ON THE APPLICATION OF MICROWAVE CALIBRATION-INDEPENDENT MEASUREMENTS FOR NONINVASIVE THICKNESS EVALUATION OF MEDIUM- OR LOW-LOSS SOLID MATERIALS

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Abstract—Microwave methods require some sort of calibration before physical (thickness, flaw, etc.) and electrical (permittivity, permeability, etc.) measurements of materials. It is always attractive to devise a method which not only eliminates this necessity but also saves time before measurements. Microwave calibration-independent measurements can be utilized for this goal. However, in the literature, these measurements are only applied for electrical measurements of materials. In this research paper, we investigate the performance of microwave calibration-independent measurements for thickness evaluation of dielectric materials to increase the potential of available microwave techniques for thickness evaluation of dielectric materials. We derive an explicit expression for thickness estimation of dielectric materials from calibration-independent measurements for the adopted calibration-independent technique. We also propose a criterion for increasing the performance of measurements. We conducted thickness measurements of six dielectric specimens with different lengths to validate the derived expressions and the proposed criterion for thickness measurements.

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1. INTRODUCTION

Noninvasive and nondestructive measurements of thickness and monitoring its variation for low-loss or medium-loss materials are important in many areas of industry [1, 2]. There are several nondestructive evaluation (NDE) methods such as radiography, ultrasonics, microwaves, eddy current, etc. [2] for this goal. It is shown that microwave NDE methods have better advantages over other NDE methods in terms of low cost, contactless feature of the sensor (antenna), good spatial resolution, superior penetration in nonmetallic materials [3], and identification of dissimilar layers [4, 5]. In addition, the accuracy and sensitivity of ultrasonic and eddy current techniques are not good for measurements of physical and electrical properties of relatively thick and lossy dielectric materials since these materials cannot be modeled as either a good transmission medium for elastic waves or a good conductor for eddy currents [3, 4].

There are numerous papers written on the detection of any flaw, disbond, and delamination in layered dielectric slabs terminated by an infinite half free-space or by a conductor [6–9] as well as the thickness evaluation of lossy materials terminated by a conductor [4, 5, 8, 9] by microwave signals. These are some of the representative examples in the literature for demonstrating the potential and efficiency of microwaves for these applications. Although microwave methods are very promising and highly accurate, the requirement of a suitable calibration for these methods before measurements decreases their potential. This situation renders researchers and experimentalists searching for new methods not only in the sense of eliminating the necessity for calibration but also for saving time and making measurements resistant to unpredictable environmental conditions.

Recent advances made it possible to evaluate the electrical properties of materials using microwave calibration-independent methods [10–19]. In addition to the advantage that they remove the need for calibration, these methods as well have wide frequency coverage so that they are ready candidates for broadband applications. However, minimal attention is given to the evaluation and detection of physical properties (disbond, thickness, delamination, etc.) of materials by these methods. In this paper, we measure the thickness of medium- or low-materials in a rectangular waveguide using uncalibrated scattering (S-) parameter measurements. It is well known that waveguide measurements have much better accuracy and resolution than other NDE methods. Although waveguide measurements are destructive (samples must be cut and fitted into the waveguide), the proposed approach in this study can be applied
to free-space nondestructive measurements provided that horn-lens
antenna combination is utilized to focus the wave on samples. In
addition, it may seem that the use of microwave measurements for
thickness determination of laboratory specimens appears somewhat
artificial. This is because the dimensions of specimens can be easily
measured during their machining process. However, uncalibrated S-
parameter measurements for thickness evaluation of samples fitted
into waveguides will give an insight on the accuracy and performance
and demonstrate the potential of these measurements in other NDE
methods. The motivation of this work is to analyze the efficiency of
uncalibrated S-parameter measurements for thickness measurements
of dielectric medium- and low-loss materials fitted into a waveguide
and thus increase the potential of microwave methods for thickness
measurements.

The organization of the paper is as follows: Firstly, a
theoretical background is given and then the employed method for the
potential evaluation of microwave calibration-independent thickness
measurements is introduced in Section 2. Next, we derive a criterion for
optimizing the thickness evaluation and a correction term for resistive
losses for improving the accuracy of measurements by the employed
method. Finally, in Section 3, thickness measurements at X-band
(8.2–12.4 GHz) of six specimens obtained from two different dielectric
materials are presented to demonstrate the efficiency and potential of
microwave calibration-independent measurements.

2. THE METHOD

2.1. Background

The problem for thickness evaluation of a dielectric sample, which
is positioned into a waveguide section, using calibration-independent
measurements by the employed method [11] is shown in Fig. 1. It is
assumed that the sample is isotropic, symmetric, and homogenous. We
also assume that only the dominant mode (TE\(_{10}\)) is present inside the
waveguide and that the sample is fitted precisely into its measurement
cell (a waveguide section) in Fig. 1.

The two ports referred to as \(X\) and \(Y\) in Fig. 1 are used as
transitions between a vector network analyzer (VNA) and the cell.
These ports include source and load match errors, tracking (frequency)
errors, hardware imperfection of VNA (VNA IF mixer conversion
factor), etc. In implementing the method, firstly the uncalibrated
(raw) \(S\)-parameters of the configuration in Fig. 1(a), which refers to
sample’s original position in the cell, are measured. Then, the sample
is shifted to the right (or left) by a distance \(L_{02}\) (Fig. 1(b)) and raw
$S$-parameters for this new configuration are measured \cite{11}. Finally, the sample thickness, $L$, is determined using these measurements as follows.

Microwave networks consisting of a cascade connection of two or more two-port networks (e.g., $X$ and $Y$ in Fig. 1) can conveniently be analyzed by either ABCD matrix \cite{11, 20, 21} or wave cascading matrix (WCM) presentations \cite{22} of such microwave networks. For the mathematical analysis, we will utilize the wave cascading matrix (WCM) since it is useful in calibration/error correction problems \cite{15}. We denote the two-port WCM matrices, $T_X$, $T_Y$, $T_{01}$, $T_{02}$, $T_{03}$, and $T_S$, respectively, for modeling the transitions $X$ and $Y$, air regions with lengths $L_{01}$, $L_{02}$, and $L_{03}$ in the cell (Fig. 1), and the sample. It is assumed that $X$ and $Y$ are unequal and are unchanged for each measurement configuration in Fig. 1. Then, we write the following for the measurement configurations in Fig. 1

$$M_a = T_X T_{01} T_S T_{02} T_{03} T_Y, \quad M_b = T_X T_{01} T_{02} T_S T_{03} T_Y$$

(1)

where

$$M_i = \frac{1}{S_{21i}} \begin{bmatrix} (S_{12i}, S_{21i} - S_{11i}, S_{22i}) & S_{11i} \\ -S_{22i} & 1 \end{bmatrix}, \quad i = a, b,$$

(2)

and $S_{km}$ parameters ($k, m = 1, 2$) are raw $S$-parameters and the subscripts ‘a’ and ‘b’ in (1) and (2), accordingly, correspond to the measurement configurations in Figs. 1(a) and (b). For each $T_X, T_Y, T_{01}, T_{02}, T_{03}$ and $T_S$ matrix, we will have an expression similar to that in (2). However, for these matrices, this expression will contain theoretical $S$-parameters of each two-port network instead of measured

Figure 1. The measurement steps for thickness evaluation by the proposed method.
S-parameters. For an isotropic and reflection-symmetric sample and for an isotropic and non-reflecting line of length $L_{02}$, using (2), we express $T_{02}$ and $T_S$ as [23]

$$T_S = \frac{1}{(1 - \Gamma^2)^T} \begin{bmatrix} T^2 - \Gamma^2 & \Gamma (1 - T^2) \\ -\Gamma (1 - T^2) & 1 - \Gamma^2 T^2 \end{bmatrix}, \quad T_{02} = \begin{bmatrix} \alpha & 0 \\ 0 & 1/\alpha \end{bmatrix},$$  

(3)

where

$$\Gamma = \frac{\gamma_0 - \gamma}{\gamma_0 + \gamma}, \quad T = e^{-\gamma L}, \quad \alpha = e^{-\gamma_0 L_{02}},$$  

(4)

$$\gamma = j \frac{2\pi}{\lambda_0} \sqrt{\varepsilon - \left(\frac{\lambda_0}{\lambda_c}\right)^2}, \quad \gamma_0 = j \frac{2\pi}{\lambda_0} \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}. \quad \text{(5)}$$

The unknowns $T_X, T_Y, T_{01}$, and $T_{03}$ will be eliminated from (1) by the proposed method. Therefore, their expressions are not needed to be known. In (3)–(5), $\gamma$ and $\gamma_0$ are, respectively, the propagation constants of the sample- and air-filled cells; $\Gamma$ and $T$ are, accordingly, the first reflection and transmission coefficients of the sample-filled cell; $\varepsilon = \varepsilon' - j\varepsilon''$ is the relative complex permittivity of the sample; $\lambda_0 = c/f$ and $\lambda_c = c/f_c$ correspond to the free-space and cut-off wavelengths; and $f, f_c$, and $c$ are the operating and cut-off frequencies and the speed of light, respectively.

### 2.2. Thickness Evaluation

Using (1), we find the following

$$M_b M_a^{-1} = T_X T_{01} T_{02} T_S T_{02}^{-1} T_S^{-1} (T_X T_{01})^{-1},$$  

(6)

where it is clear that the effects of $T_Y$ and $T_{03}$ on measurements are already eliminated. It is obvious from (6) that $M_b M_a^{-1}$ and $T_{02} T_S T_{02}^{-1} T_S^{-1}$ are similar matrices [23]. It is well known that similar matrices have the same trace, which is defined as the sum of diagonal elements in a square matrix [23]. Using this fact and the expressions in (3), we derive [23]

$$Tr (M_b^{-1} M_a) = \frac{2 (T^2 - \Gamma^2) (1 - \Gamma^2 T^2) + \Lambda_1 \Gamma^2 (1 - T^2)^2}{(1 - \Gamma^2)^2 T^2},$$  

(7)

$$\Lambda_1 = \alpha^2 + 1/\alpha^2,$$  

(8)

where $Tr(\cdot)$ denotes the trace of the square matrix $\cdot^{-1}$. By the last step we eliminated $T_X$ and $T_{01}$ from (6). It is seen from (3)–(5) that while $T$ is a function of $\varepsilon$ and $L$, $\Gamma$ is a function of only $\varepsilon$. Therefore, for
a known or pre-measured $\varepsilon$, $L$ can be determined from $T$ using (7).

Arranging (7) in polynomials of $T$, we find

$$(\Lambda_1 - 2) T^4 + \Lambda_2 T^2 + (\Lambda_1 - 2) \Gamma^2 = 0,$$

where

$$\Lambda_2 = (2 - Tr (M_b^{-1} M_a)) (1 - \Gamma^2)^2 - 2 \Gamma^2 (\Lambda_1 - 2).$$

Then, using (9) the explicit expression for $L$ will be

$$L_{(1, 2)} = -\frac{1}{2\gamma} \ln \left( \frac{-\Lambda_2 \mp \sqrt{\Lambda_2^2 - 4 (\Lambda_1 - 2)^2 \Gamma^2}}{2 (\Lambda_1 - 2)} \right).$$

The correct root for $L$ from (11) can be picked up by evaluating (11) at two frequencies and comparing them. The average of the two which result in approximately the same values from (11) will reflect the actual $L$. It is noted from (11) that the advantage of the employed method is that it does not require the original location of the sample inside its cell for its thickness evaluation [11].

2.3. Optimization of the Evaluation

At this point, it is important to analyze whether there is any possible sample shifting which degrades or increases the performance of thickness evaluation using (11). It is for sure that if $M_a = M_b$, the expression in (11) will not work for our purpose. In this circumstance, $Tr (M_b^{-1} M_a) = 2$, which is the sum of diagonal elements in a unit matrix. Substituting $Tr (M_b^{-1} M_a) = 2$ into (7) will result in

$$\Lambda_1 = 2 \Rightarrow \alpha = 1/\alpha, \quad |\alpha| = 1.$$

As a result, we derive the necessary condition for accurate thickness evaluation based on shifting distance, $L_{02}$, as

$$L_{02} \neq (\lambda_0 n) \sqrt{1 - (\lambda_0 \lambda_c)^2}, \quad n = 0, 1, 2, \ldots$$

Based on (13) for increasing the performance and accuracy of thickness evaluation, one should do the followings. First, with respect to the operating frequency, the value of $L_{02}$ should be selected such that it is the mean of any two different values given in (13); i.e.,

$$L_{02} \cong (k\lambda_0) \sqrt{2 \sqrt{1 - (\lambda_0 / \lambda_c)^2}}, \quad k = 1, 2, 3, \ldots$$
Next, the second frequency, which is necessary for exact thickness evaluation from (11), should be optimized so that, at that frequency, $L_{02}$ also meets the condition in (14). Therefore, for increasing the accuracy of measurements, one must satisfy the following conditions

$$\frac{k\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_c)^2}} \neq \frac{m\lambda_{02}}{\sqrt{1 - (\lambda_{02}/\lambda_c)^2}}, \quad (15)$$

$$\left| \frac{k\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_c)^2}} - \frac{m\lambda_{02}}{\sqrt{1 - (\lambda_{02}/\lambda_c)^2}} \right| \gg 0, \quad m = 1, 2, 3, \ldots, \quad (16)$$

where $\lambda_{02}$ corresponds to the wavelength for the second frequency, $f_2$, and $|·|$ in (16) denotes the magnitude of ‘·’. A close investigation for the conditions in (15) and (16) reveals that assuring the condition in (16) allows one to meet that in (15). As a result, the condition in (16) is the dominant and fundamental requirement for $\lambda_{02}$ for high-performance measurements.

2.4. The Effect of Resistive Losses on Measurements

Some calibration techniques based on measurements of the line standard, such as the thru-reflect-line (TRL) method [24] and the line-reflect-line (LRL) method [25], will eliminate resistive (conductor) losses of the line standard. This is an important feature since measured electrical properties of samples, which are positioned into the line standard, will directly reflect their dielectric losses. However, for calibration-independent measurements, a correction term for resistive losses for improving the accuracy of measurement should be provided. Measured $S$-parameters of the cell in which there is no sample and of the thru connection (e.g., $X$ and $Y$ are connected back-to-back) can be used for this purpose as

$$M_GM_T^{-1} = T_XT_GT_X^{-1}, \quad (17)$$

where $M_T$ and $M_G$ are the measured WCM of the thru and the empty cell; $T_G$ is given by

$$T_G = \begin{bmatrix} \alpha_2 & 0 \\ 0 & 1/\alpha_2 \end{bmatrix}, \quad \alpha_2 = e^{-\gamma_0L_g}, \quad (18)$$
and $L_g$ is the length of the waveguide section. As a result, using (18), we derive a correction term for resistive losses as

$$e^{-\alpha_c} = e^{-\gamma_0} \left[ \frac{\text{Tr} (M_G M_T^{-1}) \mp \sqrt{\text{Tr} (M_G M_T^{-1})^2 - 4}}{2} \right]^{1/L_g},$$

(19)

where $\alpha_c$ is the attenuation constant for the wave propagation inside the cell. Because the measurements in Figs. 1(a) and (b) are conducted using the cell, the correction term in (19) can readily be employed in (4) for $\alpha$ and (11) for $L$.

3. MEASUREMENTS

We constructed a general purpose waveguide measurement set-up for validation of the proposed method [26]. The set-up has a HP8720C VNA connected as a source and measurement equipment, and operates at X-band (8.2–12.4 GHz). The waveguide used in measurements has a width of 22.86 ± 5% mm and a cut-off frequency of $f_c \cong 6.555$ GHz. We employed two additional waveguide sections with lengths greater than $2\lambda_0$ at X-band between the cell and coaxial-to-waveguide adapters to filter out any higher order modes [27, 28].

We used six dielectric specimens with different lengths cut from PTFE and Plexiglas materials ($L = 10$ mm, 14 mm, and 18 mm for the Plexiglas specimen; and $L = 10$ mm, 15 mm, and 20 mm for the PTFE specimen) to validate the proposed method. The thicknesses of these samples were measured by a high precision micrometer. We prepared them with no scratches, nicks, or cracks, and machined them to fit precisely into the cell to decrease measurement uncertainties and reduce the effect of air gaps [27, 28]. The $\varepsilon$ of the Plexiglas and PTFE samples are, respectively, approximately $2.59 - j0.0174$ and $2.04 - j0.003$ at 10 GHz [29]. Because electrical properties of these specimens are almost constant over X-band (8.2–12.4 GHz), we assumed that $\varepsilon \cong 2.59 - j0.02$ for each Plexiglas and $\varepsilon \cong 2.04 - j0.003$ for each PTFE specimen over X-band.

Before validation of the proposed method, we employed the self-checking technique [23, 30] in order to ensure that the accuracy of raw S-parameters is sufficient for correct thickness measurements. This technique was employed for monitoring the performance of raw S-parameters for measurements of extra cells (similar to the measurement cell in Fig. 1) and $\varepsilon$ of materials. Since it does not depend on sample thickness as well as the $\varepsilon$ of samples, in this study, we utilize it for checking the performance of raw S-parameters for thickness
measurements. For example, Fig. 2 demonstrates the dependency of this metric function, $F_{c2}$, (Eqn. (29) in [23]) over X-band for 10 mm long Plexiglas and PTFE samples.

![Graph showing the dependency of $F_{c2}$ over X-band for two samples.]

**Figure 2.** The dependency of $F_{c2}$ over X-band for ensuring the performance of measurements for validation of the proposed method.

It is expected that the value of $F_{c2}$ should be equal to zero [29]. It is seen from Fig. 2 that the maximum and minimum levels of this metric function for two samples throughout X-band are around 0.015. This shows a very good agreement with the theory and the measurements. However, there is no specific range for values of $F_{c2}$ which assures that the accuracy of measurements is sufficient. A maximum value of $F_{c2}$ which is as closer as to zero throughout the band is desirable. Assuming that the samples perfectly fit into the waveguide, the level of measured $F_{c2}$ in Fig. 2 demonstrates how flat the surfaces of the samples are prepared. This is because the measured WCM matrices for the configurations in Fig. 1 contain both reflection and transmission $S$-parameters and reflection measurements are mainly affected by surface roughness more than transmission measurements [31, 32]. In addition, the relatively smooth dependency of the measured $F_{c2}$ in Fig. 2 exhibits that the samples are fairly homogeneous. We also observed similar behaviour for all other prepared samples. It is pointed out that the value of metric function $F_{c2}$ can severely be altered by the surface roughness.

We located all samples into a waveguide section (the cell) one by one and shifted each by a different distance from its original position inside the cell. The original position of all samples is set constant, which is measured from the left terminal of the cell. Then, we measured the thickness of each sample at 9 GHz and 10 GHz.
using (11) for three different cases to examine: a) the effect of small shifts on measurements, b) the effect of large shifts (different \( k \) values) on measurements, and c) the effect of a second frequency on measurements. To better interpret the measurement results, we will use the following equation as a measure tool

\[
\psi \left| \frac{k\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_c)^2}} - \frac{m\lambda_{02}}{\sqrt{1 - (\lambda_{02}/\lambda_c)^2}} \right| \approx N, \tag{20}
\]

where \( N \) is a number in mm and \( \psi \) is a term used for denoting small shifts from the original position of each specimen. On the basis of this convention or the equation in (20), Table 1 shows the measured thickness of each specimen for different \( \psi \) values where \( N = 2 \) mm and \( k = m = 1 \), while Table 2 illustrates the measured thickness of each specimen for different \( k \) values where \( m = k, \psi = 0.5, \) and \( N = 2 \) mm. Finally, Table 3 demonstrates the measured thickness of each specimen for different \( N \) values where \( k = m = 1 \) and \( \psi = 0.5 \). The second frequencies, which are needed for exact thickness evaluation, for each case in Tables 1–3, are computed using (20) for given \( \psi, N, k, \) and \( m \) values.

### Table 1. Measured thicknesses of Plexiglas and PTFE specimens at 9 GHz and 10 GHz for different \( \psi \) values (\( N = 2 \) mm and \( k = m = 1 \)).

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>9</th>
<th>10</th>
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</thead>
<tbody>
<tr>
<td>( \psi )</td>
<td></td>
<td></td>
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<tr>
<td>( \approx0.25 )</td>
<td>9.83</td>
<td>9.85</td>
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<tr>
<td>( \approx0.50 )</td>
<td>9.90</td>
<td>9.85</td>
</tr>
<tr>
<td>( \approx0.75 )</td>
<td>9.82</td>
<td>10.14</td>
</tr>
<tr>
<td>Plex. Thickness (mm)</td>
<td>10 ±5%</td>
<td>14 ±5%</td>
</tr>
<tr>
<td>10 ±5%</td>
<td>14.13</td>
<td>14.18</td>
</tr>
<tr>
<td>14 ±5%</td>
<td>14.08</td>
<td>14.15</td>
</tr>
<tr>
<td>18 ±5%</td>
<td>14.15</td>
<td>14.10</td>
</tr>
<tr>
<td>PTFE Thickness (mm)</td>
<td>10 ±5%</td>
<td>14 ±5%</td>
</tr>
<tr>
<td>10 ±5%</td>
<td>9.85</td>
<td>14.87</td>
</tr>
<tr>
<td>14 ±5%</td>
<td>9.89</td>
<td>14.92</td>
</tr>
<tr>
<td>16 ±5%</td>
<td>19.89</td>
<td>20.10</td>
</tr>
<tr>
<td>20 ±5%</td>
<td>19.94</td>
<td>20.05</td>
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</table>

The main results obtained from measurements are as follows:

1) The accuracy of measurements decreases if \( \psi \) is different from 0.5 for a constant \( N \) and the same \( k \) and \( m \) (Table 1). This is completely in agreement with the condition in (14), which is expected to increase the accuracy and sensitivity of measurements.

2) The accuracy of measurements increases with \( k \) for a constant \( \psi \) and \( N \) and the same \( k \) and \( m \) (Table 2). This is because
Table 2. Measured thicknesses of Plexiglas and PTFE specimens at 9 GHz and 10 GHz for different $k$ values ($k = m$, $\psi = 0.5$, and $N = 2$ mm).

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>9</th>
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<tbody>
<tr>
<td>$k$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>10 $\pm$ 5%</td>
<td>9.90</td>
<td>9.91</td>
</tr>
<tr>
<td>14 $\pm$ 5%</td>
<td>14.08</td>
<td>14.07</td>
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<tr>
<td>18 $\pm$ 5%</td>
<td>18.05</td>
<td>18.04</td>
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<tr>
<td>10 $\pm$ 5%</td>
<td>9.89</td>
<td>9.90</td>
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<tr>
<td>15 $\pm$ 5%</td>
<td>14.92</td>
<td>14.93</td>
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<tr>
<td>20 $\pm$ 5%</td>
<td>19.94</td>
<td>19.95</td>
</tr>
</tbody>
</table>

Table 3. Measured thicknesses of Plexiglas and PTFE specimens at 9 GHz and 10 GHz for different $N$ values ($k = m = 1$ and $\psi = 0.5$).

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>9</th>
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<tbody>
<tr>
<td>$N$ (mm)</td>
<td>2</td>
<td>3</td>
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<tr>
<td>10 $\pm$ 5%</td>
<td>9.90</td>
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measurements conducted at two widely separated frequencies are more inter-dependent from one another than those at two closely separated frequencies. It is known that the uncertainty in frequency measurements increases if measured frequencies are closer to each other. However, at some measurements given in Table 2, increasing $k$ does not much improve the accuracy of measurements. This can be interpreted as that the inter-dependency between measured quantities at different frequencies saturates, and increasing the separation of frequency difference does not affect much the accuracy.

3) The accuracy of measurements increases with $N$ for the same $k$ and $m$ and constant $\psi$ (Table 3). The reason for this is similar to that given in (2); that is, measurements performed with higher $N$ will
yield large frequency separation $\Delta f = |f_2 - f|$ than those with lower $N$.

4) It is noted from Tables 1–3 that the accuracy of measurements decreases with $f$. This can be because the dynamic range for measurements decreases. The dynamic range for selecting different $\psi$, $DR_\psi$, can be defined as the difference between any consecutive $L_{02}$ values in (13) where $\psi$ is kept constant. That is,

$$DR_\psi = \frac{\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_c)^2}}.$$  

(21)

It is obvious from (21) that $DR_\psi$ is inversely proportional with $f$. As a result, measurements carried out at lower frequencies are more reliable and sensitive than those at higher frequencies.

5) It is noted from Tables 1–3 that the accuracy of measurements increases for thicker samples. There can be two reasons for this. First, the accuracy of physical measurements increases for thicker samples. Second, the ability of any measurement instrument to resolve or detect measurements at sequentially and linearly separated frequencies for thicker samples is greater than for thinner samples. This fact can be monitored from (11) where $\varepsilon$ is assumed constant.

6) We observe that the correction term derived for eliminating the resistive losses over waveguide walls in (19) did not much improve the measurement results given in Tables 1–3. This is because the specimens used in measurements are thin, so we did not record a considerable improvement, although the loss tangent (the ratio between imaginary and real parts of the permittivity) of the specimens is low. Nevertheless, the correction term for resistive losses in (19) is general and can be applied to any complex permittivity measurement by calibration-independent methods.

In order to investigate the effectiveness of the proposed method for thickness measurement of materials with surface roughness, we scratched a small portion of the surfaces of the $L = 10.02$ PTFE sample and $L = 9.98$ mm Plexiglas sample. Then, we measured their thicknesses as 9.75 mm and 9.78 mm for PTFE and Plexiglas samples, respectively, by the proposed method when $N = 2$ mm, $\psi = 0.5$, $f = 10$ GHz and $k = m = 1$. This shows that although the proposed method accurately measures the lengths of medium- or low-loss materials, its accuracy considerably suffers from surface roughness.

The accuracy and performance of thickness measurements by the proposed method should have been evaluated for dispersive medium- or low-loss materials. However, in our laboratory, at the time of measurements, we have not had dispersive materials to test our
method. We expect that the accuracy and performance of thickness measurements of dispersive medium- or low-loss materials will almost be equal to those of nondispersive materials. However, the proposed method may not be feasible for highly dispersive materials such as negative index metamaterials since it requires precise permittivity data of materials over the frequency band. In addition, it is instructive to discuss the accuracy of thickness measurements of medium- or low-loss materials with higher dielectric constants. Since the proposed method uses $T$ in (9)–(11) for thickness evaluation of a material for its known/given permittivity, it is expected that the resolution and thus the accuracy of thickness measurements will increase for samples with higher dielectric constants. If the thickness measurements were carried out inside a non-standard or non-uniform cell [33, 34] ($X$ and $Y$ regions in Fig. 1), we expect that the accuracy and performance of thickness measurements did not considerably change since the difference between the two configurations in Fig. 1 is the presence of the sample [30].

4. CONCLUSIONS

The performance and accuracy of microwave calibration-independent measurements for thickness evaluation of low-loss dielectric materials has been investigated. It has been shown that these measurements, which are employed for permittivity evaluation, are also very promising for thickness estimation of materials. We have derived an explicit equation for thickness evaluation of dielectric materials from these measurements for the analyzed calibration-independent technique. In addition, we have derived a criterion for thickness evaluation for increasing the precision of measurements. The effect of resistive losses of the cell is eliminated from measurements. We conducted thickness measurements of some dielectric specimens to assess the derived expressions and proposed criterion.

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