtained by two approaches are relatively close (the absolute discrepancy is about 0.318).

4.8. Example 8

(h) A thin disk coil (pancake) and a filamentary circular coil with parallel axes

This coil configuration can be obtained from the general case (4) by replacing $h_P = a = b = 0$, $N_2 = 1$ and omitting the three sums for variables l, g and p.

Table 2. Mutual inductance calculation (disk coil — filament coil).

d (m)	M(nH) This work (12)	M(nH) [17] (Conway)
0.00	526.0592	526.0592
0.02	473.52272	473.52272
0.04	327.4543	327.4543
0.06	152.6697	152.6697
0.08	35.16857	35.16857
0.10	−11.05914	−11.05914
0.12	−20.9420	−20.9420
0.20	−9.8126	−9.816

The mutual inductance between a pancake/disk coil and a circular filament coil with parallel axes was calculated using Bessel functions, (Conway) [17].

Assume that $R_P = 0.05\,\mathrm{m}$, $R_S = 0.02\,\mathrm{m}$, $h_P = 0.02\,\mathrm{m}$, $c = 0.05\,\mathrm{cm}$, and the number of turns $N_1 = 100$.

In Table 2, we show the values of the mutual inductance obtained by this approach (4) and the method described in [17] for different distances between the parallel axes. The number of subdivisions was $n = 150$.

4.9. Example 9

(i) A solenoid (thin wall) and a filamentary circular coil with parallel axes

This coil configuration can be obtained from the general case (5) by replacing $h_P = h_S = a = 0$, $N_2 = 1$, and omitting the three sums for variables h, l and p.

We find the mutual inductance between circular filament and wall solenoid with parallel axes, for which the given constants are: $R_P = 10\,\mathrm{cm}$, $R_S = 10\,\mathrm{cm}$, $a = 12\,\mathrm{cm}$, $c = 20\,\mathrm{cm}$, $d = 20\,\mathrm{cm}$ and $N_2 = 100$, (Grover) [3].

According to Grover [3] the mutual inductance is,

$$M = 0.307417\,\mu\mathrm{H}$$

Applying formula (4) the mutual inductance is,

$$M = 0.307150\,\mu\mathrm{H}$$

The number of subdivision was $m = 400$.

Using the software *FastHenry* [21] the mutual inductance is,

$$M_{FastHenry} = 0.307217\,\mu\mathrm{H}$$

We can see that in each calculation all results are in a very good agreement with already published data.

5. CONCLUSION

In this paper we present a lucid, easy and accurate approach for the calculation of the mutual inductance of non-coaxial circular coils with parallel axes. This approach is based on the filament method. We used Grovers formula for calculating the mutual inductance between two filamentary circular coils with parallel axes and developed an approach that can be a good alternative to modern numerical methods, such as FEM and BEM. In this paper, we treated all combinations of two non-coaxial circular coils either with rectangular cross section or with negligible cross sections. According to our knowledge the presented approach is an easy and fast method for the calculation of the mutual inductance between these types of coils, so engineers can use it immediately. The many examples presented confirm this statement.

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